

Standard Code Library

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一切的开始

宏定义

- 需要 C++11

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 using LL = long long;
4 #define FOR(i, x, y) for (decay<decltype(y)>::type i = (x), _##i = (y); i < _##i; ++i)
5 #define FORD(i, x, y) for (decay<decltype(x)>::type i = (x), _##i = (y); i > _##i; --i)
6 #ifdef DEBUG
7 #ifndef ONLINE_JUDGE
8 #define zerol
9 #endif
10 #endif
11 #ifndef zerol
12 #define dbg(x...) do { cout << "\033[32;1m" << #x << " -> "; err(x); } while (0)
13 void err() { cout << "\033[39;0m" << endl; }
14 template<template<typename...> class T, typename t, typename... A>
15 void err(T<t> a, A... x) { for (auto v: a) cout << v << ' '; err(x...); }
16 template<typename T, typename... A>
17 void err(T a, A... x) { cout << a << ' '; err(x...); }
18 #else
19 #define dbg(...)
20 #define err(...)
21 #endif
22 // -----
```

- 调试时添加编译选项 -DDEBUG, 提交时注释
- 注意检查判题系统编译选项, 修改 #ifndef ONLINE_JUDGE
- FOR ++ 循环 FOR(循环变量名称, 循环变量起始值, 循环变量结束值 (不含))
- FORD - 循环
- err() 调试时输出 (支持单层迭代)
- dbg() 变色输出变量名和变量值 (支持单层迭代)
- 黄色 33, 蓝色 34, 橙色 31

对拍

- Linux

```
1 #!/usr/bin/env bash
2 g++ -o r main.cpp -O2 -std=c++11
3 g++ -o std std.cpp -O2 -std=c++11
4 while true; do
5     python gen.py > in
6     ./std < in > stdout
7     ./r < in > out
8     if test $? -ne 0; then
9         exit 0
10    fi
11    if diff stdout out; then
12        printf "AC\n"
13    else
14        printf "GG\n"
15        exit 0
16    fi
17 done
```

- Windows

```
1 @echo off
2 setlocal enabledelayedexpansion
3
4 g++ -o r main.cpp -O2 -std=c++11
5 g++ -o std std.cpp -O2 -std=c++11
6
7 :loop
8 python gen.py > in
9 if !errorlevel! neq 0 exit /b
```

```

10
11 std.exe < in > stdout
12 if !errorlevel! neq 0 exit /b
13
14 r.exe < in > out
15 if !errorlevel! neq 0 exit /b
16
17 fc /b stdout out > nul
18 if !errorlevel! equ 0 (
19     echo AC
20 ) else (
21     echo GG
22     exit /b
23 )
24
25 goto loop

```

快速编译运行（配合无插件 VSC）

- Linux

```

1 #!/bin/bash
2 g++ $1.cpp -o $1 -O2 -std=c++14 -Wall -Dzerol -g
3 if $? -eq 0; then
4     ./$1
5 fi

```

- Windows

```

@echo off
:: 参数为文件名（不含.cpp后缀）
g++ %1.cpp -o %1 -O2 -std=c++14 -Wall -Dzerol -g
if %errorlevel% equ 0 (
    %1.exe
)

```

数据结构

ST 表

- 一维

```

1 #define M 10
2
3 struct RMQ {
4     int f[22][M];
5     inline int highbit(int x) { return 31 - __builtin_clz(x); }
6     void init(int* v, int n) {
7         FOR (i, 0, n) f[0][i] = v[i];
8         FOR (x, 1, highbit(n) + 1)
9             FOR (i, 0, n - (1 << x) + 1)
10                 f[x][i] = min(f[x - 1][i], f[x - 1][i + (1 << (x - 1))]);
11     }
12     int get_min(int l, int r) {
13         assert(l <= r);
14         int t = highbit(r - l + 1);
15         return min(f[t][l], f[t][r - (1 << t) + 1]);
16     }
17 };

```

- 二维

```

1 #define maxn 10
2 LL n, m, a[maxn][maxn];
3
4 struct RMQ2D{
5     int f[maxn][maxn][10][10];
6     inline int highbit(int x) { return 31 - __builtin_clz(x); }

```

```

7 inline int calc(int x, int y, int xx, int yy, int p, int q) {
8     return max(
9         max(f[x][y][p][q], f[xx - (1 << p) + 1][yy - (1 << q) + 1][p][q]),
10        max(f[xx - (1 << p) + 1][y][p][q], f[x][yy - (1 << q) + 1][p][q])
11    );
12 }
13 void init() {
14     FOR (x, 0, highbit(n) + 1)
15     FOR (y, 0, highbit(m) + 1)
16     FOR (i, 0, n - (1 << x) + 1)
17     FOR (j, 0, m - (1 << y) + 1) {
18         if (!x && !y) { f[i][j][x][y] = a[i][j]; continue; }
19         f[i][j][x][y] = calc(
20             i, j,
21             i + (1 << x) - 1, j + (1 << y) - 1,
22             max(x - 1, 0), max(y - 1, 0)
23         );
24     }
25 }
26 inline int get_max(int x, int y, int xx, int yy) {
27     return calc(x, y, xx, yy, highbit(xx - x + 1), highbit(yy - y + 1));
28 }
29 };

```

线段树

朴素线段树

- 默认为最大值，可自行修改 struct Q struct P P operator &
- 注意建树时的下标问题 (1-based)

```

1 const LL INF = LONG_LONG_MAX;
2 #define maxn 10
3 LL n;
4
5 namespace SGT {
6     struct Q {
7         LL setv;
8         explicit Q(LL setv = -1): setv(setv) {}
9         void operator += (const Q& q) { if (q.setv != -1) setv = q.setv; }
10    };
11    struct P {
12        LL max;
13        explicit P(LL max = -INF): max(max) {}
14        void up(Q& q) { if (q.setv != -1) max = q.setv; }
15    };
16    template<typename T>
17    P operator & (T&& a, T&& b) {
18        return P(max(a.max, b.max));
19    }
20    P p[maxn << 2];
21    Q q[maxn << 2];
22    #define lson o * 2, l, (l + r) / 2
23    #define rson o * 2 + 1, (l + r) / 2 + 1, r
24    void up(int o, int l, int r) {
25        if (l == r) p[o] = P();
26        else p[o] = p[o * 2] & p[o * 2 + 1];
27        p[o].up(q[o]);
28    }
29    void down(int o, int l, int r) {
30        q[o * 2] += q[o]; q[o * 2 + 1] += q[o];
31        q[o] = Q();
32        up(lson); up(rson);
33    }
34    template<typename T>
35    void build(T&& f, int o = 1, int l = 1, int r = n) {
36        if (l == r) q[o] = f(l);
37        else { build(f, lson); build(f, rson); q[o] = Q(); }
38        up(o, l, r);
39    }
40    P query(int ql, int qr, int o = 1, int l = 1, int r = n) {

```

```

41     if (ql > r || l > qr) return P();
42     if (ql <= l && r <= qr) return p[o];
43     down(o, l, r);
44     return query(ql, qr, lson) & query(ql, qr, rson);
45 }
46 void update(int ql, int qr, const Q& v, int o = 1, int l = 1, int r = n) {
47     if (ql > r || l > qr) return;
48     if (ql <= l && r <= qr) q[o] += v;
49     else {
50         down(o, l, r);
51         update(ql, qr, v, lson); update(ql, qr, v, rson);
52     }
53     up(o, l, r);
54 }
55 }
56
57 // -----
58 void solve(){
59     vector<LL> arr = {1, 5, 7, 4, 2, 8, 3, 6, 10, 9};
60     n = arr.size();
61     SGT::build([&](int idx){
62         return SGT::Q(arr[idx-1]);
63     });
64     for(LL i=1; i<=n; i++){
65         dbg(SGT::query(1, i).max);
66     }
67     SGT::update(2, 4, SGT::Q(-3));
68     cout << "MODIFIED\n";
69     for(LL i=1; i<=n; i++){
70         dbg(SGT::query(1, i).max);
71     }
72 }

```

- 区间修改，区间累加，查询区间和、最大值、最小值。

