Real analysis Quiz 1

August 2024

1. Verify that for all $n \geq 1$, the sum of the squares of the first $2n$ positive integers is given by the formula

$$1^2 + 2^2 + 3^2 + ... + (2n)^2 = \frac{n(2n+1)(4n+1)}{3}$$

2. Prove that

$$(A \cup B)^c = A^c \cap B^c \quad \text{and} \quad (A \cap B)^c = A^c \cup B^c$$

3. Use induction to prove that if $1+x > 0$, then $(1+x)^n \geq 1+nx$ for all natural numbers n.