Graph models and hypothesis testing

Leto Peel
l.peel@maastrichtuniversity.nl
@PiratePeel

Why Graph models?

Why Graph models?

Graph models allow us to generate synthetic networks

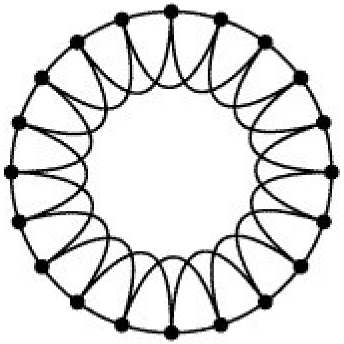
They allow us to capture and model properties observed in real networks

Graph models can serve as hypotheses for mechanisms of network formation

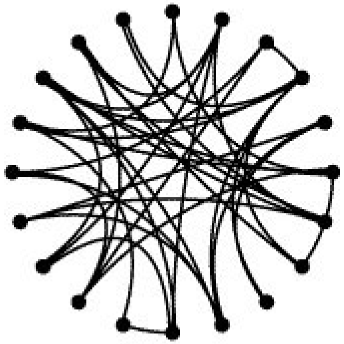
Allows us to explore how “similar” networks behave

Regular and random graphs

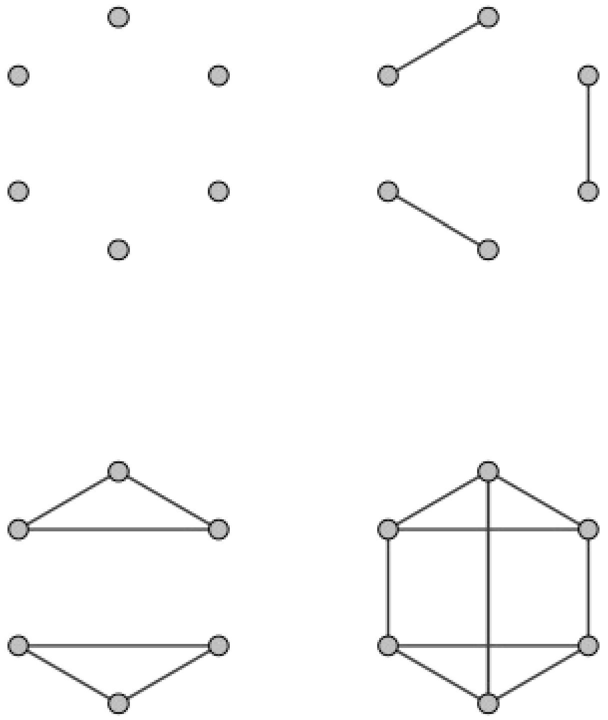
Regular



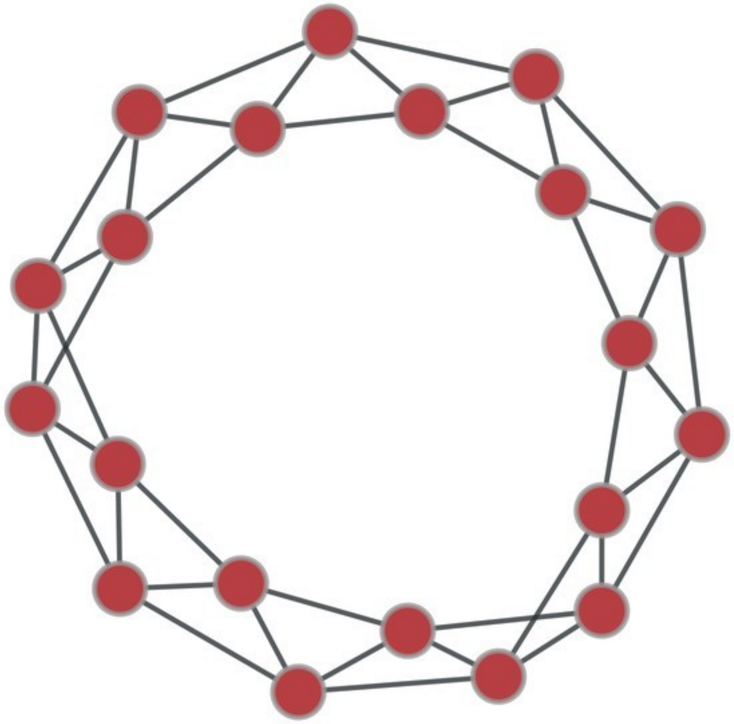
Random



Regular graphs

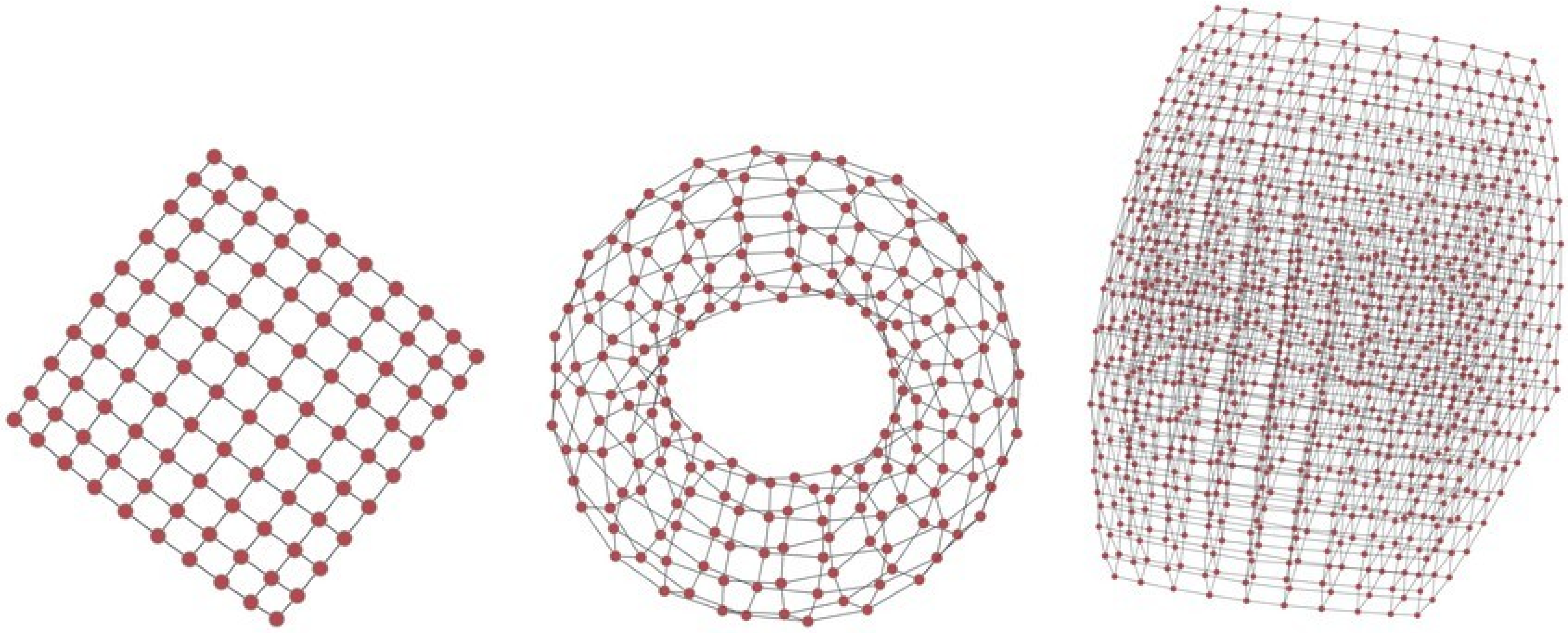


Regular graphs with 0 – 3 degree nodes



Regular Ring Lattice

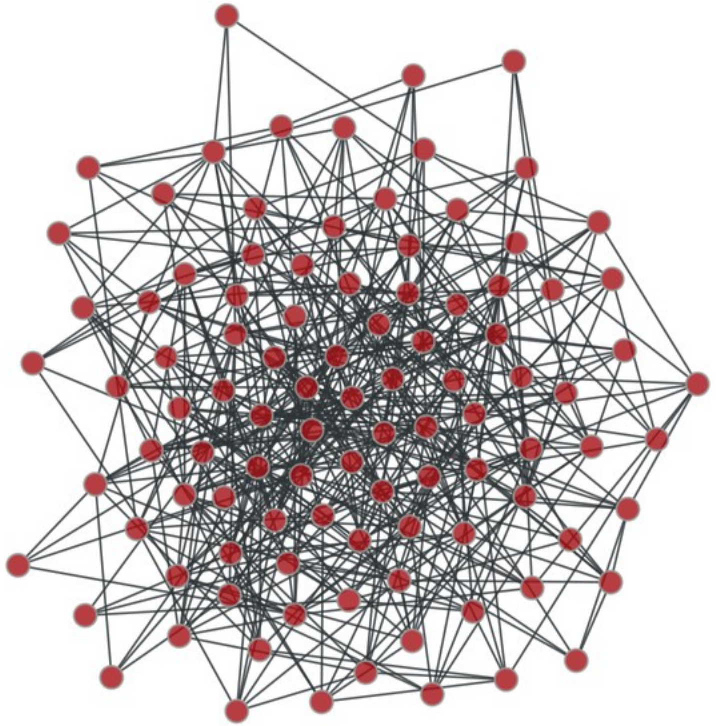
Lattice graphs



Every time we generate regular/lattice graphs we get the same output

Random graphs

The Erdos-Renyi model



Specified by a number of nodes, n, and either:

- a number of edges, m
- a probability of connection, p

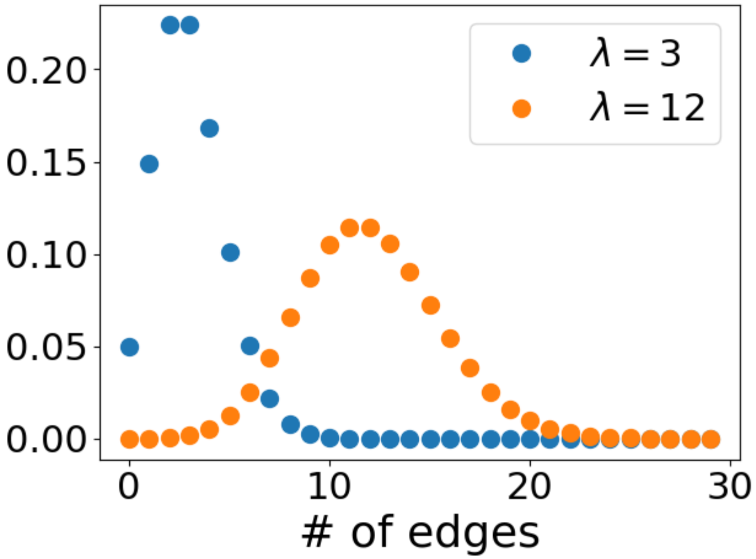
All edges are equally likely to exist

Properties of random graphs

- Locally tree-like
- Large connected component for mean degree greater than 1
- Short path lengths

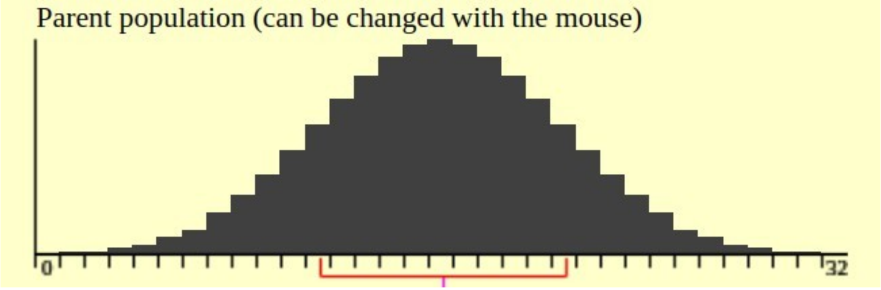
When is the ER not so good?

- Degree distribution
- Clustering



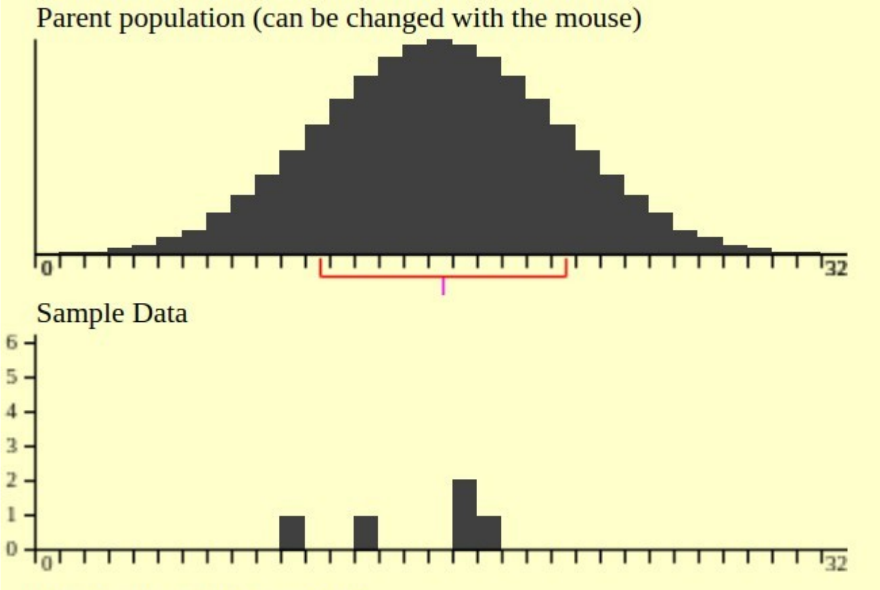
Null hypothesis testing

Population distribution



Mean = 16.00
Sd = 5.00

Population distribution



Mean = 16.00
Sd = 5.00

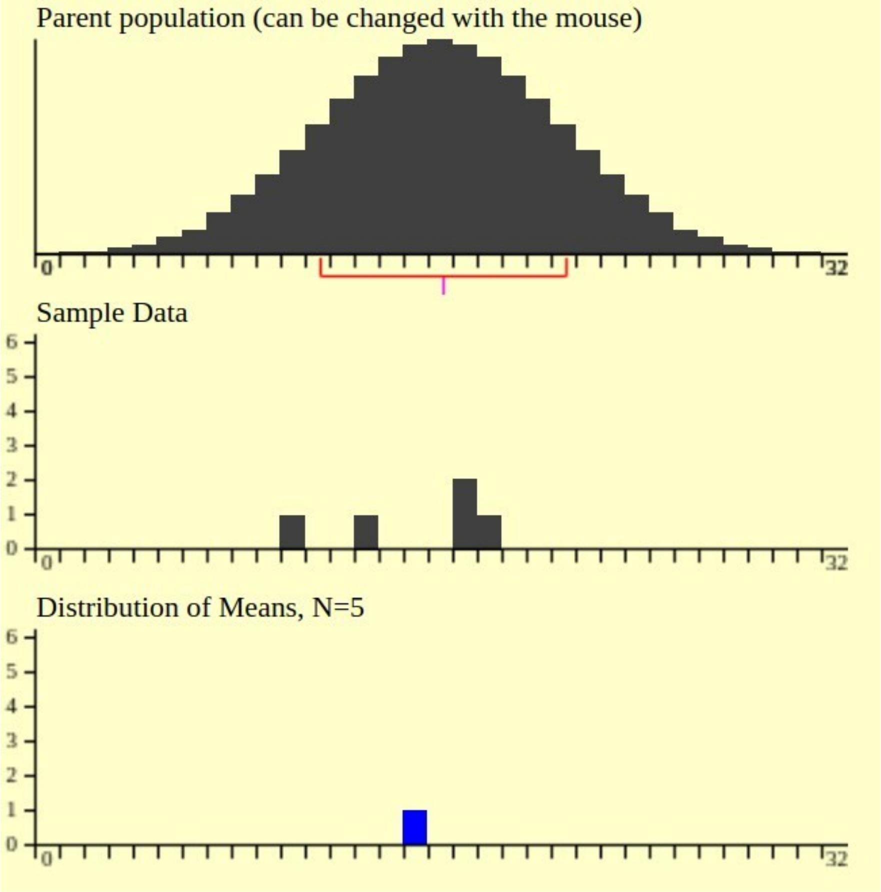
One sample

(size = 5)

