

# Camera Parameters and Calibration

Paulo Dias





- Basic definitions
- The image processing pipeline
- Image parameters
- Camera parameters
- Basic optics
- Camera models
- Camera calibration



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## Luminance

Luminance is normally defined as a measurement of the **photometric luminous intensity** per **unit area** of light travelling in a given direction.

Luminance describes the **amount of light** that goes through, or is emitted from, a particular area, and falls within a given solid angle.

The SI unit for luminance is candela per square meter ( $\text{cd/m}^2$ ).



## Chrominance

Chrominance describes the way a certain amount of light is **distributed among the visible spectrum**.

A **grayscale** image viewed by a human image has **no color information** which means that its color information is zero.

Any RGB triplet in which the value of  $R=G=B$  has no chrominance information.

Chrominance has **no luminance** information but is **used together** with it to describe a **colored** image defined, for instance, by an RGB triplet.



## Separating Luminance from Chrominance

Given an RGB triplet, we can define a derived triplet in which luminance and chrominance can be separated:

$$\begin{aligned}
 Y &= W_r R + W_g G + W_b B && \text{Luminance} \\
 U &= U_{\max} \frac{B - Y}{1 - W_b} \approx 0.492(B - Y) \\
 V &= V_{\max} \frac{R - Y}{1 - W_r} \approx 0.877(R - Y) && \text{Chrominance}
 \end{aligned}$$

where

$$\begin{aligned}
 W_r &= 0.299 \\
 W_B &= 0.114 \\
 W_G &= 0.587 \\
 U_{\max} &= 0.436 \\
 V_{\max} &= 0.615
 \end{aligned}$$

This values originally derivates from the general model of the human visual system and had a significant impact on the ability to develop a television color system compatible with the previous B&W television systems.

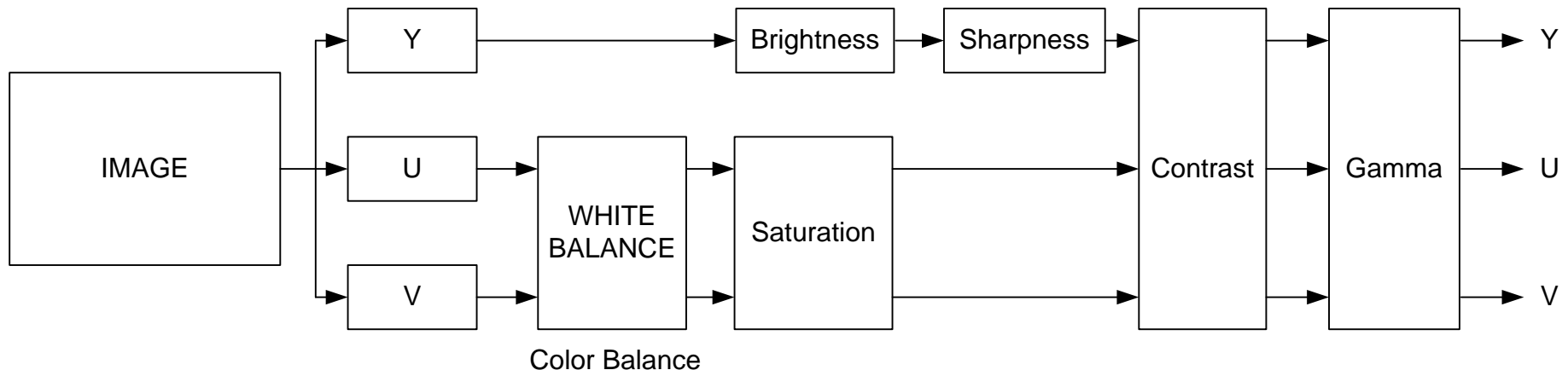
A symetric operation can be performed in order to recover the original RGB triple.



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## Image processing pipeline

A typical image processing pipeline (inside the image device) for a tri-stimulus system is shown bellow. This processing can be performed on the YUV or RGB components depending on the system. This should be understood as a mere example.

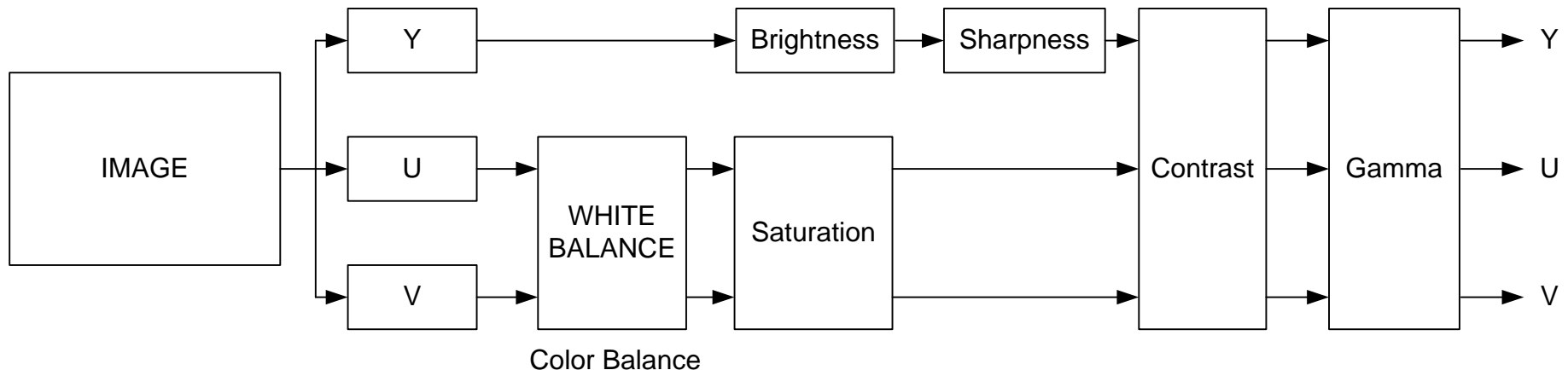






## Image processing pipeline

Depending on the system, more or less image parameters may be available for the user to control. Also, some of these parameters (namely brightness, contrast and saturation) are also intrinsic original image characteristics apart from being externally controllable parameters.





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## Brightness (as an intrinsic image characteristic)

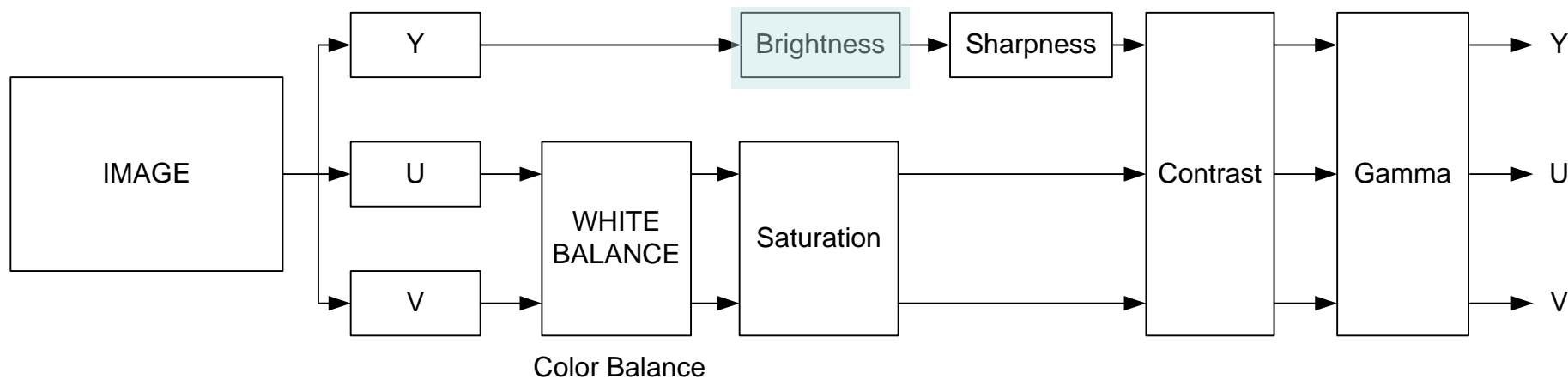
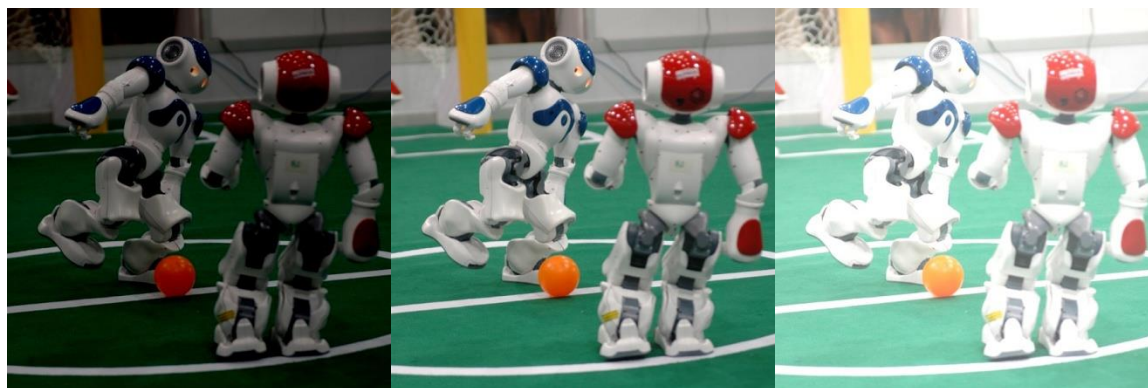
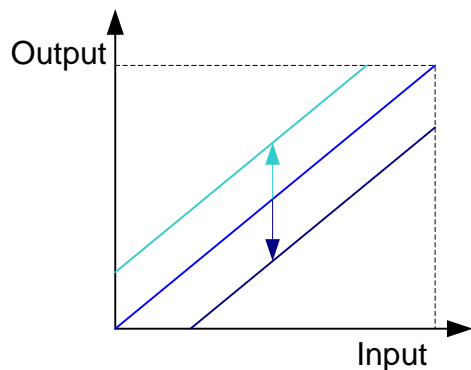
**Brightness** is one on **the intrinsic original image characteristics**. It represents a measure of the **average amount of light** that is integrated over the image during the exposure time. Exposure time (that is, the period of time during which the sensor receives light while forming the image, may or may not be a controllable parameter of the image device).

If the brightness is **too high overexposure** may occur which will white saturate part or the totality of the image.



## Brightness (as a controllable parameter)

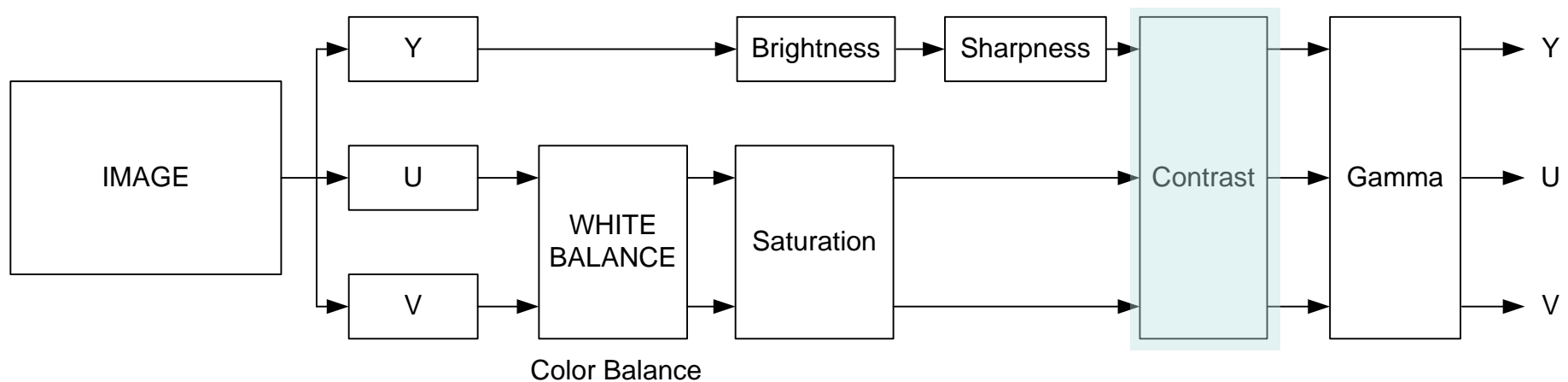
The **brightness** parameter is basically a **constant** (or offset) that can be **added** (subtracted) from the **luminance** component of the image.



