



Universiteit Utrecht

*summerschool*  
**UTRECHT**

# Diffusion in Networks

Jiamin Ou

Vincent Buskens

Day 5 of the summerschool: Network Science

# Introducing ourselves

- Vincent Buskens: prof. of sociology; interested in effects of social networks on behavior: cooperation, trust etc.
- Jiamin Ou: assistant professor, interested in dynamics of sustainable behavior, energy/emission models



# Simple contagion

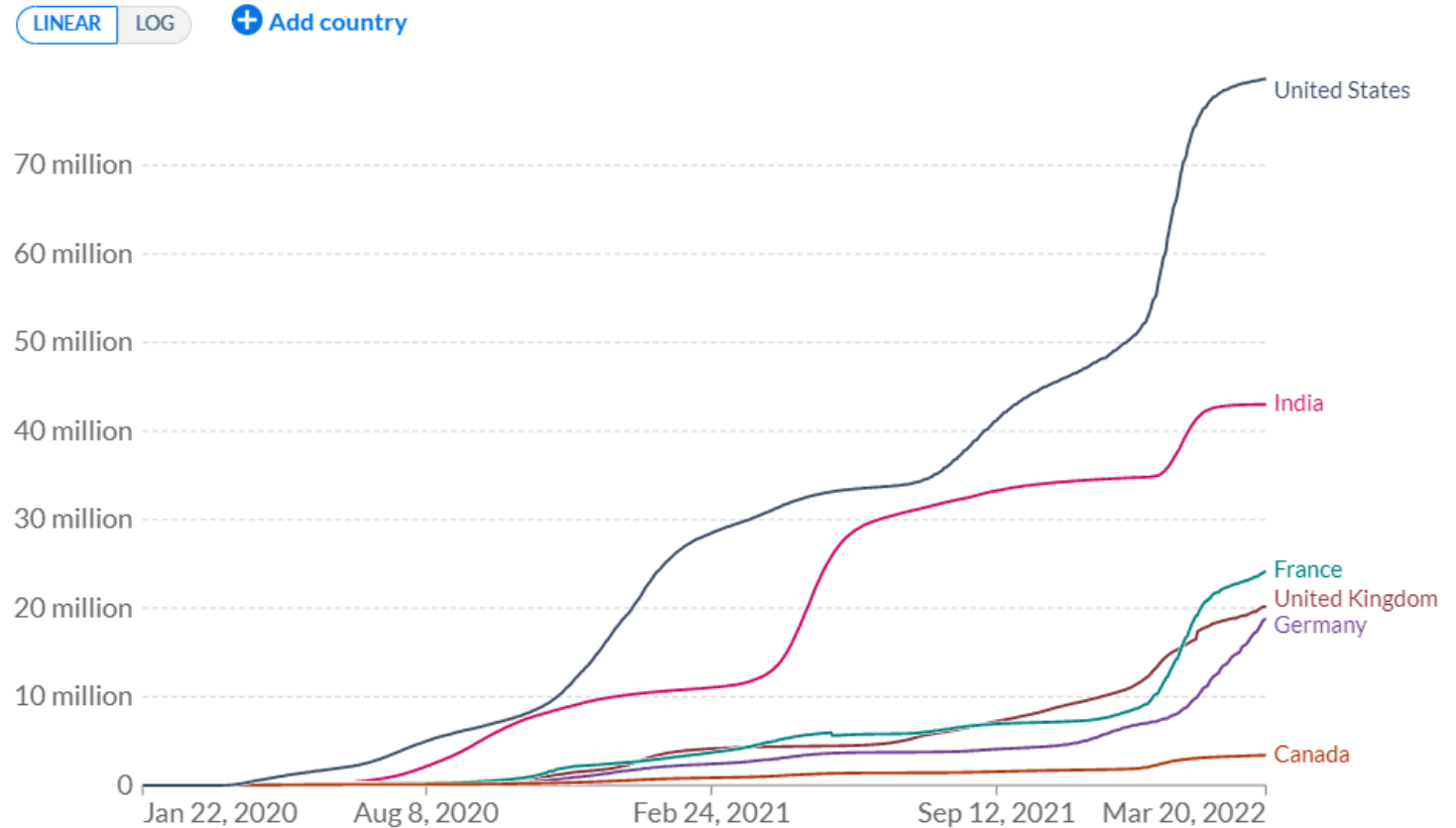
## The spread of COVID in a human network

- **COVID 19:** From patient 0 in Dec 2019 to 470 million cases till now

### Cumulative confirmed COVID-19 cases

Due to limited testing, the number of confirmed cases is lower than the true number of infections.

Our World  
in Data



Source: Johns Hopkins University CSSE COVID-19 Data

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▶ Jan 22, 2020 ◯ Mar 20, 2022

Nicholas Christakis realizing the importance of networks



# The spread of obesity in social network of 12,067 people from 1971 to 2003 (“Framingham Heart Study”)

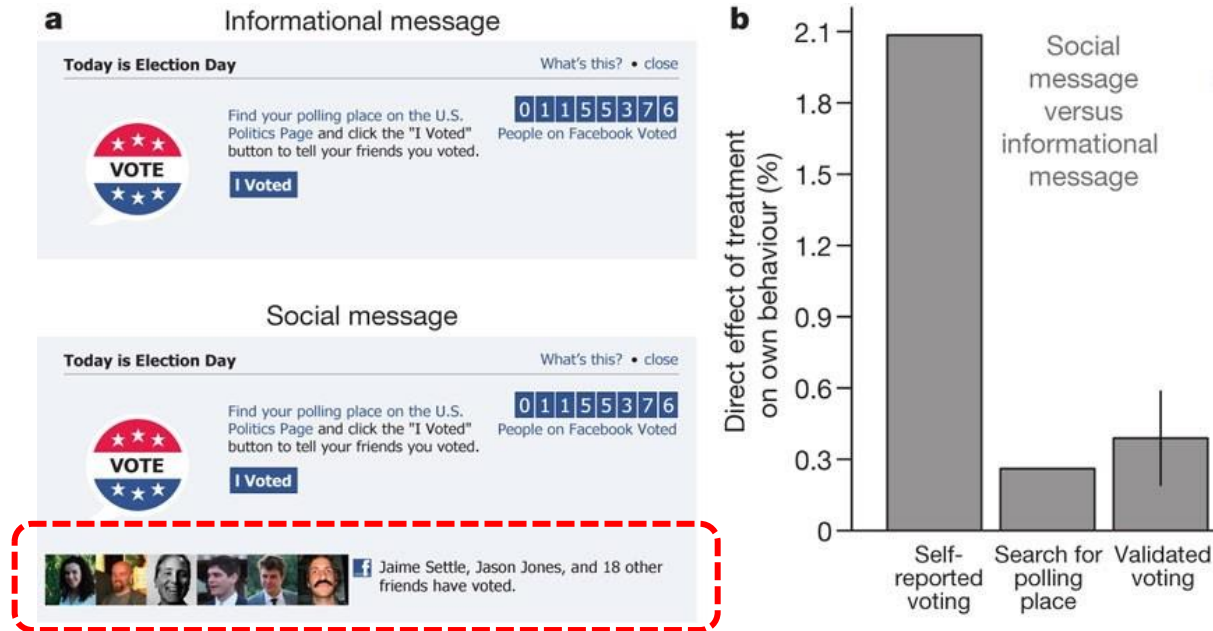
- A person’s chance of being obese increased by **57%** (95% confidence interval [CI], 6 to 123) if he or she has a **friend** who is obese.
- If one **spouse** became obese, the likelihood that the other spouse would become obese increased by **37%** (95% CI, 7 to 73).
- These effects were **not seen among neighbours** in the immediate geographic location.
- Persons of the **same sex** had relatively **greater influence** on each other as compared with those of the opposite sex.