Population distribution

One sample
(size = 5)

Distribution of means
(1 sample)

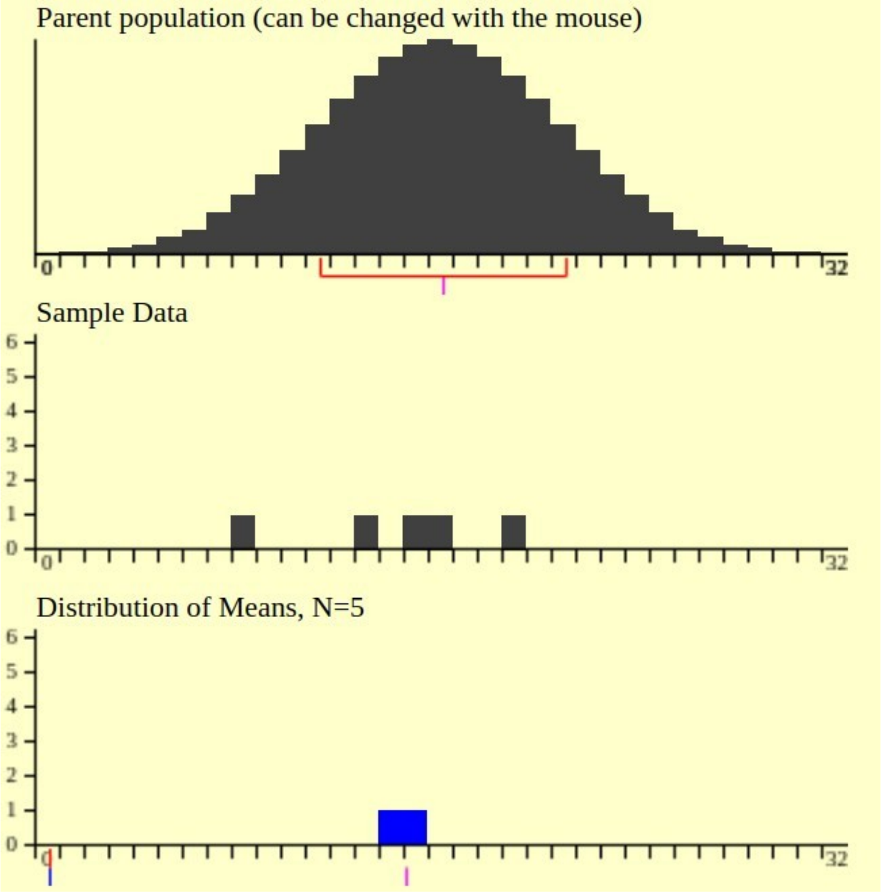


Mean = 16.00
Sd = 5.00

Population distribution

One sample
(size = 5)

Distribution of means
(2 samples)



Mean = 16.00
Sd = 5.00

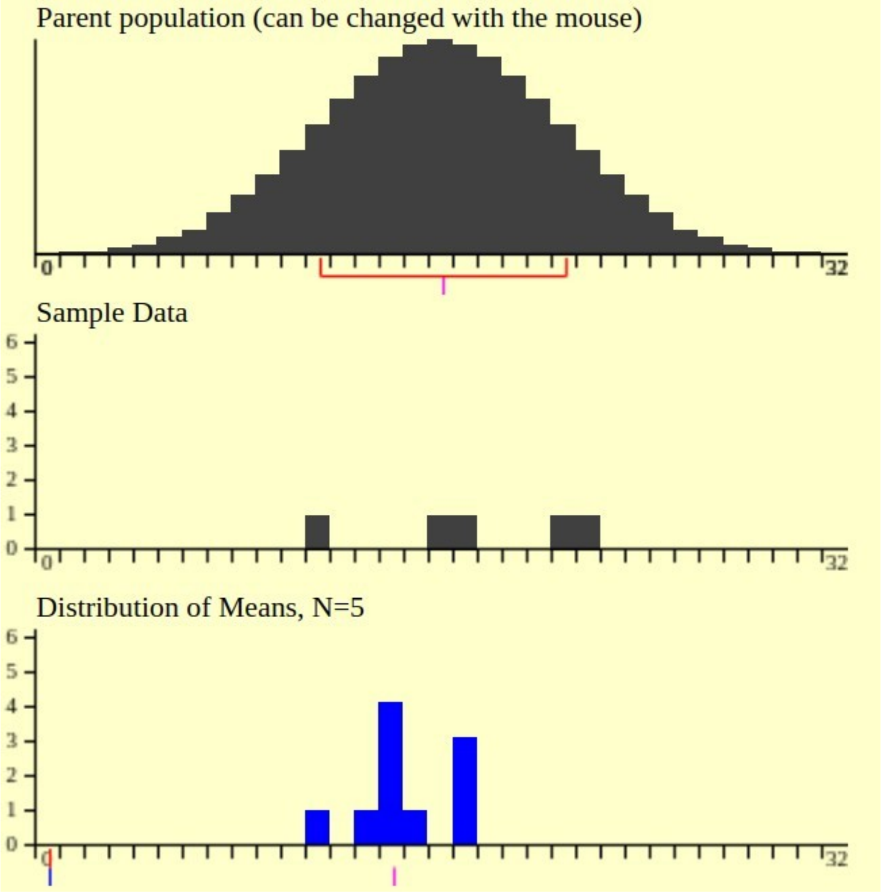
Population distribution

One sample

(size = 5)

Distribution of means

(10 samples)



Mean = 16.00

Sd = 5.00

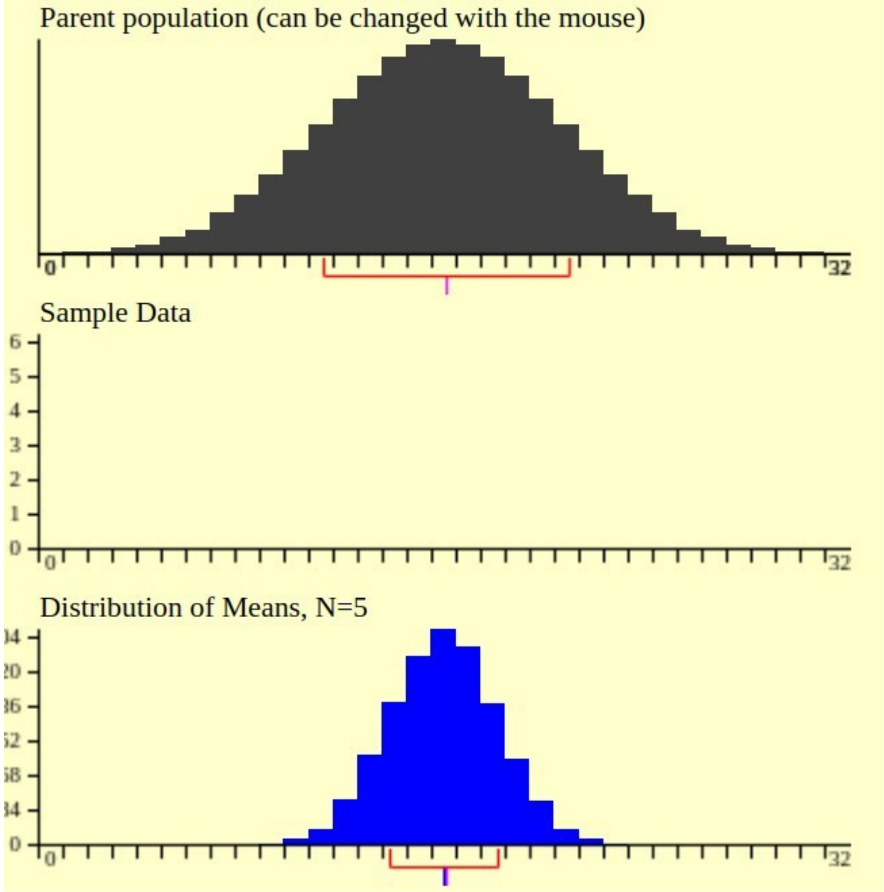
Population distribution

One sample

(size = 5)

Distribution of means

(10,000 samples)



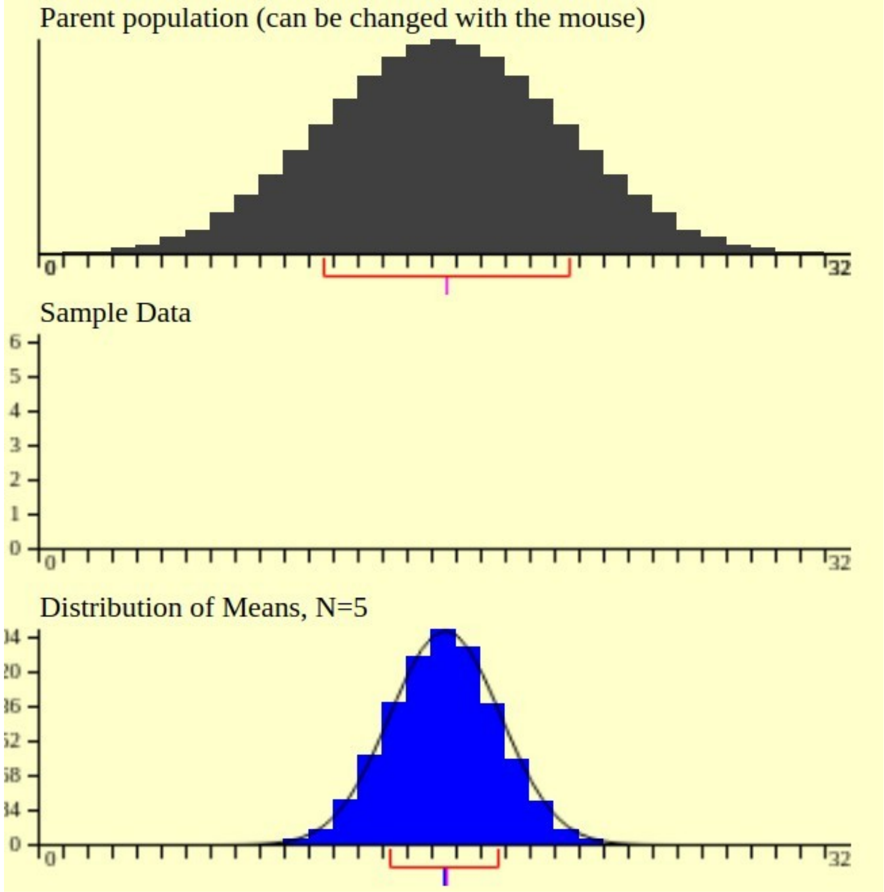
Mean = 16.00
Sd = 5.00

Mean = 15.99
Sd = 2.25

Population distribution

One sample
(size = 5)

Distribution of means
(10,000 samples)



Mean = 16.00
Sd = 5.00

Mean = 15.99
Sd = 2.25

Central limit theorem

The infamous P-value

P-VALUE

The probability, computed assuming that H_0 is true, that the test statistic would take a value as extreme or more extreme than that actually observed is called the ***P*-value** of the test. The smaller the P-value, the stronger the evidence against H_0 provided by the data.

Definition, pg 405
Introduction to the Practice of Statistics, Fifth Edition

The infamous P-value

P-VALUE

The probability, computed assuming that H_0 is true, that the test statistic would take a value as extreme or more extreme than that actually observed is called the ***P*-value** of the test. The smaller the *P*-value, the stronger the evidence against H_0 provided by the data.

Definition, pg 405
Introduction to the Practice of Statistics, Fifth Edition

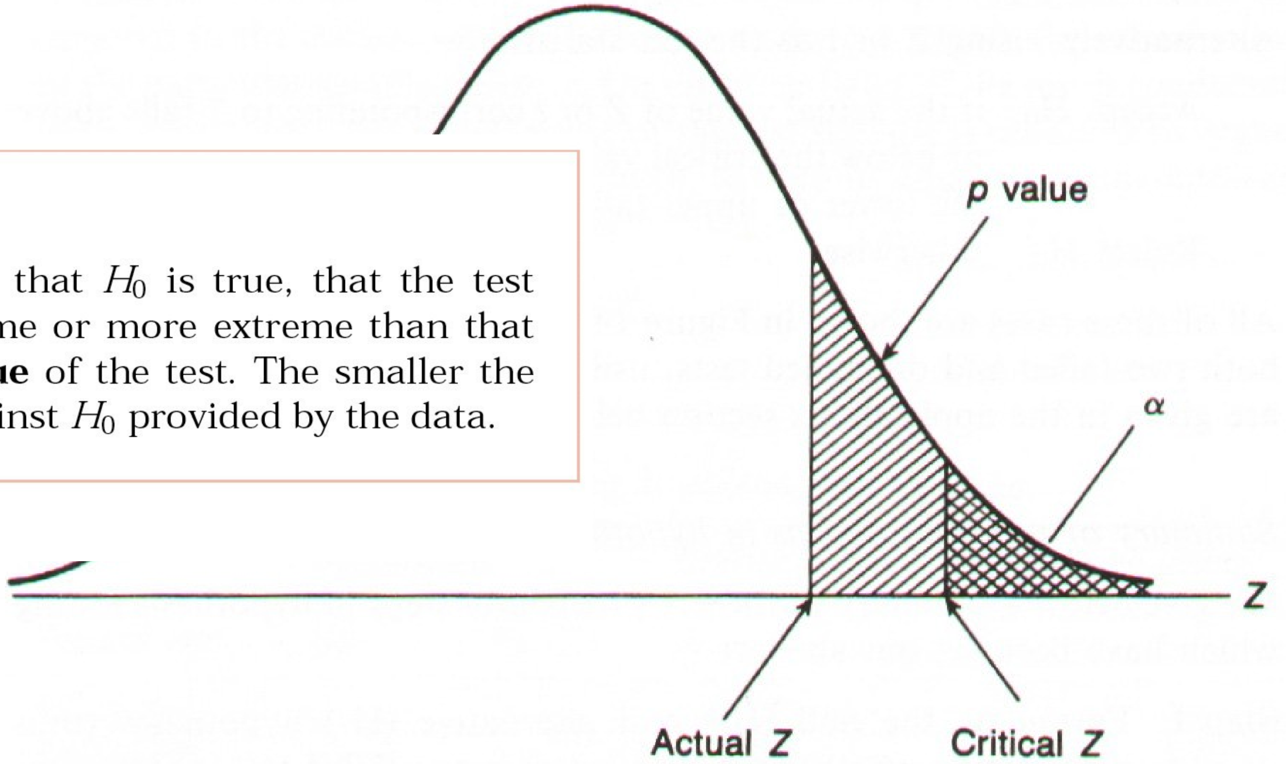


Figure 14.2 Comparison of *p* values and critical values of *Z* in a one-tailed test

The infamous P-value

If p-value < α,
then we reject the
null hypothesis

P-VALUE

The probability, computed assuming that H_0 is true, that the test statistic would take a value as extreme or more extreme than that actually observed is called the ***P*-value** of the test. The smaller the P-value, the stronger the evidence against H_0 provided by the data.