```

1  #define maxn 100005
2  #define INF LONG_LONG_MAX
3  LL a[maxn];
4
5  struct IntervalTree {
6      #define ls o * 2, l, m
7      #define rs o * 2 + 1, m + 1, r
8      static const LL M = maxn * 4, RS = 1E18 - 1;
9      LL addv[M], setv[M], minv[M], maxv[M], sumv[M];
10     int n;
11     void init() {
12         memset(addv, 0, sizeof addv);
13         fill(setv, setv + M, RS);
14         memset(minv, 0, sizeof minv);
15         memset(maxv, 0, sizeof maxv);
16         memset(sumv, 0, sizeof sumv);
17     }
18     void maintain(LL o, LL l, LL r) {
19         if (l < r) {
20             LL lc = o * 2, rc = o * 2 + 1;
21             sumv[o] = sumv[lc] + sumv[rc];
22             minv[o] = min(minv[lc], minv[rc]);
23             maxv[o] = max(maxv[lc], maxv[rc]);
24         } else sumv[o] = minv[o] = maxv[o] = 0;
25         if (setv[o] != RS) { minv[o] = maxv[o] = setv[o]; sumv[o] = setv[o] * (r - l + 1); }
26         if (addv[o]) { minv[o] += addv[o]; maxv[o] += addv[o]; sumv[o] += addv[o] * (r - l + 1); }
27     }
28     void build(LL o, LL l, LL r) {
29         if (l == r) addv[o] = a[l];
30         else {
31             LL m = (l + r) / 2;
32             build(ls); build(rs);
33         }
34         maintain(o, l, r);
35     }
36     void pushdown(LL o) {
37         LL lc = o * 2, rc = o * 2 + 1;

```



```

38     if (setv[o] != RS) {
39         setv[lc] = setv[rc] = setv[o];
40         addv[lc] = addv[rc] = 0;
41         setv[o] = RS;
42     }
43     if (addv[o]) {
44         addv[lc] += addv[o]; addv[rc] += addv[o];
45         addv[o] = 0;
46     }
47 }
48 void update(LL p, LL q, LL o, LL l, LL r, LL v, LL op) {
49     if (p <= r && l <= q) {
50         if (p <= l && r <= q) {
51             if (op == 2) { setv[o] = v; addv[o] = 0; }
52             else addv[o] += v;
53         } else {
54             pushdown(o);
55             LL m = (l + r) / 2;
56             update(p, q, ls, v, op); update(p, q, rs, v, op);
57         }
58     }
59     maintain(o, l, r);
60 }
61 void query(LL p, LL q, LL o, LL l, LL r, LL add, LL& ssum, LL& smin, LL& smax) {
62     if (p > r || l > q) return;
63     if (setv[o] != RS) {
64         LL v = setv[o] + add + addv[o];
65         ssum += v * (min(r, q) - max(l, p) + 1);
66         smin = min(smin, v);
67         smax = max(smax, v);
68     } else if (p <= l && r <= q) {
69         ssum += sumv[o] + add * (r - l + 1);
70         smin = min(smin, minv[o] + add);
71         smax = max(smax, maxv[o] + add);
72     } else {
73         LL m = (l + r) / 2;
74         query(p, q, ls, add + addv[o], ssum, smin, smax);
75         query(p, q, rs, add + addv[o], ssum, smin, smax);
76     }
77 }
78 // 简化接口
79 void build(int _n) {
80     n = _n;
81     build(1, 1, n);
82 }
83
84 void range_add(int l, int r, int val) {
85     update(l, r, 1, 1, n, val, 1);
86 }
87
88 void range_set(int l, int r, int val) {
89     update(l, r, 1, 1, n, val, 2);
90 }
91
92 void range_query(int l, int r, LL& sum, LL& min_val, LL& max_val) {
93     sum = 0;
94     min_val = INF;
95     max_val = -INF;
96     query(l, r, 1, 1, n, 0, sum, min_val, max_val);
97 }
98 } IT;
99 // -----
100 void solve(){
101     IT.init();
102
103     LL n = 5;
104     vector<int> data = {1, 3, 5, 7, 9};
105     for (int i = 0; i < n; i++) {
106         a[i + 1] = data[i]; // 注意: 线段树从 1 开始索引
107     }
108 }

```

```

109     IT.build(n);
110
111     LL sum, min_val, max_val;
112     IT.range_query(1, 5, sum, min_val, max_val);
113     cout << " " << sum << " " << min_val << " " << max_val << endl;
114
115     IT.range_add(2, 4, 2);
116     IT.range_query(1, 5, sum, min_val, max_val);
117     cout << " " << sum << " " << min_val << " " << max_val << endl;
118
119     IT.range_set(3, 5, 10);
120     IT.range_query(1, 5, sum, min_val, max_val);
121     cout << " " << sum << " " << min_val << " " << max_val << endl;
122
123     IT.range_query(2, 4, sum, min_val, max_val);
124     cout << " " << sum << " " << min_val << " " << max_val << endl;
125 }

```

动态开点

```

1  namespace SGT{
2      const LL N = 3e5 + 10, INF = LONG_LONG_MAX;
3      LL sum[N << 2], lazy[N << 2];
4      // LL minn[N << 2], lazy2[N << 2];
5      LL lson[N << 2], rson[N << 2], tot = 0, root = 0;
6
7      inline void push_up(LL rt){
8          sum[rt] = sum[lson[rt]] + sum[rson[rt]];
9      // minn[rt] = min(minn[lson[rt]], minn[rson[rt]]);
10     }
11     inline void push_down(LL rt, LL m){
12         if(!lazy[rt]) return;
13         if(!lson[rt]){
14             lson[rt] = ++tot;
15             // minn[lson[rt]] = INF;
16         }
17         if(!rson[rt]){
18             rson[rt] = ++tot;
19             // minn[rson[rt]] = INF;
20         }
21         lazy[lson[rt]] += lazy[rt], lazy[rson[rt]] += lazy[rt];
22         sum[lson[rt]] += lazy[rt] * (m - (m >> 1));
23         sum[rson[rt]] += lazy[rt] * (m >> 1);
24         lazy[rt] = 0;
25
26         // lazy2[lson[rt]] = min(lazy2[lson[rt]], lazy2[rt]);
27         // lazy2[rson[rt]] = min(lazy2[rson[rt]], lazy2[rt]);
28         // minn[lson[rt]] = min(minn[lson[rt]], lazy2[rt]);
29         // minn[rson[rt]] = min(minn[rson[rt]], lazy2[rt]);
30         // lazy2[rt] = INF;
31     }
32
33     static void add_range(LL &rt, LL l, LL r, LL L, LL R, LL val){
34         if(!rt){
35             rt = ++tot;
36             // minn[rt] = INF;
37             // lazy2[rt] = INF;
38         }
39         if(l >= L && r <= R){
40             lazy[rt] += val;
41             sum[rt] += val * (r - l + 1);
42             // minn[rt] = min(minn[rt], val);
43             // lazy2[rt] = min(lazy2[rt], val);
44             return;
45         }
46         push_down(rt, r - l + 1);
47         LL mid = l + r >> 1;
48         if(mid >= L) add_range(lson[rt], l, mid, L, R, val);
49         if(mid < R) add_range(rson[rt], mid + 1, r, L, R, val);
50         push_up(rt);
51     }

```

```

52
53 static void add_point(LL &rt, LL l, LL r, LL pos, LL val){
54     if(!rt){
55         rt = ++tot;
56         minn[rt] = INF;
57     }
58     if(l == r){
59         sum[rt] += val;
60         minn[rt] = min(minn[rt], val);
61         return;
62     }
63     LL mid = l + r >> 1;
64     if(mid >= pos) add_point(lson[rt], l, mid, pos, val);
65     else add_point(rson[rt], mid + 1, r, pos, val);
66     push_up(rt);
67 }
68
69 static LL query(LL rt, LL l, LL r, LL L, LL R){
70     if(!rt) return 0;
71     if(!rt) return INF;
72     if(l >= L && r <= R) return sum[rt];
73     if(l >= L && r <= R) return minn[rt];
74     push_down(rt, r - l + 1);
75     LL mid = l + r >> 1;
76     LL ans = 0;
77     LL ans = INF;
78     if(mid >= L) ans += query(lson[rt], l, mid, L, R);
79     if(mid < R) ans += query(rson[rt], mid + 1, r, L, R);
80     if(mid >= L) ans = min(ans, query(lson[rt], l, mid, L, R));
81     if(mid < R) ans = min(ans, query(rson[rt], mid + 1, r, L, R));
82     return ans;
83 }
84 };
85 // ----- Template End -----
86 // !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!! Tester Start !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
87
88 void solve(){
89     LL rt = 0, l = -1e9, r = 1e9; // 根（必需为 0）和值域（操作范围）
90     SGT::root = rt;
91     SGT::add_point(rt, l, r, 1e4, 1e4);
92     SGT::add_range(rt, l, r, 100000, 100010, 100);
93     cout << SGT::query(rt, l, r, -1e9, 0) << '\n';
94     cout << SGT::query(rt, l, r, 0, 1e5) << '\n';
95     cout << SGT::query(rt, l, r, 1e5, 1e6) << '\n';
96     cout << SGT::query(rt, l, r, 1e6, 1e10) << '\n';
97 }

```

树状数组

- 单点修改，区间查询
- 频次统计下的 k 小值
- 维护差分数组时的区间修改，单点查询