## Contrast (as an intrinsic image characteristic)

There is not a unique definition of contrast. One of the most used is that contrast is the **difference in luminance (or color) along the 2D space** that makes an object distinguishable. In visual perception of the real world, contrast is determined by the difference in the color and brightness of the object and other objects within the same field of view. The faster and higher the luminance (or color) changes along the space the higher the contrast is.

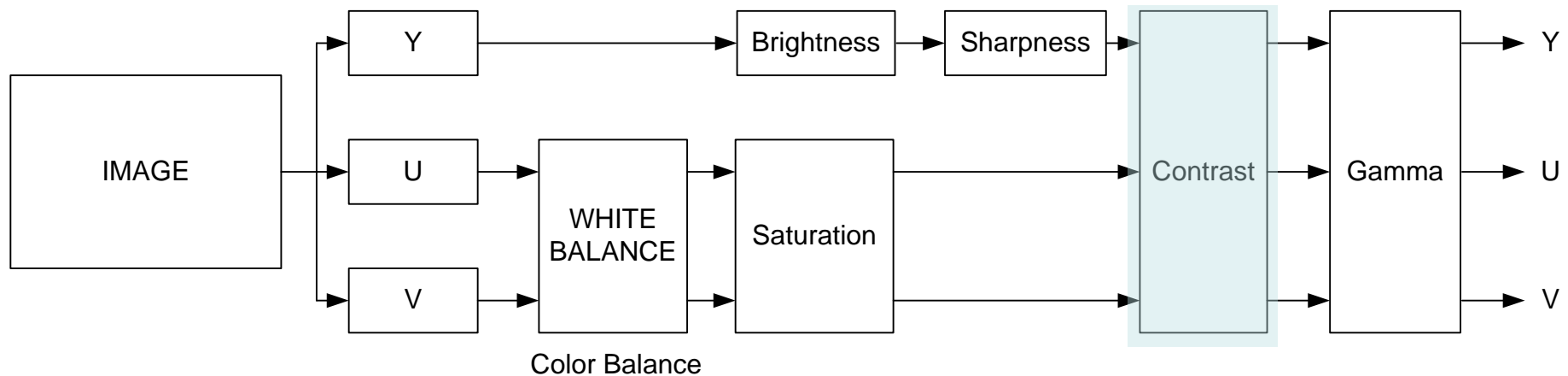
The maximum possible contrast of an image is also denominated contrast ratio or dynamic range.



## Contrast (as an intrinsic image characteristic)

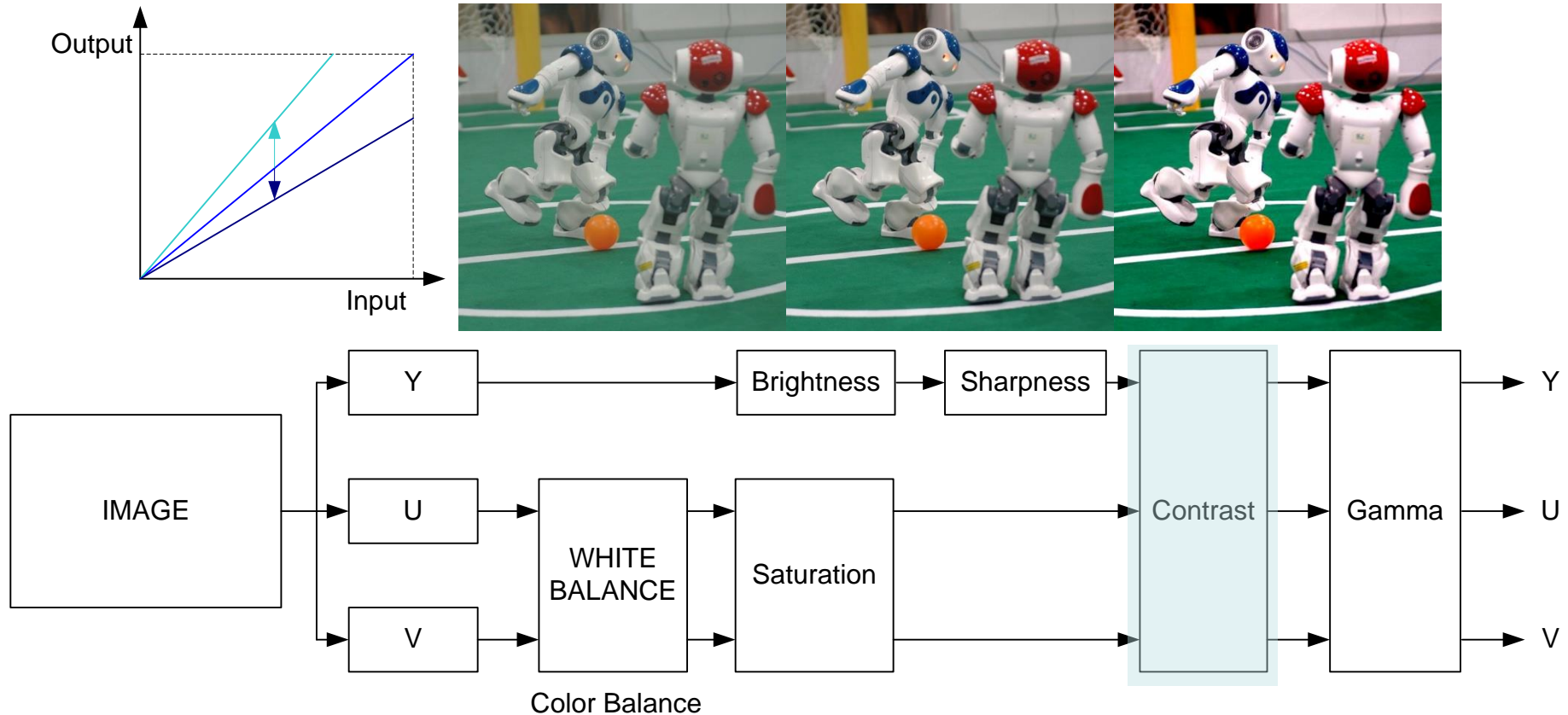
One of the possible definitions of contrast is given by the expression

$$\frac{\text{Luminance difference}}{\text{Average luminance}}$$



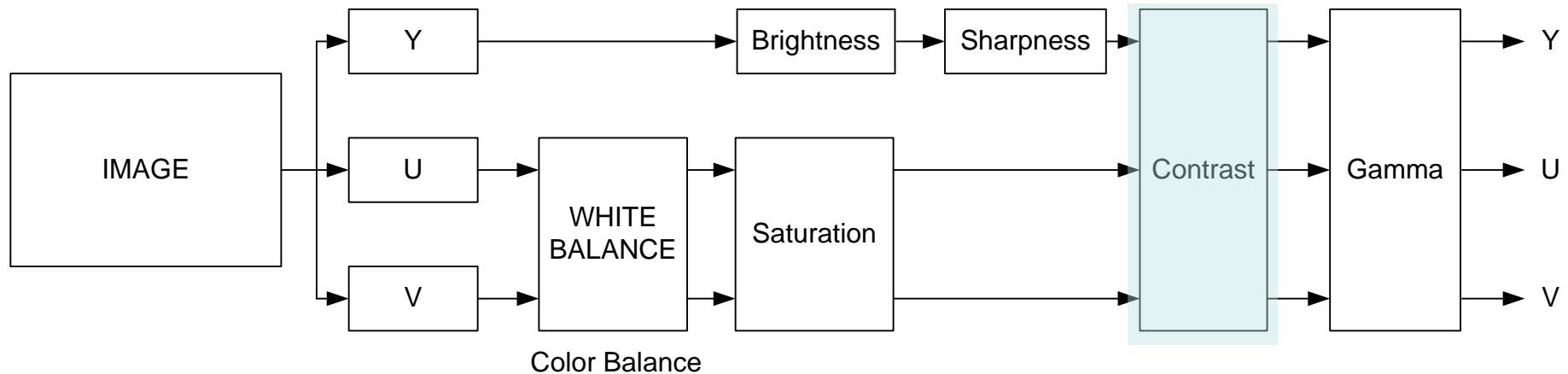
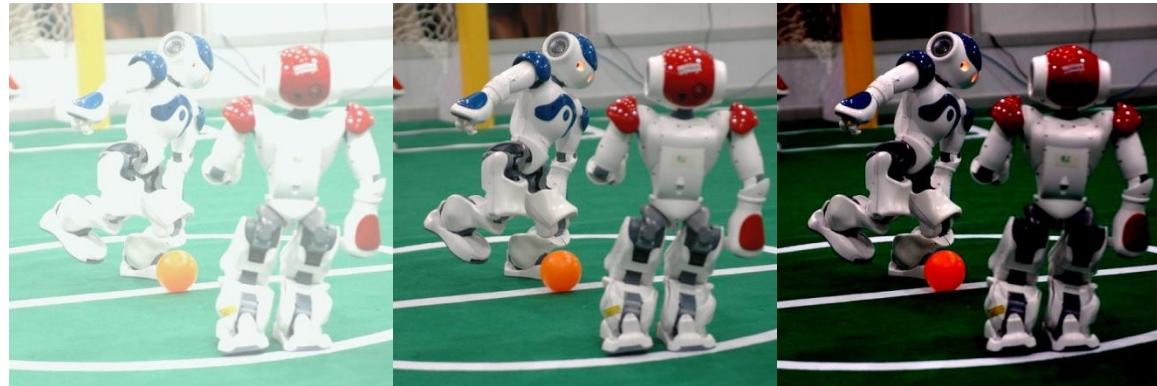
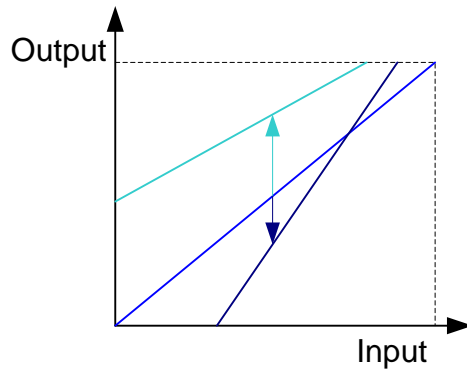
## Contrast (as a controllable parameter)

The **contrast** parameter is basically a variation in the **gain** control function of the **luminance** component of the image.



## Contrast + Brightness<sub>(as controllable parameters)</sub>

It is common that **contrast and brightness** are actually a **combined** single transfer function.