Definition, pg 405
Introduction to the Practice of Statistics, Fifth Edition

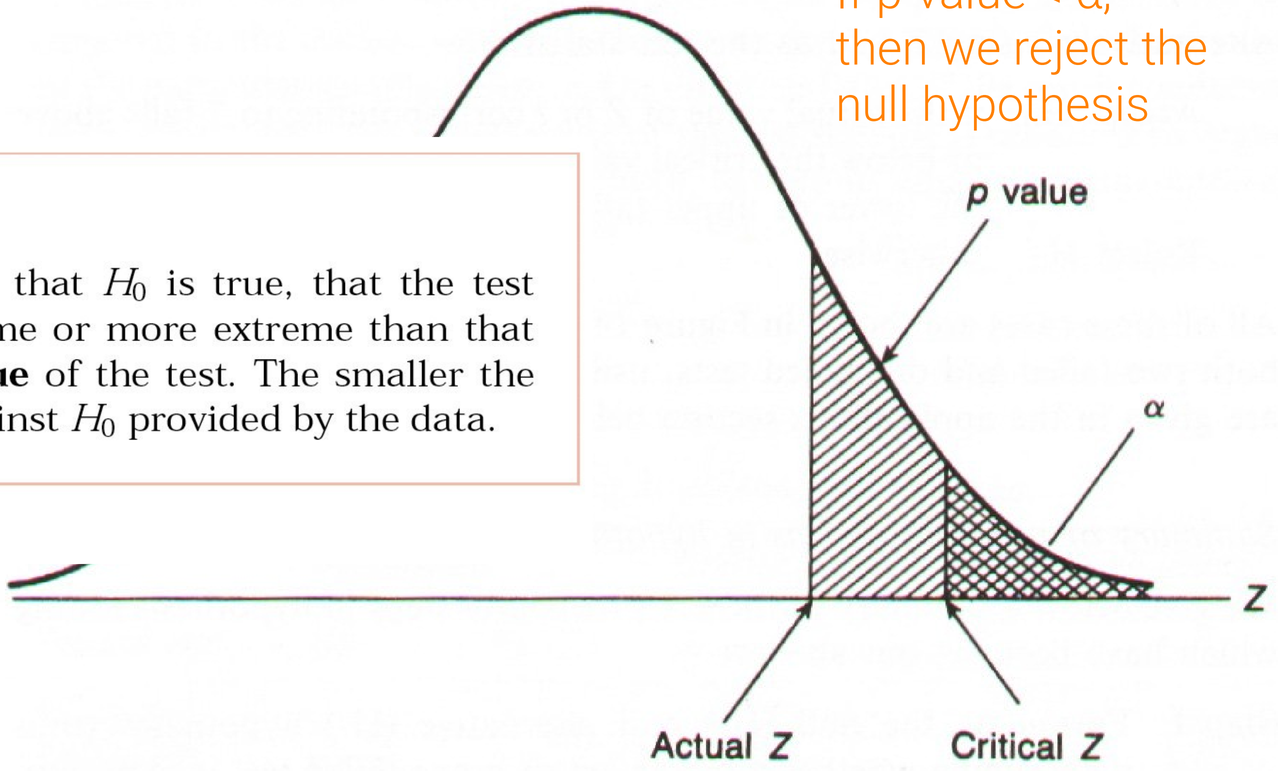


Figure 14.2 Comparison of *p* values and critical values of *Z* in a one-tailed test

For networks we can use graph models as a null model

- Does the network appear to be significantly different from a random graph?
- Can specific properties of the observed network (e.g., clustering coefficient) be explained by a particular generative process?
- Two approaches to create samples:
  - permute edges/nodes in a way that is consistent with the graph model
  - “fit” a model to an observed network and generate networks from it

Practical Part 1

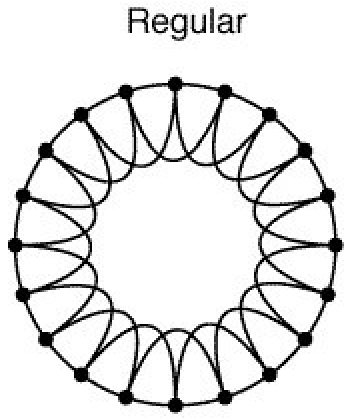
Small-world networks

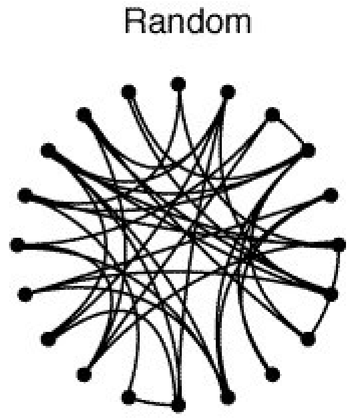
It's a network after all

Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of 'small-world' networks. Nature, 393(6684), 440-442.

Small-world networks

It’s a network after all



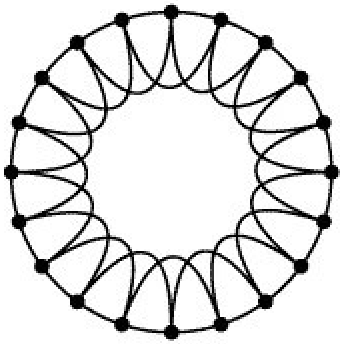


Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of ‘small-world’ networks. Nature, 393(6684), 440-442.

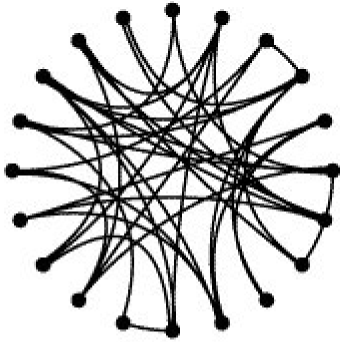
Small-world networks

It’s a network after all

Regular



Random

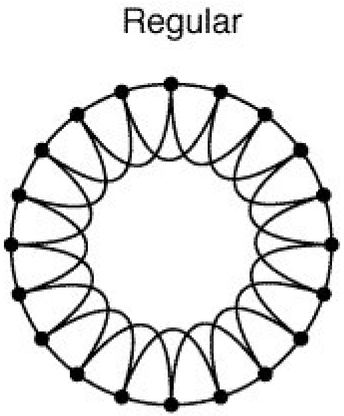


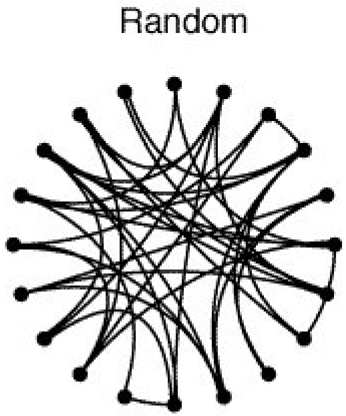
(triangles) High **clustering coefficient** Low **clustering coefficient**

Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of ‘small-world’ networks. Nature, 393(6684), 440-442.

Small-world networks

It’s a network after all



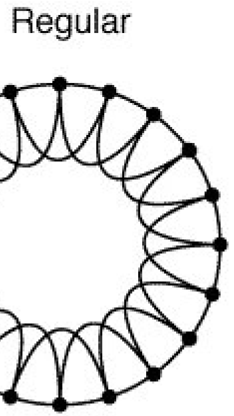


(triangles)	High **clustering coefficient**	Low **clustering coefficient**
(shortest paths)	High mean **geodesic path**	Low mean **geodesic path**

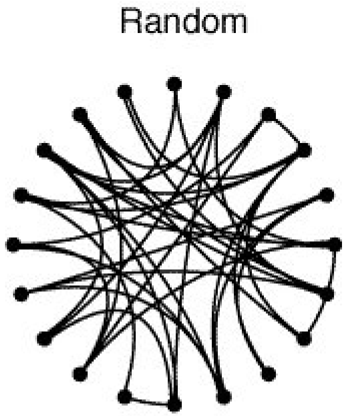
Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of ‘small-world’ networks. Nature, 393(6684), 440-442.

Small-world networks

It's a network after all



Real-world networks

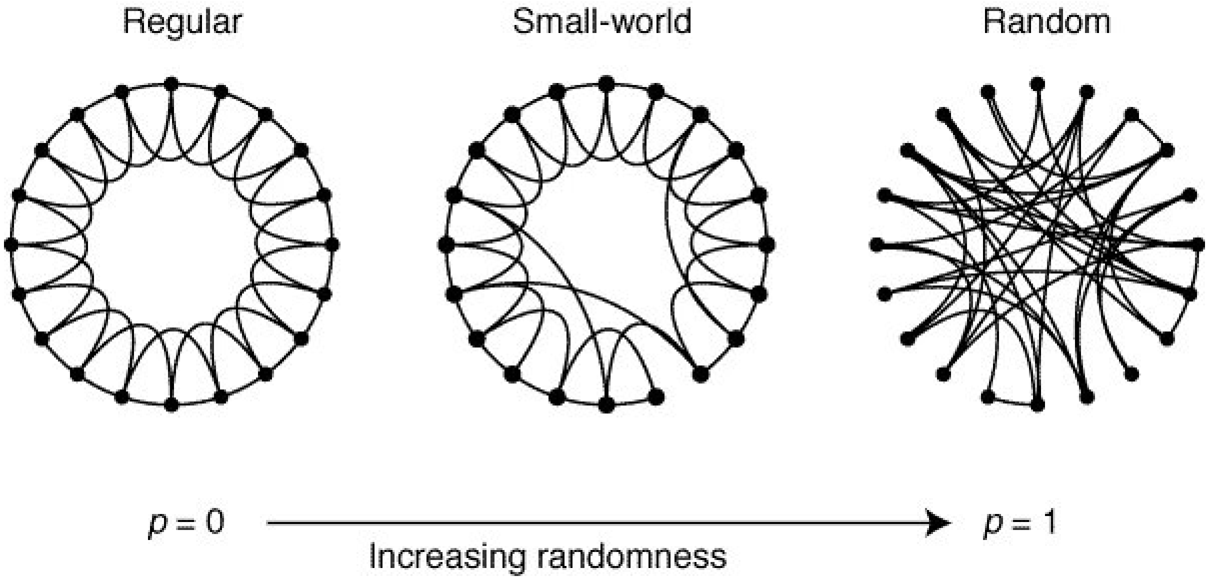


	Regular	Random
(triangles)	High **clustering coefficient**	Low **clustering coefficient**
(shortest paths)	High mean **geodesic path**	Low mean **geodesic path**

Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of 'small-world' networks. Nature, 393(6684), 440-442.

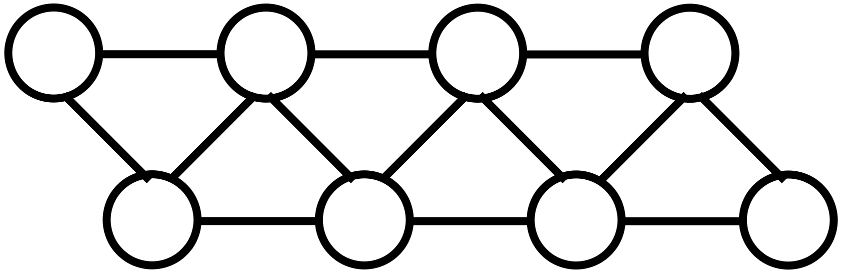
Small-world networks

It's a network after all



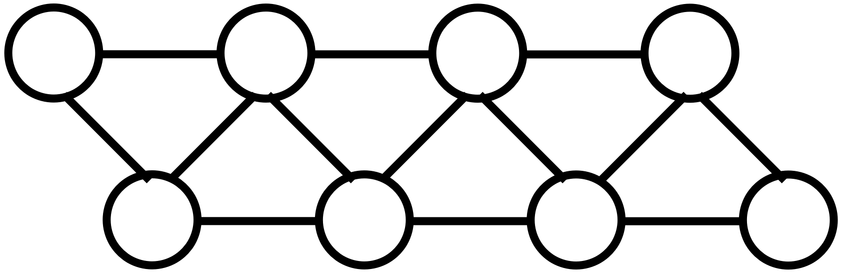
Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of 'small-world' networks. Nature, 393(6684), 440-442.