```

1  #define M 100005
2
3  namespace BIT {
4      LL c[M]; // 注意初始化开销
5      inline int lowbit(int x) { return x & -x; }
6      void add(int x, LL v) { // 单点加
7          for (int i = x; i < M; i += lowbit(i))
8              c[i] += v;
9      }
10     LL sum(int x) { // 前缀和
11         LL ret = 0;
12         for (int i = x; i > 0; i -= lowbit(i))
13             ret += c[i];
14         return ret;
15     }
16     int kth(LL k) { // 频次统计下从小到大第 k 个，详见应用
17         int p = 0;

```

```

18     for (int lim = 1 << 20; lim; lim /= 2)
19         if (p + lim < M && c[p + lim] < k) {
20             p += lim;
21             k -= c[p];
22         }
23     return p + 1;
24 }
25 LL sum(int l, int r) { return sum(r) - sum(l - 1); } // 区间和
26 // 区间加 (此时树状数组为差分数组, sum(x) 为第 x 个数的值)
27 void add(int l, int r, LL v) { add(l, v); add(r + 1, -v); }
28 }
29 // -----
30 void solve(){
31     vector<LL> a={9, 9, 9, 9, 5, 3, 3, 3, 1, 1};
32     LL n = a.size(), i;
33     for(i=1; i<=n; i++) BIT::add(a[i-1], 1);
34     // 1 1 3 3 3 5 9 9 9 9
35     for(i=1; i<=n; i++) cout << BIT::kth(i) << ' ';
36 }

```

● 区间修改、区间查询

```

1  #define maxn 100005
2
3  namespace BIT {
4      int n;
5      int c[maxn], cc[maxn];
6      inline int lowbit(int x) { return x & -x; }
7      void init(int siz){ // 初始化
8          n = siz;
9          for(LL i=0; i<=n; i++){
10             c[i] = cc[i] = 0;
11         }
12     }
13     void add(int x, int v) { // 不要用这个
14         for (int i = x; i <= n; i += lowbit(i)) {
15             c[i] += v; cc[i] += x * v;
16         }
17     }
18     void add(int l, int r, int v) { add(l, v); add(r + 1, -v); } // 区间修改
19     int sum(int x) { // 前缀和
20         int ret = 0;
21         for (int i = x; i > 0; i -= lowbit(i))
22             ret += (x + 1) * c[i] - cc[i];
23         return ret;
24     }
25     int sum(int l, int r) { return sum(r) - sum(l - 1); } // 区间和
26 }
27 // -----
28 void solve(){
29     LL i, n=8;
30     BIT::init(n);
31     BIT::add(2, 4, 2);
32     for(i=1; i<=n; i++) cout << BIT::sum(i, i) << ' ';
33     cout << '\n';
34     cout << BIT::sum(5) << '\n';
35     cout << BIT::sum(2, 3) << '\n';
36 }

```

● 三维

```

1  #define maxn 105
2
3  namespace BIT{
4      int n;
5      LL c[maxn][maxn][maxn];
6      inline int lowbit(int x) { return x & -x; }
7      void init(int siz){
8          n = siz;
9          for(int i=0; i<=n; i++){
10             for(int j=0; j<=n; j++){
11                 for(int k=0; k<=n; k++){

```

```

12         c[i][j][k] = 0;
13     }
14 }
15 }
16 }
17 void update(int x, int y, int z, int d) {
18     for (int i = x; i <= n; i += lowbit(i))
19         for (int j = y; j <= n; j += lowbit(j))
20             for (int k = z; k <= n; k += lowbit(k))
21                 c[i][j][k] += d;
22 }
23 LL query(int x, int y, int z) {
24     LL ret = 0;
25     for (int i = x; i > 0; i -= lowbit(i))
26         for (int j = y; j > 0; j -= lowbit(j))
27             for (int k = z; k > 0; k -= lowbit(k))
28                 ret += c[i][j][k];
29     return ret;
30 }
31 LL solve(int x, int y, int z, int xx, int yy, int zz) {
32     return query(xx, yy, zz)
33         - query(xx, yy, z - 1)
34         - query(xx, y - 1, zz)
35         - query(x - 1, yy, zz)
36         + query(xx, y - 1, z - 1)
37         + query(x - 1, yy, z - 1)
38         + query(x - 1, y - 1, zz)
39         - query(x - 1, y - 1, z - 1);
40 }
41 }

```

数学

快速乘

```

1 LL mul(LL a, LL b, LL m) {
2     LL ret = 0;
3     while (b) {
4         if (b & 1) {
5             ret += a;
6             if (ret >= m) ret -= m;
7         }
8         a += a;
9         if (a >= m) a -= m;
10        b >>= 1;
11    }
12    return ret;
13 }

```

• $O(1)$

```

1 LL mul(LL u, LL v, LL p) {
2     return (u * v - LL((long double) u * v / p) * p + p) % p;
3 }
4 LL mul(LL u, LL v, LL p) { // 卡常
5     LL t = u * v - LL((long double) u * v / p) * p;
6     return t < 0 ? t + p : t;
7 }

```

高斯消元

- n 是方程个数, m 是未知量个数, $a[n][m+1]$ 是增广矩阵
- $x[m]$ 是每个未知量的解 (如果有), $free_x[m]$ 是每个未知量是否为自由变量。

```

1 typedef double LD;
2 const LD eps = 1E-10;
3 const int maxn = 2000 + 10;
4
5 int n, m;

```

```

6 LD a[maxn][maxn], x[maxn];
7 bool free_x[maxn];
8
9 inline int sgn(LD x) { return (x > eps) - (x < -eps); }
10
11 int gauss(LD a[maxn][maxn], int n, int m) {
12 //int gauss() {
13     memset(free_x, 1, sizeof free_x); memset(x, 0, sizeof x);
14     int r = 0, c = 0;
15     while (r < n && c < m) {
16         int m_r = r;
17         FOR (i, r + 1, n)
18             if (fabs(a[i][c]) > fabs(a[m_r][c])) m_r = i;
19         if (m_r != r)
20             FOR (j, c, m + 1)
21                 swap(a[r][j], a[m_r][j]);
22         if (!sgn(a[r][c])) {
23             a[r][c] = 0;
24             ++c;
25             continue;
26         }
27         FOR (i, r + 1, n)
28             if (a[i][c]) {
29                 LD t = a[i][c] / a[r][c];
30                 FOR (j, c, m + 1) a[i][j] -= a[r][j] * t;
31             }
32         ++r; ++c;
33     }
34     FOR (i, r, n)
35         if (sgn(a[i][m])) return -1;
36     if (r < m) {
37         FORD (i, r - 1, -1) {
38             int f_cnt = 0, k = -1;
39             FOR (j, 0, m)
40                 if (sgn(a[i][j]) && free_x[j]) {
41                     ++f_cnt;
42                     k = j;
43                 }
44             if (f_cnt > 0) continue;
45             LD s = a[i][m];
46             FOR (j, 0, m)
47                 if (j != k) s -= a[i][j] * x[j];
48             x[k] = s / a[i][k];
49             free_x[k] = 0;
50         }
51         return m - r;
52     }
53     FORD (i, m - 1, -1) {
54         LD s = a[i][m];
55         FOR (j, i + 1, m)
56             s -= a[i][j] * x[j];
57         x[i] = s / a[i][i];
58     }
59     return 0;
60 }

```

快速幂

- 如果模数是素数，则可在函数体内加上 $n \% = \text{MOD} - 1$ ；（费马小定理）。

```

1 LL bin(LL x, LL n, LL MOD) {
2     LL ret = MOD != 1;
3     for (x %= MOD; n; n >>= 1, x = x * x % MOD)
4         if (n & 1) ret = ret * x % MOD;
5     return ret;
6 }

```

- 防爆 LL
- 前置模板：快速乘

```

1 LL bin(LL x, LL n, LL MOD) {

```

```

2     LL ret = MOD != 1;
3     for (x %= MOD; n; n >>= 1, x = mul(x, x, MOD))
4         if (n & 1) ret = mul(ret, x, MOD);
5     return ret;
6 }

```

高精度

- https://github.com/Baobaobear/MiniBigInteger/blob/main/bigint_tiny.h, 带有压位优化
- 按需实现

