- Throughout each question, we follow the same notation and terminology, in all parts.

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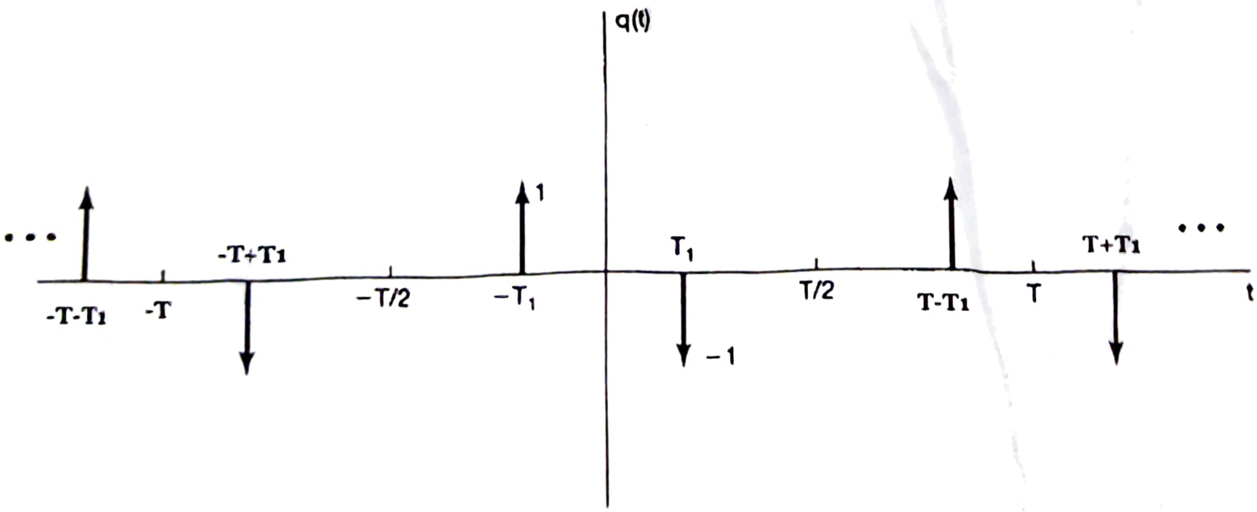
1. (14 marks) Let T be some fixed positive real number. Consider a system with the input $x(t)$ and output $y(t)$ being related as follows.

$$y(t) = \sum_{n \in \mathbb{Z}} x(nT)\delta(t - nT).$$

   (a) (2.5+2.5=5 marks) Prove or disprove that this system is linear and time-invariant. [Note that within the summation, $x(nT)$ refers to the value of the signal x at time instant nT.]

   (b) (2+7) After finding the Laplace transform (LT) of the signal $\delta(t - t_0)$ for some constant time-shift $t_0 \in \mathbb{R}$, use this to determine the LT of the output of the above system, for the input being $e^{-t}u(t)$ (note that you have to obtain the ROC of the LT of the output, also).

Figure 1: The signal $q(t)$



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2. (16 marks)

   (a) (2+2=4 marks) Define precisely the fundamental period of a periodic signal. Use this definition to precisely argue that the fundamental period of the signal $q(t)$ given in Fig. 1 is indeed T.

(b) (6 marks) Find the Fourier series coefficients of $q(t)$.

(c) (3 marks) Derive the relationship between the Fourier series coefficients of an arbitrary periodic signal $x(t)$ and its derivative $\frac{d}{dt}(x(t))$.

(d) (1.5+1.5 = 3 marks) Let $r(t) = \int_{\tau=-\infty}^{t} q(\tau)d\tau$. Plot $r(t)$. Note that by definition $\frac{d}{dt}(r(t)) = q(t)$. Using this relationship and part (c), obtain the Fourier series coefficients of $r(t)$.

3. (10 marks) Let $s = \sigma + j\omega$ be an arbitrary complex number, where $\sigma, \omega \in \mathbb{R}$. Consider an LTI system with system function $H(s)$ as follows.

$$H(s) = \begin{cases} \dfrac{1}{s-2} - \dfrac{1}{s+3}, & \text{if } |\omega| \leq 1000\pi. \\ 0, & \text{otherwise,} \end{cases}$$

with $\text{ROC}(H(s))$ being $\{s \in \mathbb{C} : -3 < \sigma < 2\}$.

(a) (7 marks) What is the response (i.e., the output time-signal) of this system to the input signal $x(t) = sin(300\pi t) + cos(500\pi t) + cos(2000\pi t)$? (Note: You can try to simplify your expressions, however, there is no expectation of expressing your answer in the most simple form). [Hint: Recall the idea that LTI systems have complex exponentials as their eigen-functions.]

(b) (3 marks) Can you sketch a technique to obtain the impulse response of this system? (This means, provide some ideas on how this can be done, as per your learnings. Some equations/expressions are expected. But, actual final calculation is not required).