Exercise



Calculate the **clustering coefficient** and mean **shortest path**

Exercise



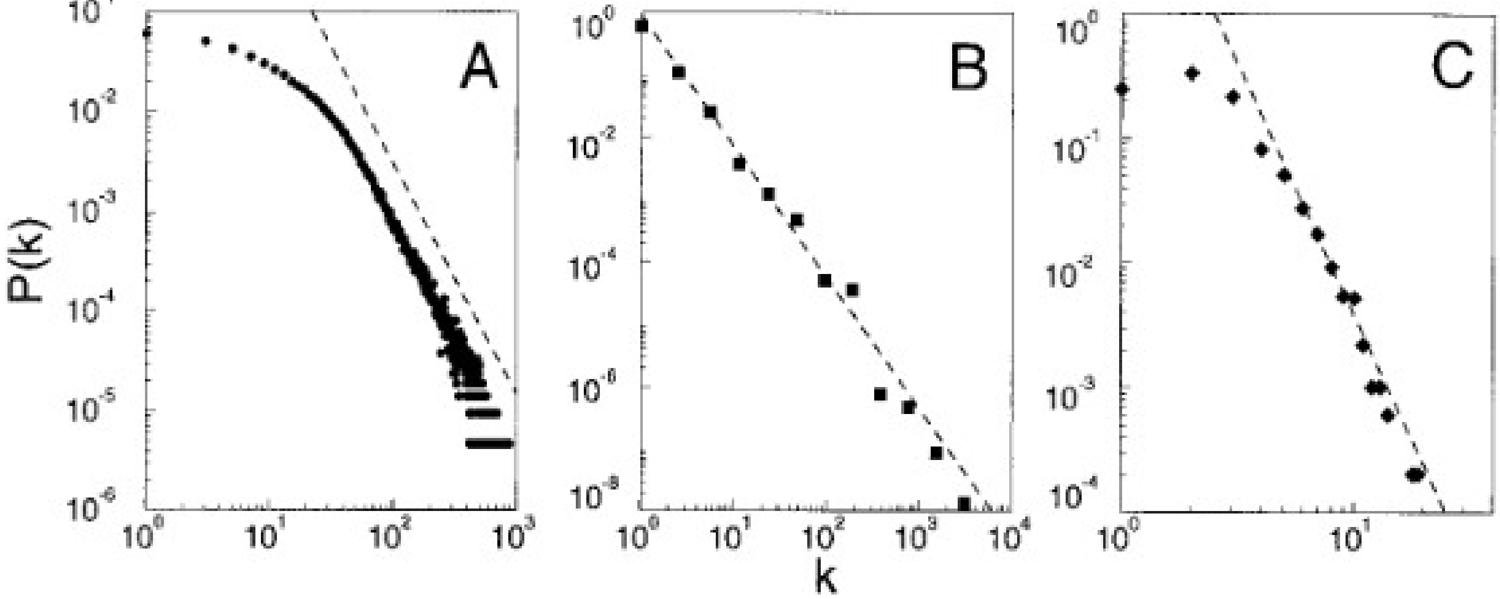
Calculate the **clustering coefficient** and mean **shortest path**

Now choose a pair of edges to randomly rewire and recalculate

"Scale-free" networks

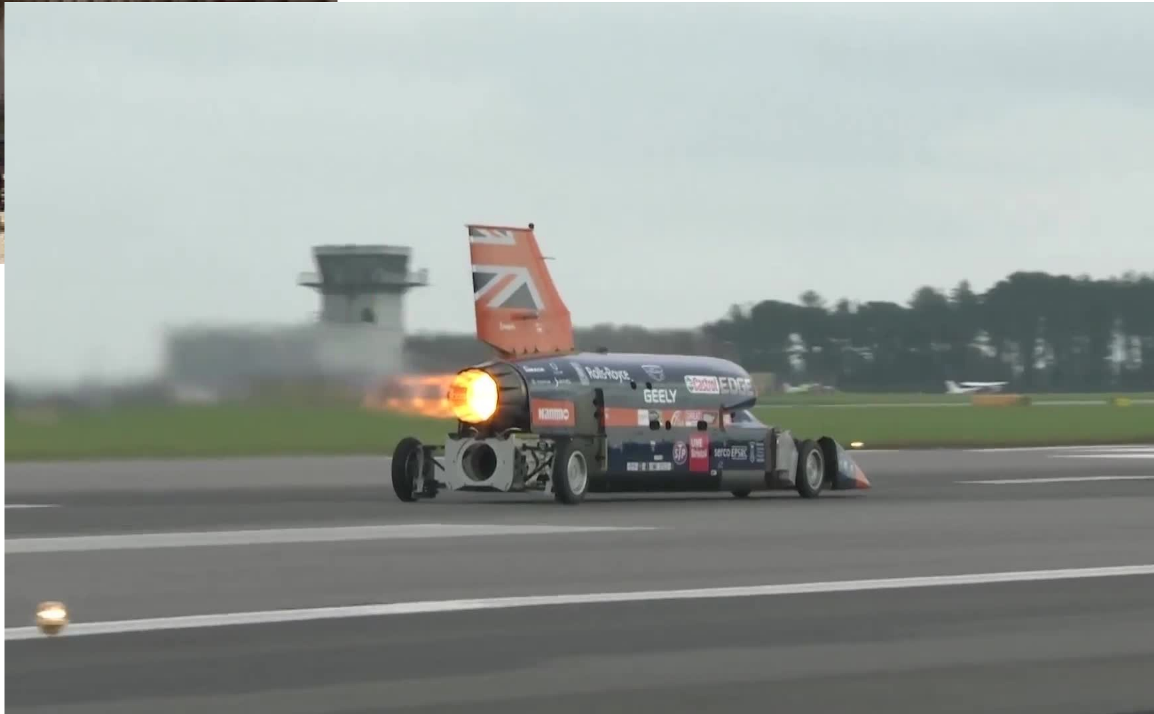
Barabási, A. L., & Albert, R. (1999). Emergence of scaling in random networks. Science, 286(5439), 509-512.

"Scale-free" networks



Barabási, A. L., & Albert, R. (1999). Emergence of scaling in random networks. Science, 286(5439), 509-512.


UCKER PARK


GEELY

The Price model

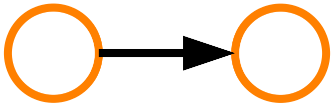
For undirected networks, this model is known as the Barabasi-Albert model

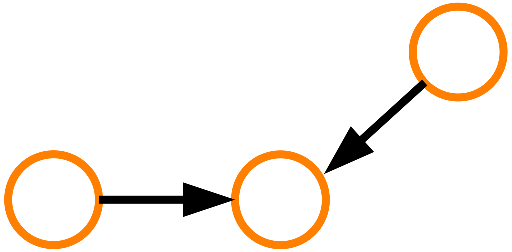
Add nodes to a network one at a time.

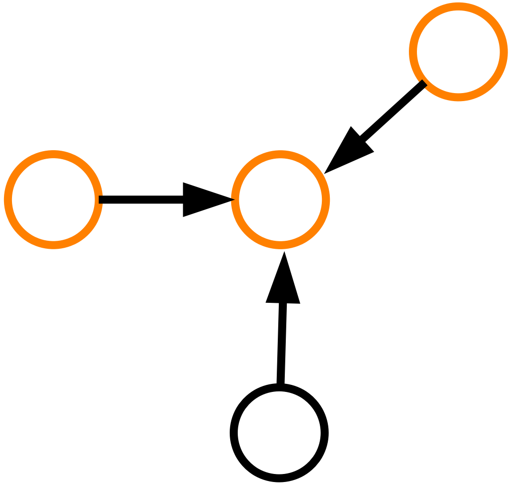
Connect to existing nodes with probability **proportional to their degree**

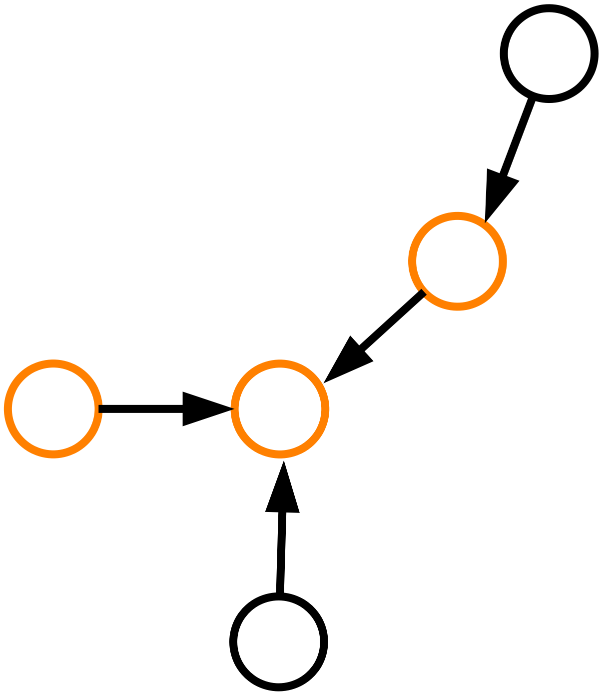
$$p_i = \frac{k_i}{\sum_j k_j},$$

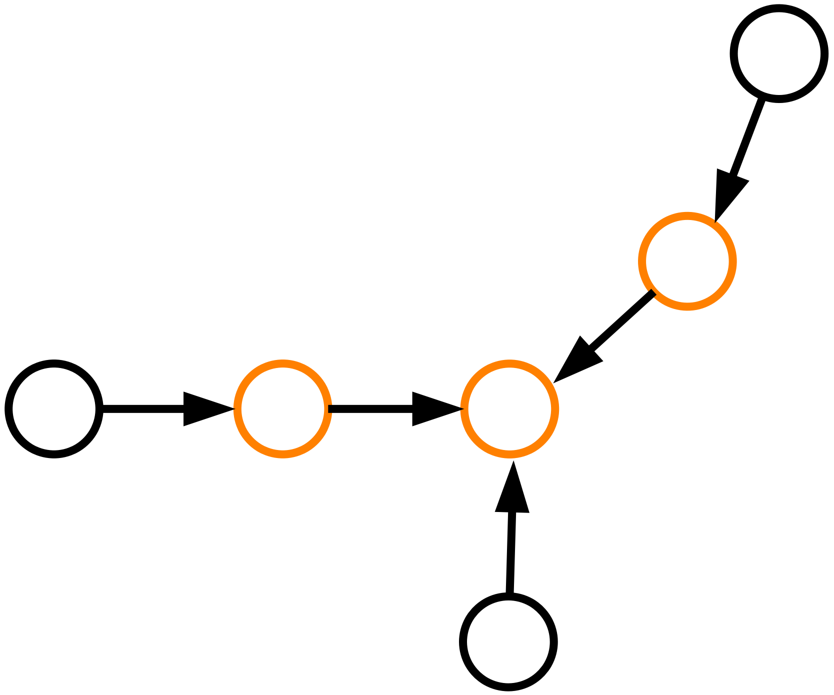
Price, D. D. S. (1976). A general theory of bibliometric and other cumulative advantage processes. Journal of the American society for Information science, 27(5), 292-306.

