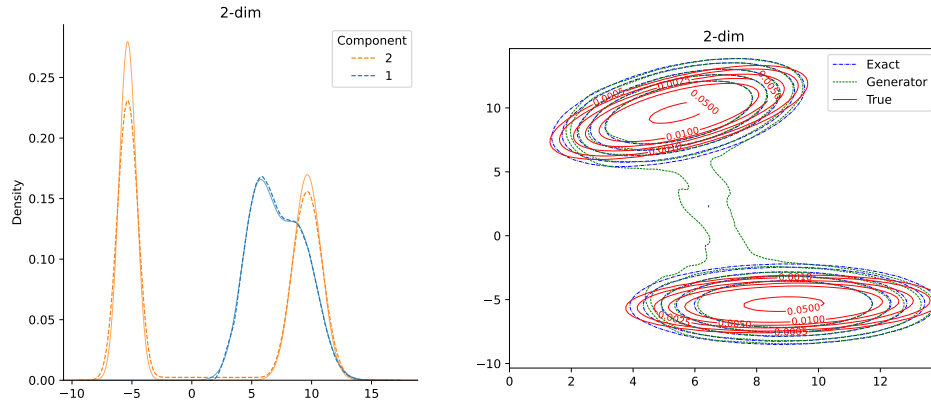


# Plots and tables for article “Generative neural networks for characteristic functions”

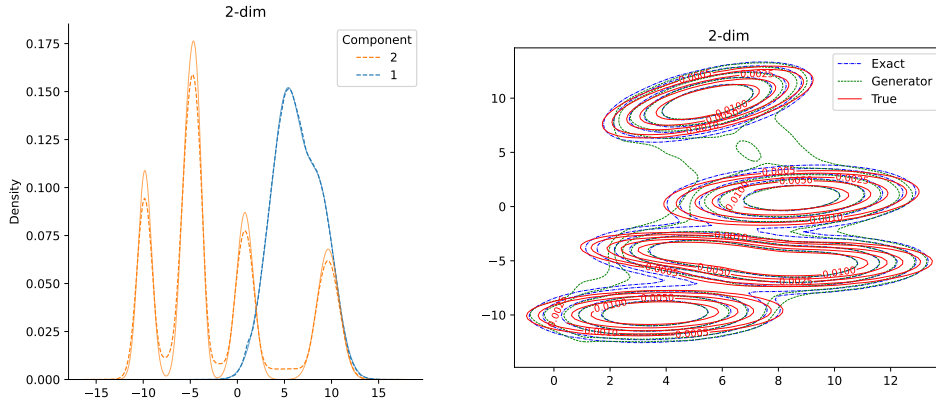
Florian Brück

September 13, 2024

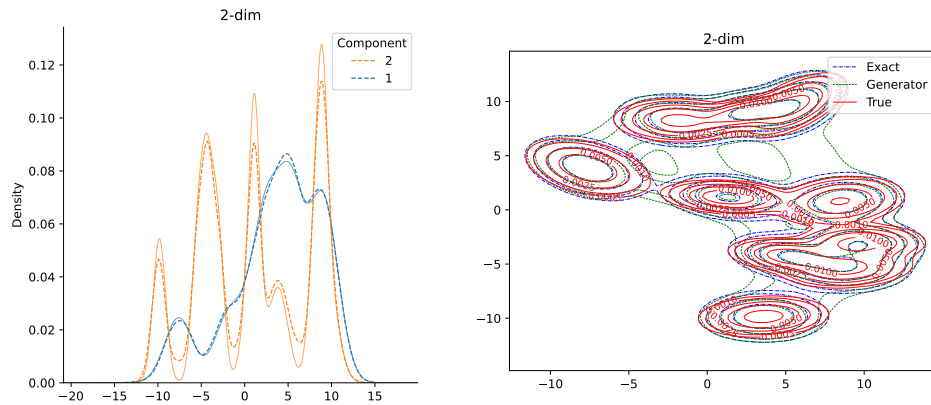
## 1 Plots for Gaussian mixture distributions



