Real analysis
Quiz 2 (Fall 2024)
Duration: 1 hour

Question 1: (8 marks) Define what it means for a function to be uniformly continuous on a set.

Question 2: (9 marks) Give examples, with justification, of each of the following.

1. A bounded sequence (x_n) for which $\lim_{n\to\infty} \sup x_n \neq \lim_{n\to\infty} \inf x_n$.
2. A function $f : [0,1] \to \mathbb{R}$ which is discontinuous at each $x \in [0,1]$.
3. A continuous function which is not uniformly continuous.

Question 3: (8 marks)
Show the following statements:

1. A bounded monotone sequence is convergent.
2. Every sequence has a monotone subsequence.

Question 4 (6 marks)
Let us say that a sequence $(c_n)_{n=1}^{\infty}$ of real numbers "*cervonges* to c" (where $c \in \mathbb{R}$) if and only if there is an $N \in \mathbb{N}$ such that, for all $n > N$ and all $\epsilon > 0$, we have $|c_n - c| < \epsilon$.

1. If a sequence (c_n) cervonges to c, does (c_n) converge to c? Explain, and if not, give an example.
2. If a sequence (c_n) converges to c, does (c_n) cervonge to c? Explain, and if not, give an example.

Question 5: (9 marks)
Let X be a metric space such that $X \subseteq Y$, where Y is a complete metric space. Let (x_n) be a Cauchy sequence in X such that (x_n) contains a convergent subsequence in X. Then (x_n) converges in X.

Question 6: (10 marks)
Let Z be a metric space and let Y be a dense subset of Z. Suppose that every Cauchy sequence in Y converges in Z. Prove that Z is complete.