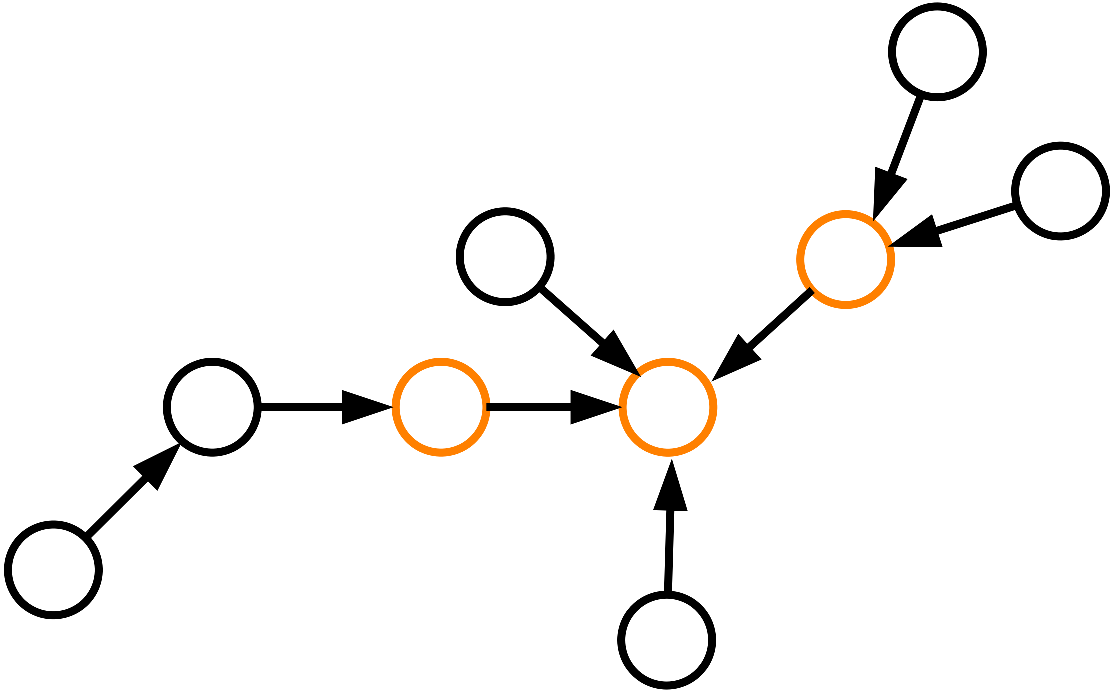


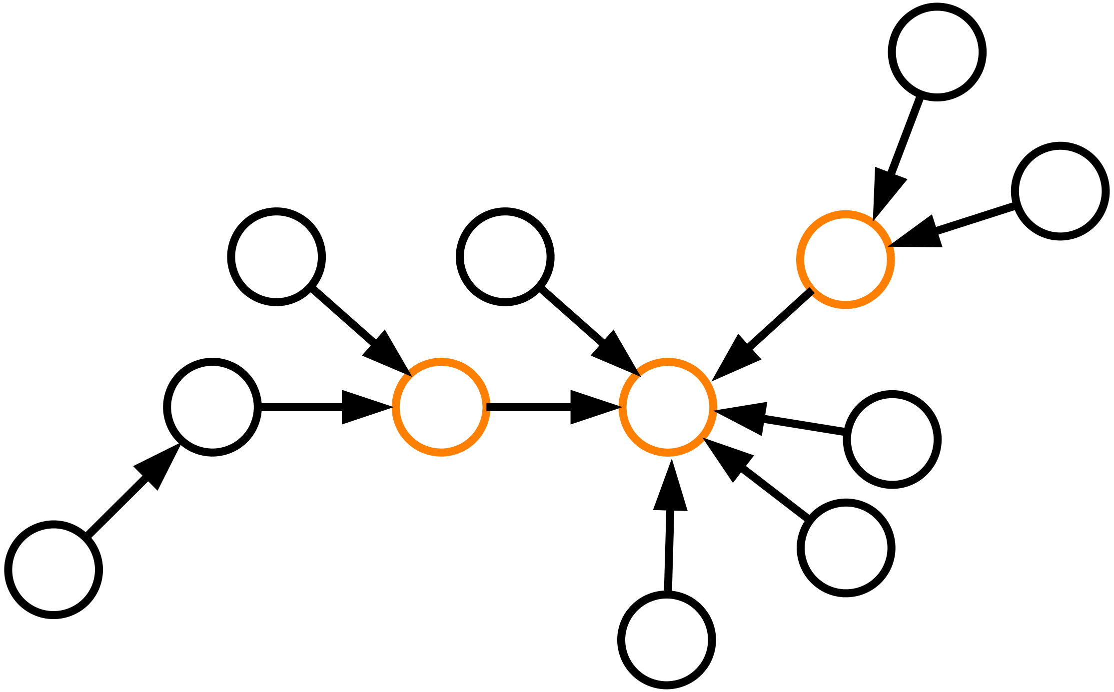


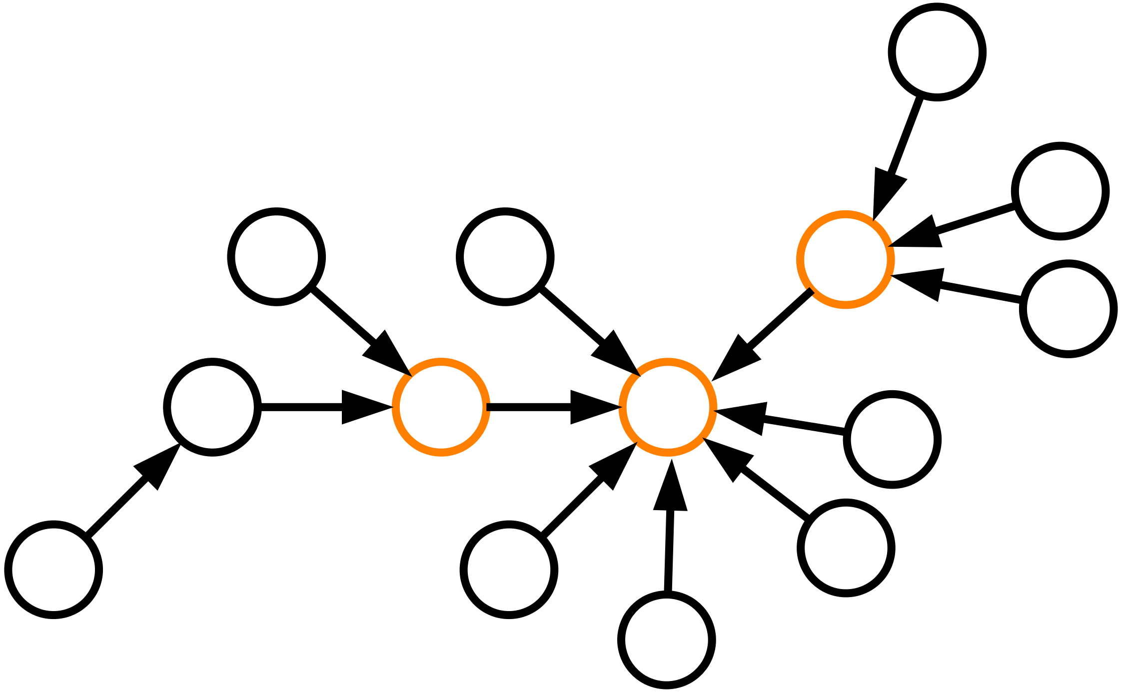




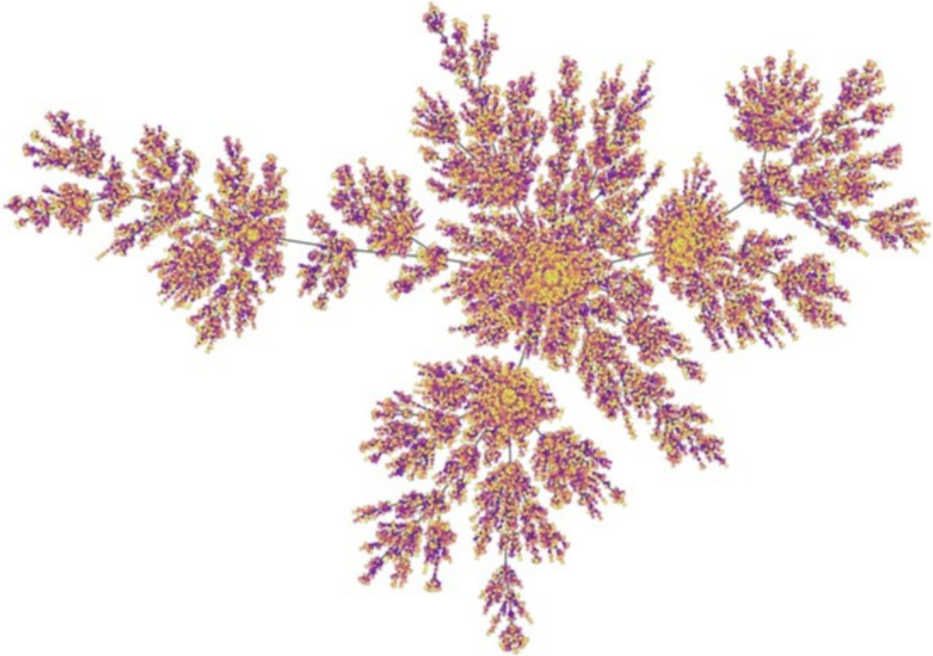


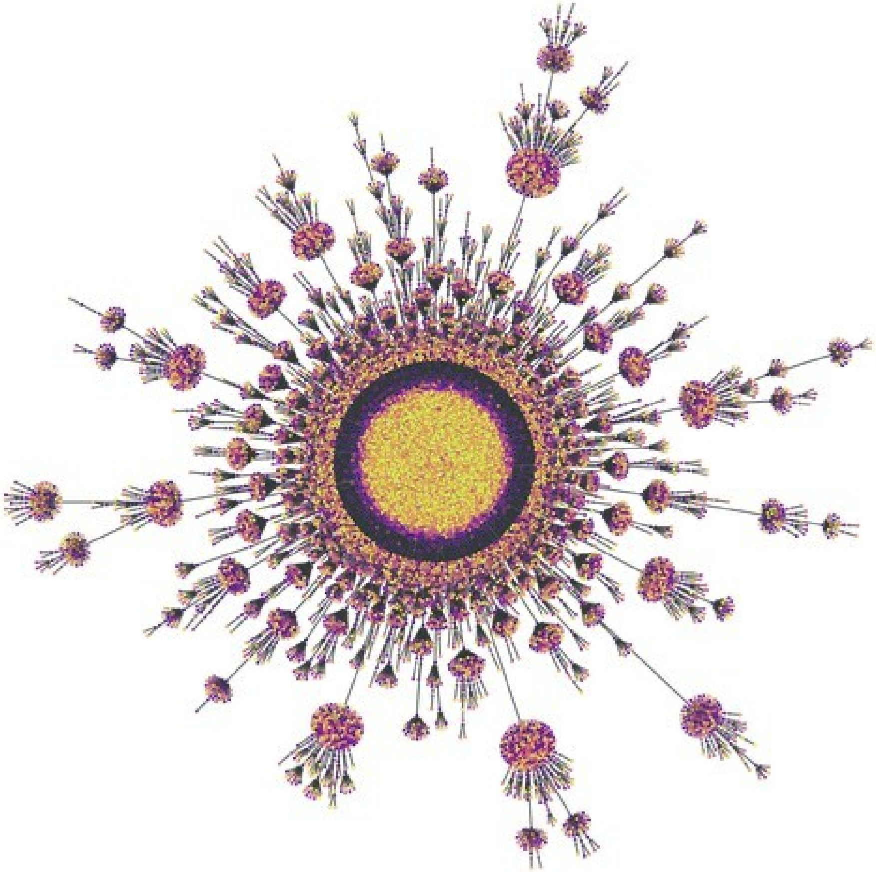






Nodes that join the network earlier have higher degree





Configuration model

- More realistic than ER
- Edges are random conditional on a specified degree sequence
- Unlike the ER model, probability of the degree of a randomly selected node is not the same as the probability of the degree of its neighbours

The friendship paradox

Your friends are more popular than you are

Feld, S. L. (1991). Why your friends have more friends than you do. American Journal of Sociology, 96(6), 1464-1477.

The friendship paradox

Your friends are more popular than you are

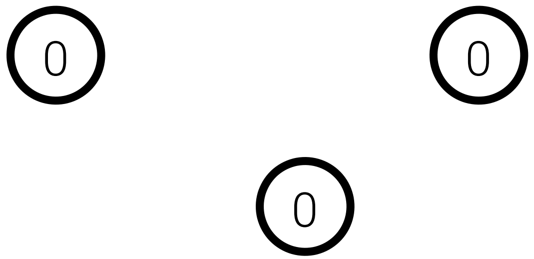
Some people have **no friends.**

Feld, S. L. (1991). Why your friends have more friends than you do. American Journal of Sociology, 96(6), 1464-1477.

The friendship paradox

Your friends are more popular than you are

Some people have **no friends.** But because **they appear in nobody's friendship circles**, they're not making anyone else feel popular.



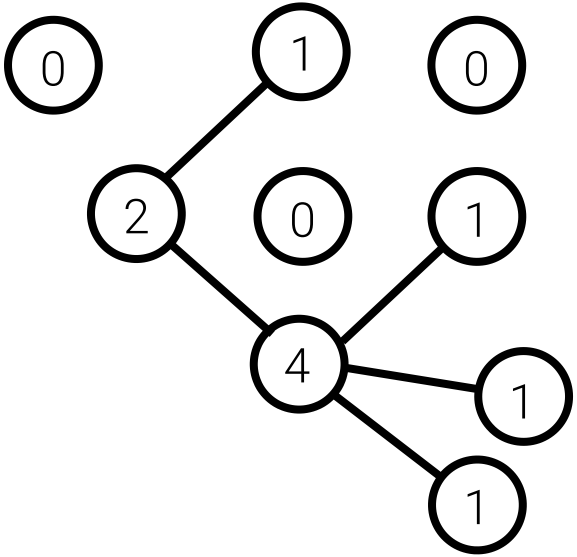
Feld, S. L. (1991). Why your friends have more friends than you do. American Journal of Sociology, 96(6), 1464-1477.

The friendship paradox

Your friends are more popular than you are

Some people have **no friends.** But because **they appear in nobody's friendship circles**, they're not making anyone else feel popular.

The same applies to other people: the more friends you have, the likelier you are to be represented in people's friendship circles.



Feld, S. L. (1991). Why your friends have more friends than you do. American Journal of Sociology, 96(6), 1464-1477.

The friendship paradox

Your friends are more popular than you are

average friend
(count node proportional
to their degree)

average person
(count each node once)

So **popular people are oversampled as friends**. Hence the paradox.

Feld, S. L. (1991). Why your friends have more friends than you do. American Journal of Sociology, 96(6), 1464-1477.

Practical Part II

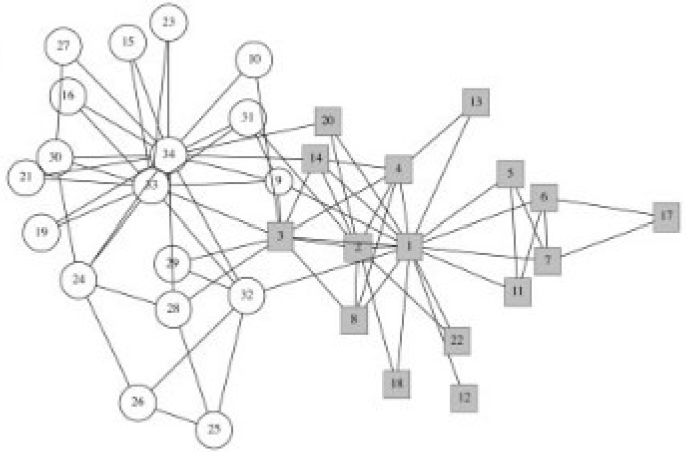
Community structure

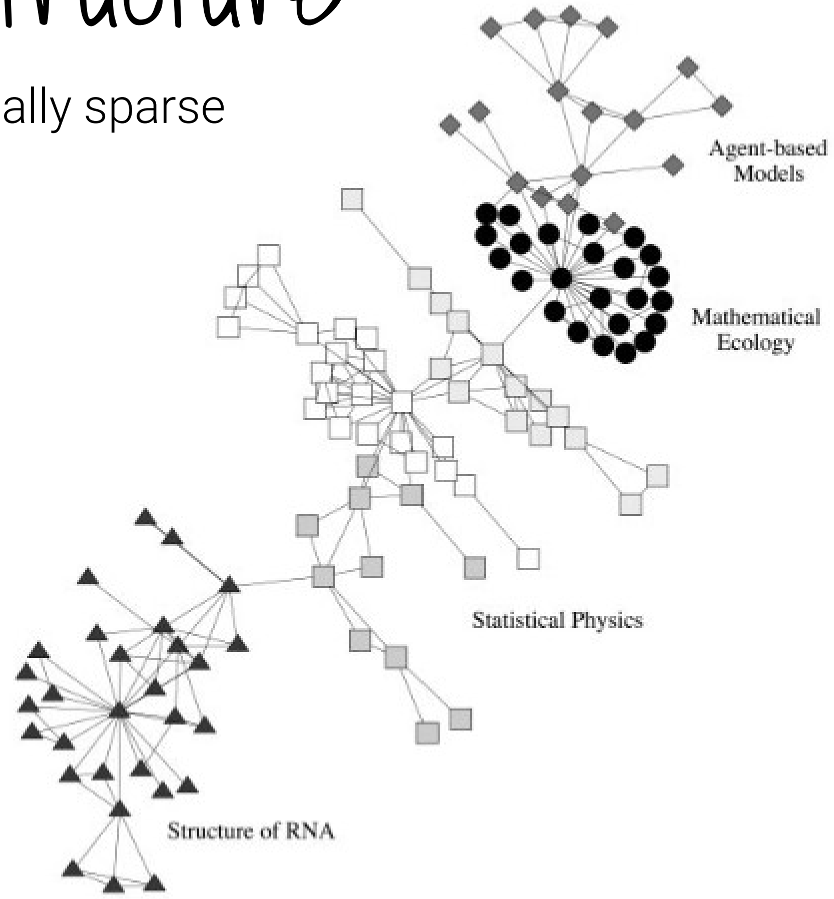
Locally dense, globally sparse

Girvan, M., & Newman, M. E. (2002). Community structure in social and biological networks. PNAS, 99(12), 7821-7826.