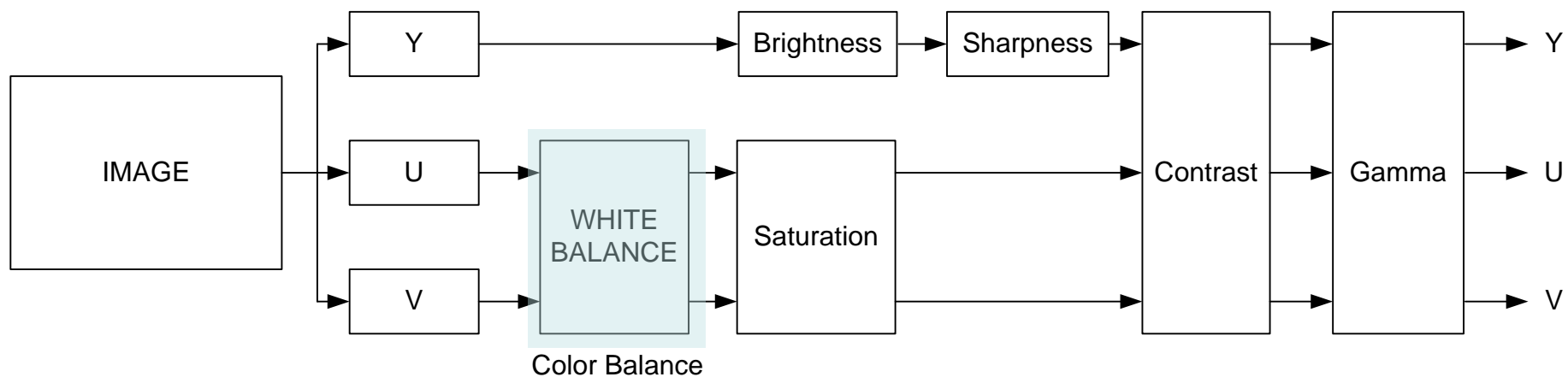




## White Balance<sub>(as controllable parameters)</sub>

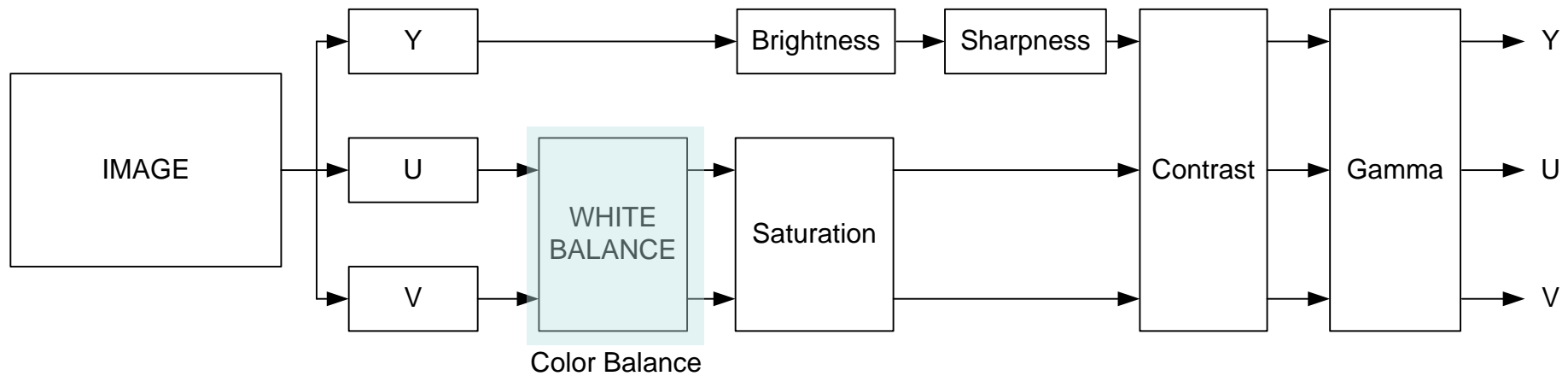
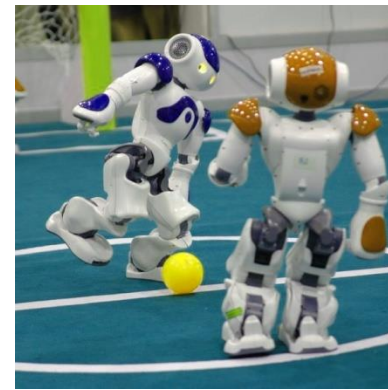
**White balance** is the global **adjustment** of the intensities of the **colors** (typically red, green, and blue primary colors).

An important goal of this adjustment is to **render specific colors** – particularly neutral colors – correctly; hence, the general method is sometimes called gray balance, neutral balance, or white balance.



## White Balance

Examples

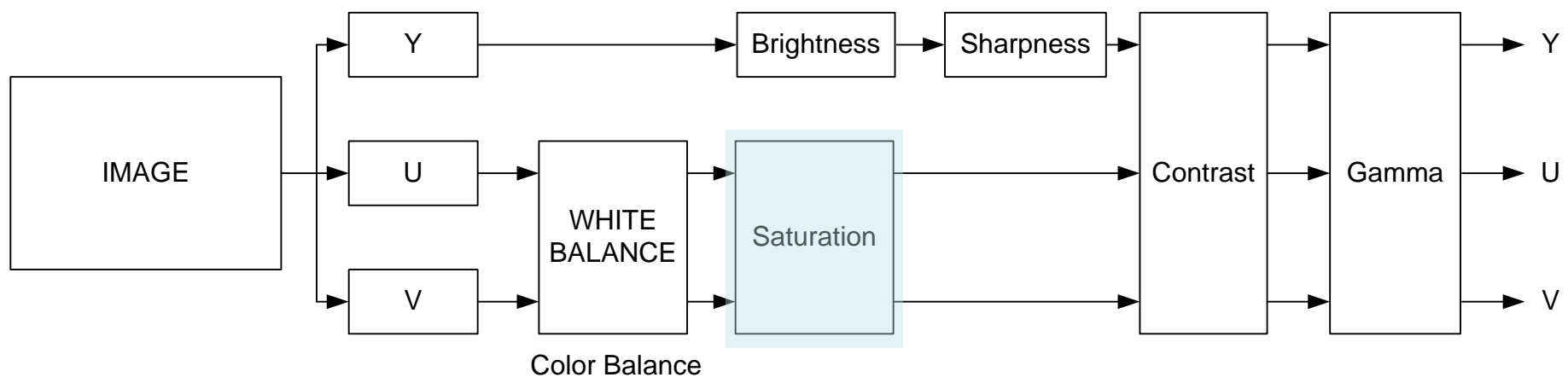


## Saturation (as an intrinsic image characteristic)

The saturation of a color is related to the **distribution of light across the spectrum** of different wavelengths. The **purest (most saturated)** color is obtained when using a **single wavelength** at a high intensity (laser light is a good example).

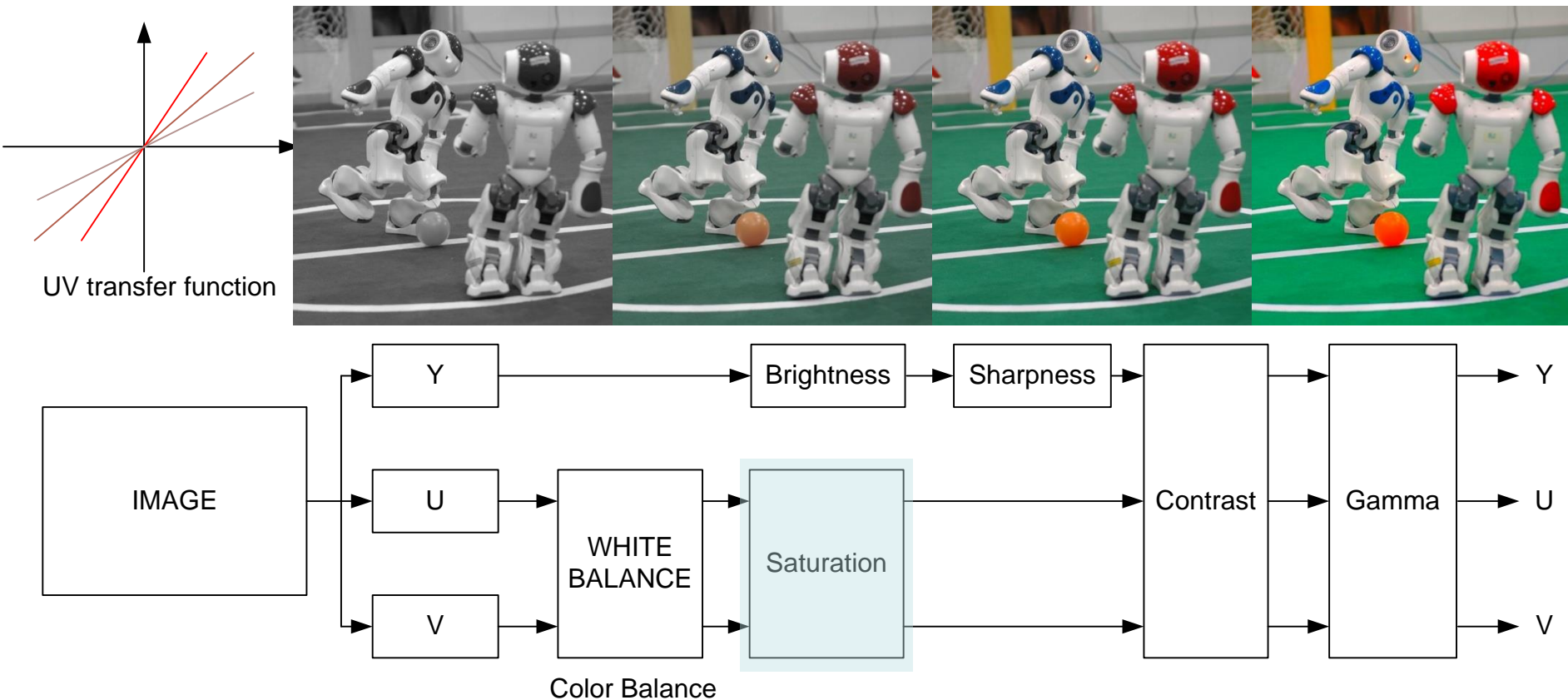
If the light intensity declines, then, as a result, the saturation also decline.

Saturation is sometimes also defined as the **amount of white you have blended into a pure color**.



## Saturation (as a controllable parameter)

To reduce the saturation of an image we can add white to the original colors. In fact, this is the same as changing the gain of the U and V chromatic components.



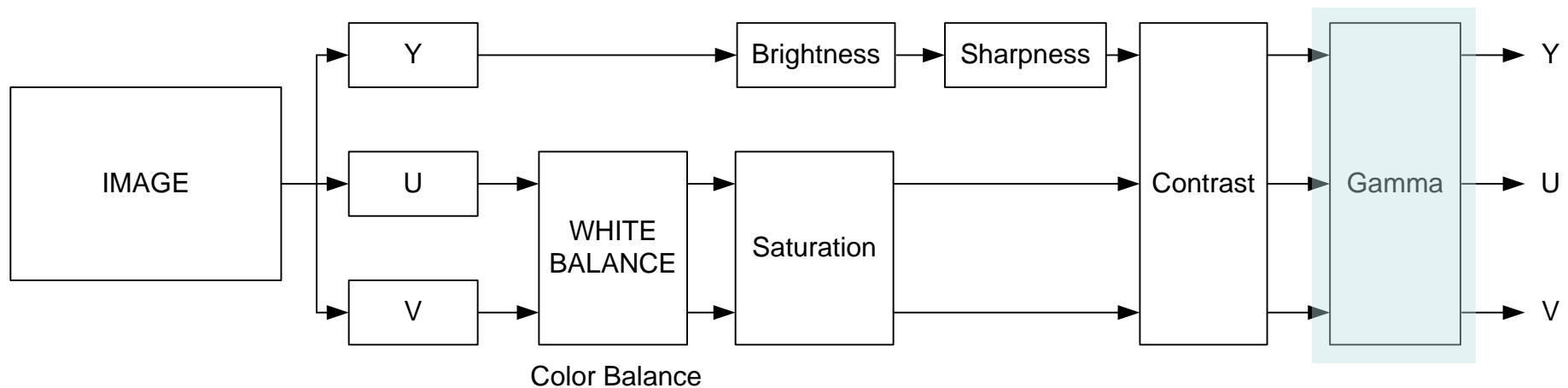
# Gamma

Gamma correction is the name of a nonlinear operation used to code and decode luminance values. In the simplest cases gamma is defined by the power-law expression:

$$V_{out} = AV_{in}^{\delta}$$

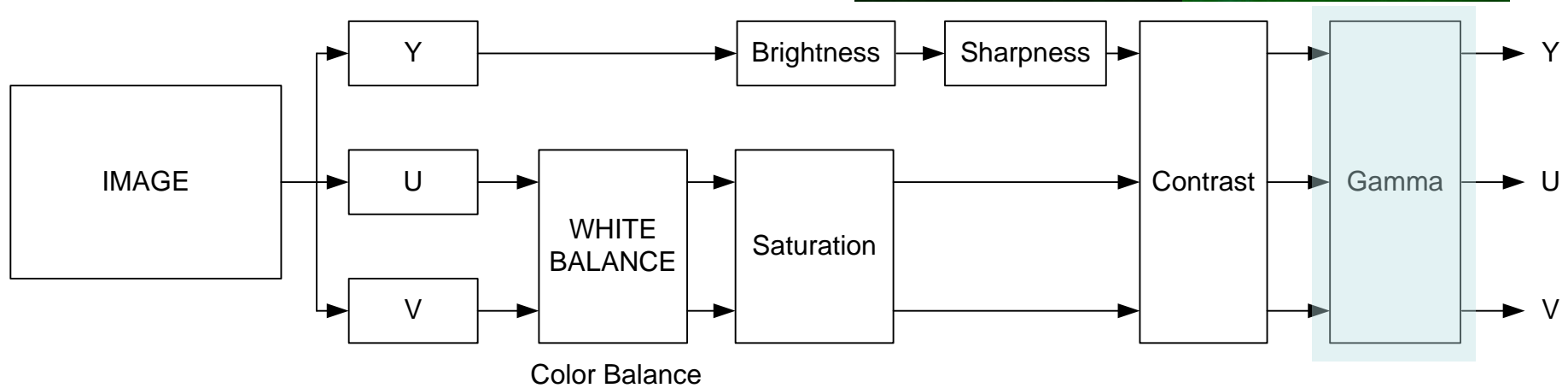
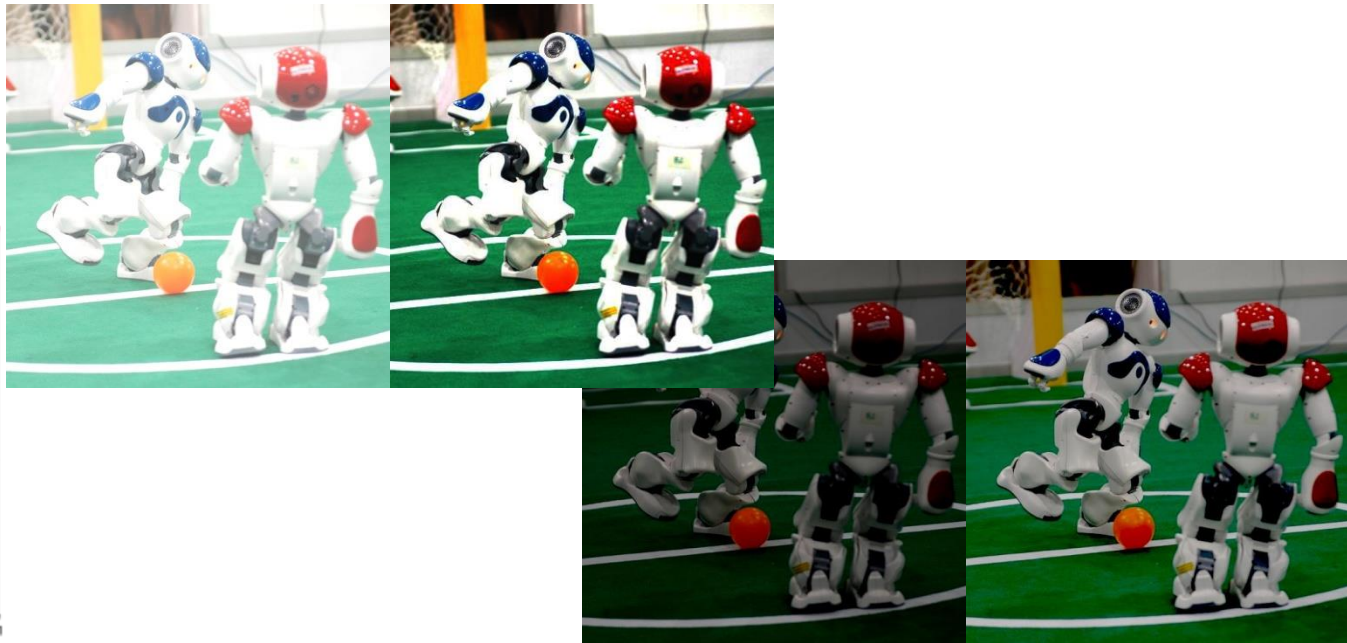
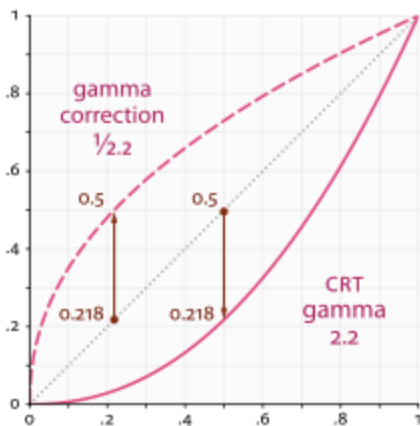
where  $A$  is a constant and the input and output values are non-negative real values.

In most cases  $A = 1$ , and inputs and outputs are typically in the range 0–1.



## Gamma

### Examples

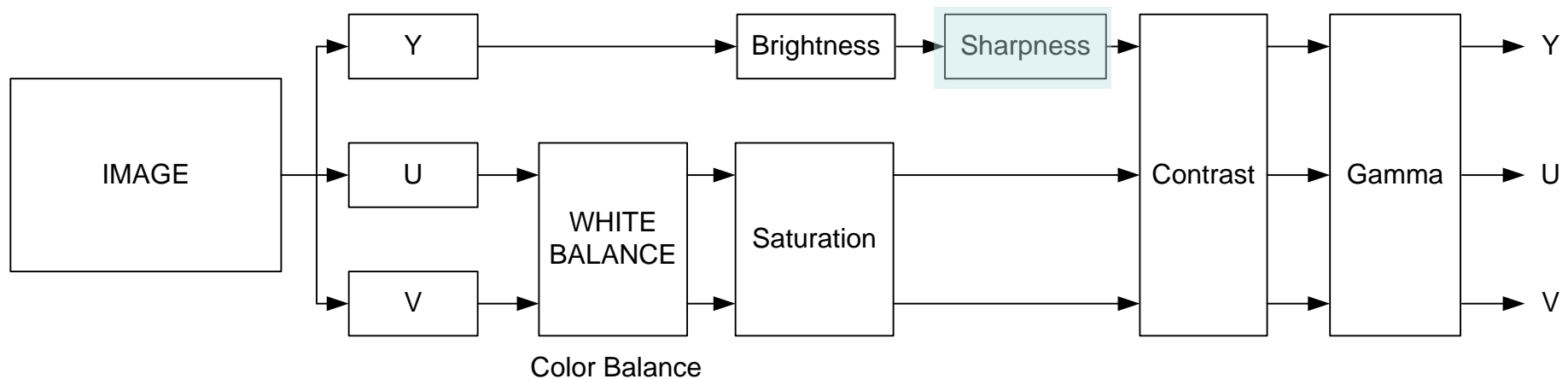


## Sharpness (as a controllable parameter)

**Sharpness** is a measure of the **energy frequency spatial distribution** over the image.

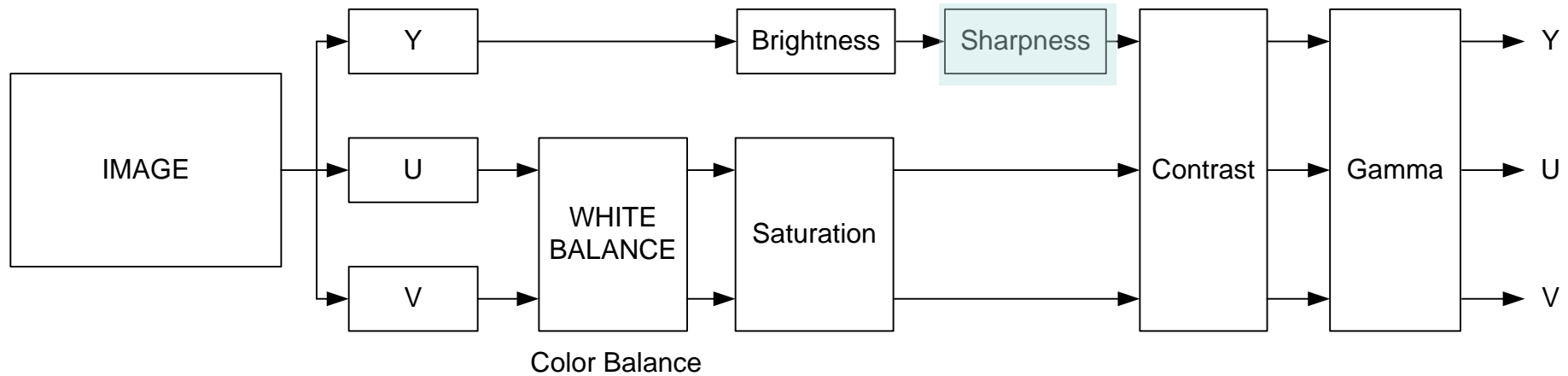
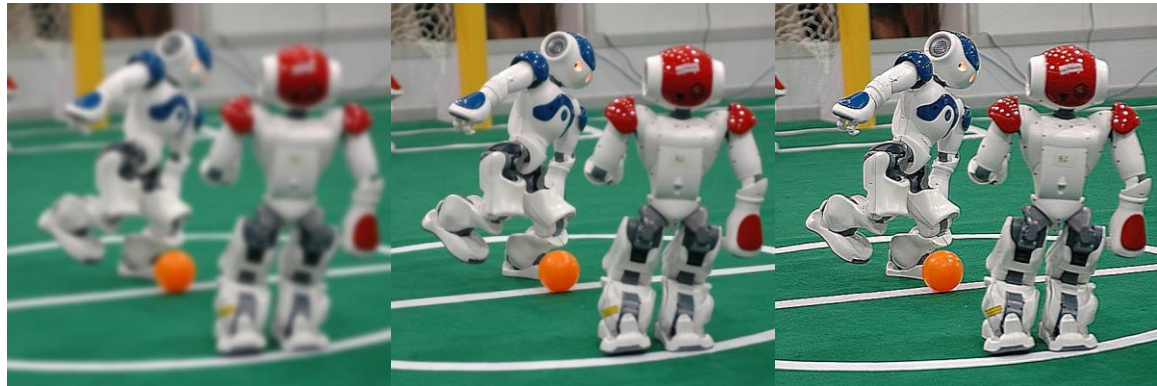
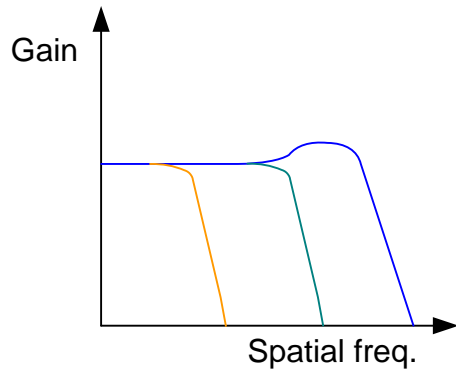
Not all devices provide access to this parameter.

Sharpness basically allows the **control of the cut-off frequency of a low pass** spatial filter. This may be very useful if the image is afterward intended to be decimated, since it allows to prevent spatial aliases artifacts.



## Sharpness (as a controllable parameter)

Examples.







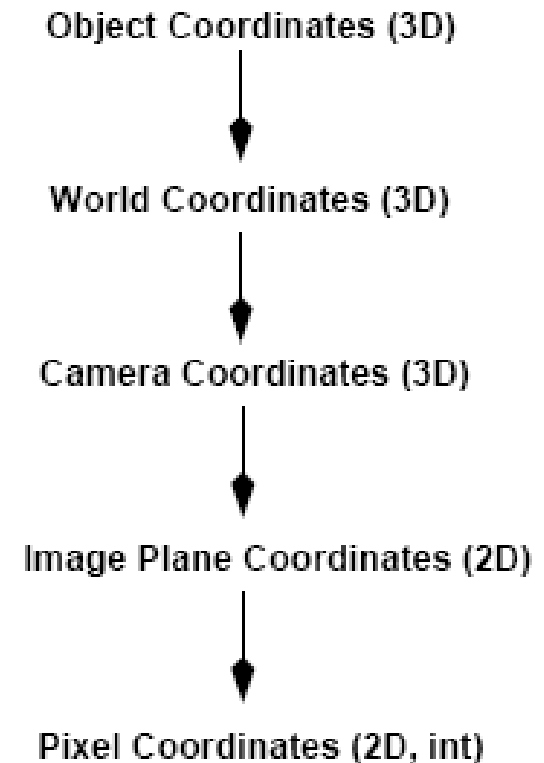
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One of the many applications of computer vision is to extract, from the frame image, **information** regarding the **external world** from which it has a limited perspective.

These applications may include, for instance, the determination of a certain **distance of a point** represented in the image when evaluated in real **world coordinate system**.