(a) Gaussian mixture distribution 2-dim with 2 mixture components

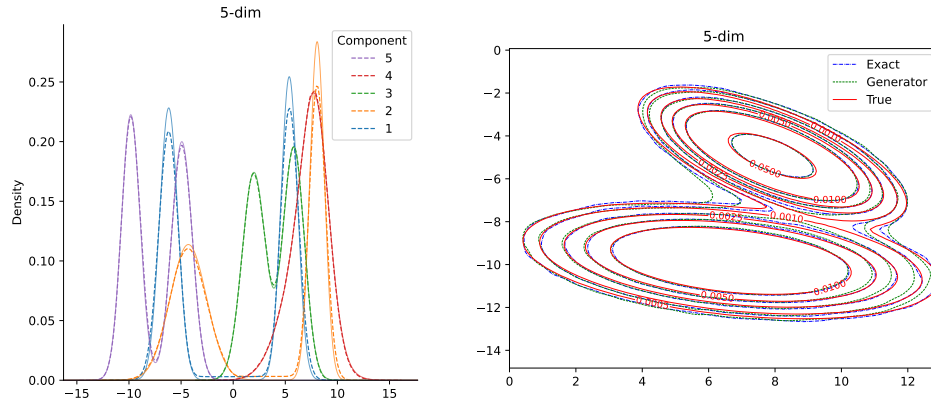


(b) Gaussian mixture distribution 2-dim with 5 mixture components

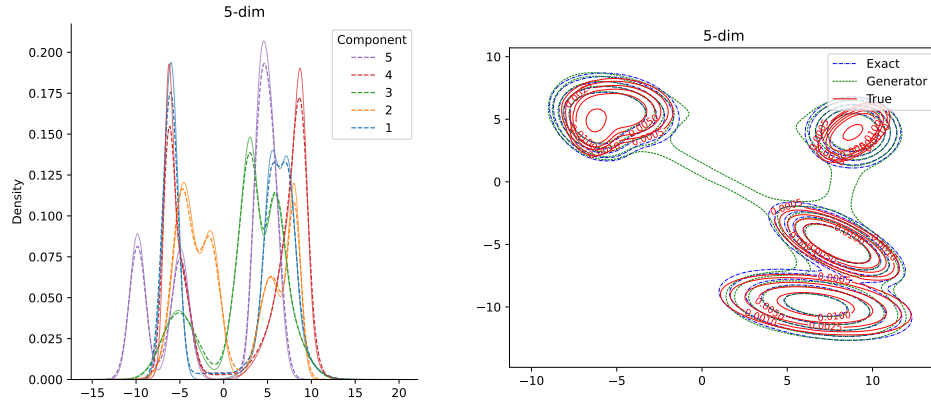


(c) Gaussian mixture distribution 2-dim with 10 mixture components

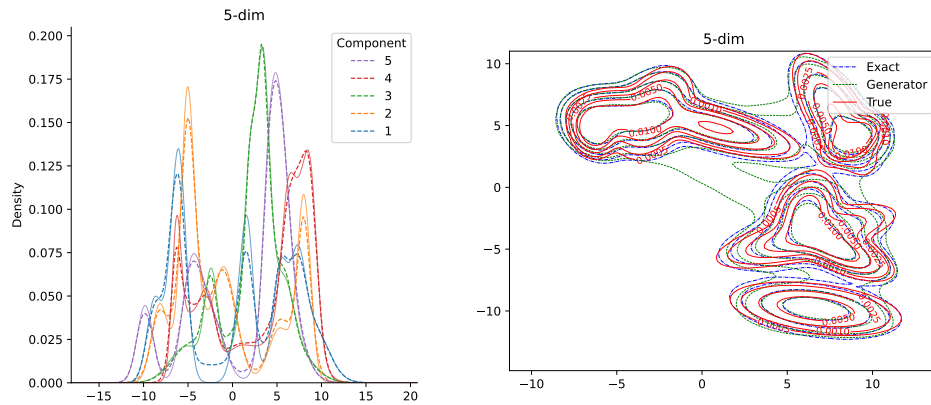
Figure 1: Estimated marginal (left) and bivariate (right) densities of the Gaussian mixture distribution in dimensions 2 with 2, 5, 10 mixture components. The solid red lines show the true densities and the dashed green and dash-dotted blue lines correspond to estimated densities from the generator and the exact simulation algorithm.



