

# Algorithmique et Recherche Opérationnelle

## TD2 - Correction Exercice 2

### Méthode du simplexe à deux phases.

$$\begin{array}{rcll}
 \max & x_2 & + & 3x_1 & \text{s.c.} \\
 & x_2 & + & 2x_1 & \leq 4 \\
 & -x_2 & - & x_1 & \leq -3 \\
 & -x_2 & + & x_1 & \leq -1 \\
 & & & x_1, x_2 & \geq 0
 \end{array} \tag{1}$$

$$\begin{array}{rcll}
 \min & x_0 & \text{s.c.} \\
 x_3 & + & x_2 & + & 2x_1 & - & x_0 & = & 4 \\
 x_4 & - & x_2 & - & x_1 & - & x_0 & = & -3 \\
 x_5 & - & x_2 & + & x_1 & - & x_0 & = & -1 \\
 & & & & x_0, \dots, x_5 & \geq & 0
 \end{array} \tag{1 Aux.}$$

DICTIONNAIRE 1 AUX.I:

$$\begin{array}{rcll}
 x_3 & = & 4 & - & x_2 & - & 2x_1 & + & x_0 \\
 x_4 & = & -3 & + & x_2 & + & x_1 & + & x_0 \\
 x_5 & = & -1 & + & x_2 & - & x_1 & + & x_0 \\
 \hline
 z & = & x_0
 \end{array}$$

Solution 1:  $(0, 0, 0, 4, -3, -1)$ .

$$\text{variable entrante } x_0 \Rightarrow \begin{cases} 4 + x_0 \geq 0 & (x_3) \\ -3 + x_0 \geq 0 & (x_4) \\ -1 + x_0 \geq 0 & (x_5) \end{cases} \Rightarrow \begin{array}{l} \text{variable sortante } x_4 \text{ car} \\ x_0 \geq 3 \text{ est la contrainte la} \\ \text{plus restrictive.} \end{array}$$

DICTIONNAIRE 1 AUX.II:

$$\begin{array}{rcll}
 x_0 & = & 3 & - & x_2 & - & x_1 & + & x_4 \\
 x_3 & = & 4 & - & x_2 & - & 2x_1 & + & (3 - x_2 - x_1 + x_4) & = & 7 & + & x_4 & - & 2x_2 & - & 3x_1 \\
 x_5 & = & -1 & + & x_2 & - & x_1 & + & (3 - x_2 - x_1 + x_4) & = & 2 & + & x_4 & & & - & 2x_1 \\
 \hline
 z & = & 3 & - & x_2 & - & x_1 & + & x_4
 \end{array}$$

Solution 2:  $(3, 0, 0, 7, 0, 2)$ .

$$\text{variable entrante } x_1 \Rightarrow \begin{cases} 3 - x_1 \geq 0 & (x_0) \\ 7 - 3x_1 \geq 0 & (x_3) \\ 2 - 2x_1 \geq 0 & (x_5) \end{cases} \Rightarrow \begin{array}{l} \text{variable sortante } x_5 \text{ car} \\ x_1 \leq 1 \text{ est la contrainte la} \\ \text{plus restrictive.} \end{array}$$

DICTIONNAIRE 1 AUX.II:

$$\begin{array}{rclclcl} x_0 & = & 2 & - & x_2 & + & (1/2)x_4 & + & (1/2)x_5 \\ x_1 & = & 1 & & & + & (1/2)x_4 & - & (1/2)x_5 \\ x_3 & = & 4 & - & 2x_2 & - & (1/2)x_4 & + & (3/2)x_5 \\ \hline z & = & 2 & - & x_2 & + & (1/2)x_4 & + & (1/2)x_5 \end{array}$$

Solution 3: (2, 1, 0, 4, 0, 0).

$$\text{variable entrante } x_2 \Rightarrow \begin{cases} 2 - x_2 \geq 0 & (x_0) \\ 1 \geq 0 & (x_1) \\ 4 - 2x_2 \geq 0 & (x_3) \end{cases} \Rightarrow \begin{array}{l} \text{variable sortante } x_0 \text{ car on} \\ \text{a le choix entre } x_0 \text{ et } x_3 \\ \text{et on applique la règle de} \\ \text{Bland.} \end{array}$$