```

1  #include <algorithm>
2  #include <cstdio>
3  #include <string>
4  #include <vector>
5
6  struct BigIntTiny {
7      int sign;
8      std::vector<int> v;
9
10     BigIntTiny() : sign(1) {}
11     BigIntTiny(const std::string &s) { *this = s; }
12     BigIntTiny(int v) {
13         char buf[21];
14         sprintf(buf, "%d", v);
15         *this = buf;
16     }
17     void zip(int unzip) {
18         if (unzip == 0) {
19             for (int i = 0; i < (int)v.size(); i++)
20                 v[i] = get_pos(i * 4) + get_pos(i * 4 + 1) * 10 + get_pos(i * 4 + 2) * 100 + get_pos(i * 4 + 3) * 1000;
21         } else
22             for (int i = (v.resize(v.size() * 4), (int)v.size() - 1), a; i >= 0; i--)
23                 a = (i % 4 >= 2) ? v[i / 4] / 100 : v[i / 4] % 100, v[i] = (i & 1) ? a / 10 : a % 10;
24         setsign(1, 1);
25     }
26     int get_pos(unsigned pos) const { return pos >= v.size() ? 0 : v[pos]; }
27     BigIntTiny &setsign(int newsign, int rev) {
28         for (int i = (int)v.size() - 1; i > 0 && v[i] == 0; i--)
29             v.erase(v.begin() + i);
30         sign = (v.size() == 0 || (v.size() == 1 && v[0] == 0)) ? 1 : (rev ? newsign * sign : newsign);
31         return *this;
32     }
33     std::string to_str() const {
34         BigIntTiny b = *this;
35         std::string s;
36         for (int i = (b.zip(1), 0); i < (int)b.v.size(); ++i)
37             s += char(*(b.v.rbegin() + i) + '0');
38         return (sign < 0 ? "-" : "") + (s.empty() ? std::string("0") : s);
39     }
40     bool absless(const BigIntTiny &b) const {
41         if (v.size() != b.v.size()) return v.size() < b.v.size();
42         for (int i = (int)v.size() - 1; i >= 0; i--)
43             if (v[i] != b.v[i]) return v[i] < b.v[i];
44         return false;
45     }
46     BigIntTiny operator-() const {
47         BigIntTiny c = *this;
48         c.sign = (v.size() > 1 || v[0]) ? -c.sign : 1;
49         return c;
50     }
51     BigIntTiny &operator=(const std::string &s) {
52         if (s[0] == '-')
53             *this = s.substr(1);
54         else {
55             for (int i = (v.clear(), 0); i < (int)s.size(); ++i)
56                 v.push_back(*(s.rbegin() + i) - '0');
57             zip(0);
58         }
59         return setsign(s[0] == '-' ? -1 : 1, sign = 1);
60     }

```

```

60     }
61     bool operator<(const BigIntTiny &b) const {
62         return sign != b.sign ? sign < b.sign : (sign == 1 ? absless(b) : b.absless(*this));
63     }
64     bool operator==(const BigIntTiny &b) const { return v == b.v && sign == b.sign; }
65     BigIntTiny &operator+=(const BigIntTiny &b) {
66         if (sign != b.sign) return *this = (*this) - -b;
67         v.resize(std::max(v.size(), b.v.size()) + 1);
68         for (int i = 0, carry = 0; i < (int)b.v.size() || carry; i++) {
69             carry += v[i] + b.get_pos(i);
70             v[i] = carry % 10000, carry /= 10000;
71         }
72         return setsign(sign, 0);
73     }
74     BigIntTiny operator+(const BigIntTiny &b) const {
75         BigIntTiny c = *this;
76         return c += b;
77     }
78     void add_mul(const BigIntTiny &b, int mul) {
79         v.resize(std::max(v.size(), b.v.size()) + 2);
80         for (int i = 0, carry = 0; i < (int)b.v.size() || carry; i++) {
81             carry += v[i] + b.get_pos(i) * mul;
82             v[i] = carry % 10000, carry /= 10000;
83         }
84     }
85     BigIntTiny operator-(const BigIntTiny &b) const {
86         if (b.v.empty() || b.v.size() == 1 && b.v[0] == 0) return *this;
87         if (sign != b.sign) return (*this) + -b;
88         if (absless(b)) return -(b - *this);
89         BigIntTiny c;
90         for (int i = 0, borrow = 0; i < (int)v.size(); i++) {
91             borrow += v[i] - b.get_pos(i);
92             c.v.push_back(borrow);
93             c.v.back() -= 10000 * (borrow >= 31);
94         }
95         return c.setsign(sign, 0);
96     }
97     BigIntTiny operator*(const BigIntTiny &b) const {
98         if (b < *this) return b * *this;
99         BigIntTiny c, d = b;
100         for (int i = 0; i < (int)v.size(); i++, d.v.insert(d.v.begin(), 0))
101             c.add_mul(d, v[i]);
102         return c.setsign(sign * b.sign, 0);
103     }
104     BigIntTiny operator/(const BigIntTiny &b) const {
105         BigIntTiny c, d;
106         BigIntTiny e=b;
107         e.sign=1;
108
109         d.v.resize(v.size());
110         double db = 1.0 / (b.v.back() + (b.get_pos((unsigned)b.v.size() - 2) / 1e4) +
111             (b.get_pos((unsigned)b.v.size() - 3) + 1) / 1e8);
112         for (int i = (int)v.size() - 1; i >= 0; i--) {
113             c.v.insert(c.v.begin(), v[i]);
114             int m = (int)((c.get_pos((int)e.v.size()) * 10000 + c.get_pos((int)e.v.size() - 1)) * db);
115             c = c - e * m, c.setsign(c.sign, 0), d.v[i] += m;
116             while (!(c < e))
117                 c = c - e, d.v[i] += 1;
118         }
119         return d.setsign(sign * b.sign, 0);
120     }
121     BigIntTiny operator%(const BigIntTiny &b) const { return *this - *this / b * b; }
122     bool operator>(const BigIntTiny &b) const { return b < *this; }
123     bool operator<=(const BigIntTiny &b) const { return !(b < *this); }
124     bool operator>=(const BigIntTiny &b) const { return !(*this < b); }
125     bool operator!=(const BigIntTiny &b) const { return !(*this == b); }
126 };

```


中位数

```
1 struct Median{
2     const LL inf = LONG_LONG_MAX;
3     multiset<LL> smaller,bigger;
4     LL mid = -inf;
5     int sz = 2;
6
7     void init(){
8         smaller.clear();bigger.clear();
9         mid = -inf;
10        smaller.insert(-inf);
11        bigger.insert(inf);
12        sz = 2;
13    }
14
15    LL query(){
16        return mid;
17    }
18
19    LL query_size(){
20        return sz - 2;
21    }
22
23    void add(LL val){
24        if(sz & 1){ // odd add
25            if(val >= mid){
26                smaller.insert(mid);
27                bigger.insert(val);
28            }else{
29                smaller.insert(val);
30                bigger.insert(mid);
31            }
32            mid = *smaller.rbegin();
33        }
34        else{ // add even
35            if(val >= *smaller.rbegin() && val <= *bigger.begin()){
36                mid = val;
37            }else if(val < *smaller.rbegin()){
38                smaller.insert(val);
39                mid = *smaller.rbegin();
40                smaller.erase(smaller.find(mid));
41            }else{
42                bigger.insert(val);
43                mid = *bigger.begin();
44                bigger.erase(bigger.find(mid));
45            }
46        }
47        sz++;
48    }
49
50    void erase(LL val){
51        if(sz & 1){
52            if(val == mid){
53                mid = *smaller.rbegin();
54                sz --;
55                return;
56            }
57            if(val <= *smaller.rbegin())
58            {
59                smaller.erase(smaller.find(val) );
60                smaller.insert(mid);
61                mid = *smaller.rbegin();
62            }else if(val >= *bigger.begin()){
63                bigger.erase(bigger.find(val) );
64                bigger.insert(mid);
65                mid = *smaller.rbegin();
66            }
67        }else{ //erase even
68            if(val <= *smaller.rbegin()){
69                smaller.erase(smaller.find(val) );
```

```

70         mid = *bigger.begin();
71         bigger.erase(bigger.find(mid));
72     }else if(val >= *bigger.begin()){
73         bigger.erase(bigger.find(val));
74         mid = *smaller.rbegin();
75         smaller.erase(smaller.find(mid));
76     }
77 }
78 sz--;
79 }
80 };

```

矩阵运算