(a) Gaussian mixture distribution 5-dim with 2 mixture components

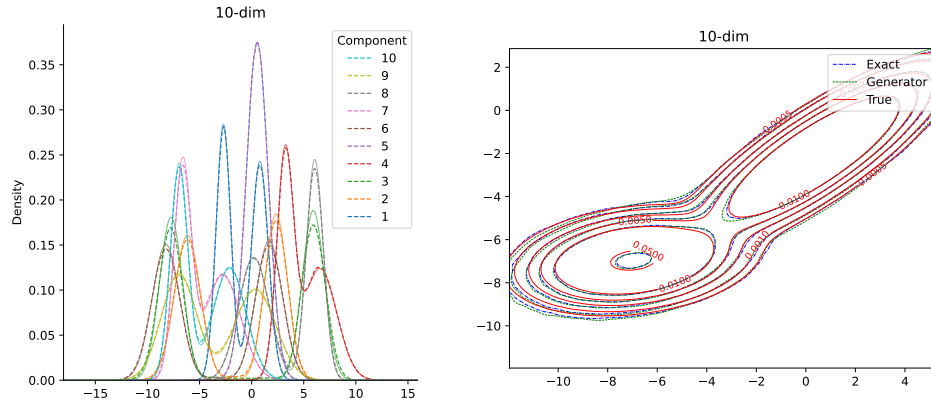


(b) Gaussian mixture distribution 5-dim with 5 mixture components

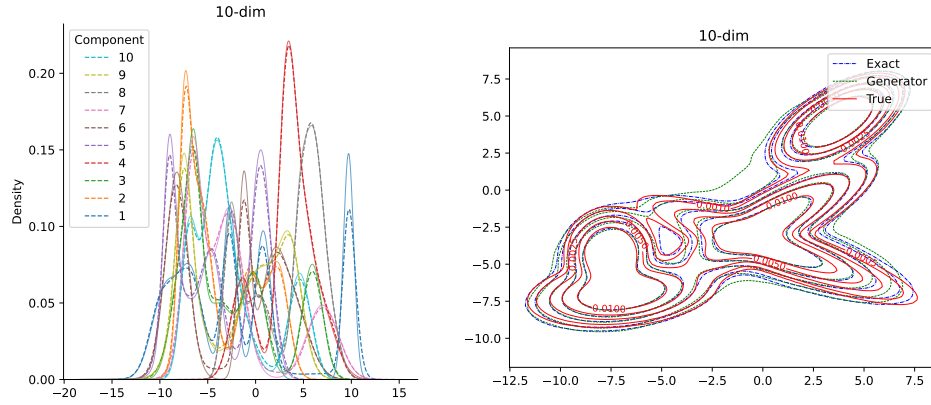


(c) Gaussian mixture distribution 5-dim with 10 mixture components

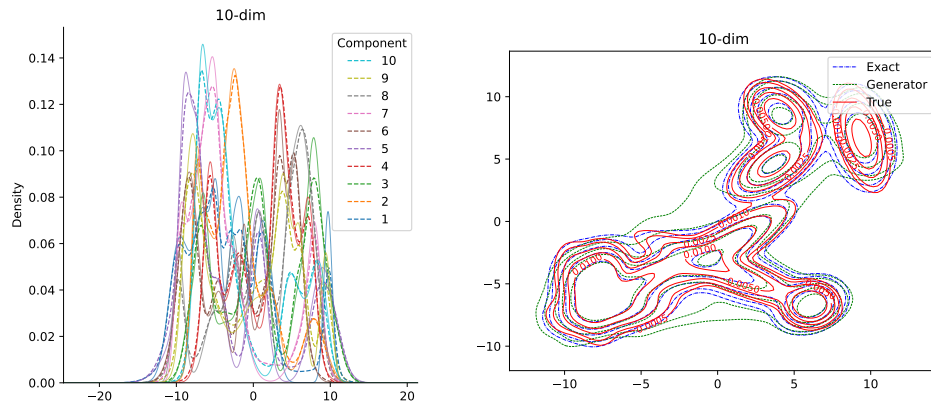
Figure 2: Estimated marginal (left) and bivariate (right) densities of the Gaussian mixture distribution in dimension 5 with 2, 5, 10 mixture components. The bivariate densities correspond to the last two components of the corresponding random vector. The solid red lines show the true densities and the dashed green and dash-dotted blue lines correspond to estimated densities from the generator and the exact simulation algorithm.



(a) Gaussian mixture distribution 10-dim with 2 mixture components



(b) Gaussian mixture distribution 10-dim with 5 mixture components



(c) Gaussian mixture distribution 10-dim with 10 mixture components

Figure 3: Estimated marginal (left) and bivariate (right) densities of the Gaussian mixture distribution in dimension 10 with 2, 5, 10 mixture components. The bivariate densities correspond to the last two components of the corresponding random vector. The solid red lines show the true densities and the dashed green and dash-dotted blue lines correspond to estimated densities from the generator and the exact simulation algorithm.

