Time duration: **45 min.** **Total marks: 10**

General Instructions: Answer all the following questions. Notations and assumptions are as discussed in the lectures. Show all the steps, **make appropriate assumptions wherever (and only if) necessary**. The marks for each question is indicated at the right.

Problem 1. Consider a 3×4 coefficient matrix A whose row-reduced echelon form has only non-zero rows. Prove that the linear system $AX = Y$, where $Y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, must have infinitely many solutions. Do mention the size of matrix X that consists of unknown scalars. **[2 marks]**

Problem 2. Consider a 3×3 matrix $S = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 2 \\ 3 & 7 & 8 \end{bmatrix}$. Using row-reduced echelon form, prove whether or not S is invertible. **[3 marks]**

Problem 3. Consider a coefficient matrix A. If the linear system $AX = Y$ has a unique solution and the linear system $AX = Z$ is consistent (i.e., there exists at least one value of X such that $AX = Z$ is true). Then, prove that the linear system $AX = Y + Z$ must have a unique solution. **[3 marks]**

Problem 4. Prove one of the following: **[2 marks]**

- Suppose A is a 2×1 matrix and B is a 1×2 matrix. Show that, $C = AB$ is not invertible.
- If $ad - bc \neq 0$, then the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ must be invertible.