**Geometrical transformations** must be performed in order to accommodate the **camera internal parameters and geometry** and its **position and posture** in the real world.





Camera parameters are normally divided in two groups:

- **Extrinsic:**

Parameters that define the *location and orientation* of the camera reference frame with respect to a known world reference frame.

- **Intrinsic:**

Parameters necessary to *link* the *image pixel coordinates* with the *corresponding coordinates in the camera* reference frame.

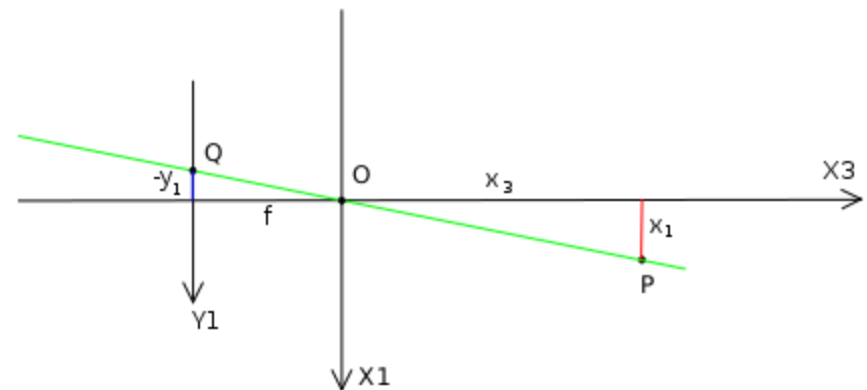
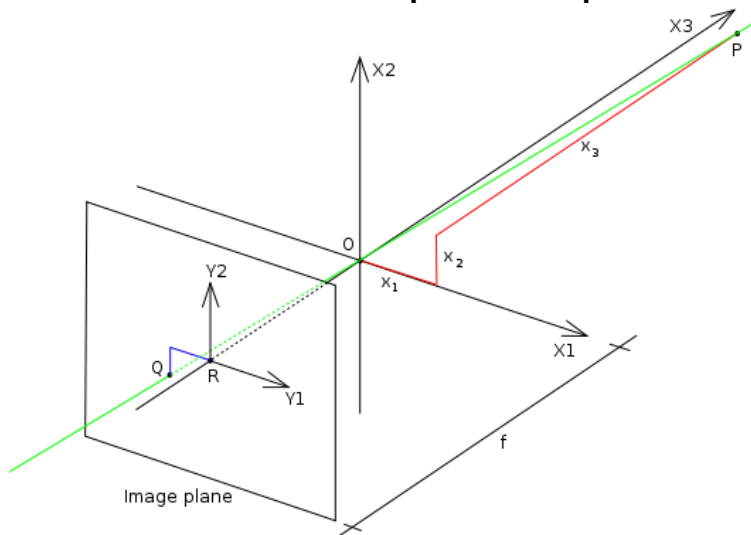
## The pinhole model

In most digital image processing tasks, the **optical system** can be **approximated** by the **pinhole model**.

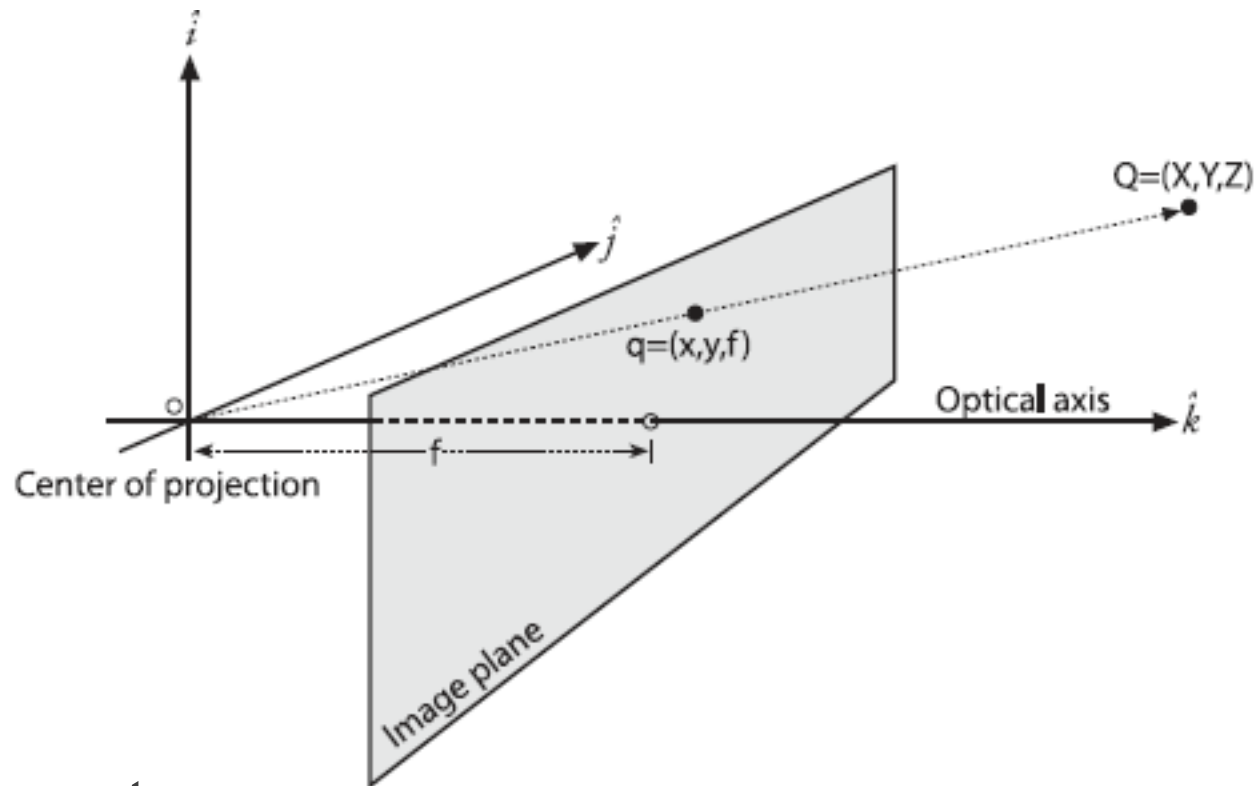
In this model, the lens is replaced by a **very narrow opening** (pinhole) through which lights go through directly into the image acquisition plane.

The point that is stroked by a light ray going through the pin hole in the direction of the lens main axis is called the **image plane** origin

In this model the acquisition plane lies at the **focal distance** from the pinhole.



- Pinhole camera model



- We get

$$x = f \frac{X}{Z}$$

$$y = f \frac{Y}{Z}$$



- Image optical centre and sensor centre are normally not aligned:

$$x_{screen} = f_x \frac{X}{Z} + c_x$$
$$y_{screen} = f_y \frac{Y}{Z} + c_y$$

- $f_x$  e  $f_y$  are derived from focal distance but take in consideration pixel size of the sensor (typically rectangular).



Homogenous Coordinates, in comparison to Cartesian coordinates, add an extra coordinate and define an equivalence relationship

$$(x, y) \rightarrow (kx, ky, k)$$

$$(X, Y, Z) \rightarrow (wX, wY, wZ, w)$$

This implies that any point in a 3D space can be represented by a multitude of equivalent matrixes, since the cartesian coordinates can be recovered from the homogenous coordinates by dividing each coordinate by the factor  $w$ .

In fact, this even allows us to represent any point in a plane which is at an infinite distance from the origin. Such representation will have its  $w = 0$



**Rotation** of a vector defined by two points in a Cartesian system can be obtained from:

$$\begin{bmatrix} \mathbf{d}_x \\ \mathbf{d}_y \\ \mathbf{d}_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_x & -\sin \theta_x \\ 0 & \sin \theta_x & \cos \theta_x \end{bmatrix} \begin{bmatrix} \cos \theta_y & 0 & \sin \theta_y \\ 0 & 1 & 0 \\ -\sin \theta_y & 0 & \cos \theta_y \end{bmatrix} \begin{bmatrix} \cos \theta_z & -\sin \theta_z & 0 \\ \sin \theta_z & \cos \theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \left( \begin{bmatrix} \mathbf{a}_x \\ \mathbf{a}_y \\ \mathbf{a}_z \end{bmatrix} - \begin{bmatrix} \mathbf{c}_x \\ \mathbf{c}_y \\ \mathbf{c}_z \end{bmatrix} \right)$$

$\mathbf{a}_n$ : coordinates of the point to be projected,

$\mathbf{c}_n$ : the pinhole coordinates

$\mathbf{D}_n$ : resulting rotated vector.

**Translation** vector  $\mathbf{T}_v$ , for the point  $\mathbf{p}$  can, on the other hand, be defined in a homogenous form by

$$\mathbf{T}_v \mathbf{p} = \begin{bmatrix} 1 & 0 & 0 & v_x \\ 0 & 1 & 0 & v_y \\ 0 & 0 & 1 & v_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} = \begin{bmatrix} p_x + v_x \\ p_y + v_y \\ p_z + v_z \\ 1 \end{bmatrix} = \mathbf{p} + \mathbf{v}$$



# Homogenous Coordinates



One of the most interesting things in the use of **homogenous coordinates** is that allows to **combine a rotation matrix and a translation matrix into a single homogeneous matrix**.

$$\begin{array}{c} \text{Rotation} \\ \text{Matrix} \end{array} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} - & - & - \\ - & R & - \\ - & - & - \\ 0 & 0 & 0 \end{bmatrix} \begin{array}{c} \text{Translation} \\ \text{Matrix} \end{array} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$



$$x_{screen} = f_x \frac{X}{Z} + c_x$$
$$y_{screen} = f_y \frac{Y}{Z} + c_y$$

- In homogeneous coordinates, previous equations can be written as

- $q = MQ$

with

- $q = \begin{bmatrix} x \\ y \\ w \end{bmatrix}, M = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}, Q = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$

- Normalizing with  $w=1$ , we get the same equations.

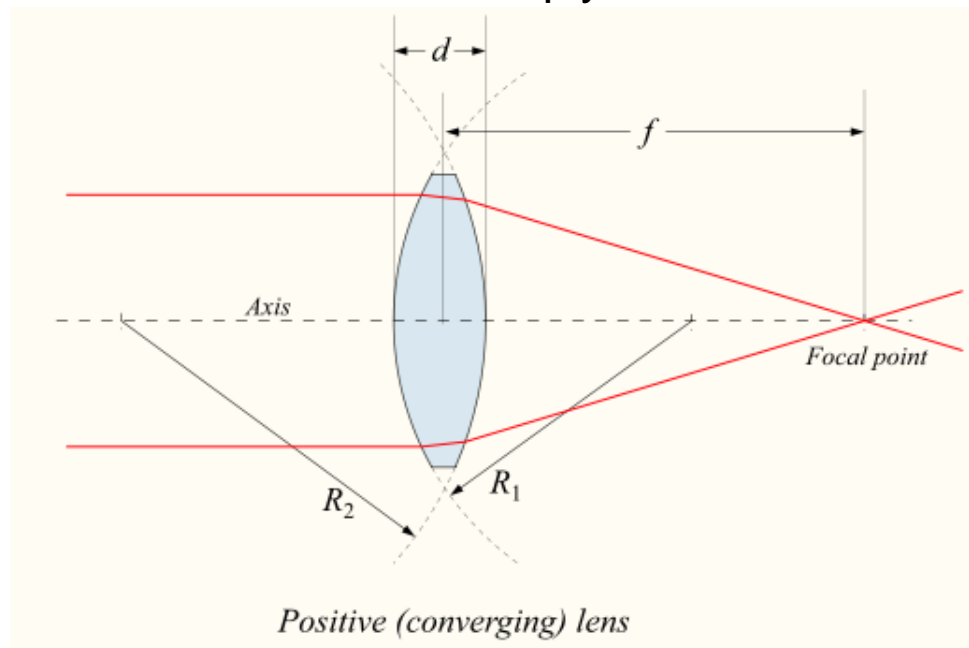


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## Optical lenses

The **optical element** of an image acquisition device is one of the most important elements of these devices. They can be made of complex groups of lenses, in particular when variable zooming is desirable. Optical component study goes far beyond the aim of this course.

Since **most simple image acquisition** systems, however, use a **single converging lens** as its optical interface, we will look simply into this case.