DICTIONNAIRE 1 AUX.III:

$$\begin{array}{rclclcl} x_1 & = & 1 & & & + & (1/2)x_4 & - & (1/2)x_5 \\ x_2 & = & 2 & - & x_0 & + & (1/2)x_4 & + & (1/2)x_5 \\ x_3 & = & & & 2x_0 & - & (3/2)x_4 & + & (1/2)x_5 \\ \hline z & = & & & x_0 & & & & \end{array}$$

**Fin du programme auxiliaire.**

DICTIONNAIRE 1.I:

$$\begin{array}{rclclcl} x_1 & = & 1 & + & (1/2)x_4 & - & (1/2)x_5 \\ x_2 & = & 2 & + & (1/2)x_4 & + & (1/2)x_5 \\ x_3 & = & & & -(3/2)x_4 & + & (1/2)x_5 \\ \hline z & = & 5 & + & 2x_4 & - & x_5 & (= x_2 + 3x_1) \end{array}$$

Le programme auxiliaire nous donne la solution admissible (1, 2, 2, 0, 0).

$$\text{variable entrante } x_4 \Rightarrow \begin{cases} 1 + (1/2)x_4 \geq 0 & (x_1) \\ 2 + (1/2)x_4 \geq 0 & (x_2) \\ -(3/2)x_4 \geq 0 & (x_3) \end{cases} \Rightarrow \begin{array}{l} \text{variable sortante } x_3 \text{ car} \\ x_4 \leq 0 \text{ est la contrainte la} \\ \text{plus restrictive.} \end{array}$$

DICTIONNAIRE 1.II:

$$\begin{array}{rclclcl} x_1 & = & 1 & - & (1/3)x_3 & - & (1/3)x_5 \\ x_2 & = & 2 & - & (1/3)x_3 & + & (2/3)x_5 \\ x_4 & = & & & -(2/3)x_3 & + & (1/3)x_5 \\ \hline z & = & 5 & - & (4/3)x_3 & - & (1/3)x_5 \end{array}$$

**Tous les coefficients de l'objectif sont négatifs, on ne peut plus entrer de variable dans la base.**  
**Solution finale:** (1, 2, 0, 0, 0) et  $z = 5$ .

$$\begin{array}{rclcl} \max & x_2 & + & 3x_1 & \text{s.c.} \\ & x_2 & + & 2x_1 & \leq 2 \\ & -x_2 & - & x_1 & \leq -3 \\ & -x_2 & + & x_1 & \leq -1 \\ & & & x_1, x_2 & \geq 0 \end{array} \quad (2)$$

$$\begin{array}{llllll}
\min & x_0 & \text{s.c.} & & & \\
x_3 & + & x_2 & + & 2x_1 & - & x_0 & = & 2 \\
x_4 & - & x_2 & - & x_1 & - & x_0 & = & -3 \\
x_5 & - & x_2 & + & x_1 & - & x_0 & = & -1 \\
& & & & x_0, \dots, x_5 & \geq & 0 & & 
\end{array} \quad (2 \text{ Aux.})$$

DICTIONNAIRE 2 AUX.I:

$$\begin{array}{rcl}
x_3 & = & 2 - x_2 - 2x_1 + x_0 \\
x_4 & = & -3 + x_2 + x_1 + x_0 \\
x_5 & = & -1 + x_2 - x_1 + x_0 \\
\hline
z & = & x_0
\end{array}$$

Solution 1:  $(0, 0, 0, 2, -3, -1)$ .

$$\text{variable entrante } x_0 \quad \Rightarrow \quad \begin{cases} 2 + x_0 \geq 0 & (x_3) \\ -3 + x_0 \geq 0 & (x_4) \\ -1 + x_0 \geq 0 & (x_5) \end{cases} \quad \Rightarrow \quad \begin{array}{l} \text{variable sortante } x_4 \text{ car} \\ x_0 \geq 3 \text{ est la contrainte la} \\ \text{plus restrictive.} \end{array}$$