Community structure

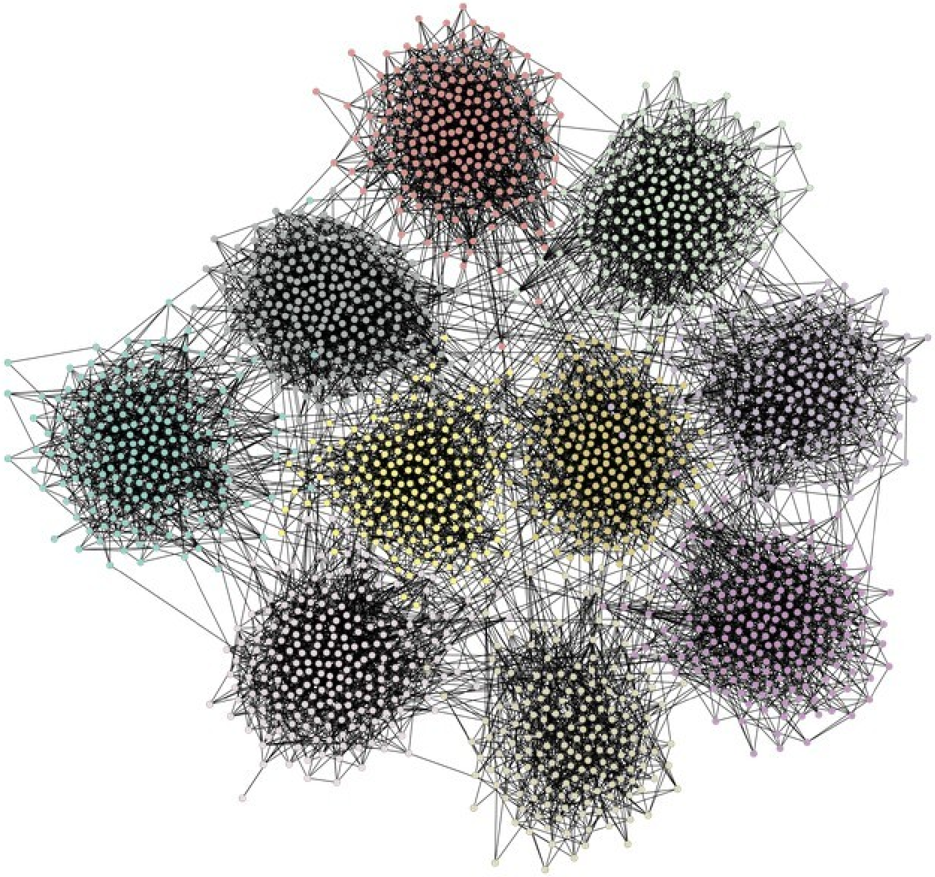
Locally dense, globally sparse





Girvan, M., & Newman, M. E. (2002). Community structure in social and biological networks. PNAS, 99(12), 7821-7826.

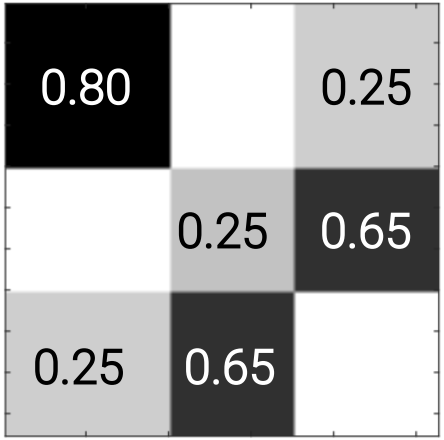
Stochastic Block Models



Generating a network using the SBM

Step 1 : Assign each node to a group

Generating a network using the SBM

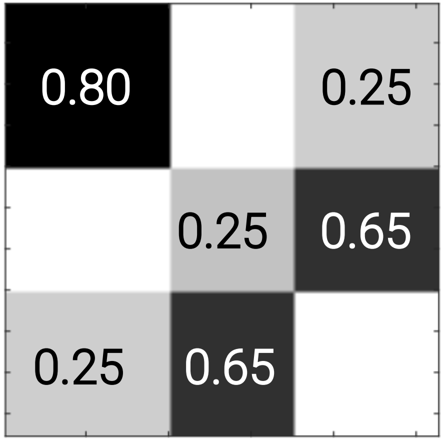


Mixing Matrix

Step 1 : Assign each node to a group

- **Step 2** : Select some connection probabilities (mixing matrix)

Generating a network using the SBM

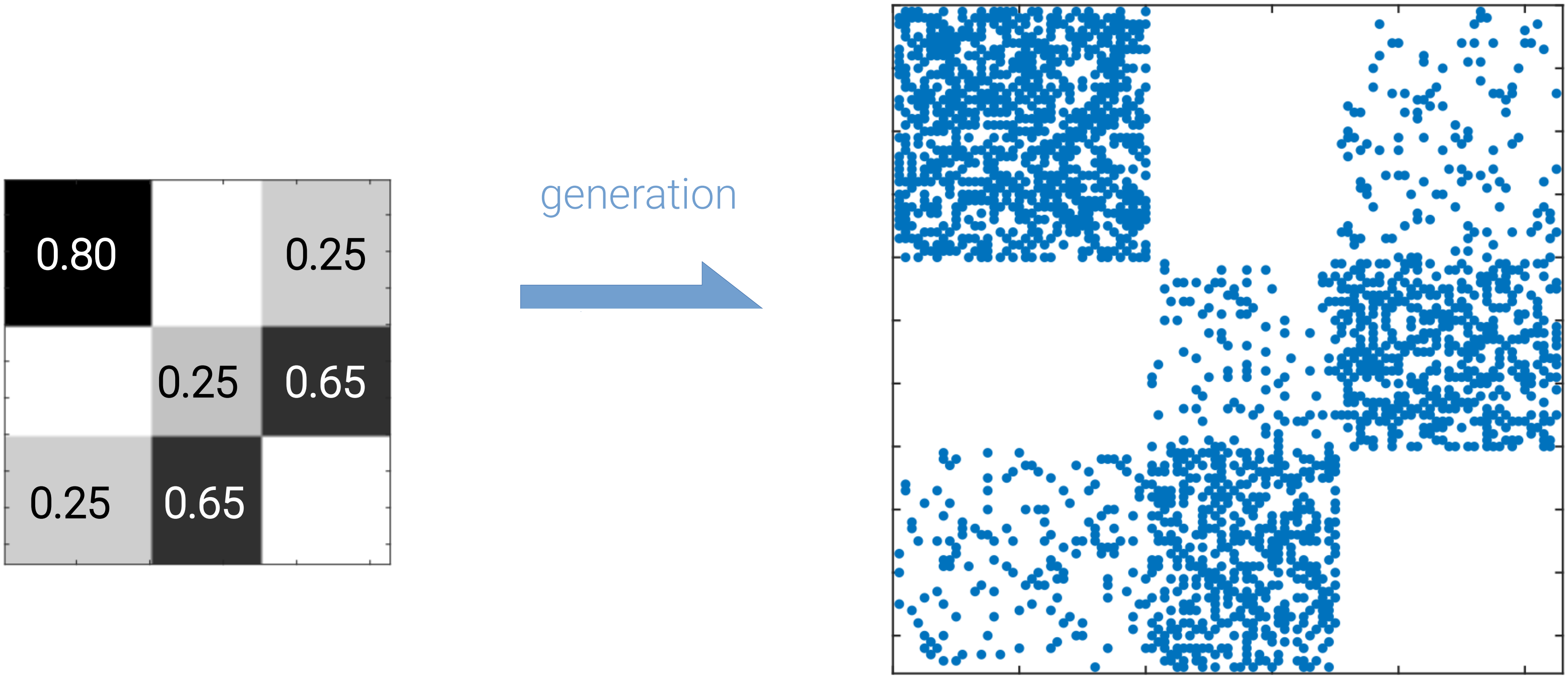


Mixing Matrix

Step 1 : Assign each node to a group

- Step 2 : Select some connection probabilities (mixing matrix)
- Step 3 : For each pair of nodes, add an edge with probability according to the group memberships

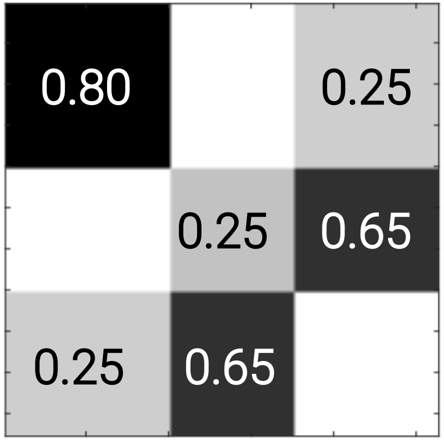
Generating a network using the SBM



Mixing Matrix

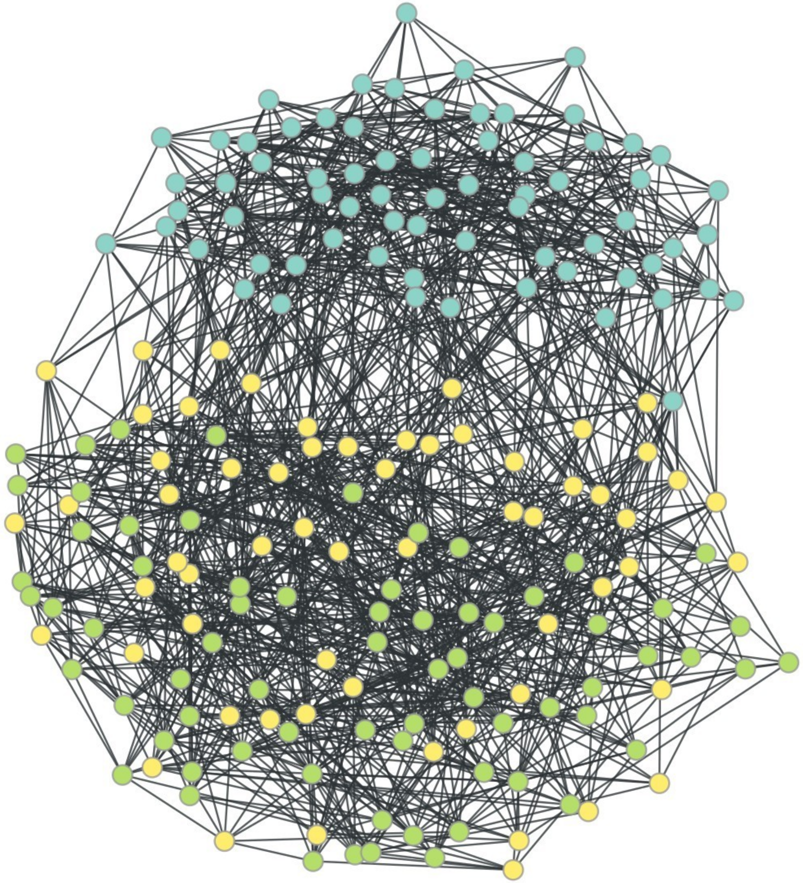
Adjacency Matrix

Generating a network using the SBM

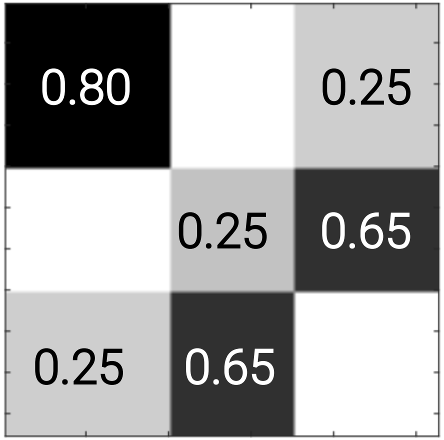


Mixing Matrix

generation



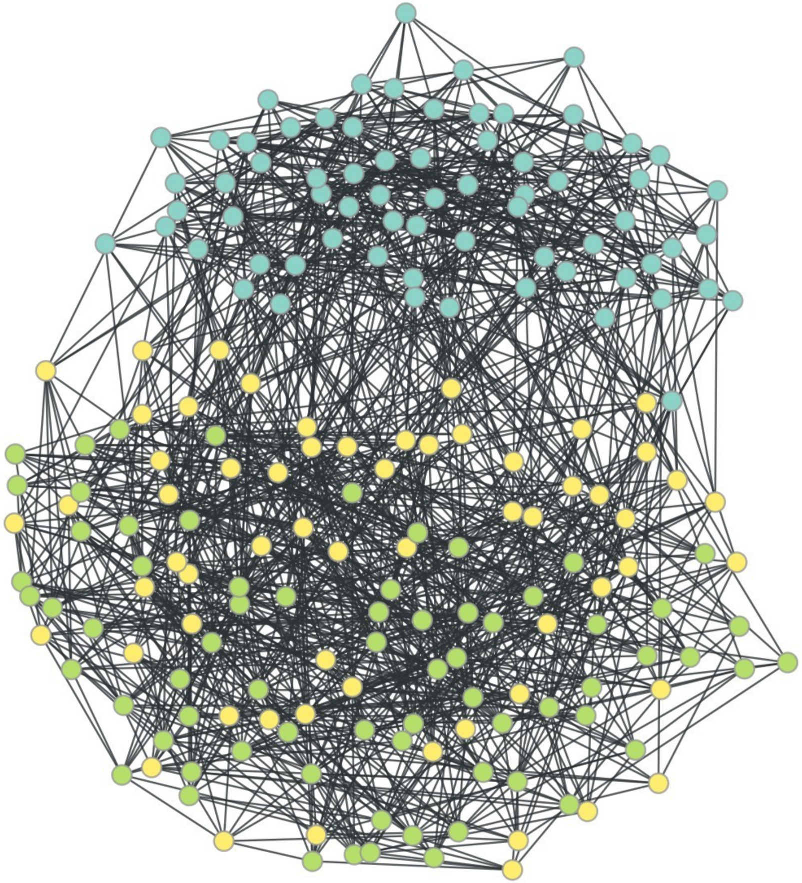
Generating a network using the SBM



Mixing Matrix

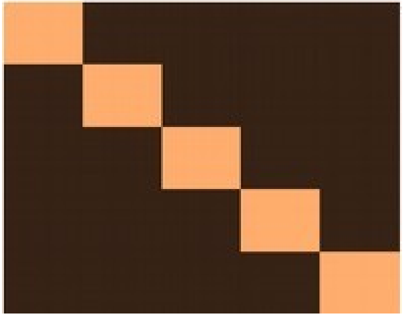
generation

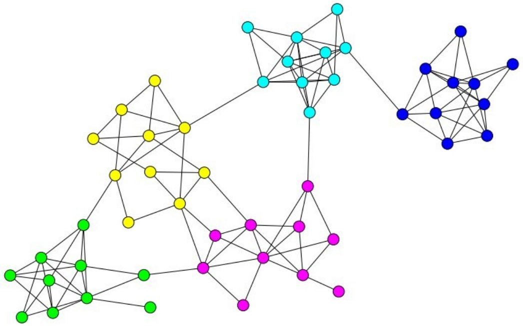
inference
(tomorrow)