## 2 Tables for stable distributions

	Quantile	0.05	0.1	0.3	0.5	0.7	0.9	0.95
b_{1}	Generator	-19.75	-5.71	1.49	2.05	2.86	18.24	55.78
	True	-51.38	-9.87	1.33	2.00	2.67	13.93	55.65
	Approximate	-51.88	-9.87	1.34	2.00	2.67	14.01	55.82
b_{2}	Generator	-29.84	-8.85	-0.56	-0.02	0.46	8.02	26.86
	True	-40.49	-9.00	-0.51	0.00	0.51	9.02	40.57
	Approximate	-40.22	-9.03	-0.50	0.00	0.51	9.05	40.18
b_{3}	Generator	-35.70	-10.51	0.30	1.06	2.01	17.98	58.48
	True	-65.42	-13.77	0.17	1.00	1.84	15.81	67.58
	Approximate	-65.04	-13.80	0.17	1.00	1.82	15.76	67.28

Table 1: Estimated quantiles of the projection of  $\langle \mathbf{u}_i, \mathbf{Y} \rangle$  for  $\alpha = 1/2$ ,  $d = 2$  and  $\Sigma = \text{id}_d + 0.5 * \mathbf{1}_d$ .

	Quantile	0.05	0.1	0.3	0.5	0.7	0.9	0.95
b_{1}	Generator	-43.03	-10.95	4.10	4.98	5.69	14.96	34.76
	True	-57.33	-8.86	4.22	5.00	5.78	18.89	67.48
	Approximate	-57.39	-8.92	4.22	5.00	5.77	18.88	67.07
b_{2}	Generator	-59.37	-22.69	0.16	0.94	1.26	4.07	8.66
	True	-36.12	-7.25	0.54	1.00	1.47	9.28	38.25
	Approximate	-35.88	-7.22	0.54	1.00	1.46	9.31	38.32
b_{3}	Generator	-9.07	-3.12	0.65	1.09	2.28	34.97	79.83
	True	-53.05	-11.02	0.32	1.00	1.68	13	54.99
	Approximate	-53.22	-11.03	0.34	1.00	1.67	12.93	54.13

Table 2: Estimated quantiles of the projection of  $\langle \mathbf{u}_i, \mathbf{Y} \rangle$  for  $\alpha = 1/2$ ,  $d = 5$  and  $\Sigma = \text{id}_d + 0.5 * \mathbf{1}_d$ .