```

1  #define MOD 998244353
2  #define M 10
3
4  struct Mat {
5      LL m;
6      LL v[M][M];
7      Mat(int siz=2) {
8          m = siz;
9          for(int i=0; i<=m; i++){
10             for(int j=0; j<=m; j++){
11                 v[i][j] = 0;
12             }
13         }
14     }
15     void eye() { FOR (i, 0, m) v[i][i] = 1; }
16     LL* operator [] (LL x) { return v[x]; }
17     const LL* operator [] (LL x) const { return v[x]; }
18     Mat operator * (const Mat& B) {
19         const Mat& A = *this;
20         Mat ret;
21         FOR (k, 0, m)
22             FOR (i, 0, m) if (A[i][k])
23                 FOR (j, 0, m)
24                     ret[i][j] = (ret[i][j] + A[i][k] * B[k][j]) % MOD;
25         return ret;
26     }
27     Mat pow(LL n) const {
28         Mat A = *this, ret; ret.eye();
29         for (; n >= 1, A = A * A)
30             if (n & 1) ret = ret * A;
31         return ret;
32     }
33     Mat operator + (const Mat& B) {
34         const Mat& A = *this;
35         Mat ret;
36         FOR (i, 0, m)
37             FOR (j, 0, m)
38                 ret[i][j] = (A[i][j] + B[i][j]) % MOD;
39         return ret;
40     }
41     void pprint() const {
42         FOR (i, 0, m)
43             FOR (j, 0, m)
44                 printf("%lld%c", (*this)[i][j], j == m - 1 ? '\n' : ' ');
45     }
46 };
47 // -----
48 void solve(){
49     Mat mat1, mat2;
50     mat1.eye();
51     mat1[1][0] = 2; // 0-based
52     mat2.eye();
53     mat2[1][1] = 4;
54     Mat mat3 = mat1 * mat2;
55     mat3.pprint();
56 }

```

数论分块

$f(i) = \lfloor \frac{n}{i} \rfloor = v$ 时 i 的取值范围是 $[l, r]$ 。

```
1 void sqrt_decomposition(LL n){
2     for (LL l = 1, v, r; l <= n; l = r + 1) {
3         v = n / l; r = n / v;
4         printf("%lld / [%lld, %lld] = %lld\n", n, l, r, v);
5     }
6 }
```

质数筛

- $\mathcal{O}(n)$

```
1 const LL p_max = 1E6 + 100;
2 LL pr[p_max], p_sz;
3 void get_prime() {
4     static bool vis[p_max];
5     FOR (i, 2, p_max) {
6         if (!vis[i]) pr[p_sz++] = i;
7         FOR (j, 0, p_sz) {
8             if (pr[j] * i >= p_max) break;
9             vis[pr[j] * i] = 1;
10            if (i % pr[j] == 0) break;
11        }
12    }
13 }
```

欧拉函数

朴素

```
1 int phi(int x)
2 {
3     int res = x;
4     for (int i = 2; i <= x / i; i++)
5         if (x % i == 0)
6         {
7             res = res / i * (i - 1);
8             while (x % i == 0) x /= i;
9         }
10    if (x > 1) res = res / x * (x - 1);
11
12    return res;
13 }
```

筛法求欧拉函数

- 前置模板：质数筛

```
1 const LL p_max = 1E5 + 100;
2 LL phi[p_max];
3 void get_phi() {
4     phi[1] = 1;
5     static bool vis[p_max];
6     static LL prime[p_max], p_sz, d;
7     FOR (i, 2, p_max) {
8         if (!vis[i]) {
9             prime[p_sz++] = i;
10            phi[i] = i - 1;
11        }
12        for (LL j = 0; j < p_sz && (d = i * prime[j]) < p_max; ++j) {
13            vis[d] = 1;
14            if (i % prime[j] == 0) {
15                phi[d] = phi[i] * prime[j];
16                break;
17            }
18            else phi[d] = phi[i] * (prime[j] - 1);
19        }
20    }
```

```

20     }
21 }

```

素性测试

试除法

- $\mathcal{O}(\sqrt{n})$

```

1 bool is_prime(int x)
2 {
3     if (x < 2) return false;
4     for (int i = 2; i <= x / i; i++)
5         if (x % i == 0)
6             return false;
7     return true;
8 }

```

Miller-Rabin

- 前置：快速幂
- $\mathcal{O}(k \times \log^3 n)$

```

1 bool miller_rabin(LL n) {
2     static vector<LL> tester = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
3     if (n < 3 || n % 2 == 0) return n == 2;
4     if (n % 3 == 0) return n == 3;
5     LL u = n - 1, t = 0;
6     while (u % 2 == 0) u /= 2, ++t;
7     for (auto nt: tester) {
8         if (nt >= n) continue;
9         LL v = bin(nt, u, n);
10        if (v == 1) continue;
11        LL s;
12        for (s = 0; s < t; ++s) {
13            if (v == n - 1) break;
14            v = v * v % n;
15        }
16        if (s == t) return false;
17    }
18    return true;
19 }

```

质因数分解

朴素质因数分解

- 前置模板：素数筛
- 带指数
- $\mathcal{O}(\frac{\sqrt{N}}{\ln N})$

```

1 LL factor[30], f_sz, factor_exp[30];
2 void get_factor(LL x) {
3     f_sz = 0;
4     LL t = sqrt(x + 0.5);
5     for (LL i = 0; pr[i] <= t; ++i)
6         if (x % pr[i] == 0) {
7             factor_exp[f_sz] = 0;
8             while (x % pr[i] == 0) {
9                 x /= pr[i];
10                ++factor_exp[f_sz];
11            }
12            factor[f_sz++] = pr[i];
13        }
14    if (x > 1) {
15        factor_exp[f_sz] = 1;
16        factor[f_sz++] = x;
17    }
18 }

```

- 不带指数

```

1 LL factor[30], f_sz;
2 void get_factor(LL x) {
3     f_sz = 0;
4     LL t = sqrt(x + 0.5);
5     for (LL i = 0; pr[i] <= t; ++i)
6         if (x % pr[i] == 0) {
7             factor[f_sz++] = pr[i];
8             while (x % pr[i] == 0) x /= pr[i];
9         }
10    if (x > 1) factor[f_sz++] = x;
11 }

```

Pollard-Rho

- 前置：素数测试

```

1 mt19937 mt(time(0));
2 LL pollard_rho(LL n, LL c) {
3     LL x = uniform_int_distribution<LL>(1, n - 1)(mt), y = x;
4     auto f = [&](LL v) { LL t = mul(v, v, n) + c; return t < n ? t : t - n; };
5     while (1) {
6         x = f(x); y = f(f(y));
7         if (x == y) return n;
8         LL d = gcd(abs(x - y), n);
9         if (d != 1) return d;
10    }
11 }
12
13 LL fac[100], fcnt;
14 void get_fac(LL n, LL cc = 19260817) {
15     if (n == 4) { fac[fcnt++] = 2; fac[fcnt++] = 2; return; }
16     if (miller_rabin(n)) { fac[fcnt++] = n; return; }
17     LL p = n;
18     while (p == n) p = pollard_rho(n, --cc);
19     get_fac(p); get_fac(n / p);
20 }
21
22 void go_fac(LL n) { fcnt = 0; if (n > 1) get_fac(n); }

```

原根

- 前置模板：质因数分解、快速幂
- 要求 p 为质数
- 别忘了调用质因数分解的函数

```

1 LL find_smallest_primitive_root(LL p) {
2     get_factor(p - 1);
3     FOR (i, 2, p) {
4         bool flag = true;
5         FOR (j, 0, f_sz)
6             if (bin(i, (p - 1) / factor[j], p) == 1) {
7                 flag = false;
8                 break;
9             }
10        if (flag) return i;
11    }
12    // assert(0);
13    return -1;
14 }

```

欧几里得

- 朴素

```

1 int gcd(int a, int b)
2 {
3     return b ? gcd(b, a % b) : a;
4 }

```

- 卡常

```

1 inline int ctz(LL x) { return __builtin_ctzll(x); }
2 LL gcd(LL a, LL b) {
3     if (!a) return b; if (!b) return a;
4     int t = ctz(a | b);
5     a >>= ctz(a);
6     do {
7         b >>= ctz(b);
8         if (a > b) swap(a, b);
9         b -= a;
10    } while (b);
11    return a << t;
12 }

```

扩展欧几里得

- 求 $ax + by = \gcd(a, b)$ 的一组解
- 如果 a 和 b 互素, 那么 x 是 a 在模 b 下的逆元
- 注意 x 和 y 可能是负数

```

1 LL ex_gcd(LL a, LL b, LL &x, LL &y) {
2     if (b == 0) { x = 1; y = 0; return a; }
3     LL ret = ex_gcd(b, a % b, y, x);
4     y -= a / b * x;
5     return ret;
6 }

```

二次剩余

- 求解二次同余方程
- 给定 a, p , 求一组 x 满足 $x^2 \equiv a \pmod{p}$
- 前置模板: 快速幂

```

1 LL q1, q2, w;
2 struct P { // x + y * sqrt(w)
3     LL x, y;
4 };
5
6 P pmul(const P& a, const P& b, LL p) {
7     P res;
8     res.x = (a.x * b.x + a.y * b.y % p * w) % p;
9     res.y = (a.x * b.y + a.y * b.x) % p;
10    return res;
11 }
12
13 P bin(P x, LL n, LL MOD) {
14     P ret = {1, 0};
15     for (; n; n >>= 1, x = pmul(x, x, MOD))
16         if (n & 1) ret = pmul(ret, x, MOD);
17     return ret;
18 }
19 LL Legendre(LL a, LL p) { return bin(a, (p - 1) >> 1, p); }
20
21 LL equation_solve(LL b, LL p) {
22     if (p == 2) return 1;
23     if ((Legendre(b, p) + 1) % p == 0)
24         return -1;
25     LL a;
26     while (true) {
27         a = rand() % p;
28         w = ((a * a - b) % p + p) % p;
29         if ((Legendre(w, p) + 1) % p == 0)
30             break;
31     }
32     return bin({a, 1}, (p + 1) >> 1, p).x;
33 }
34 // !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!! Tester Start !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
35 void solve(){
36     LL a, p; cin >> a >> p;

```