Different types of structure

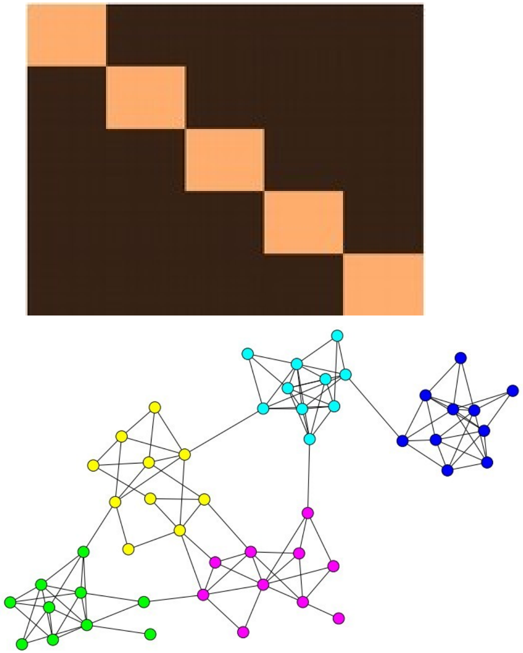
assortative



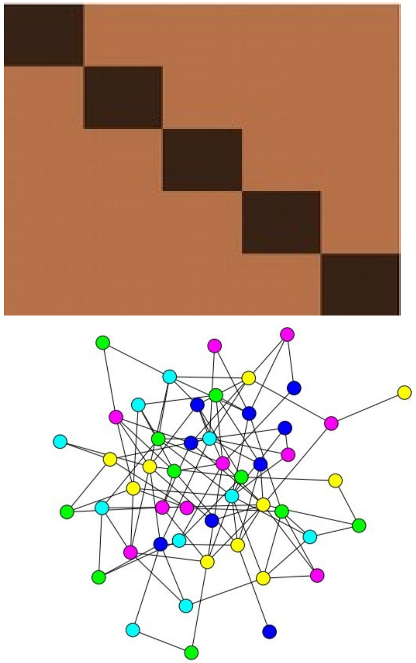


Different types of structure

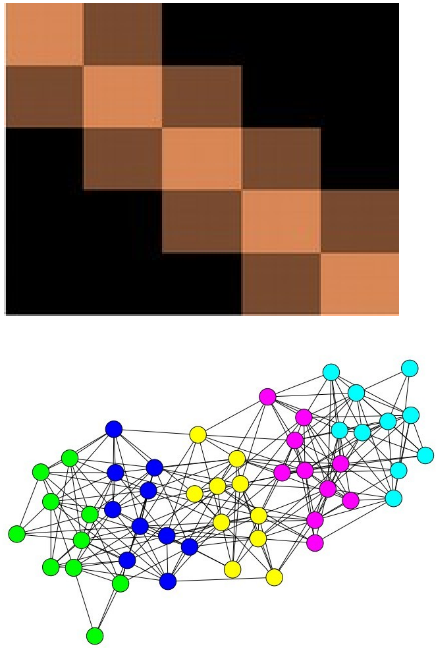
assortative



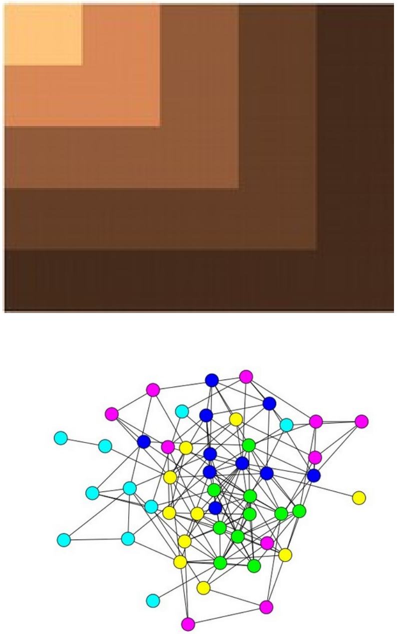
disassortative



ordered

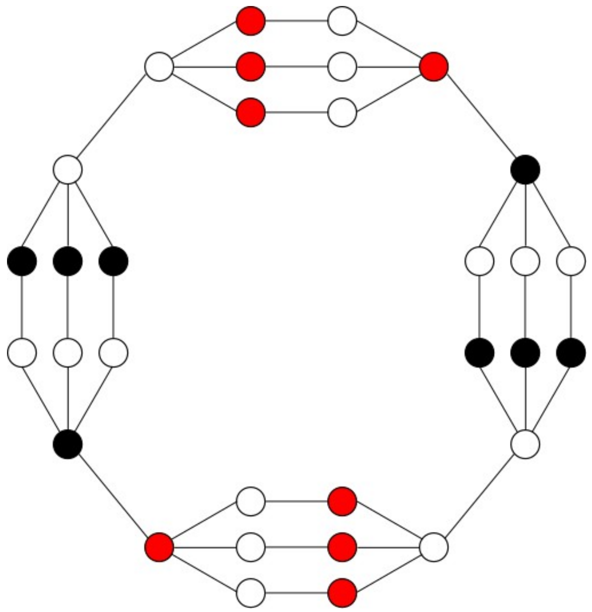


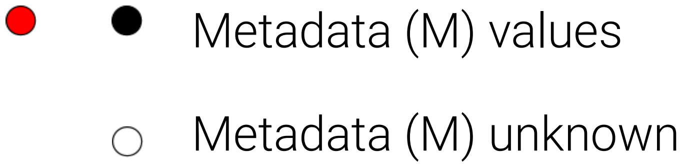
core-periphery



Practical Part III

Network nodes can have properties or attributes (metadata)





social networks *age, sex, ethnicity, race, etc.*
food webs *feeding mode, species body mass, etc.*
internet *data capacity, physical location, etc.*
protein interactions *molecular weight, association with cancer, etc.*

Blockmodel Entropy Significance Test

How well do the metadata explain the network?

Peel, Larremore, Clauset, "The ground truth about metadata and community detection in networks". Science Advances 3 (5), e1602548 (2017)

Blockmodel Entropy Significance Test

How well do the metadata explain the network?

1. Divide the network G into groups according to metadata labels M.

Peel, Larremore, Clauset, "The ground truth about metadata and community detection in networks". Science Advances 3 (5), e1602548 (2017)

Blockmodel Entropy Significance Test

How well do the metadata explain the network?

1. Divide the network G into groups according to metadata labels M.

2. Fit the parameters of an SBM and compute the entropy **H**(G,M)

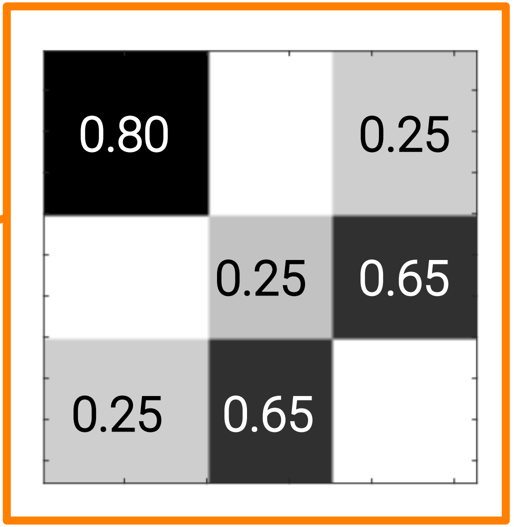
Peel, Larremore, Clauset, "The ground truth about metadata and community detection in networks". Science Advances 3 (5), e1602548 (2017)

Blockmodel Entropy Significance Test

How well do the metadata explain the network?

1. Divide the network G into groups according to metadata labels M.

2. Fit the parameters of an SBM and compute the entropy **H**(G,M)



Peel, Larremore, Clauset, "The ground truth about metadata and community detection in networks". Science Advances 3 (5), e1602548 (2017)

Blockmodel Entropy Significance Test

How well do the metadata explain the network?

1. Divide the network G into groups according to metadata labels M.

2. Fit the parameters of an SBM and compute the $\boxed{\text{entropy } \mathbf{H}(G,M)}$

Test statistic

Peel, Larremore, Clauset, "The ground truth about metadata and community detection in networks". Science Advances 3 (5), e1602548 (2017)

Blockmodel Entropy Significance Test

How well do the metadata explain the network?

1. Divide the network G into groups according to metadata labels M.

2. Fit the parameters of an SBM and compute the entropy **H**(G,M)

3. Compare this entropy to a distribution of entropies of networks partitioned using random permutations of the metadata labels.

Peel, Larremore, Clauset, "The ground truth about metadata and community detection in networks". Science Advances 3 (5), e1602548 (2017)

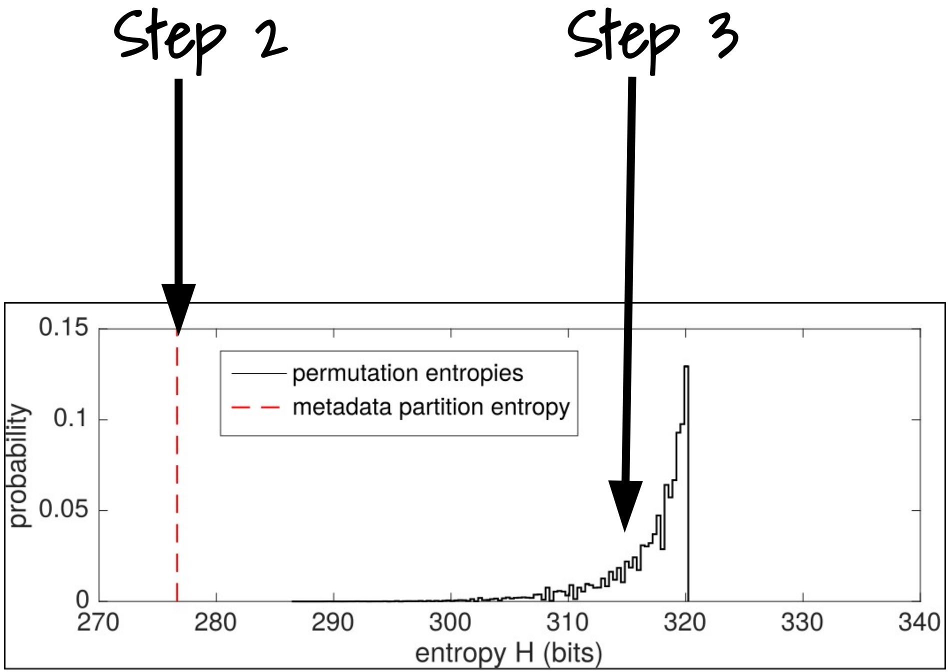
Blockmodel Entropy Significance Test

How well do the metadata explain the network?

1. Divide the network G into groups according to metadata labels M.

2. Fit the parameters of an SBM and compute the entropy **H**(G,M)

3. Compare this entropy to a distribution of entropies of networks partitioned using random permutations of the metadata labels.



Peel, Larremore, Clauset, "The ground truth about metadata and community detection in networks". Science Advances 3 (5), e1602548 (2017)

Blockmodel Entropy Significance Test

How well do the metadata explain the network?

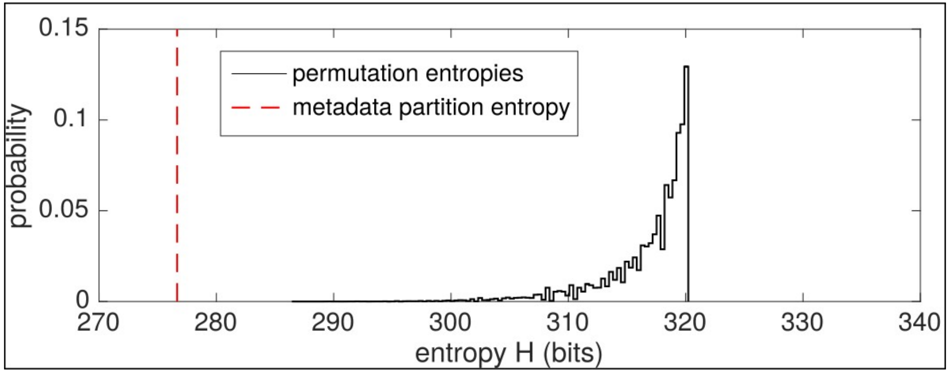
1. Divide the network G into groups according to metadata labels M.

2. Fit the parameters of an SBM and compute the entropy **H**(G,M)

3. Compare this entropy to a distribution of entropies of networks partitioned using random permutations of the metadata labels.

metadata is randomly assigned
→ model gives no explanation, high **H**

metadata correlates with structure
→ model gives good explanation, low **H**



Peel, Larremore, Clauset, "The ground truth about metadata and community detection in networks". Science Advances 3 (5), e1602548 (2017)

Multiple networks; multiple metadata attributes

Network	Status	Gender	Office	Practice	Law School
Friendship	$< 10^{-6}$	0.034	$< 10^{-6}$	0.033	0.134
Cowork	$< 10^{-3}$	0.094	$< 10^{-6}$	$< 10^{-6}$	0.922
Advice	$< 10^{-6}$	0.010	$< 10^{-6}$	$< 10^{-6}$	0.205

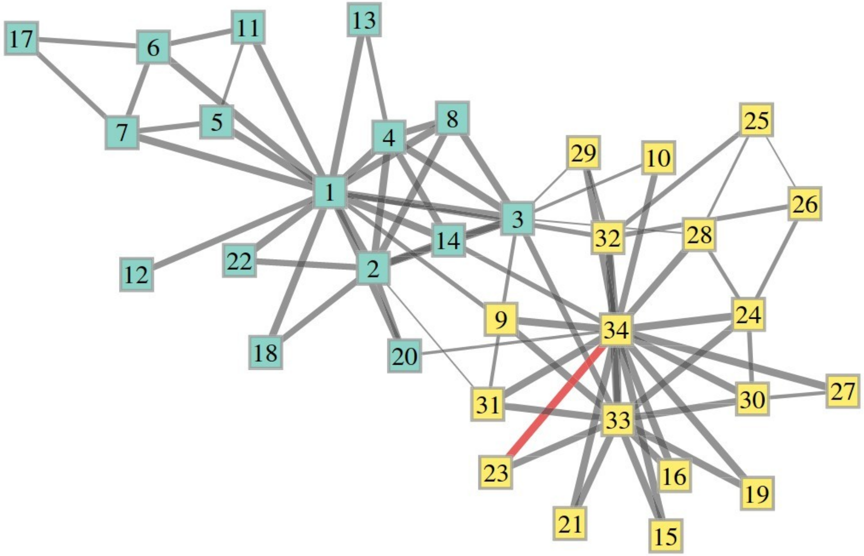
Multiple sets of metadata provide a significant explanation for multiple networks.

Lazega, The Collegial Phenomenon: The Social Mechanisms of Cooperation Among Peers in a Corporate Law Partnership, Oxford University Press (2001).

Practical Part III

Errors in networks and reconstruction

Zachary's Karate Club

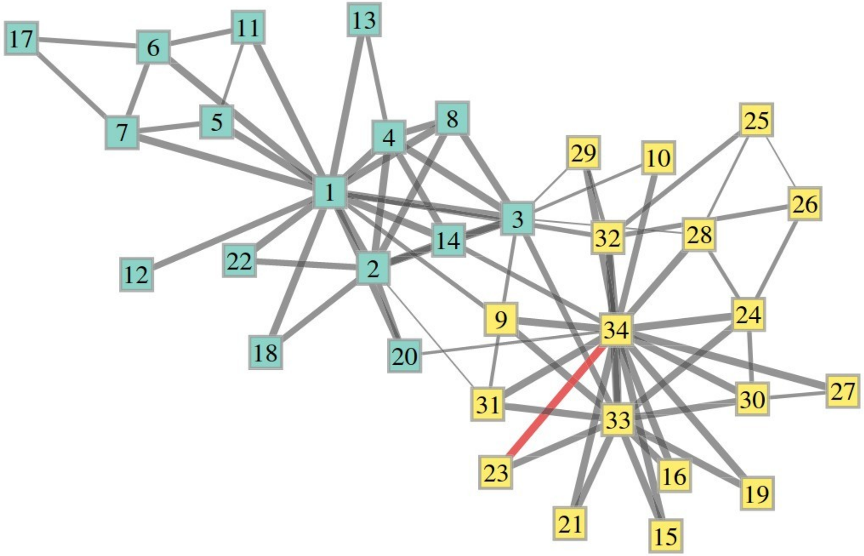


Zachary, W. W. An Information Flow Model for Conflict and Fission in Small Groups. J. Anthropological Res. 33, 452-473 (1977).

