$$\text{Min } -y_1+y_2+6y_3+6y_4-3y_5+6y_6$$

$$-3y_1+y_2-2y_3+9y_4-5y_5+7y_6 \geq -1$$

$$y_1-y_2+7y_3-4y_4+2y_5-3y_6 \geq -2$$

$$y_1, \dots, y_6 \geq 0.$$

$$|||$$

$$\text{Max } \quad y_1-y_2-6y_3-6y_4+3y_5-6y_6$$

$$3y_1-y_2+2y_3-9y_4+5y_5-7y_6 \leq 1$$

$$-y_1+y_2-7y_3+4y_4-2y_5+3y_6 \leq 2$$

$$y_1, y_2, \dots, y_6 \geq 0$$

$$\text{Max } \quad z = y_1-y_2-6y_3-6y_4+3y_5-6y_6$$

$$e_1 = 1-3y_1+y_2-2y_3+9y_4-5y_5+7y_6$$

$$e_2 = 2+y_1-y_2+7y_3-4y_4+2y_5-3y_6$$

$$y_1, \dots, y_6, e_1, e_2 \geq 0$$

y_5 entre en base

$$\begin{cases} (e_1) & \boxed{1-5y_5 \geq 0} \\ (e_2) & 2+2y_5 \geq 0 \end{cases} \Rightarrow y_5 \leq \frac{1}{5}$$

la contrainte la + restrictive donc (e_1) sort de la base.

$$y_5 = \frac{1}{5} - \frac{3}{5}y_1 + \frac{1}{5}y_2 - \frac{2}{5}y_3 + \frac{9}{5}y_4 - \frac{1}{5}e_1 + \frac{7}{5}y_6$$

$$e_2 = 2 + y_1 - y_2 + 7y_3 - 4y_4 \quad -3y_6$$

$$+2\left(\frac{1}{5} - \frac{3}{5}y_1 + \frac{1}{5}y_2 - \frac{2}{5}y_3 + \frac{9}{5}y_4 - \frac{1}{5}e_1 + \frac{7}{5}y_6\right)$$

$$e_2 = \frac{12}{5} - \frac{1}{5}y_1 - \frac{3}{5}y_2 + \frac{31}{5}y_3 - \frac{2}{5}y_4 - \frac{2}{5}e_1 - \frac{1}{5}y_6$$

$$z = y_1 - y_2 - 6y_3 - 6y_4 \quad -6y_6$$

$$+3\left(\frac{1}{5} - \frac{3}{5}y_1 + \frac{1}{5}y_2 - \frac{2}{5}y_3 + \frac{9}{5}y_4 - \frac{1}{5}e_1 + \frac{7}{5}y_6\right)$$

$$z = \frac{3}{5} - \frac{4}{5}y_1 - \frac{2}{5}y_2 - \frac{36}{5}y_3 - \frac{3}{5}y_4 - \frac{3}{5}e_1 - \frac{9}{5}y_6$$

$$\text{Max } z = \frac{3}{5} - \frac{4}{5}y_1 - \frac{2}{5}y_2 - \frac{36}{5}y_3 - \frac{3}{5}y_4 - \frac{3}{5}e_1 - \frac{9}{5}y_6$$

$$e_2 = \frac{12}{5} - \frac{1}{5}y_1 - \frac{3}{5}y_2 + \frac{31}{5}y_3 - \frac{2}{5}y_4 - \frac{2}{5}e_1 - \frac{1}{5}y_6$$

$$y_5 = \frac{1}{5} - \frac{3}{5}y_1 + \frac{1}{5}y_2 - \frac{2}{5}y_3 + \frac{9}{5}y_4 - \frac{1}{5}e_1 + \frac{7}{5}y_6$$

$$z^* = \frac{3}{5} \qquad x_1^* = \frac{3}{5} \qquad x_2^* = 0$$

$$y_1^* = y_2^* = y_3^* = y_4^* = y_6^* = 0.$$

$$z' = -\frac{3}{5} \qquad y_5^* = \frac{1}{5}$$

$$w^* = -\frac{3}{5} - 2\times 0 = -\frac{3}{5}$$

$$\begin{cases} -3\times\frac{3}{5} = -\frac{9}{5} \leq -1 \ ✓ \\ \frac{3}{5} \leq 1 \ ✓ \\ -2\times\frac{3}{5} \leq 6 \ ✓ \\ 9\times\frac{3}{5} \leq 6 \ ✓ \\ -5\times\frac{3}{5} = -3 \leq -3 \ ✓ \\ 7\times\frac{3}{5} = \frac{21}{5} \leq 6 \ ✓ \end{cases}$$

Exercice 2

(a) Min $4y_1 + 3y_2 + 5y_3 + y_4$

s.c.

