Universität Konstanz
Fachbereich Mathematik und Statistik
Jun.-Prof. Dr. Gabriele Ciaramella
Christian Jäkle

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Iterative Methods

Project 1 - Reduced formulation

Stationary

- Derive the optimality system
- Implement a function `FD_Laplacian` which creates the Finite Difference matrix representation of the Laplace operator (in sparse).
- Implement the fast Poisson solver.
- Solve the system
$$(\nu I + A^{-2})\underline{u} = A^{-1}(\underline{y_d} - A^{-1}\underline{f}) \tag{1}$$
by the stationary iteration
$$\underline{u}^{n+1} = -\frac{1}{\nu}A^{-2}\underline{u}^n + \frac{1}{\nu}A^{-1}(\underline{y_d} - A^{-1}\underline{f}) \tag{2}$$
and analyze its convergence behavior (for different h and ν). To do so:
    1. Compute the exact eigenvalues and eigenvectors of the one-dimensional discrete Laplace. Use them for the fast Poisson solver.
    2. Compute also the exact eigenvalues and eigenvectors of the two-dimensional discrete Laplace.
- Solve the system
$$(\nu A^2 + I)\underline{u} = A(\underline{y_d} - A^{-1}\underline{f}) = A\underline{y_d} - \underline{f} \tag{3}$$
using the damped-Jacobi iteration. Study the convergence behavior (different ν, h, ω).

Krylov

- Solve (1) and (3) with matrix free pcg (matlab or python). Analyze the convergence for different ν and h numerically.
- Study the convergence by computing the condition number of $\nu I + A^{-2}$ and $\nu A^2 + I$.

Multigrid

- Solve (1) by the Multigrid method using (2) as smoother.
- Solve (3) by the Multigrid method using damped-Jacobi as smoother.

Write a short Report about your findings.