Zachary's Karate Club

Individual Number

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
1	0	4	5	3	3	3	3	2	2	0	2	3	2	3	0	0	0	2	0	2	0	2	0	0	0	0	0	0	0	0	0	2	0	0
2	4	0	6	3	0	0	0	4	0	0	0	0	0	5	0	0	0	1	0	2	0	2	0	0	0	0	0	0	0	0	2	0	0	0
3	5	6	0	3	0	0	0	4	5	1	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	3	0
4	3	3	3	0	0	0	0	3	0	0	0	0	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	3	0	0	0	0	0	2	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	3	0	0	0	0	0	5	0	0	0	3	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	3	0	0	0	2	5	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	2	4	4	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	2	0	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	4	3
10	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
11	2	0	0	0	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	3	5	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	2
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	4
17	0	0	0	0	0	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2
20	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	1
22	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	4	0	2	0	0	5	4
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	3	0	0	0	2	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	2	0	0	0	0	0	0	7	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	2
28	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	3	0	0	0	0	0	0	0	0	4
29	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	2
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	4	0	0	0	0	0	3	2
31	0	2	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3
32	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	7	0	0	2	0	0	0	4	4
33	0	0	2	0	0	0	0	0	3	0	0	0	0	0	3	3	0	0	1	0	3	0	2	5	0	0	0	0	0	4	3	4	0	5
34	0	0	0	0	0	0	0	0	4	2	0	0	0	3	2	4	0	0	2	1	1	0	3	4	0	0	2	4	2	2	3	4	5	0

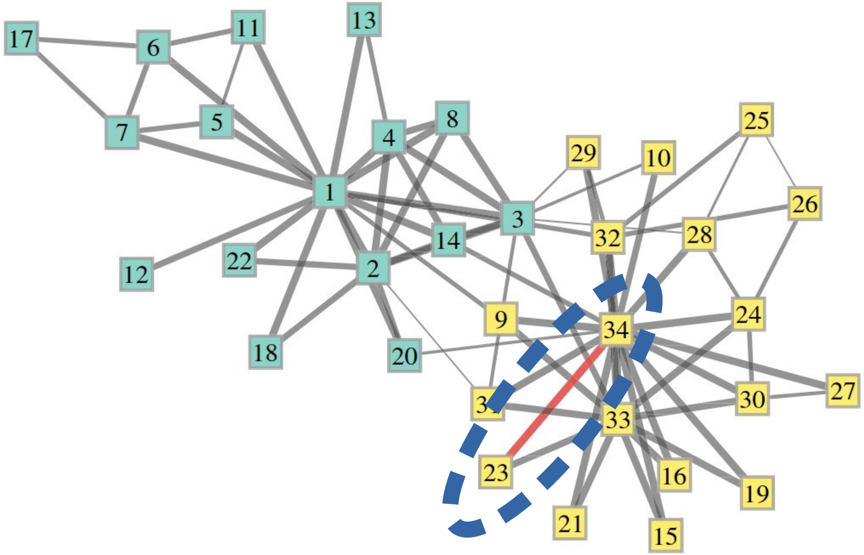


Zachary, W. W. An Information Flow Model for Conflict and Fission in Small Groups. J. Anthropological Res. 33, 452-473 (1977).

Zachary's Karate Club

Individual Number

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
1	0	4	5	3	3	3	3	2	2	0	2	3	2	3	0	0	0	2	0	2	0	2	0	0	0	0	0	0	0	0	0	2	0	0
2	4	0	6	3	0	0	0	4	0	0	0	0	0	5	0	0	0	1	0	2	0	2	0	0	0	0	0	0	0	0	2	0	0	0
3	5	6	0	3	0	0	0	4	5	1	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	3	0
4	3	3	3	0	0	0	0	3	0	0	0	0	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	3	0	0	0	0	0	2	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	3	0	0	0	0	0	5	0	0	0	3	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	3	0	0	0	2	5	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	2	4	4	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	2	0	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	4	3
10	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
11	2	0	0	0	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	3	5	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	2
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	4
17	0	0	0	0	0	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2
20	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	1
22	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	[illegible]	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	4	0	2	0	0	[illegible]	[illegible]
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	3	0	0	0	2	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	2	0	0	0	0	0	0	7	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	2
28	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	3	0	0	0	0	0	0	0	0	4
29	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	2
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	4	0	0	0	0	0	3	2
31	0	2	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3
32	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	7	0	0	2	0	0	0	4	4
33	0	0	2	0	0	0	0	0	3	0	0	0	0	0	3	3	0	0	1	0	3	0	[illegible]	5	0	0	0	0	0	4	3	4	0	5
34	0	0	0	0	0	0	0	0	4	2	0	0	0	3	2	4	0	0	2	1	1	[illegible]	3	[illegible]	0	0	2	4	2	2	3	4	5	0

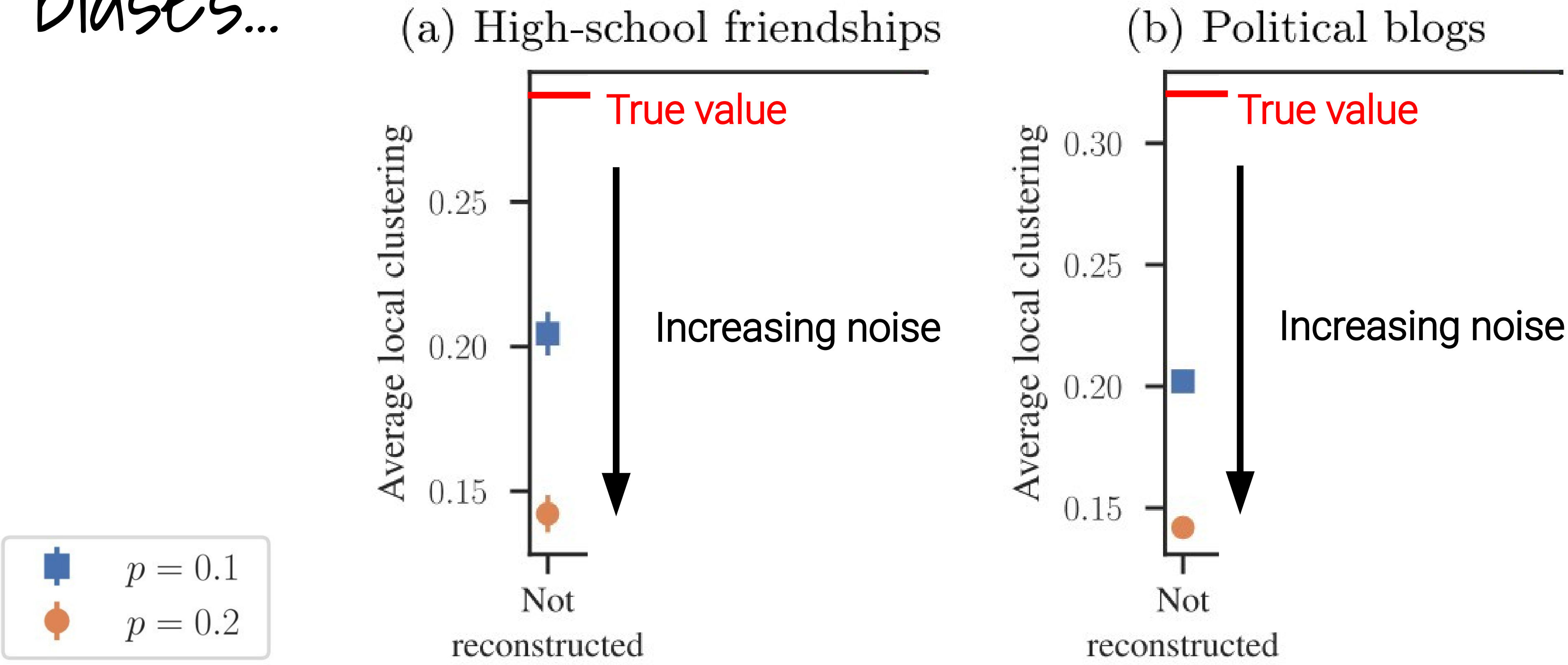


Does this edge exist?

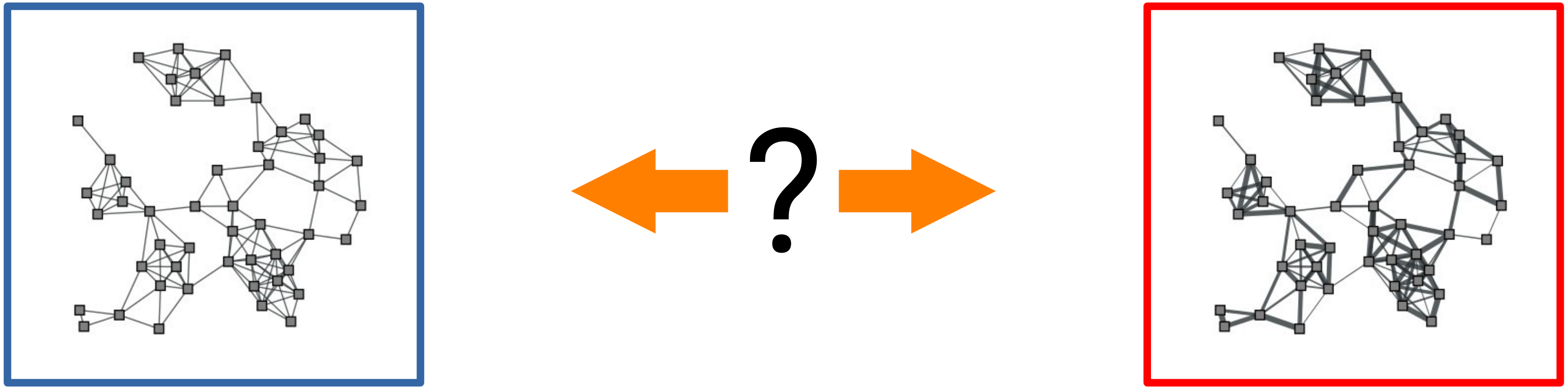
Zachary, W. W. An Information Flow Model for Conflict and Fission in Small Groups. J. Anthropological Res. 33, 452-473 (1977).

Errors in network data create systematic biases...

Errors in network data create systematic biases...



Peel, L., Peixoto, T. P., & De Domenico, M. (2022). Statistical inference links data and theory in network science. Nature Communications, 13(1), 6794.



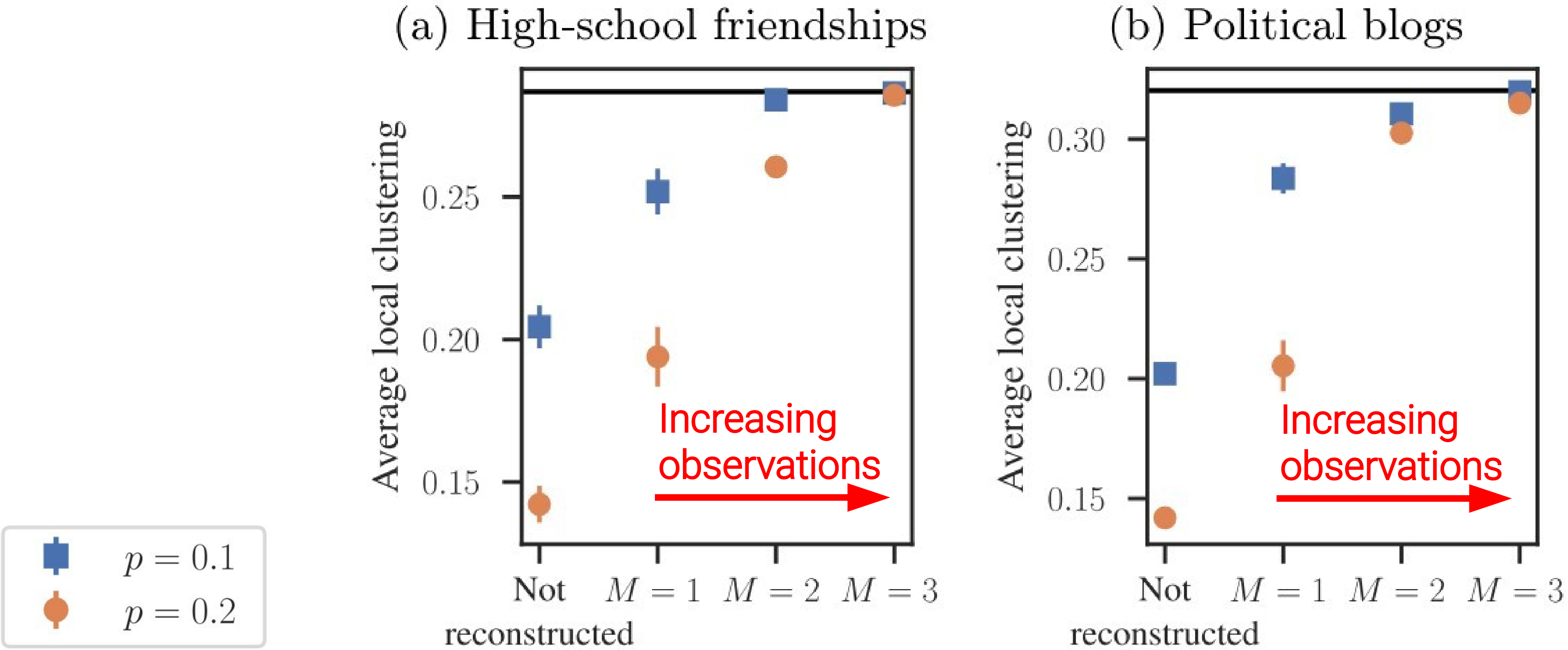
True Network

Reconstructed Network

We don't know if the network represents the system

$$P(\boldsymbol{A}|\boldsymbol{D}) = \frac{P(\boldsymbol{D}|\boldsymbol{A})P(\boldsymbol{A})}{P(\boldsymbol{D})} \ .$$

Bayesian inference



Peel, L., Peixoto, T. P., & De Domenico, M. (2022). Statistical inference links data and theory in network science. Nature Communications, 13(1), 6794.

Practical Part IV

Summary...

We can use graph models to simulate networks and better understand the effects of network structure

We can use graph models to:

- test hypotheses
- reconstruct uncertain networks