	Quantile	0.05	0.1	0.3	0.5	0.7	0.9	0.95
b_{\{1\}}	Generator	-427.47	-211.99	6.85	9.71	10.27	13.69	18.57
	True	-68.69	-7.50	9.01	10.00	10.98	27.45	88.48
	Approximate	-68.49	-7.48	9.03	10.00	10.97	27.44	88.83
b_{\{2\}}	Generator	-79.24	-39.10	-1.18	-0.10	0.16	1.88	4.31
	True	-34.27	-7.62	-0.43	0.00	0.43	7.68	34.54
	Approximate	-34.22	-7.67	-0.42	0.00	0.42	7.66	34.63
b_{\{3\}}	Generator	-104.76	-49.69	3.07	4.83	5.33	9.04	14.41
	True	-60.02	-9.46	4.18	5.00	5.81	19.41	69.81
	Approximate	-60.13	-9.52	4.20	5.00	5.8	19.41	69.85

Table 3: Estimated quantiles of the projection of  $\langle \mathbf{u}_i, \mathbf{Y} \rangle$  for  $\alpha = 1/2$ ,  $d = 10$  and  $\Sigma = \text{id}_d + 0.5 * \mathbf{1}_d$ .

	Quantile	0.05	0.1	0.3	0.5	0.7	0.9	0.95
b_{\{1\}}	Generator	-50.22	-12.54	1.27	1.97	2.48	9.08	22.21
	True	-45.24	-8.51	1.41	2.00	2.59	12.46	49.04
	Approximate	-44.70	-8.58	1.41	2.00	2.58	12.54	49.24
b_{\{2\}}	Generator	-31.10	-10.06	-0.68	-0.04	0.52	9.33	29.36
	True	-47.06	-10.46	-0.59	0.00	0.59	10.51	47.27
	Approximate	-47.11	-10.50	-0.59	0.00	0.58	10.5	47.25
b_{\{3\}}	Generator	-55.90	-16.85	0.08	1.04	1.97	14.79	42.55
	True	-73.53	-15.57	0.07	1.00	1.94	17.61	75.68
	Approximate	-73.46	-15.62	0.06	1.00	1.92	17.58	75.59

Table 4: Estimated quantiles of the projection of  $\langle \mathbf{u}_i, \mathbf{Y} \rangle$  for  $\alpha = 1/2$ ,  $d = 2$  and  $\Sigma = \text{id}_d$ .

	Quantile	0.05	0.1	0.3	0.5	0.7	0.9	0.95
b_{\{1\}}	Generator	-12.71	-2.00	4.51	5.01	5.56	15.29	33.8
	True	-36.93	-4.33	4.47	5.00	5.52	14.28	46.75
	Approximate	-36.41	-4.27	4.49	5.00	5.52	14.36	46.91
b_{\{2\}}	Generator	-85.78	-34.42	-0.04	0.93	1.26	4.06	8.36
	True	-40.78	-8.29	0.48	1.00	1.53	10.35	43.04
	Approximate	-40.76	-8.32	0.48	1.00	1.52	10.27	42.94
b_{\{3\}}	Generator	-9.87	-3.48	0.61	1.11	2.54	56.51	136.11
	True	-60.90	-12.76	0.23	1.00	1.78	14.83	63.18
	Approximate	-61.07	-12.73	0.23	1.00	1.76	14.81	62.67

Table 5: Estimated quantiles of the projection of  $\langle \mathbf{u}_i, \mathbf{Y} \rangle$  for  $\alpha = 1/2$ ,  $d = 5$  and  $\Sigma = \text{id}_d$ .