When your close ones gain weight, so will you 😞



# A 61-million-person experiment of social influence in Facebook

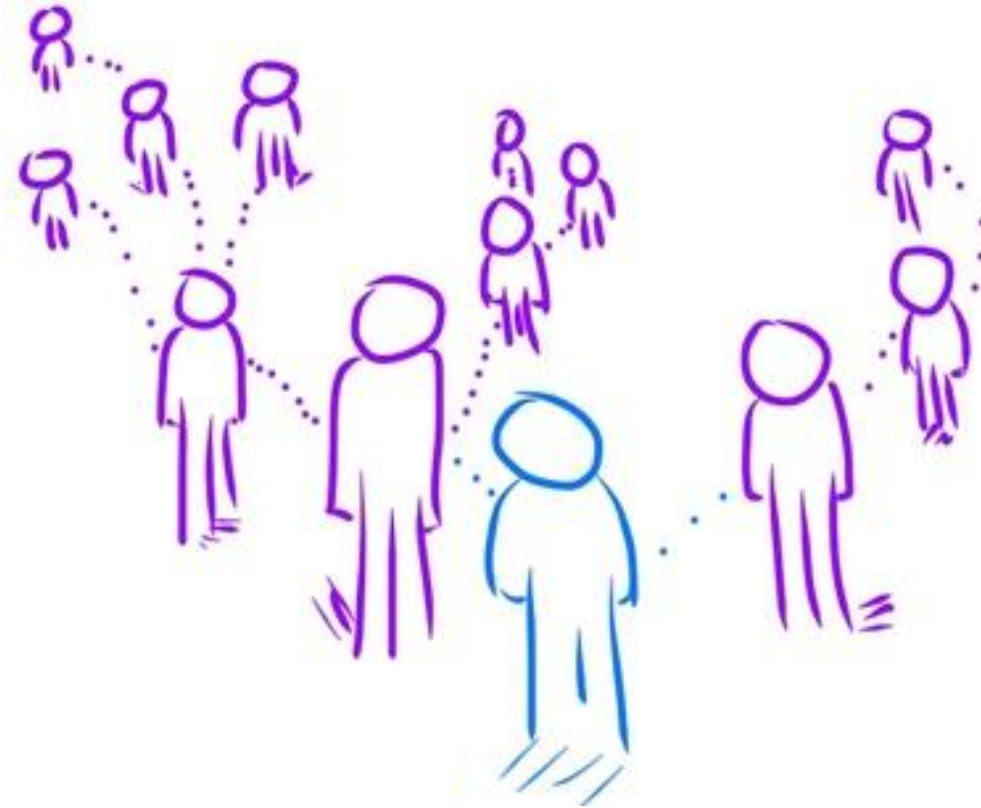
A controlled experiment for US Facebook users during the 2010 US congressional elections



Bond, R., Fariss, C., Jones, J. *et al.* A 61-million-person experiment in social influence and political mobilization. *Nature* **489**, 295–298 (2012).



We are highly social creatures that make belief/behavior/health condition contagious...



“Tell me who your friends are and I’ll tell you who you are.”

Many mechanism at work that cause similarities between connected people

- Contagion
- Selection
- Common context / third variables
- Social dynamics



We will focus on the contagion mechanism today



But why some new ideas/news can go viral quickly in the network?

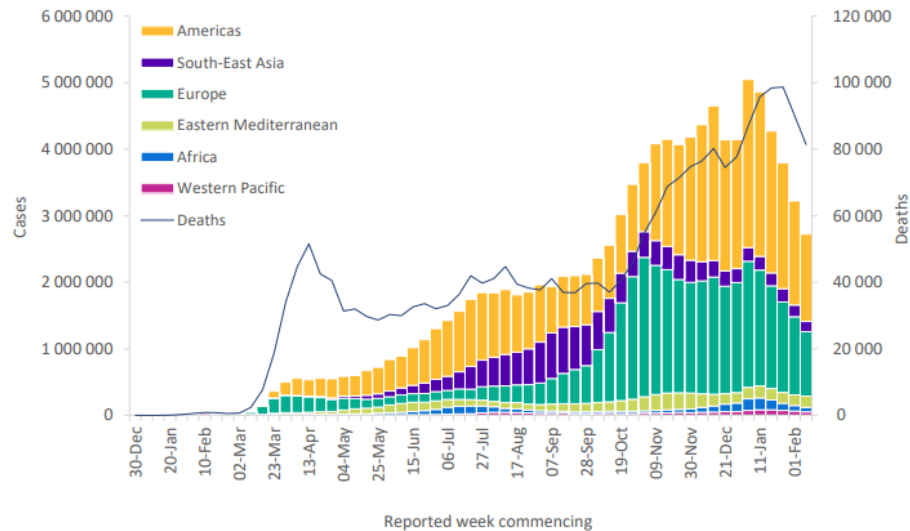
While some social innovations that can benefit society often fail to diffuse?

### Spread of COVID

VS

### Spread of hygiene, mask-wearing behaviour

Figure 1: COVID-19 cases reported weekly by WHO Region, and global deaths, as of 14 February 2021\*\*



Let's try to find out:

Do virus, information, behavioral change spread in the same way in social networks?

Can we depict the contagion process in numerical models?

Can we deduce how contagion depends on network structure?