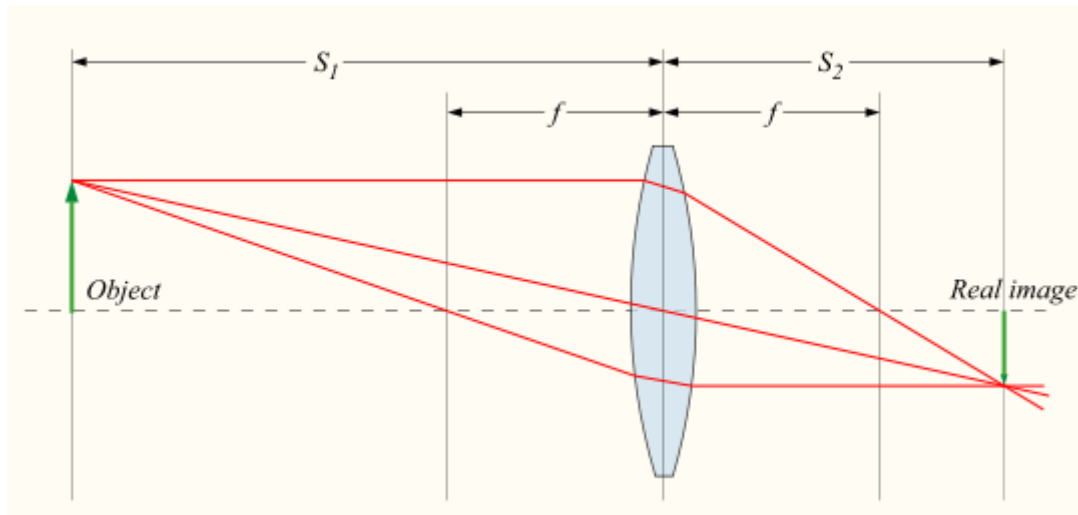


## Formation of image

If the distances from the object to the lens and from the lens to the image are  $S_1$  and  $S_2$  respectively, for a lens of negligible thickness, the distances are related by the **thin lens formula** which is an acceptable approximation to the full optical equation

$$\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}$$

If  $S_1 \gg S_2$  then  $S_2 \approx f$



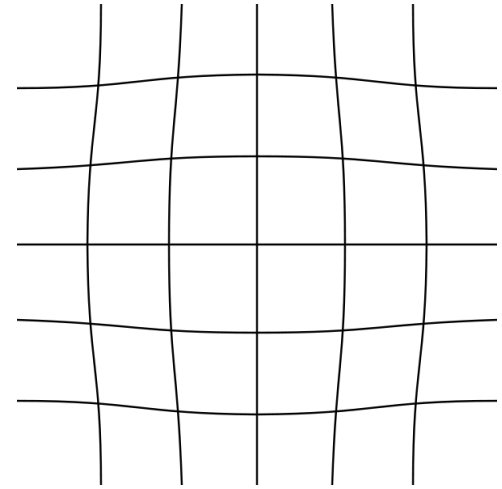
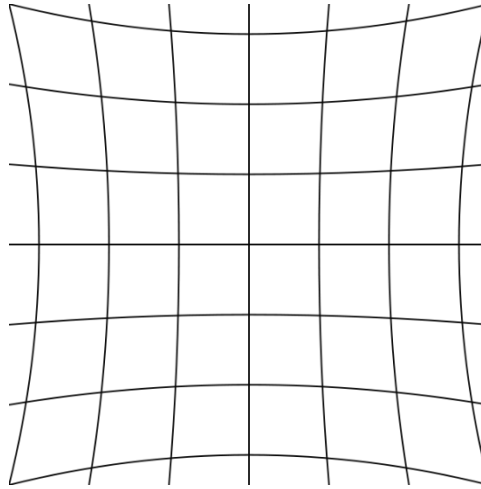
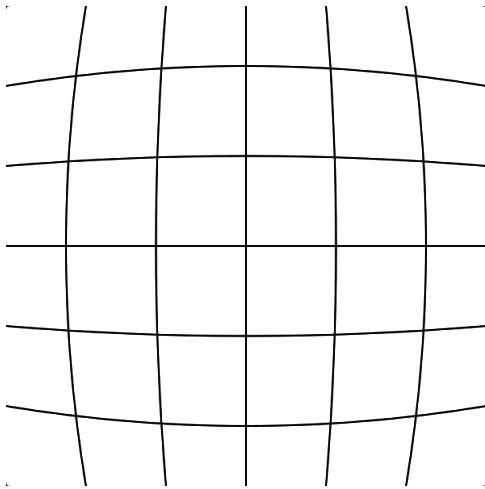


## Spherical Aberration

Spherical Aberration is an **image deformation** resulting from the fact that most lenses have a **spherical surface** cut which is easier to make than its proper shape.

Spherical Aberration can show as a **barrel** form (left side), **pincushion** effect (center) and **Mustache** distortion (right).

The **barrel** distortion is by far the **most common** one and normally increases with diminishing focal distance.



## Spherical Aberration

Spherical Aberration results in **multiple focus points** depending on the distance at which each ray of light enters the lens when referred to its major axis, and can be corrected by can be corrected using the **simplified Brown's distortion model**

$$x_u = (x_d - x_o)(1 + K_1 r^2 + K_2 r^4 + \dots)$$

$$y_u = (y_d - y_o)(1 + K_1 r^2 + K_2 r^4 + \dots)$$

where

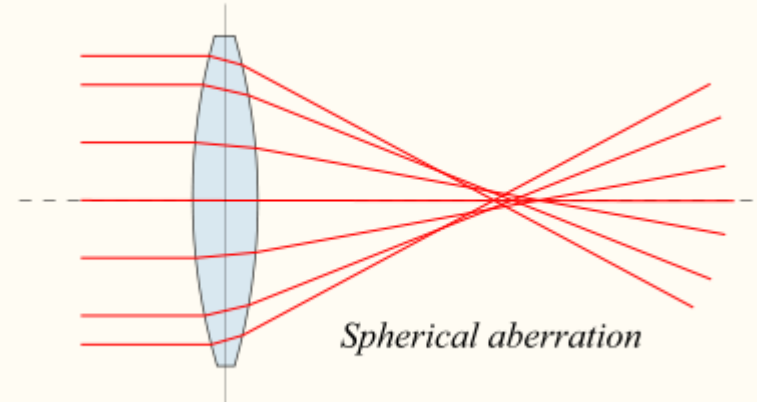
$(x_d, y_d)$  - distorted image point as projected on image plane using specified lens

$(x_u, y_u)$  - undistorted image point as projected by an ideal pin - hole camera

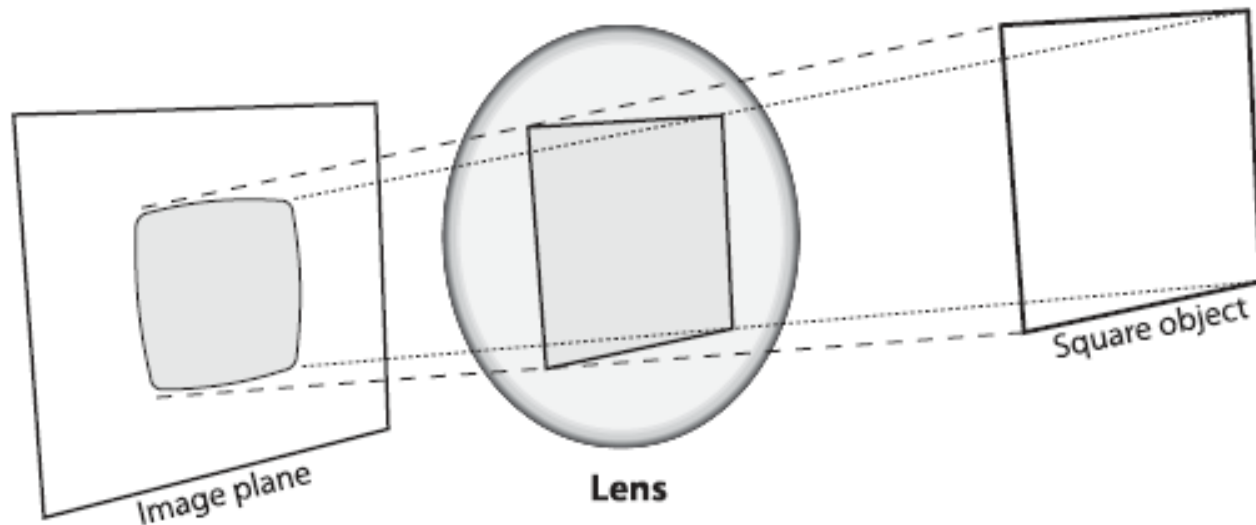
$(x_o, y_o)$  - distortion center (assumed to be the principal point)

$K_n = n^{th}$  - radial distortion coefficient

$$r = \sqrt{(x_d - x_o)^2 + (y_d - y_o)^2}$$



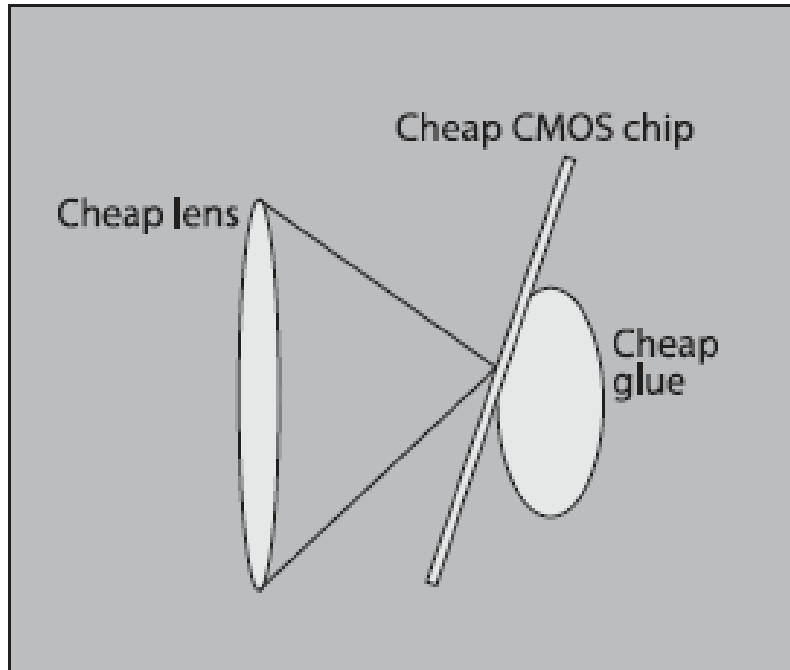
# Lens distortion – Radial Distortion



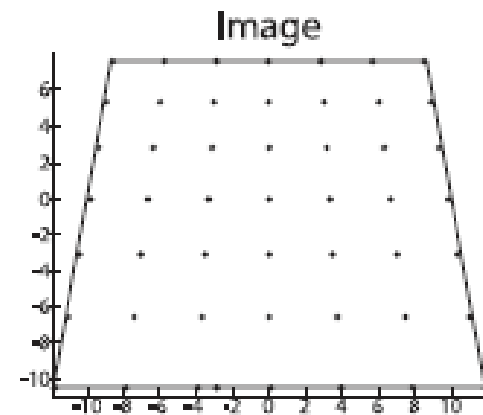
- $x_{corrected} = x(1 + k_1r^2 + k_2r^4 + k_3r^6)$
- $y_{corrected} = y(1 + k_1r^2 + k_2r^4 + k_3r^6)$



## – Tangential



Cheap camera



- $x_{corrected} = x + [2p_1y + p_2(r^2 + 2x^2)]$
- $y_{corrected} = y + [p_1(r^2 + 2y^2) + 2p_2x]$



- Necessary also to find out the Pose (rotation and translation relative to a given coordinate system) of cameras
  - **Rotation** – vector with **3 rotation angles**  $(r_x, r_y, r_z)$  that might be composed in a **3x3 rotation matrix**.
  - **Translation** – vector with 3 values  $(t_x, t_y, t_z)$



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# Parameters of the general model



Referring to the pinhole camera model, a camera matrix can therefore be used to denote a general projective mapping 3D world coordinates with 2D homogeneous coordinates of pixels in images.

$$z_c \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = A \begin{bmatrix} R & T \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