	Quantile	0.05	0.1	0.3	0.5	0.7	0.9	0.95
b_{\{1\}}	Generator	-175.44	-73.96	8.55	9.88	10.16	12.13	14.72
	True	-30.24	1.05	9.50	10.00	10.5	18.93	50.15
	Approximate	-30.53	1.02	9.50	10.00	10.49	18.87	49.96
b_{\{2\}}	Generator	-6.91	-3.03	-0.28	0.06	0.98	31.54	75.32
	True	-40.13	-8.92	-0.50	0.00	0.51	8.98	40.39
	Approximate	-39.80	-8.87	-0.50	0.00	0.5	8.99	40.5
b_{\{3\}}	Generator	-197.06	-81.57	3.18	4.87	5.37	9.09	14.13
	True	-58.60	-9.14	4.20	5.00	5.8	19.16	68.66
	Approximate	-59.48	-9.26	4.21	5.00	5.78	19.05	68.35

Table 6: Estimated quantiles of the projection of  $\langle \mathbf{u}_i, \mathbf{Y} \rangle$  for  $\alpha = 1/2$ ,  $d = 10$  and  $\Sigma = \text{id}_d$ .

	Quantile	0.05	0.1	0.3	0.5	0.7	0.9	0.95
b_{\{1\}}	Generator	-4.44	-1.11	1.26	1.99	2.7	4.94	7.8
	True	-4.30	-1.07	1.27	2.00	2.73	5.08	8.32
	Approximate	-4.27	-1.05	1.31	2.00	2.69	5.05	8.25
b_{\{2\}}	Generator	-5.03	-2.45	-0.58	-0.01	0.57	2.4	4.83
	True	-5.02	-2.45	-0.58	-0.00	0.58	2.44	5
	Approximate	-5.02	-2.42	-0.54	0.00	0.54	2.42	5.02
b_{\{3\}}	Generator	-6.97	-2.96	0.06	1.00	1.95	4.97	9.13
	True	-7.17	-2.98	0.06	1.00	1.94	4.97	9.15
	Approximate	-7.14	-2.94	0.11	1.00	1.88	4.94	9.16

Table 7: Estimated quantiles of the projection of  $\langle \mathbf{u}_i, \mathbf{Y} \rangle$  for  $\alpha = 1$ ,  $d = 2$  and  $\Sigma = \text{id}_d + 0.5 * \mathbf{1}_d$ .

	Quantile	0.05	0.1	0.3	0.5	0.7	0.9	0.95
b_{\{1\}}	Generator	-1.71	1.53	4.16	5.03	5.94	9.09	13.47
	True	-2.61	1.29	4.13	5.00	5.87	8.69	12.57
	Approximate	-2.58	1.34	4.17	5.00	5.82	8.67	12.59
b_{\{2\}}	Generator	-3.06	-0.99	0.50	1.02	1.58	3.63	7.01
	True	-3.74	-1.31	0.45	1.00	1.55	3.33	5.77
	Approximate	-3.76	-1.30	0.49	1.00	1.51	3.29	5.74
b_{\{3\}}	Generator	-8.12	-2.92	0.15	0.96	1.73	3.96	6.57
	True	-5.91	-2.37	0.20	1.00	1.8	4.37	7.91
	Approximate	-5.87	-2.33	0.26	1.00	1.75	4.36	7.93

Table 8: Estimated quantiles of the projection of  $\langle \mathbf{u}_i, \mathbf{Y} \rangle$  for  $\alpha = 1$ ,  $d = 5$  and  $\Sigma = \text{id}_d + 0.5 * \mathbf{1}_d$ .

	Quantile	0.05	0.1	0.3	0.5	0.7	0.9	0.95
b_{1}	Generator	-2.50	4.64	8.78	9.92	11.01	14.11	17.61
	True	0.37	5.30	8.89	10.00	11.11	14.7	19.62
	Approximate	0.39	5.36	8.95	10.00	11.05	14.65	19.59
b_{2}	Generator	-3.77	-2.04	-0.50	0.00	0.51	2.26	5.04
	True	-4.46	-2.18	-0.51	-0.00	0.51	2.18	4.46
	Approximate	-4.45	-2.14	-0.48	-0.00	0.48	2.16	4.46
b_{3}	Generator	-5.21	0.56	4.01	4.95	5.89	8.57	11.56
	True	-3.25	0.98	4.05	5.00	5.95	9.03	13.27
	Approximate	-3.23	1.01	4.11	5.00	5.89	8.97	13.21

Table 9: Estimated quantiles of the projection of  $\langle \mathbf{u}_i, \mathbf{Y} \rangle$  for  $\alpha = 1$ ,  $d = 10$  and  $\Sigma = \text{id}_d + 0.5 * \mathbf{1}_d$ .

