CS 47

Midterm Review

Topics

- Integer Arithmetic
- Floating Point
- Machine Instructions

Integer Arithmetic

Addition is associative and commutative

Multiplication is associative and commutative and distributes over addition.

Answers are computed modulo 2^w, where w is the word size, 32 for most machines, often 8 or less for hand computation.

Compute the right answer mathematically, and adjust it by a multiple of 2^w to bring it into the proper range.

Unsigned range = [0 to 2^w-1]

Signed range = [-2^{w-1} to 2^{w-1}-1]

Integer Arithmetic

Most tricky behavior happens around the wrap-around values ($2^w \rightarrow 0$) for unsigned and ($2^{w-1} \rightarrow -2^{w-1}$) for two's complement.

Two's complement: -x = ~x + 1 = ~(x - 1) = invert all bits except the last 1.

Most ordering properties of integers can fail mod 2^w

x > y	**⇒**	**-x < -y**	**(y = TMIN)**
x * x >= 0			**(x = Ceil(Sqrt(TMAX)))**

Most algebraic properties hold

$(x - y)^2 == x^2 – 2 * x * y + y^2$	**(True)**

Problem 2.75 [2.49]

The problem asks us to compute the full 2w-bit product of w-bit unsigned int's x' and y' on a machine that can do 2w-bit products only in two's complement form. Let x = (int) x' and y = (int) y'.

We already know the low-order w bits of the product x' * y' are the same as the low-order w bits of x * y.

By Equation 2.16 we have $x' = x + x_{w-1}2^w$ and

$$x' \cdot y' = x \cdot y + (x_{w-1}y + y_{w-1}x)2^w + x_{w-1}y_{w-1}2^{2w}.$$

The final term has no effect on the 2w-bit representation of x' · y', but the middle term represents a correction factor that must be added to the high order w bits.

Example 1

The idea here is that the unsigned_high_prod is gotten from signed_high_prod by adding $2^w (x_{w-1}y + y_{w-1}x)$. That means if y < 0 add x and if x < 0 add y to the high order w bits, where x and y are the two's complement values.

Example 1: w = 8

x = 7C = 124

y = 83 = -125

Remembering to use sign extension, x = 007C, y = FF83

x * y = C374 = -15500

x' = 7C = 124

y' = 83 = 131

x' * y' = C374 + 7C00 = 3F74 = 16244

Example 2

Example 2: w = 8

x = FD = -3

y = 83 = -125

Remembering to use sign extension, x = FFFD, y = FF83

x * y = 0177 = 375

x' = FD = 253

y' = 83 = 131

x' * y' = 0177 + FD00 + 8300 = 8177 = 33143

C solution

This is implemented as follows:

```
unsigned unsigned_high_prod(unsigned x, unsigned y)
{
        unsigned p = (unsigned) signed_high_prod((int) x, (int) y);

        if ((int) x < 0)               /* x_{w-1} = 1 */
                p += y;
        if ((int) y < 0)               /* y_{w-1} = 1 */
                p += x;
        return p;
}
```


Floating Point Formats

s, exp, fraction

1	8	23	float
1	11	52	double

$Val = (-1)^s \times 2^E \times M$

$E = exp - bias + (M < 1)$ bias $= 2^{expdigits-1} - 1$ (127 or 1023)

M = 1 + fraction if exp != 0

M = 0 + fraction if exp == 0

0 <= fraction < 1

Floating Point Formats

s, exp, fraction

1	8	23	float

1	11	52	double

Inf = ∞ has exp = $2^{expdigits}$ -1 = all 1's (255 or 2047)

and fraction == 0.

NaN has same exp and fraction != 0

Problem 2.47 [2.33]

Bits	exp	E	frac	M	value
0 00 00	0	0	0	0	0
0 00 01	0	0	1/4	1/4	1/4
0 00 10	0	0	2/4	2/4	2/4
0 00 11	0	0	3/4	3/4	3/4
0 01 00	1	0	0	4/4	4/4
0 01 01	1	0	1/4	5/4	5/4
0 01 10	1	0	2/4	6/4	6/4
0 01 11	1	0	3/4	7/4	7/4

Problem 2.47 [2.33] (cont.)

Bits	exp	E	frac	M	value
0 10 00	2	1	0	4/4	8/4
0 10 01	2	1	1/4	5/4	10/4
0 10 10	2	1	2/4	6/4	12/4
0 10 11	2	1	3/4	7/4	14/4
0 11 00	3	-	0	-	∞
0 11 01	3	-	1/4	-	NaN
0 11 10	3	-	2/4	-	NaN
0 11 11	3	-	3/4	-	NaN

Floating Point Arithmetic

Addition is commutative but not always associative

Multiplication is commutative and not associative and does not distribute over addition.