Where  $A$  represents the camera intrinsic parameters,  $R$  and  $T$  the rotation and translation matrix  $\mathbf{x}_w$  each of the coordinates in 3D space and  $\mathbf{u}$  and  $\mathbf{v}$  the indexes of each pixel on the frame buffer.

$$A = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$



- OpenCV Camera model:
  - 4 intrinsic parameters:
    - Focal distance:  $f_x, f_y$
    - Optical centre:  $c_x, c_y$
  - 5 distortion parameters
    - Lens distortion:  $k_1, k_2, k_3, p_1, p_2$
  - 6 extrinsic parameters:
    - Rotation:  $r_x, r_y, r_z$
    - Translation:  $t_x, t_y, t_z$

Total: 15 parameters
- Other models: Tsai, Heikkila, Zhang

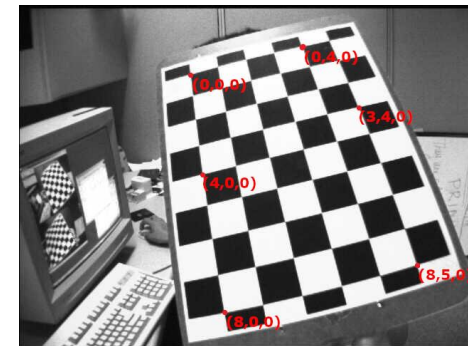
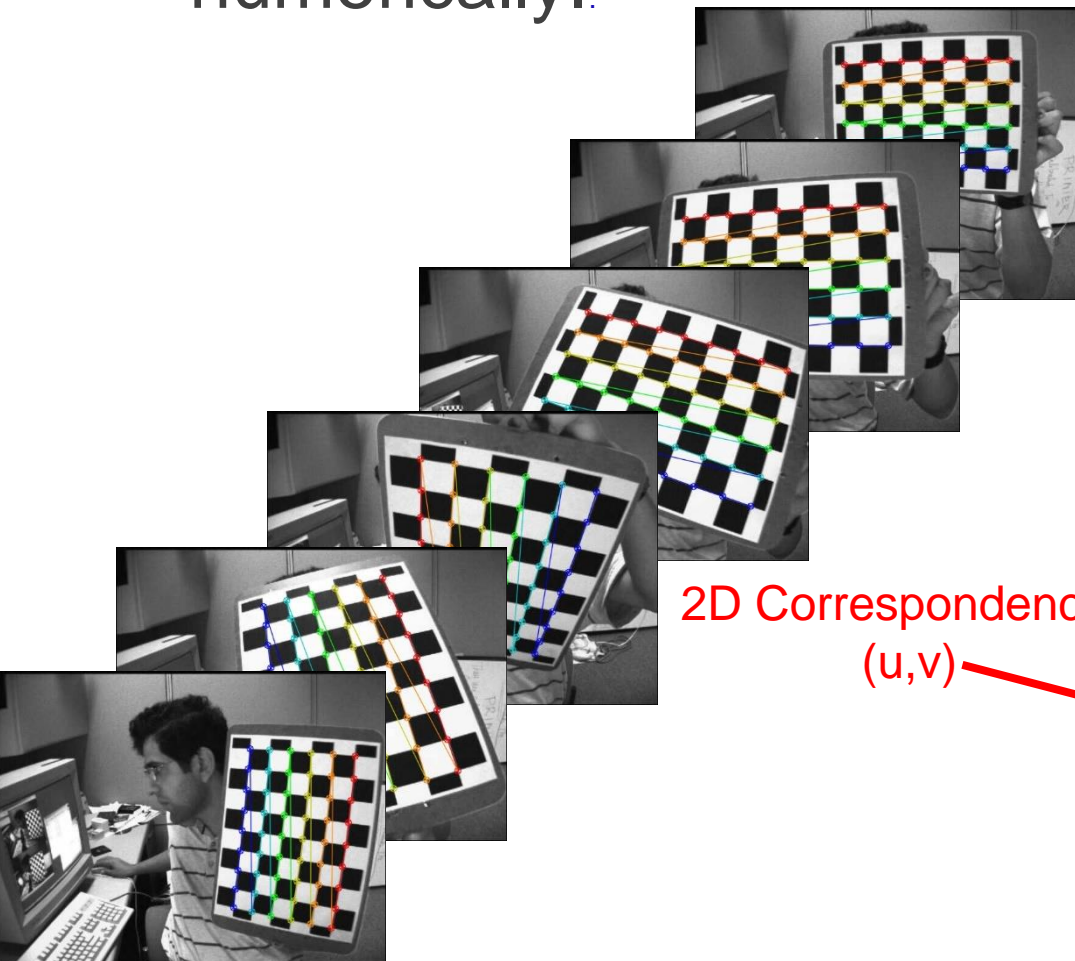


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# Single camera calibration



- Given **3D-2D correspondences**, possible to define a system of equation that might be solved numerically..



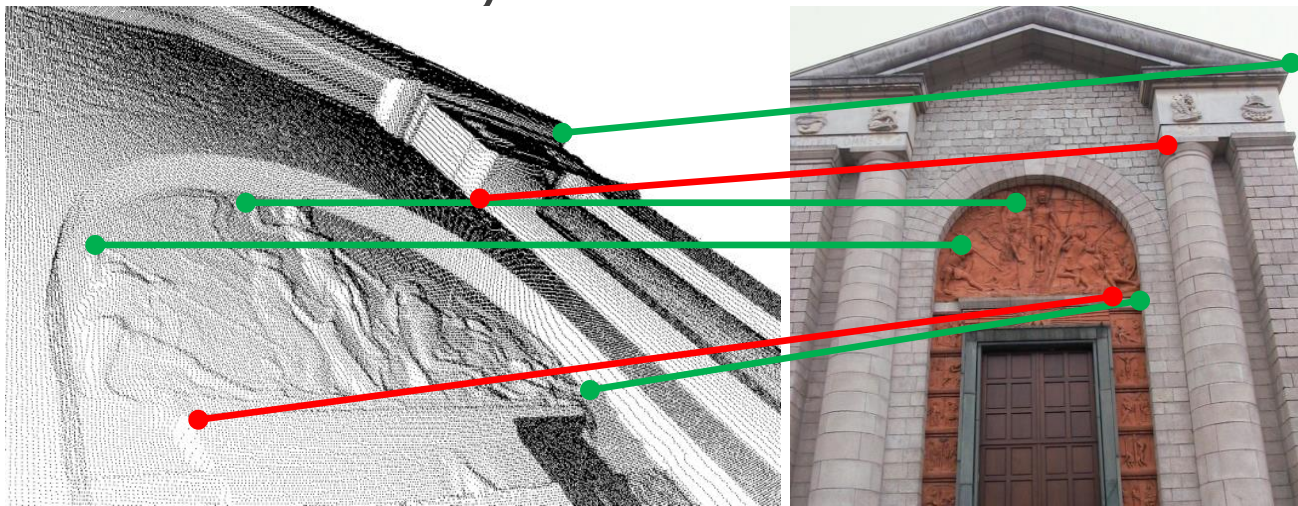
3D Coordinates  
( $x_w, y_w, z_w$ )

2D Correspondences  
( $u, v$ )

$$z_c \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = A \begin{bmatrix} R & T \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

- Given **3D-2D correspondences**, possible to define a system of equation that might be solved numerically.

**=>Problem:** Occurrence of **Outliers** - Only some matches are mutually consistent



Images from 3 Dimensional Reconstruction Of Real World Scenes Using Laser and Intensity Data. Dias P. PhD Thesis, University of Aveiro, 2003.

- RANSAC** - "RANdom SAmple Consensus" might be used to remove outliers





- Random Sample Consensus
  - Example of a “**voting**”-based fitting scheme
  - Each hypothesis gets voted on by each data point, **best hypothesis** wins
- Idea
  - All the inliers will agree with each other on the camera pose; the (hopefully small) number of **outliers** will (hopefully) **disagree** with each other

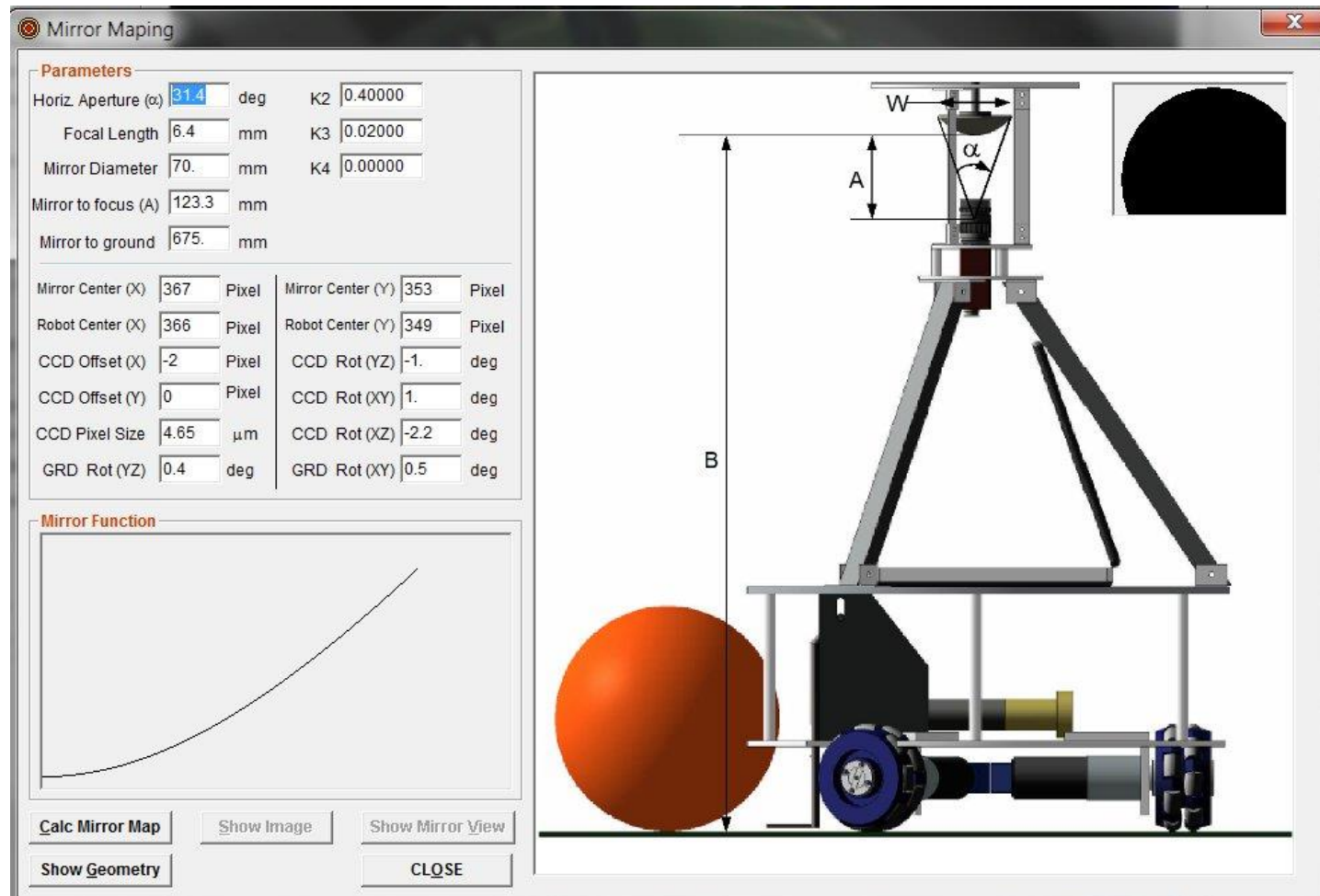


- `cvFindChessboardCorners`: internal corners of the chessboard
  - Optional: `cvFindCornerSubPix`: Refines the corner locations
- `cvCalibrateCamera2`: find intrinsic and extrinsic parameters from several views of chessboard.
- `solvePnP`: finds an object pose from 3D-2D point correspondences.