	Quantile	0.05	0.1	0.3	0.5	0.7	0.9	0.95
b_{1}	Generator	-3.59	-0.76	1.34	1.99	2.64	4.71	7.4
	True	-3.68	-0.77	1.35	2.00	2.65	4.77	7.68
	Approximate	-3.64	-0.74	1.38	2.00	2.62	4.75	7.66
b_{2}	Generator	-5.49	-2.72	-0.65	-0.00	0.66	2.77	5.6
	True	-5.70	-2.78	-0.66	-0.00	0.65	2.76	5.67
	Approximate	-5.65	-2.75	-0.62	-0.00	0.62	2.75	5.67
b_{3}	Generator	-7.91	-3.40	-0.04	1.00	2.03	5.3	9.66
	True	-8.00	-3.39	-0.04	1.00	2.03	5.38	9.99
	Approximate	-7.95	-3.34	0.02	1.00	1.98	5.35	9.98

Table 10: Estimated quantiles of the projection of  $\langle \mathbf{u}_i, \mathbf{Y} \rangle$  for  $\alpha = 1$ ,  $d = 2$  and  $\Sigma = \text{id}_d$ .

	Quantile	0.05	0.1	0.3	0.5	0.7	0.9	0.95
b_{1}	Generator	-0.19	2.42	4.38	5.00	5.62	7.63	10.28
	True	-0.29	2.42	4.39	5.00	5.61	7.58	10.29
	Approximate	-0.30	2.43	4.43	5.00	5.57	7.57	10.27
b_{2}	Generator	-4.17	-1.44	0.41	1.00	1.62	3.64	6.25
	True	-4.28	-1.57	0.39	1.00	1.61	3.57	6.28
	Approximate	-4.29	-1.57	0.43	1.00	1.57	3.55	6.28
b_{3}	Generator	-6.48	-2.81	0.10	1.00	1.9	4.73	8.61
	True	-6.87	-2.83	0.10	1.00	1.9	4.82	8.84
	Approximate	-6.88	-2.80	0.15	1.00	1.85	4.8	8.82

Table 11: Estimated quantiles of the projection of  $\langle \mathbf{u}_i, \mathbf{Y} \rangle$  for  $\alpha = 1$ ,  $d = 5$  and  $\Sigma = \text{id}_d$ .

	Quantile	0.05	0.1	0.3	0.5	0.7	0.9	0.95
b_{1}	Generator	2.68	6.96	9.35	9.97	10.57	12.19	14.23
	True	4.84	7.48	9.41	10.00	10.59	12.52	15.15
	Approximate	4.87	7.52	9.45	10.00	10.56	12.49	15.15
b_{2}	Generator	-4.55	-2.43	-0.61	-0.01	0.58	2.54	5.62
	True	-5.16	-2.52	-0.59	-0.00	0.59	2.52	5.18
	Approximate	-5.10	-2.48	-0.55	0.00	0.56	2.51	5.16
b_{3}	Generator	-4.50	0.85	4.06	4.98	5.94	8.76	11.95
	True	-3.17	1.02	4.06	5.00	5.94	8.99	13.17
	Approximate	-3.20	1.03	4.12	5.00	5.87	8.92	13.09

Table 12: Estimated quantiles of the projection of  $\langle \mathbf{u}_i, \mathbf{Y} \rangle$  for  $\alpha = 1$ ,  $d = 10$  and  $\Sigma = \text{id}_d$ .