DICTIONNAIRE 2 AUX.II:

$$\begin{array}{rcl}
x_0 & = & 3 + x_4 - x_2 - x_1 \\
x_3 & = & 5 + x_4 - 2x_2 - 3x_1 \\
x_5 & = & 2 + x_4 - 2x_1 \\
\hline
z & = & 3 + x_4 - x_2 - x_1
\end{array}$$

Solution 2:  $(3, 0, 0, 5, 0, 2)$ .

$$\text{variable entrante } x_1 \quad \Rightarrow \quad \begin{cases} 3 - x_1 \geq 0 & (x_0) \\ 5 - 3x_1 \geq 0 & (x_3) \\ 2 - 2x_1 \geq 0 & (x_5) \end{cases} \quad \Rightarrow \quad \begin{array}{l} \text{variable sortante } x_5 \text{ car} \\ x_1 \leq 1 \text{ est la contrainte la} \\ \text{plus restrictive.} \end{array}$$

DICTIONNAIRE 2 AUX.II:

$$\begin{array}{rcl}
x_0 & = & 2 + (1/2)x_5 + (1/2)x_4 - x_2 \\
x_1 & = & 1 - (1/2)x_5 + (1/2)x_4 \\
x_3 & = & 2 + (3/2)x_5 - (1/2)x_4 - 2x_2 \\
\hline
z & = & 2 + (1/2)x_5 + (1/2)x_4 - x_2
\end{array}$$

Solution 3:  $(2, 1, 0, 2, 0, 0)$ .

$$\text{variable entrante } x_2 \quad \Rightarrow \quad \begin{cases} 2 - x_2 \geq 0 & (x_0) \\ 1 \geq 0 & (x_1) \\ 2 - 2x_2 \geq 0 & (x_3) \end{cases} \quad \Rightarrow \quad \begin{array}{l} \text{variable sortante } x_3 \text{ car} \\ x_2 \leq 1 \text{ est la contrainte la} \\ \text{plus restrictive.} \end{array}$$

DICTIONNAIRE 2 AUX.III:

$$\begin{array}{rcll}
 x_0 & = & 1 & - (1/4)x_5 + (3/4)x_4 + (1/2)x_3 \\
 x_1 & = & 1 & - (1/2)x_5 + (1/2)x_4 \\
 x_2 & = & 1 & + (3/4)x_5 - (1/4)x_4 - (1/2)x_3 \\
 \hline
 z & = & 1 & - (1/4)x_5 + (3/4)x_4 + (1/2)x_3
 \end{array}$$

Solution 4: (1, 1, 1, 0, 0, 0).

$$\text{variable entrante } x_5 \quad \Rightarrow \quad \begin{cases} 1 - (1/4)x_5 \geq 0 & (x_0) \\ 1 - (1/2)x_5 \geq 0 & (x_1) \\ 1 + (3/4)x_5 \geq 0 & (x_2) \end{cases} \Rightarrow \begin{array}{l} \text{variable sortante } x_1 \text{ car} \\ x_5 \leq 2 \text{ est la contrainte la} \\ \text{plus restrictive.} \end{array}$$

DICTIONNAIRE 2 AUX.IV:

$$\begin{array}{rcll}
 x_0 & = & (1/2) & + (1/2)x_4 + (1/2)x_3 + (1/2)x_1 \\
 x_2 & = & (5/2) & + (1/4)x_4 - (1/2)x_3 - (3/2)x_1 \\
 x_5 & = & 2 & + x_4 - 2x_1 \\
 \hline
 z & = & (1/2) & + (1/2)x_4 + (1/2)x_3 + (1/2)x_1
 \end{array}$$

**Fin du programme auxiliaire. Aucune variable hors-base ne peut être entrée dans la base (que des coefficients positifs pour un problème de minimisation), mais  $z \neq 0$  pour la solution optimale (1/2, 0, 5/2, 0, 0, 2).**