# Today's programme

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- **Simple contagion**  
Mechanism and the strength of 'weak ties'
- **Diffusion model for simple contagion**  
Independent cascade model and other variants
- **Complex contagion**  
Mechanism and the strength of 'strong ties'
- **Diffusion model for complex contagion**  
Threshold model

# Mechanism of simple contagion

- Single contact sufficient for transmission

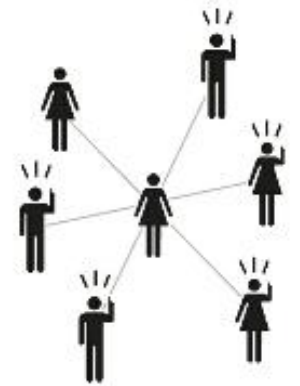
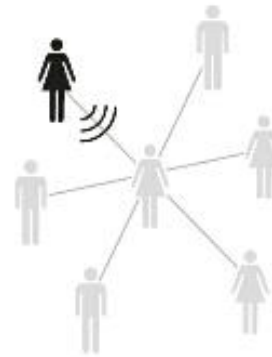


- Examples

Epidemic

Easily convincing rumors (one can costlessly repeat a story to many others)

Job information



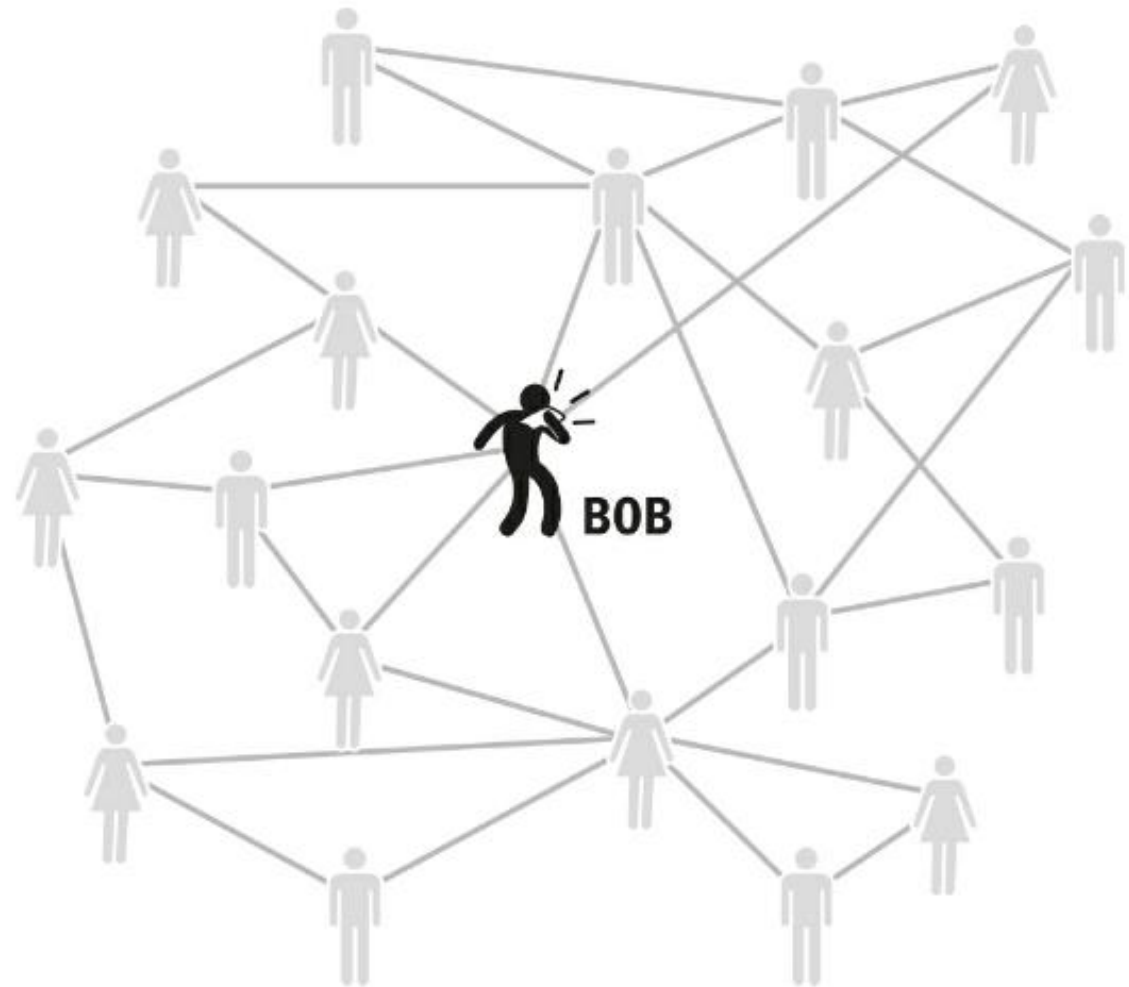
# Diffusion in social network under simple contagion

**Two states of people:**

**Inactivated**, susceptible to a contagion;

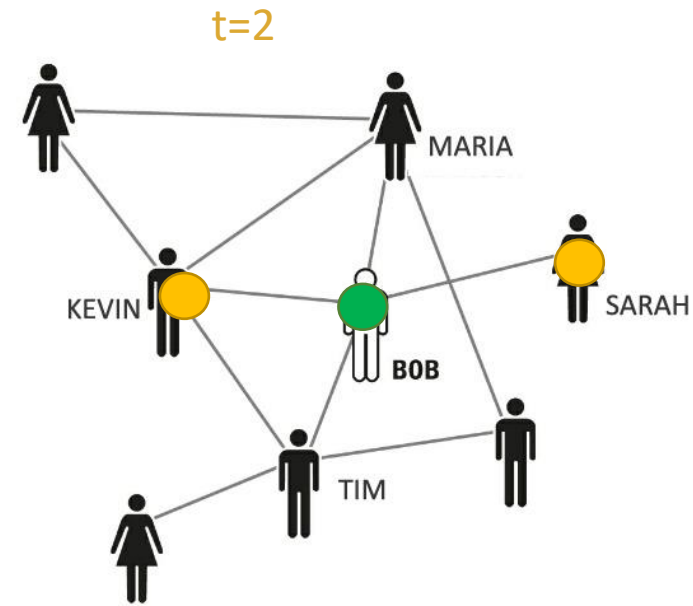
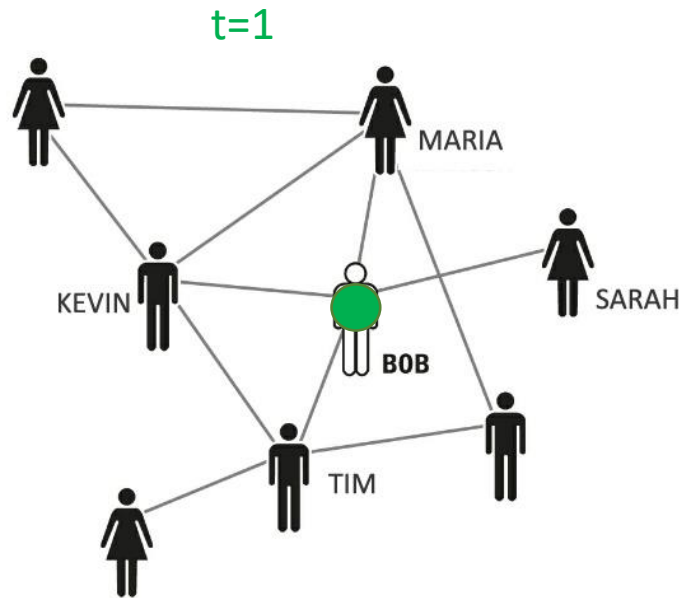
**Activated**, infected and can transmit the contagion to others