# Vision System – example



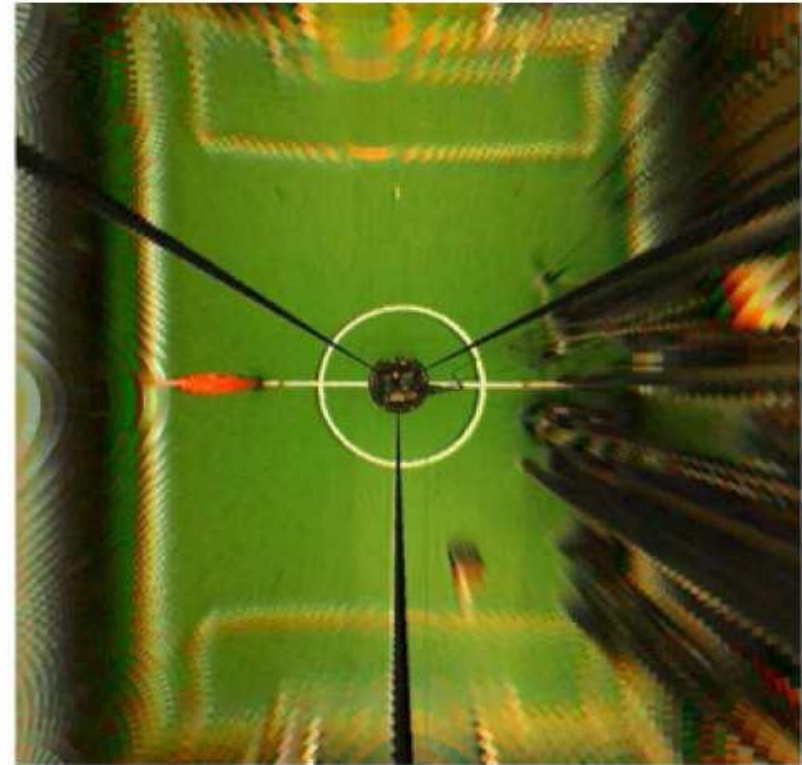
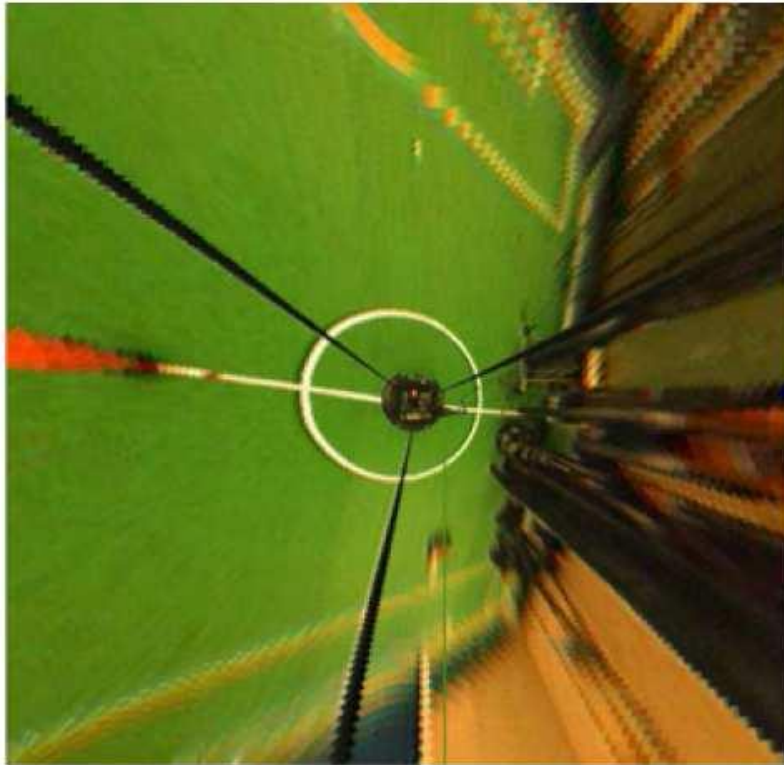
- A catadioptric system is a vision system based on one camera and one mirror system



# Vision System –example



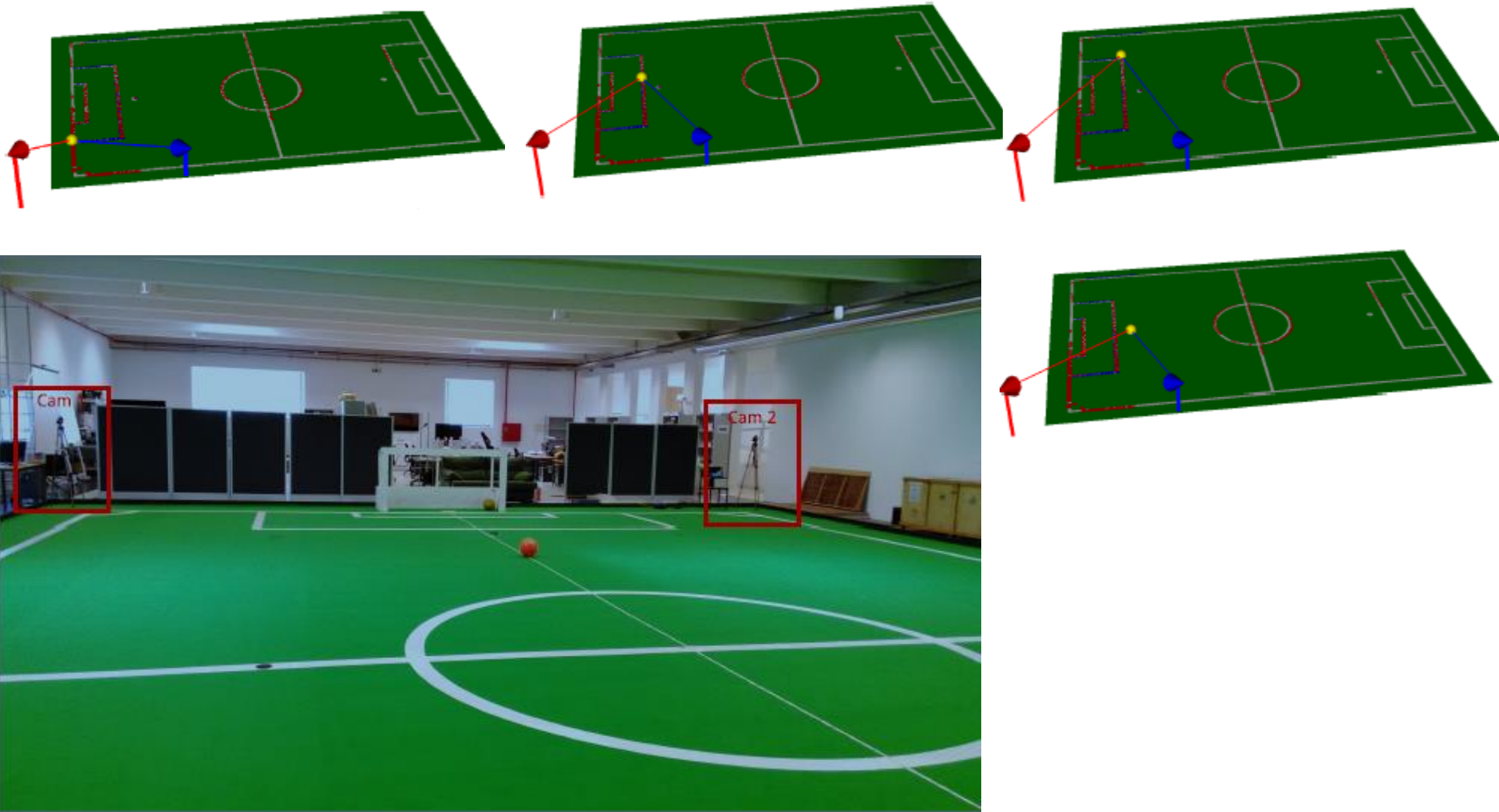
Such a system suffers from mechanical misalignments originating from several possible different reasons. By using the camera model, the catadioptric model and estimating the geometry parameters that introduce the distortion, one can correct the original image to obtain a fairly accurate distance map at the ground level



# Vision System – example



Ground truth for ball positions



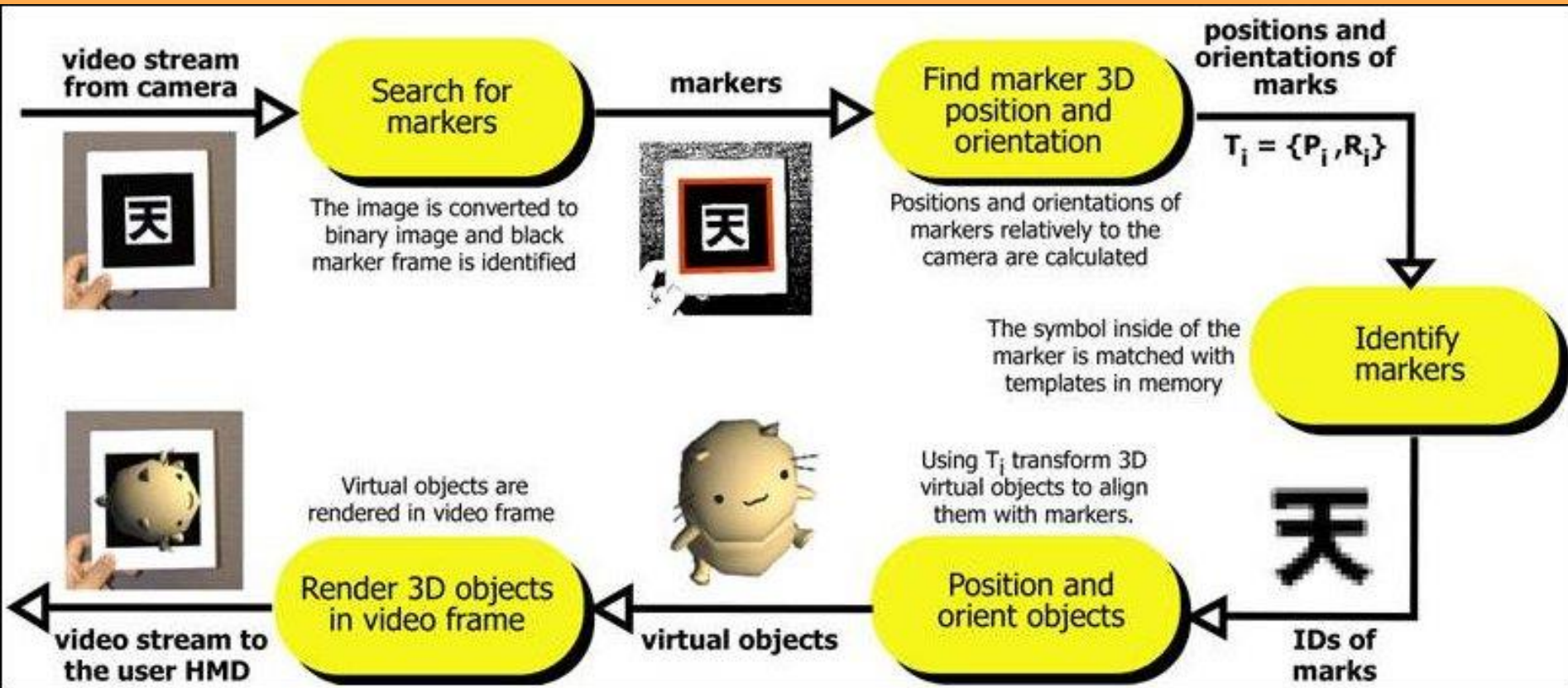


# OPENCV CAMERA CALIBRATION DEMO



- C/C++ library
- Estimates position/orientation of a camera
- Marker detection
- Camera calibration procedure
- Allow Real Time AR application
- 3D models reading
- Open-source





- captures video of the real world
- searches through each video frame for any square shapes.
- If a square is found, mathematics to calculate the position of the camera relative to the square.
- graphics model is drawn from that same position.
- model is drawn on top of the video of the real world and so appears stuck on the square marker.
- final output is shown back in the display, user looks through the display and see graphics overlaid on the real world.





- Default camera properties are contained in the camera parameter file camera\_para.dat, that is read in each time an application is started.
- should be sufficient for a wide range of different cameras.
- However using a relatively simple camera calibration technique it is possible to generate a separate parameter file for the specific cameras that are being used  
(<http://www.hitl.washington.edu/artoolkit/documentation/usercalibration.htm>)
- if camera parameters are known then the video image can be warped to remove camera distortions.



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