```

37     a = a % p;
38     LL x = equation_solve(a, p);
39     if (x == -1) {
40         puts("No root");
41     } else {
42         LL y = p - x;
43         if (x == y) {
44             cout << x << endl;
45         } else {
46             LL tx = min(x, y), ty = max(x, y);
47             cout << tx << " " << ty << endl;
48         }
49     }
50 }
51 // !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!! Tester End !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
52

```

中国剩余定理

- 求解线性同余方程组

•

$$\begin{cases} x \equiv r_1 \pmod{m_1} \\ x \equiv r_2 \pmod{m_2} \\ \vdots \\ x \equiv r_k \pmod{m_k} \end{cases}$$

- 无解返回 -1
- 前置模板：扩展欧几里得

```

1  LL CRT(LL *m, LL *r, LL n) {
2      if (!n) return 0;
3      LL M = m[0], R = r[0], x, y, d;
4      FOR (i, 1, n) {
5          d = ex_gcd(M, m[i], x, y);
6          if ((r[i] - R) % d) return -1;
7          x = (r[i] - R) / d * x % (m[i] / d);
8          // 防爆 LL
9          // x = mul((r[i] - R) / d, x, m[i] / d);
10         R += x * M;
11         M = M / d * m[i];
12         R %= M;
13     }
14     return R >= 0 ? R : R + M;
15 }

```

逆元

- 如果 p 是素数，使用快速幂（费马小定理）
- 前置模板：快速幂

```

1  inline LL get_inv(LL x, LL p) { return bin(x, p - 2, p); }

```

- 如果 p 不是素数，使用拓展欧几里得
- 前置模板：扩展欧几里得

```

1  LL get_inv(LL a, LL M) {
2      static LL x, y;
3      assert(exgcd(a, M, x, y) == 1);
4      return (x % M + M) % M;
5  }

```

- 预处理 1~n 的逆元

```

1  LL inv[N];
2  void inv_init(LL n, LL p) {
3      inv[1] = 1;
4      FOR (i, 2, n)

```

```

5         inv[i] = (p - p / i) * inv[p % i] % p;
6     }

```

- 预处理阶乘及其逆元

```

1  LL invf[M], fac[M] = {1};
2  void fac_inv_init(LL n, LL p) {
3      FOR (i, 1, n)
4          fac[i] = i * fac[i - 1] % p;
5      invf[n - 1] = bin(fac[n - 1], p - 2, p);
6      FORD (i, n - 2, -1)
7          invf[i] = invf[i + 1] * (i + 1) % p;
8  }

```

组合数

组合数预处理（递推法）

```

1  LL C[M][M];
2  void init_C(int n) {
3      FOR (i, 0, n) {
4          C[i][0] = C[i][i] = 1;
5          FOR (j, 1, i)
6              C[i][j] = (C[i - 1][j] + C[i - 1][j - 1]) % MOD;
7      }
8  }

```

预处理逆元法

- 如果数较小，模较大时使用逆元
- 前置模板：逆元-预处理阶乘及其逆元

```

1  inline LL C(LL n, LL m) { // n >= m >= 0
2      return n < m || m < 0 ? 0 : fac[n] * invf[m] % MOD * invf[n - m] % MOD;
3  }

```

Lucas 定理

- 如果模数较小，数字较大，使用 Lucas 定理
- 前置模板可选 1：求组合数（如果使用阶乘逆元，需 fac_inv_init(MOD, MOD);）

```

1  LL C(LL n, LL m) { // m >= n >= 0
2      if (m - n < n) n = m - n;
3      if (n < 0) return 0;
4      LL ret = 1;
5      FOR (i, 1, n + 1)
6          ret = ret * (m - n + i) % MOD * bin(i, MOD - 2, MOD) % MOD;
7      return ret;
8  }

```

- 前置模板可选 2：模数不固定下使用，无法单独使用。

```

1  LL Lucas(LL n, LL m) { // m >= n >= 0
2      return m ? C(n % MOD, m % MOD) * Lucas(n / MOD, m / MOD) % MOD : 1;
3  }

```

求具体值

- 分解质因数法

```

1  int primes[N], cnt; // 存储所有质数
2  int sum[N]; // 存储每个质数的次数
3  bool st[N]; // 存储每个数是否已被筛掉
4
5  void get_primes(int n) // 线性筛法求素数
6  {
7      for (int i = 2; i <= n; i++)
8      {
9          if (!st[i]) primes[cnt++] = i;
10         for (int j = 0; primes[j] <= n / i; j++)

```



```

11     {
12         st[primes[j] * i] = true;
13         if (i % primes[j] == 0) break;
14     }
15 }
16 }
17
18
19 int get(int n, int p)    // 求 n! 中的次数
20 {
21     int res = 0;
22     while (n)
23     {
24         res += n / p;
25         n /= p;
26     }
27     return res;
28 }
29
30
31 vector<int> mul(vector<int> a, int b)    // 高精度乘低精度模板
32 {
33     vector<int> c;
34     int t = 0;
35     for (int i = 0; i < a.size(); i++)
36     {
37         t += a[i] * b;
38         c.push_back(t % 10);
39         t /= 10;
40     }
41
42     while (t)
43     {
44         c.push_back(t % 10);
45         t /= 10;
46     }
47
48     return c;
49 }
50
51 get_primes(a);    // 预处理范围内的所有质数
52
53 for (int i = 0; i < cnt; i++)    // 求每个质因数的次数
54 {
55     int p = primes[i];
56     sum[i] = get(a, p) - get(b, p) - get(a - b, p);
57 }
58
59 vector<int> res;
60 res.push_back(1);
61
62 for (int i = 0; i < cnt; i++)    // 用高精度乘法将所有质因子相乘
63     for (int j = 0; j < sum[i]; j++)
64         res = mul(res, primes[i]);

```

FFT & NTT & FWT

FFT

- 计算多项式乘法，可用于高精度乘法
- $\mathcal{O}(n \log n)$

```

1 typedef double LD;
2 const LD PI = acos(-1.0);
3
4 struct Complex {
5     LD r, i;
6     Complex(LD r = 0, LD i = 0) : r(r), i(i) {}
7     Complex operator + (const Complex& other) const {
8         return Complex(r + other.r, i + other.i);
9     }

```

```

10     Complex operator - (const Complex& other) const {
11         return Complex(r - other.r, i - other.i);
12     }
13     Complex operator * (const Complex& other) const {
14         return Complex(r * other.r - i * other.i, r * other.i + i * other.r);
15     }
16 };
17
18 // 快速傅里叶变换, p=1 为正向, p=-1 为反向
19 void FFT(vector<Complex>& x, int p) {
20     int n = x.size();
21     for (int i = 0, t = 0; i < n; ++i) {
22         if (i > t) swap(x[i], x[t]);
23         for (int j = n >> 1; (t ^= j) < j; j >>= 1);
24     }
25     for (int h = 2; h <= n; h <= 1) {
26         Complex wn(cos(p * 2 * PI / h), sin(p * 2 * PI / h));
27         for (int i = 0; i < n; i += h) {
28             Complex w(1, 0);
29             for (int j = 0; j < h / 2; ++j) {
30                 Complex u = x[i + j];
31                 Complex v = x[i + j + h/2] * w;
32                 x[i + j] = u + v;
33                 x[i + j + h/2] = u - v;
34                 w = w * wn;
35             }
36         }
37     }
38     if (p == -1) {
39         for (int i = 0; i < n; ++i) {
40             x[i].r /= n;
41         }
42     }
43 }
44
45 // 计算两个多项式的卷积, 返回结果多项式的系数向量
46 vector<LD> convolution(const vector<LD>& a, const vector<LD>& b) {
47     int len = 1;
48     int n = a.size(), m = b.size();
49     while (len < n + m - 1) len <= 1;
50     vector<Complex> fa(len), fb(len);
51     for (int i = 0; i < n; ++i) fa[i] = Complex(a[i], 0);
52     for (int i = 0; i < m; ++i) fb[i] = Complex(b[i], 0);
53     FFT(fa, 1);
54     FFT(fb, 1);
55     for (int i = 0; i < len; ++i) {
56         fa[i] = fa[i] * fb[i];
57     }
58     FFT(fa, -1);
59     vector<LD> res(n + m - 1);
60     for (int i = 0; i < n + m - 1; ++i) {
61         res[i] = fa[i].r;
62     }
63     return res;
64 }

```

NTT

- 用于大整数乘法时, 位数不宜过高 (在 MOD=998244353 的情况下, 总位数不超过 12324004(3510²))
- 前置模板: 快速幂、逆元