Bob is 'patient 0' and can pass the virus/information with probability  $\beta$



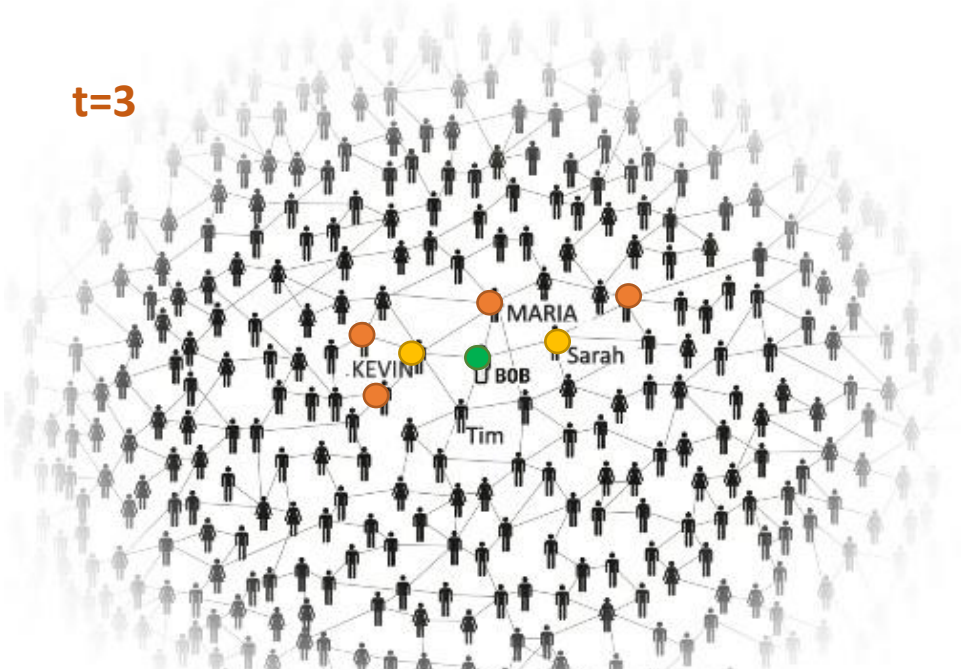
# Diffusion in social network under simple contagion