$$y_1 + 4y_2 + 2y_3 + 3y_4 \geq 7 \qquad (x_1)$$

$$3y_1 + 2y_2 + 4y_3 + y_4 \geq 6 \qquad (x_2) \quad ✓$$

$$5y_1 - 2y_2 + 4y_3 + 2y_4 \geq 5 \qquad (x_3) \quad ✓$$

$$-2y_1 + y_2 - 2y_3 - y_4 \geq -2 \qquad (x_4) \quad ✓$$

$$2y_1 + y_2 + 5y_3 - 2y_4 \geq 3 \qquad (x_5)$$

$$y_1, y_2, y_3, y_4 \geq 0.$$

Solution proposée $\left(x_1 = 0,\ x_2 = \frac{4}{3};\ x_3 = \frac{2}{3},\ x_4 = \frac{5}{3},\ x_5 = 0\right)$

$3\times\frac{4}{3} + 5\times\frac{2}{3} - 2\times\frac{5}{3} = 4$ serrée

$2\times\frac{4}{3} \cancel{+} - 2\times\frac{2}{3} + \frac{5}{3} = \frac{9}{3} = 3$ serrée

$4\times\frac{4}{3} + 4\times\frac{2}{3} - 2\times\frac{5}{3} = \frac{14}{3} < \frac{15}{3} \Rightarrow y_3 = 0.$

$\frac{4}{3} + 2\times\frac{2}{3} - \frac{5}{3} = 1$ serrée

$x_2 > 0 \Rightarrow 3y_1 + 2y_2 + y_4 = 6$

$x_3 > 0 \Rightarrow 5y_1 - 2y_2 + 2y_4 = 5.$

$x_4 > 0 \Rightarrow -2y_1 + y_2 - y_4 = -2$

$y_1 = y_2 = y_4 = 1$ ok. $\quad y_3 = 0$

$1+4+3 \geq 7$ ✓

$2+1-2 \geq 3$ ✗

La solution n'est pas optimale !

(b) $\min \quad y_1+4y_2+4y_3+5y_4+7y_5+5y_6$

s.l.c. $y_1+5y_2+4y_3 \quad -2y_5+2y_6 \geq 4$

$3y_2+y_3-y_4+y_5-3y_6 \geq 5$

$-4y_1+y_2-3y_3 \quad +y_5+2y_6 \geq 1$

$3y_1+y_2+3y_3+2y_4+y_5-y_6 \geq 3$

$y_1-y_2-4y_3+y_4+2y_5+4y_6 \geq -5$

$y_1+3y_2+y_3-5y_4+2y_5+5y_6 \geq 8$

$y_1, \cdots, y_6 \geq 0$

$(0, 0, 5/2, 7/2, 0, 1/2)$ solution optimale ?

$-4 \times 5/2 + 3 \times 7/2 + 1 \times 1/2 = 1$ serrée

$1 \times 5/2 + 0 \times 7/2 + 3 \times 1/2 = 4$ serrée

$-3 \times 5/2 + 3 \times 7/2 + 1 \times 1/2 = \frac{7}{2} < 4 \Rightarrow y_3 = 0$

$0 \times 5/2 + 2 \times 7/2 - 5 \times 1/2 = \frac{9}{2} < 5 \Rightarrow y_4 = 0$

$1 \times 5/2 + 1 \times 7/2 + 2 \times 1/2 = 14/2 = 7$ serrée

$2 \times 5/2 - 1 \times 7/2 + 5 \times 1/2 = 8/2 = 4 < 5 \Rightarrow y_6 = 0.$

$$x_3 > 0 \Rightarrow -4y_1 + y_2 + y_5 = 1 \quad (*)$$

$$x_4 > 0 \Rightarrow 3y_1 + y_5 = 3 \quad (\Delta)$$

$$x_6 > 0 \Rightarrow y_1 + 3y_2 + 2y_5 = 8$$

$$4y_2 + 4y_5 = 12$$

$$y_2 + y_5 = 3 \quad (**)$$

$$(*) - (**) \Rightarrow -4y_1 = -2 \Rightarrow y_1 = \frac{1}{2}$$

$$(\Delta) + y_1 = \frac{1}{2} \Rightarrow 3 \times \frac{1}{2} + y_5 = 3 \Rightarrow y_5 = \frac{3}{2}$$

$$(**) + y_5 = \frac{3}{2} \Rightarrow y_2 + \frac{3}{2} = 3 \Rightarrow y_2 = \frac{3}{2}$$

$$(x_1)\ \frac{1}{2} + 5 \times \frac{3}{2} - 2 \times \frac{3}{2} = \frac{10}{2} = 5 \geq 4 \ \checkmark$$

$$(x_2)\ 3 \times \frac{3}{2} + \frac{3}{2} = \frac{12}{2} = 6 \geq 5 \ \checkmark$$

$$(x_5)\ \frac{1}{2} - \frac{3}{2} + 2 \times \frac{3}{2} = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2 \geq -5 \ \checkmark$$

La solution est optimale