```

1  const int N = 1e5+10;
2  const int MOD = 998244353; // 模数
3  const int G = 3; // 原根
4
5  LL wn[N << 2], rev[N << 2];
6  int NTT_init(int n_) {
7      int step = 0; int n = 1;
8      for (; n < n_; n <= 1) ++step;
9      FOR (i, 1, n)
10         rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (step - 1));

```

```

11     int g = bin(G, (MOD - 1) / n, MOD);
12     wn[0] = 1;
13     for (int i = 1; i <= n; ++i)
14         wn[i] = wn[i - 1] * g % MOD;
15     return n;
16 }
17
18 void NTT(vector<LL> &a, int n, int f) {
19     FOR (i, 0, n) if (i < rev[i])
20         std::swap(a[i], a[rev[i]]);
21     for (int k = 1; k < n; k <= 1) {
22         for (int i = 0; i < n; i += (k < 1)) {
23             int t = n / (k < 1);
24             FOR (j, 0, k) {
25                 LL w = f == 1 ? wn[t * j] : wn[n - t * j];
26                 LL x = a[i + j];
27                 LL y = a[i + j + k] * w % MOD;
28                 a[i + j] = (x + y) % MOD;
29                 a[i + j + k] = (x - y + MOD) % MOD;
30             }
31         }
32     }
33     if (f == -1) {
34         LL ninv = get_inv(n, MOD);
35         FOR (i, 0, n)
36             a[i] = a[i] * ninv % MOD;
37     }
38 }
39
40 vector<LL> conv(vector<LL> a, vector<LL> b){
41     int len_a = a.size(), len_b = b.size();
42     int len = len_a + len_b - 1;
43     int n = NTT_init(len);
44     a.resize(n);
45     b.resize(n);
46     NTT(a, n, 1);
47     NTT(b, n, 1);
48     vector<LL> c(n);
49     for (int i = 0; i < n; ++i) {
50         c[i] = a[i] * b[i] % MOD;
51     }
52     NTT(c, n, -1);
53     vector<LL> res(len);
54     for (int i = 0; i < len; ++i) {
55         res[i] = c[i];
56     }
57     return res;
58 }

```

FWT

```

1  const LL MOD = 998244353;
2
3  template<typename T>
4  void fwt(vector<LL> &a, int n, T f) {
5      for (int d = 1; d < n; d *= 2)
6          for (int i = 0, t = d * 2; i < n; i += t)
7              FOR (j, 0, d)
8                  f(a[i + j], a[i + j + d]);
9  }
10
11 void AND(LL& a, LL& b) { a += b; }
12 void OR(LL& a, LL& b) { b += a; }
13 void XOR (LL& a, LL& b) {
14     LL x = a, y = b;
15     a = (x + y) % MOD;
16     b = (x - y + MOD) % MOD;
17 }
18 void rAND(LL& a, LL& b) { a -= b; }
19 void rOR(LL& a, LL& b) { b -= a; }
20 void rXOR(LL& a, LL& b) {

```

```

21     static LL INV2 = (MOD + 1) / 2;
22     LL x = a, y = b;
23     a = (x + y) * INV2 % MOD;
24     b = (x - y + MOD) * INV2 % MOD;
25 }
26
27 int next_power_of_two(int n) {
28     if (n <= 0) return 1;
29     // __lg(n-1) 返回 n-1 的最高位所在位置 (0-based)
30     return 1 << (__lg(n - 1) + 1);
31 }
32
33 template<typename T, typename F>
34 vector<LL> conv(vector<LL> a, vector<LL> b, T f, F inv_f){
35     LL len_a = a.size(), len_b = b.size(), len = max(len_a, len_b), n = next_power_of_two(len);
36     a.resize(n), b.resize(n);
37     fwt(a, n, f), fwt(b, n, f);
38     vector<LL> c(n);
39     for (int i = 0; i < n; i++) {
40         c[i] = a[i] * b[i] % MOD;
41     }
42     fwt(c, n, inv_f);
43     // 提取结果 (可选)
44     c.resize(len);
45     return c;
46 }

```

线性基

贪心法

可查询最大异或和

```

1 struct BasisGreedy{
2     ULL p[64];
3     BasisGreedy(){memset(p, 0, sizeof p);}
4     void insert(ULL x) {
5         for (int i = 63; ~i; --i) {
6             if (!(x >> i)) // x 的第 i 位是 0
7                 continue;
8             if (!p[i]) {
9                 p[i] = x;
10                break;
11            }
12            x ^= p[i];
13        }
14    }
15    ULL query_max(){
16        ULL ans = 0;
17        for (int i = 63; ~i; --i) {
18            ans = std::max(ans, ans ^ p[i]);
19        }
20        return ans;
21    }
22 };

```

高斯消元法

可查询任意大异或和

```

1 struct BasisGauss{
2     vector<ULL> a;
3     LL n, tmp, cnt;
4
5     BasisGauss(){a = {0};}
6
7     void insert(ULL x){
8         a.push_back(x);
9     }
10 }

```

```

11 void init(){
12     n = (LL)a.size() - 1;
13     LL k=1;
14     for(int i=63;i>=0;i--){
15         int t=0;
16         for(LL j=k;j<=n;j++){
17             if((a[j]>>i)&1){
18                 t=j;
19                 break;
20             }
21         }
22         if(t){
23             swap(a[k],a[t]);
24             for(LL j=1;j<=n;j++){
25                 if(j!=k&&(a[j]>>i)&1) a[j]^=a[k];
26             }
27             k++;
28         }
29     }
30     cnt = k-1;
31     tmp = 1LL << cnt;
32     if(cnt==n) tmp--;
33 }
34
35 LL query_xth(LL x){ // 从小到大, 若 x 为负数, 则查询倒数第几个
36     if(x<0) x = tmp + x + 1;
37     if(x>tmp) return -1;
38     else{
39         if(n>cnt) x--;
40         LL ans=0;
41         for(LL i=0; i<cnt; i++){
42             if((x>>i)&1) ans^=a[cnt-i];
43         }
44         return ans;
45     }
46 }
47 };

```

性质与公式

低阶等幂求和

- $\sum_{i=1}^n i^1 = \frac{n(n+1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n$
- $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$
- $\sum_{i=1}^n i^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2$
- $\sum_{i=1}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{1}{5}n^5 + \frac{1}{2}n^4 + \frac{1}{3}n^3 - \frac{1}{30}n$
- $\sum_{i=1}^n i^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12} = \frac{1}{6}n^6 + \frac{1}{2}n^5 + \frac{5}{12}n^4 - \frac{1}{12}n^2$

一些组合公式

- 错排公式 (对于 $1 \sim n$ 的排列 P , 满足 $P_i \neq i$): $D_1 = 0, D_2 = 1, D_n = (n-1)(D_{n-1} + D_{n-2}) = n! \left(\frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!} \right) = \lfloor \frac{n!}{e} + 0.5 \rfloor$
- 卡特兰数 (n 对括号合法方案数, n 个结点二叉树个数, $n \times n$ 方格中对角线下方的单调路径数, 凸 $n+2$ 边形的三角形划分数, n 个元素的合法出栈序列数): $C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$

互质

若整数 a 与 m 互质 (即 $\gcd(a, m) = 1$)

- 对于整数 $k = 0, 1, 2, \dots, m-1$, $ak \bmod m$ 的结果恰好是 $0, 1, 2, \dots, m-1$ 的一个排列 (每个数出现且仅出现一次)。
- 存在唯一的整数 b ($1 \leq b < m$), 使得 $ab \equiv 1 \pmod m$, 此时 b 称为 a 在模 m 下的乘法逆元 (记为 $a^{-1} \bmod m$)。

图论

最短路

朴素 djikstra 算法

- 无负权边、稠密图

```
1 int g[N][N]; // 存储每条边
2 int dist[N]; // 存储 1 号点到每个点的最短距离
3 bool st[N]; // 存储每个点的最短路是否已经确定
4
5 // 求 1 号点到 n 号点的最短路, 如果不存在则返回-1
6 int dijkstra(){
7     memset(dist, 0x3f, sizeof dist);
8     dist[1] = 0;
9     for (int i = 0; i < n - 1; i ++ ){
10         int t = -1; // 在还未确定最短路的点中, 寻找距离最小的点
11         for (int j = 1; j <= n; j ++ )
12             if (!st[j] && (t == -1 || dist[t] > dist[j]))
13                 t = j;
14         // 用 t 更新其他点的距离
15         for (int j = 1; j <= n; j ++ )
16             dist[j] = min(dist[j], dist[t] + g[t][j]);
17         st[t] = true;
18     }
19     if (dist[n] == 0x3f3f3f3f) return -1;
20     return dist[n];
21 }
```

堆优化的 djikstra

- 无负权边、稀疏图