$\beta=50\%$

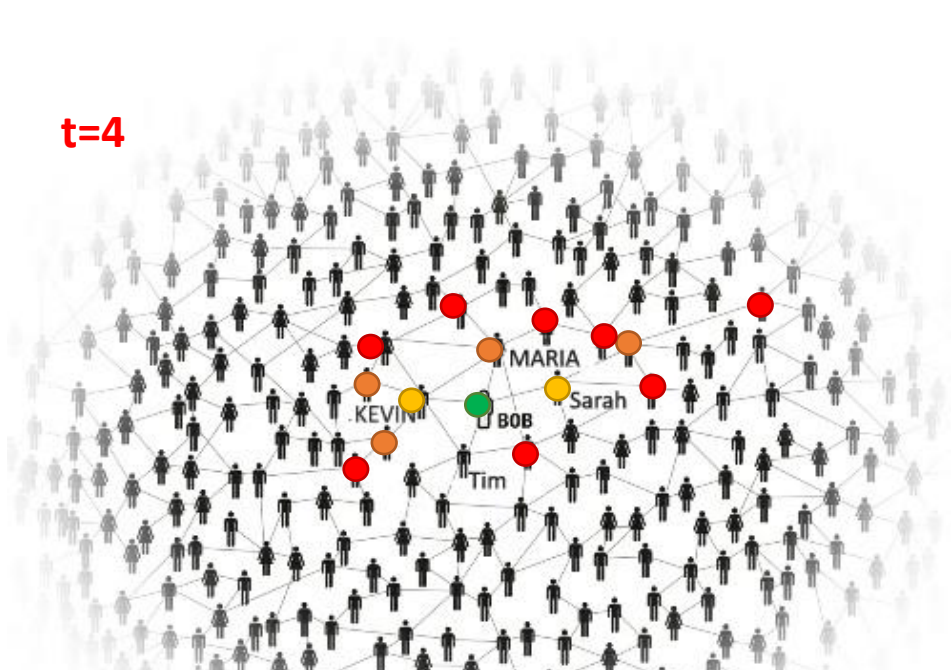




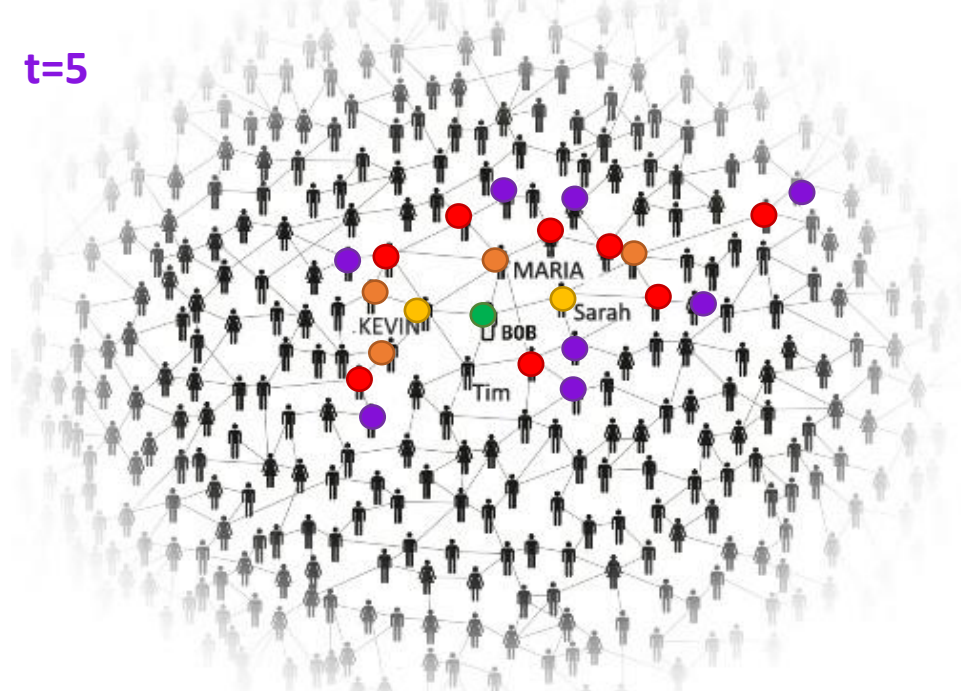
t=3



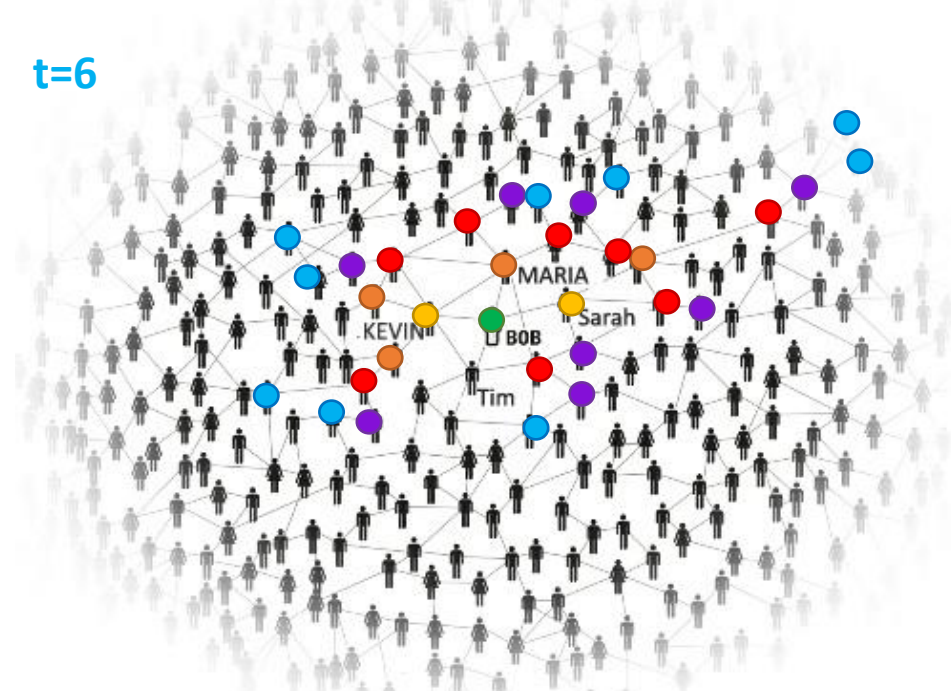
t=4



t=5



t=6



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- **Diffusion model for simple contagion**

Independent cascade model and other variants

- **Complex contagion**

Mechanism and the strength of 'strong ties'

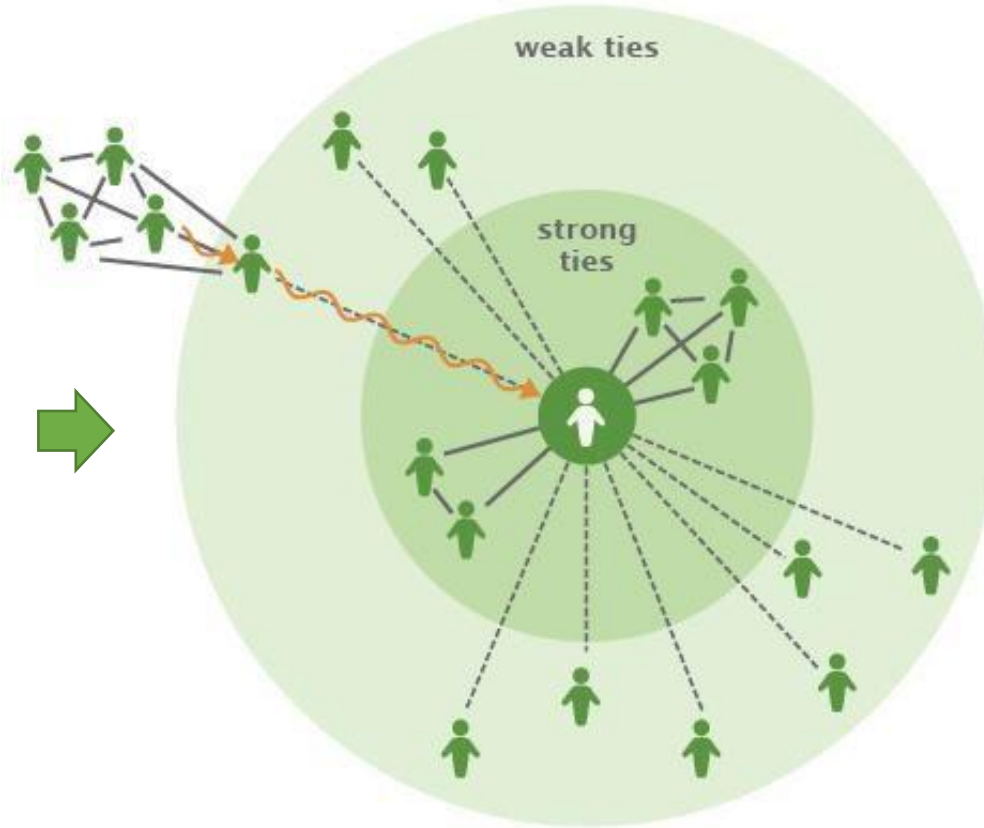
- **Diffusion model for complex contagion**

Threshold model

# Weak and strong ties

Frequency of Contact

Emotional intensity and support



## Strong ties

Family, partner, close friends (Core discussion network)

## Weak ties

Distant friends, acquaintances, neighbors, colleagues

Interact less frequently, less invested in relationship

“The strength of weak ties”, Mark Granovetter, 1973

Most people got their current jobs through acquaintances rather than close friends





# Most people got their current jobs through acquaintances rather than close friends

- **Sample:**

A random sample of recent professional, technical and managerial job changers living in a Boston suburb, 1973

- **Procedure & Result:**

How often they saw the contact who communicated job offers to them:

Often (> twice a week), 16.7%

Occasionally ( more than once a year but less than twice a week), 55.6%

Rarely (once a year or less), 27.8%



***People that you don't meet often provide more efficient access to new information***

# Is LinkedIn making you more successful?



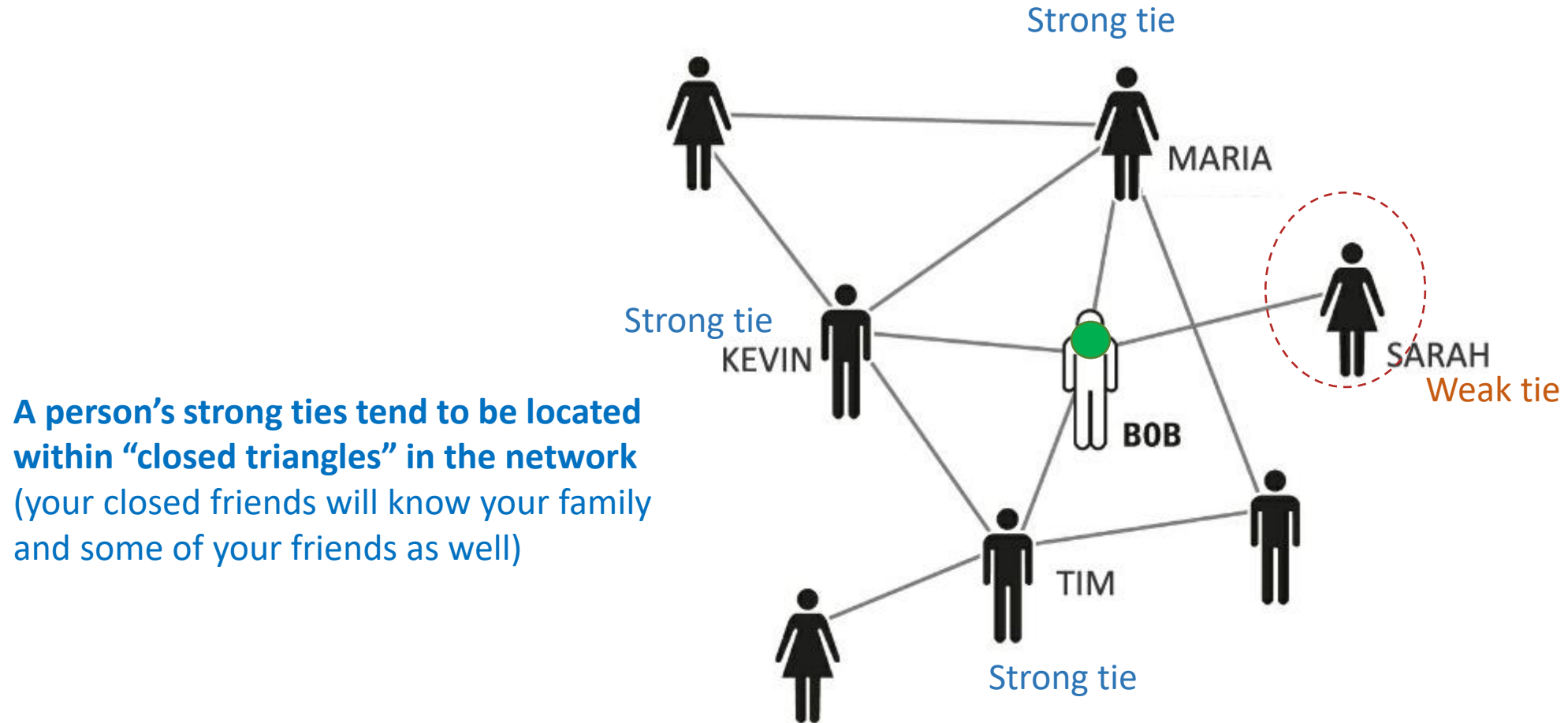
- Mission of LinkedIn “connect the world’s professionals to make them more productive and successful.”
- Major social ties in LinkedIn: acquaintances or former colleagues
- Informational benefits from the usage of LinkedIn, Twitter, Facebook among a representative sample of Dutch online users
- Using LinkedIn significantly increased informational benefits. Using Twitter also resulted in a significant increase in informational benefits.
- Using Facebook resulted in significantly lower informational benefits.

LinkedIn, Facebook and Twitter:  
Weak or strong ties?



# Why weak ties can accelerate information diffusion?

Among all the people connected to Bob, who is the weak tie?



Now Bob needs to hire a new programmer and wants to use his “word of mouth” network to spread the news of job opening

1<sup>st</sup> round: Bob

2<sup>nd</sup> round: Bob → Kevin, Maria, Tim, Sarah

3<sup>rd</sup> round:

Kevin → Mila, Maria

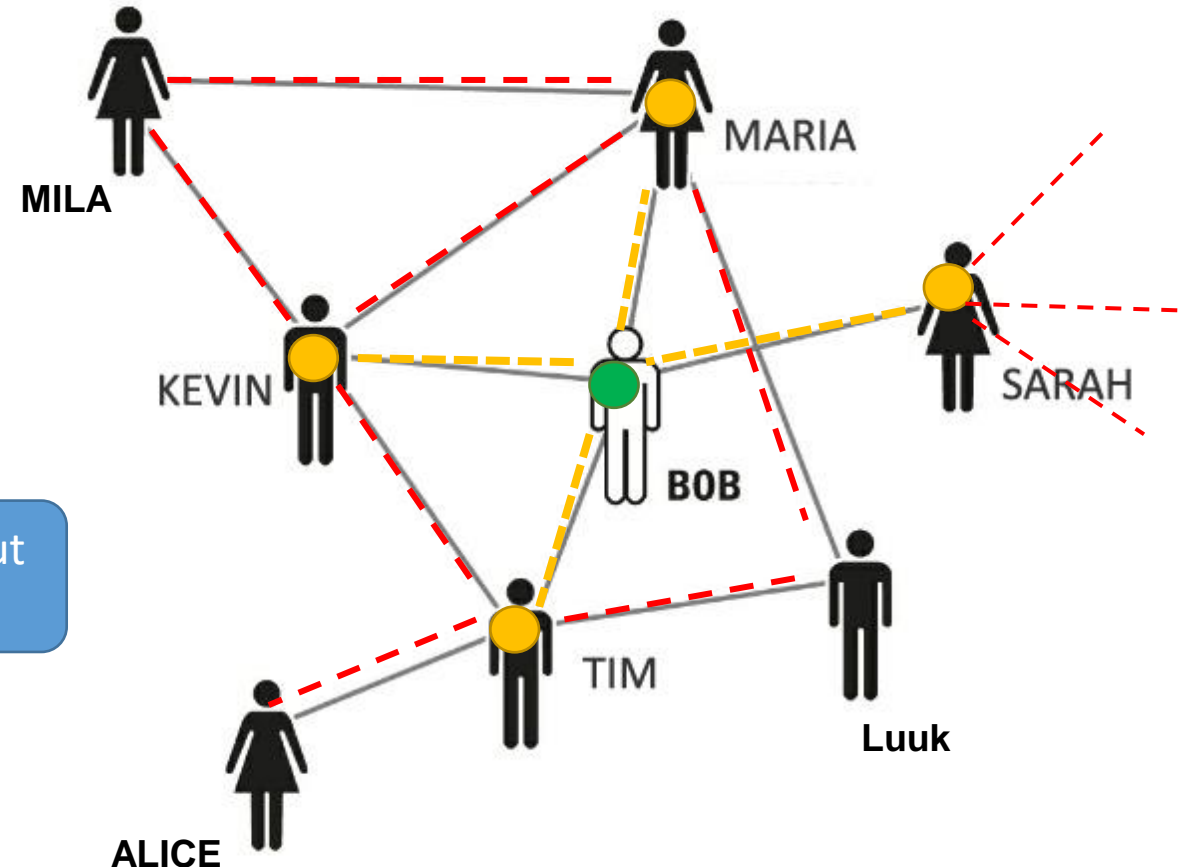
Maria → Kevin, Mila, Luuk

Tim → Kevin, Luuk, Alice

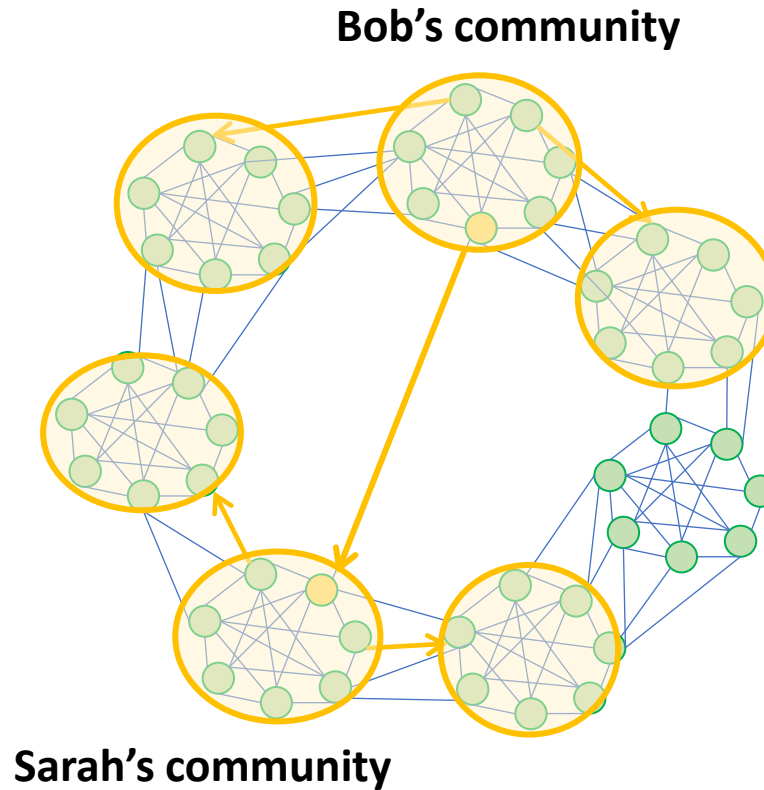
Sarah → to her other contacts

I've heard about it already!

A lot of *redundant information* within the close community of Bob, which is *not efficient*.



# Diffusion of simple contagion via weak tie



Under **simple contagion**, the only tie between Bob and Sarah **can** spread the rumor/virus to Sarah's community and fasten the diffusion process

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Threshold model

Now Bob tries out a new product and loves it.  
He shares with other friends.

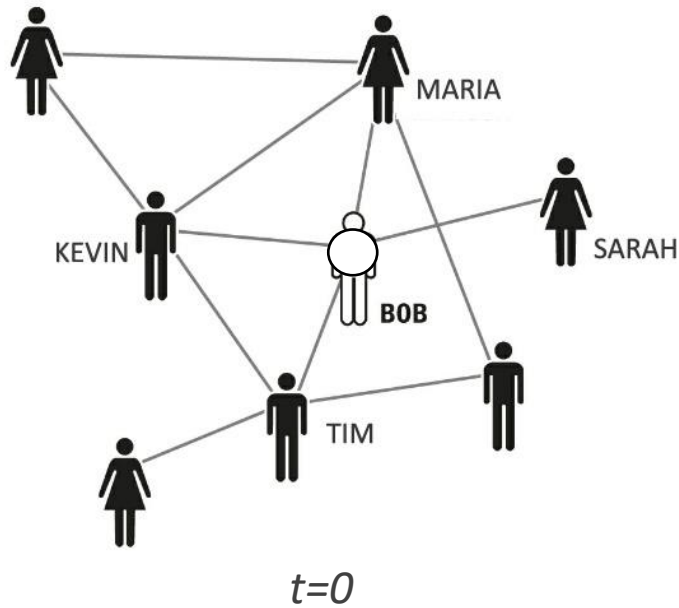


# Independent Cascade Model

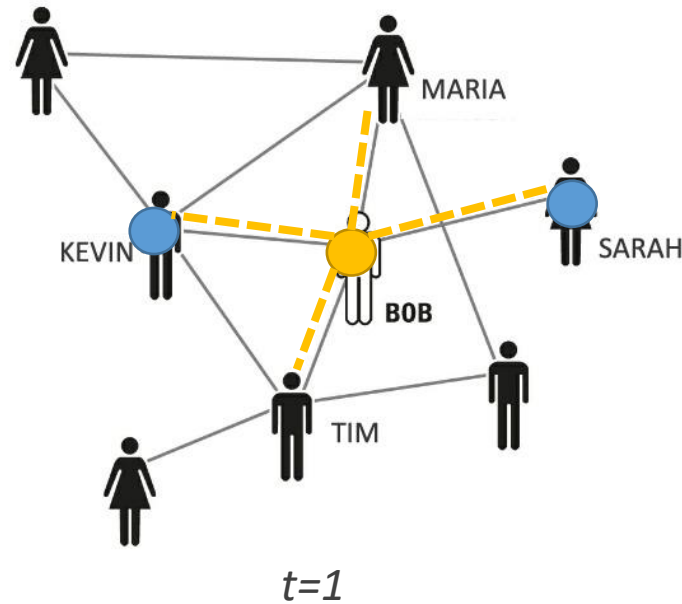
- Nodes can have two states — active ( $S=1$ ) and inactive states ( $S=0$ ); once activated, cannot be inactive again (e.g., Bob used the product)
- At time  $t=0$ ,  $k$  nodes are selected (i.e., activated). These nodes are called “*seed nodes*”
- At time  $t=1$ , seeds can activate neighboring nodes. The success depends on the probability  $p_{uv}$  assigned to the edge connecting  $u$  and  $v$ . ( $p_{uv}$  can be same for every edge or different by edges)
- At subsequent moments  $t$ , all activated nodes can activate not activated neighbors with the same probabilities
- Stop when all the nodes are activated or the number of activated nodes is saturated.



$p_{uw}=60\%$  for all edges

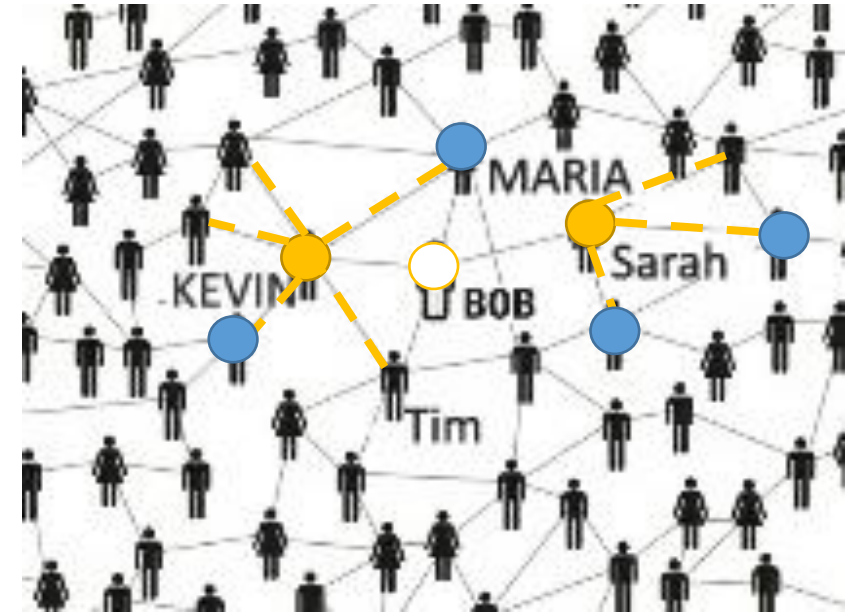


All nodes are inactive;  
Bob is activated.