Answers are rounded to the floating point value nearest to the exact answer. Round to even in case of ties.

Most tricky behavior happens when rounding errors depend on the order of the operations.

Every value has an exact negative: x = -(-x)

Floating Point Arithmetic

Most ordering properties of reals hold for floats.

d < 0.0	⇒	((d*2) < 0.0)	True
d > f	⇒	-f > -d	True
a + b + c	==	c + (a + b)	Yes! commutative

Many algebraic properties of reals don't hold for floats.

(d + f) - d	==	f	No: Not associative
a + b + c	==	c + a + b	No: Not associative

Practice Problems

Study suggestions

Chapter 2 has many Practice Problems that ask you to convert numbers between formats: binary, hexadecimal, decimal, signed, and unsigned.

There are also many problems dealing with floating point numbers. Look carefully at the ones with shortened formats (<= 8 bits) for hand calculations similar to 2.47 [2.33]. Review your builddbl and dblparts programs.

Answers are given in the back of the chapter. If you can do these problems without peeking you will be well prepared for this part of the exam.

Machine Level Programs

Topics

- instructions
  - mov, lea, push, pop
  - add, sub, imul, and, or, xor, shl, sar, slr
  - not, neg, inc, dec
- addressing
  - src, dest Imm(E_b, E_i, s) or E_b
- control
  - test, cmp, set, jmp, je, jne, jg, etc.
- procedures
  - call, ret, leave

Example of Assembly Code from C

```c
int arith
  (int x, int y, int z)
{
  int t1 = x+y;
  int t2 = z+t1;
  int t3 = x+4;
  int t4 = y * 48;
  int t5 = t3 + t4;
  int rval = t2 * t5;
  return rval;
}
```

```
arith:
   pushl %ebp
   movl %esp,%ebp

   movl 8(%ebp),%eax
   movl 12(%ebp),%edx
   leal (%edx,%eax),%ecx
   leal (%edx,%edx,2),%edx
   sall $4,%edx
   addl 16(%ebp),%ecx
   leal 4(%edx,%eax),%eax
   imull %ecx,%eax

   movl %ebp,%esp
   popl %ebp
   ret
```

Set Up

Body

Finish

Address Computation Examples

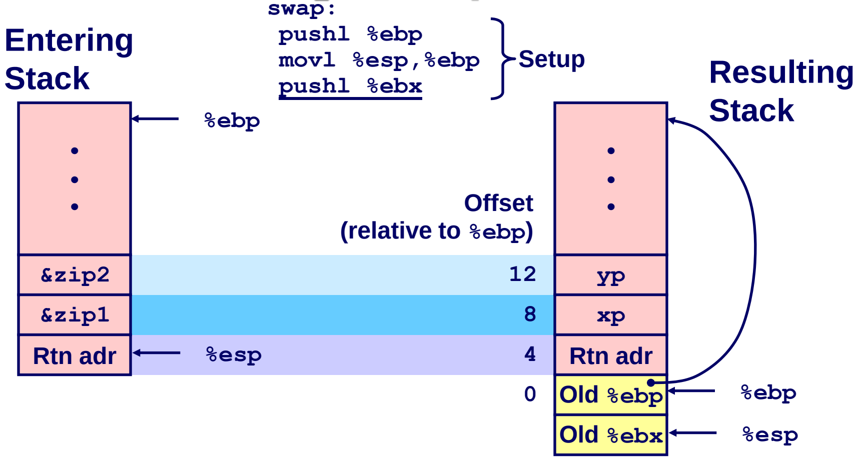
`%edx`	`0xf000`
`%ecx`	`0x100`

Expression	Computation	Address
`0x8(%edx)`	`0xf000 + 0x8`	`0xf008`
`(%edx,%ecx)`	`0xf000 + 0x100`	`0xf100`
`(%edx,%ecx,4)`	`0xf000 + 4*0x100`	`0xf400`
`0x80(,%edx,2)`	`2*0xf000 + 0x80`	`0x1e080`

Effect of swap Setup

```
swap:
 pushl %ebp
 movl %esp,%ebp
 pushl %ebx
```

Setup




```
movl 12(%ebp),%ecx # get yp
movl 8(%ebp),%edx  # get xp
. . .
```

Body

Practice Problems

Study suggestions

Chapter 3 has many Practice Problems that ask you to find registers and memory locations corresponding to C variables and machine instructions corresponding to C operators.

Sometimes you need to fill in C code for given assembly code or vice versa.

Answers are given in the back of the chapter. If you can do these problems without peeking you will be well prepared for this part of the exam.