Check out each neighbour of Bob:  
Generate a random number  $[0,1]$ , if smaller than 60%,  
activated; Otherwise, remains inactive.

Bob-Kevin: random number is 0.5, Kevin is activated;  
Bob-Tim: random number is 0.7, Tim is not activated;  
Bob-Sarah: random number is 0.1, Sarah is activated;  
Bob-Maria: random number is 0.9, Maria is not  
activated;



Bob, Kevin, Sarah can activate not yet activated  
nodes with the same probabilities. Node that  
e.g., the overall probability that Tim is activated  
now depends on multiple activation probabilities  
over several ties

# Extensions, e.g., for disease modelling: Susceptible-Infected- Recovered (SIR) Model

- People in a network can have three states:

**Susceptible (S)**: healthy people that can catch the virus with the contact of infected people, with certain probability  $\beta$ ;



**Infected (I)**: people who have been infected and are capable of infecting susceptible individuals. (*being infectious for  $1/\gamma$  time steps*)



**Recovered (R)**: people who have been infected and have either recovered from the disease and entered the removed compartment or died (cannot infect or be infected).

- Virus “strength”:  $\beta/\gamma$
- Recovery rate  
Not much you can do

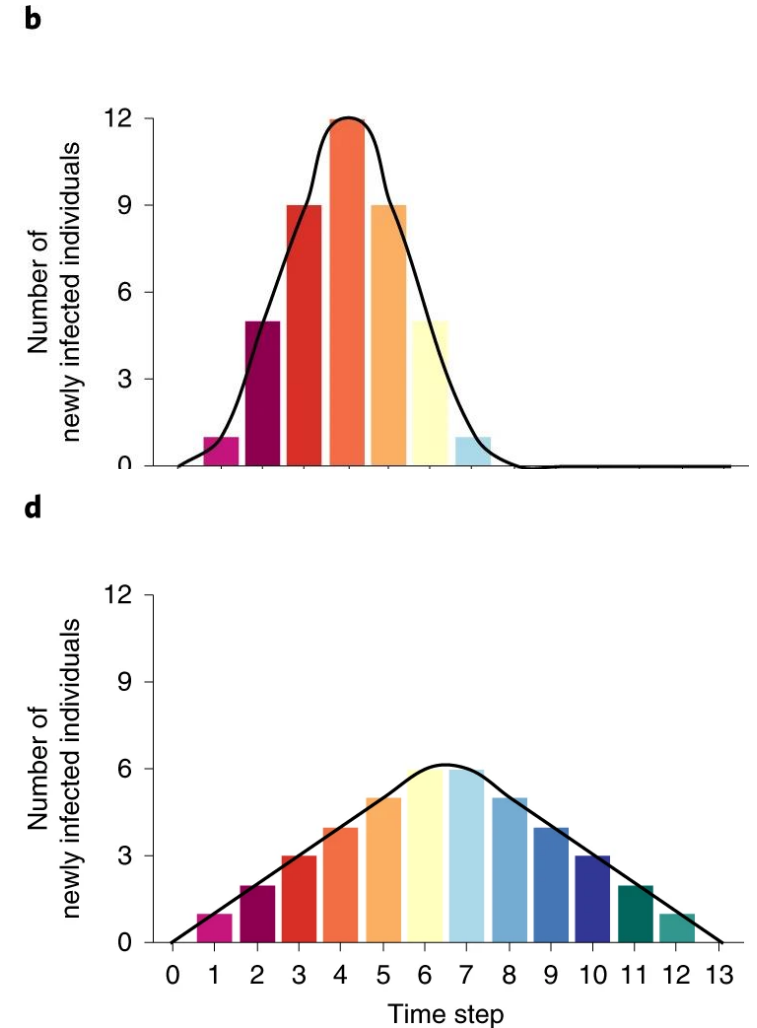
Lower transmission rate  
e.g., hand washing, mask wearing...

**Note: many epidemiological models are similar, but neglect network structure!**

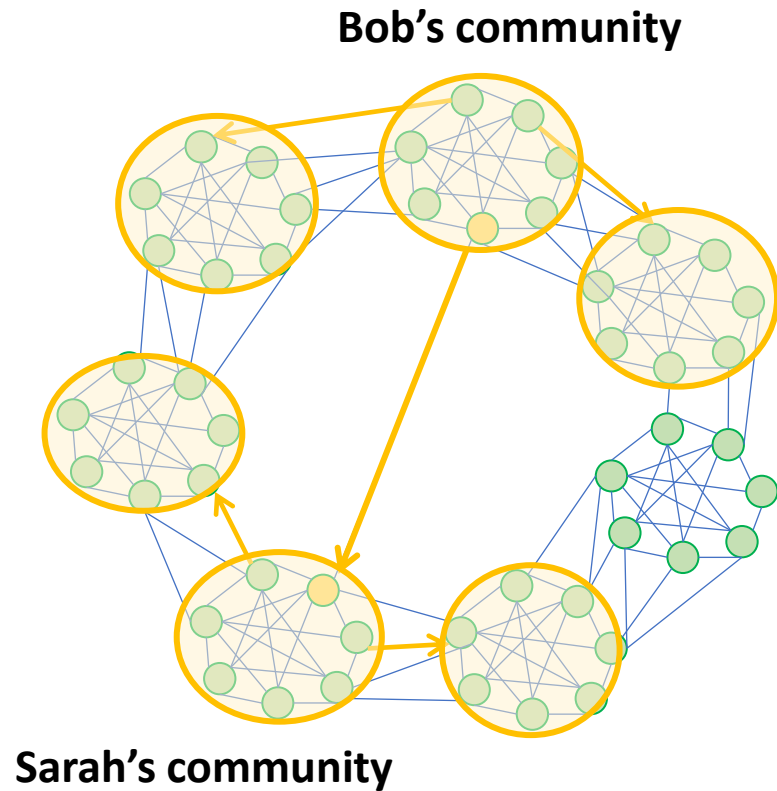
# SEIR model to simulation COVID-19 contagion in social network

A diffusion model for COVID: Susceptible (S), Expose (infected but not yet infectious) (E), Infectious (I), Recovered (R)

- Used the pre-COVID contact data to build a social network
- An infectious individual infects a healthy person when they interact, who then becomes exposed. This contagion occurs with probability  $\pi_{infection}$ .
- After a fixed number of steps  $T_{exposure}$ , an exposed individual becomes infectious.
- After becoming infectious, recovery occurs in  $T_{recovery}$  steps. Once recovered, individuals cannot be infected anymore, nor infect others.
- The process ends once there is no longer anyone exposed or infectious.



When will the strength of weak ties fail? (Are there any other mechanisms driving diffusion?)



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