```
1 typedef pair<int, int> PII;
2
3 int n; // 点的数量
4 int h[N], w[N], e[N], ne[N], idx; // 邻接表存储所有边
5 int dist[N]; // 存储所有点到 1 号点的距离
6 bool st[N]; // 存储每个点的最短距离是否已确定
7
8 // 求 1 号点到 n 号点的最短距离, 如果不存在, 则返回-1
9 int dijkstra(){
10     memset(dist, 0x3f, sizeof dist);
11     dist[1] = 0;
12     priority_queue<PII, vector<PII>, greater<PII>> heap;
13     heap.push({0, 1}); // first 存储距离, second 存储节点编号
14     while (heap.size()){
15         auto t = heap.top();
16         heap.pop();
17         int ver = t.second, distance = t.first;
18         if (st[ver]) continue;
19         st[ver] = true;
20         for (int i = h[ver]; i != -1; i = ne[i]){
21             int j = e[i];
22             if (dist[j] > distance + w[i]){
23                 dist[j] = distance + w[i];
24                 heap.push({dist[j], j});
25             }
26         }
27     }
28     if (dist[n] == 0x3f3f3f3f) return -1;
29     return dist[n];
30 }
```

Bellman-Ford 算法

- 有负权边、可以处理负环

```
1 int n, m; // n 表示点数, m 表示边数
2 int dist[N]; // dist[x] 存储 1 到 x 的最短路距离
```

```

3
4 struct Edge{ // 边, a 表示出点, b 表示入点, w 表示边的权重
5     int a, b, w;
6 }edges[M];
7
8 // 求 1 到 n 的最短路距离, 如果无法从 1 走到 n, 则返回-1。
9 int bellman_ford(){
10     memset(dist, 0x3f, sizeof dist);
11     dist[1] = 0;
12
13     // 如果第 n 次迭代仍然会松弛三角不等式, 就说明存在一条长度是 n+1 的最短路, 由抽屉原理, 路径中至少存在两个相同的点, 说明图中存在负权回路。
14     for (int i = 0; i < n; i ++ ){
15         for (int j = 0; j < m; j ++ ){
16             int a = edges[j].a, b = edges[j].b, w = edges[j].w;
17             if (dist[b] > dist[a] + w)
18                 dist[b] = dist[a] + w;
19         }
20     }
21
22     if (dist[n] > 0x3f3f3f3f / 2) return -1;
23     return dist[n];
24 }

```

spfa 算法

- 有负权边、不能有负环, 快

```

1 int n; // 总点数
2 int h[N], w[N], e[N], ne[N], idx; // 邻接表存储所有边
3 int dist[N]; // 存储每个点到 1 号点的最短路距离
4 bool st[N]; // 存储每个点是否在队列中
5
6 // 求 1 号点到 n 号点的最短路距离, 如果从 1 号点无法走到 n 号点则返回-1
7 int spfa(){
8     memset(dist, 0x3f, sizeof dist);
9     dist[1] = 0;
10    queue<int> q;
11    q.push(1);
12    st[1] = true;
13    while (q.size()){
14        auto t = q.front();
15        q.pop();
16        st[t] = false;
17        for (int i = h[t]; i != -1; i = ne[i]){
18            int j = e[i];
19            if (dist[j] > dist[t] + w[i]){
20                dist[j] = dist[t] + w[i];
21                if (!st[j]){ // 如果队列中已存在 j, 则不需要将 j 重复插入
22                    q.push(j);
23                    st[j] = true;
24                }
25            }
26        }
27    }
28    if (dist[n] == 0x3f3f3f3f) return -1;
29    return dist[n];
30 }

```

spfa 判断负环

```

1 int n; // 总点数
2 int h[N], w[N], e[N], ne[N], idx; // 邻接表存储所有边
3 int dist[N], cnt[N]; // dist[x] 存储 1 号点到 x 的最短路距离, cnt[x] 存储 1 到 x 的最短路中经过的点数
4 bool st[N]; // 存储每个点是否在队列中
5
6 // 如果存在负环, 则返回 true, 否则返回 false。
7 bool spfa(){
8     // 不需要初始化 dist 数组
9     // 原理: 如果某条最短路径上有 n 个点 (除了自己), 那么加上自己之后一共有 n+1 个点, 由抽屉原理一定有两个点相同, 所以存在环。
10    queue<int> q;
11    for (int i = 1; i <= n; i ++ ){

```

```

12     q.push(i);
13     st[i] = true;
14 }
15 while (q.size()){
16     auto t = q.front();
17     q.pop();
18     st[t] = false;
19     for (int i = h[t]; i != -1; i = ne[i]){
20         int j = e[i];
21         if (dist[j] > dist[t] + w[i]){
22             dist[j] = dist[t] + w[i];
23             cnt[j] = cnt[t] + 1;
24             if (cnt[j] >= n) return true; // 如果从 1 号点到 x 的最短路中包含至少 n 个点 (不包括自己), 则说明存在环
25             if (!st[j]){
26                 q.push(j);
27                 st[j] = true;
28             }
29         }
30     }
31 }
32 return false;
33 }

```

floyd 算法

```

1 初始化:
2     for (int i = 1; i <= n; i ++ )
3         for (int j = 1; j <= n; j ++ )
4             if (i == j) d[i][j] = 0;
5             else d[i][j] = INF;
6
7 // 算法结束后, d[a][b] 表示 a 到 b 的最短距离
8 void floyd(){
9     for (int k = 1; k <= n; k ++ )
10        for (int i = 1; i <= n; i ++ )
11            for (int j = 1; j <= n; j ++ )
12                d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
13 }

```

最小生成树

朴素 Prim 算法

- 稠密图 (m 接近于 n^2)

```

1 int n; // n 表示点数
2 int g[N][N]; // 邻接矩阵, 存储所有边
3 int dist[N]; // 存储其他点到当前最小生成树的距离
4 bool st[N]; // 存储每个点是否已经在生成树中
5 // 如果图不连通, 则返回 INF(值是 0x3f3f3f3f), 否则返回最小生成树的树边权重之和
6 int prim(){
7     memset(dist, 0x3f, sizeof dist);
8     int res = 0;
9     for (int i = 0; i < n; i ++ ){
10         int t = -1;
11         for (int j = 1; j <= n; j ++ )
12             if (!st[j] && (t == -1 || dist[t] > dist[j]))
13                 t = j;
14         if (i && dist[t] == INF) return INF;
15         if (i) res += dist[t];
16         st[t] = true;
17         for (int j = 1; j <= n; j ++ ) dist[j] = min(dist[j], g[t][j]);
18     }
19     return res;
20 }

```

Kruskal 算法

- 实现简单, 稀疏图 (m 接近 n)


```

1  int n, m;          // n 是点数, m 是边数
2  int p[N];          // 并查集的父节点数组
3  struct Edge{       // 存储边
4      int a, b, w;
5      bool operator< (const Edge &W) const{
6          return w < W.w;
7      }
8  }edges[M];
9
10 int find(int x){    // 并查集核心操作
11     if (p[x] != x) p[x] = find(p[x]);
12     return p[x];
13 }
14
15 int kruskal(){
16     sort(edges, edges + m);
17     for (int i = 1; i <= n; i ++ ) p[i] = i;    // 初始化并查集
18     int res = 0, cnt = 0;
19     for (int i = 0; i < m; i ++ ){
20         int a = edges[i].a, b = edges[i].b, w = edges[i].w;
21         a = find(a), b = find(b);
22         if (a != b){    // 如果两个连通块不连通, 则将这两个连通块合并
23             p[a] = b;
24             res += w;
25             cnt ++ ;
26         }
27     }
28     if (cnt < n - 1) return INF;
29     return res;
30 }

```

拓扑排序

- 有向图
- 别忘了存储入度
- 当 `toporder(int n)` 返回值的长度不等于 `n` 时, 不存在拓扑排序。

```

1  const int N = 1e5+10;
2  vector<int> G[N];
3  int deg[N]; // 入度
4
5  vector<int> toporder(int n) {
6      vector<int> orders;
7      queue<int> q;
8      for (int i = 1; i <= n; i++)
9          if (!deg[i]) {
10             q.push(i);
11             orders.push_back(i);
12         }
13     while (!q.empty()) {
14         int u = q.front(); q.pop();
15         for (int v: G[u])
16             if (--deg[v]) {
17                 q.push(v);
18                 orders.push_back(v);
19             }
20     }
21     return orders;
22 }

```

差分约束

一个系统 n 个变量和 m 个约束条件组成, 每个约束条件形如 $x_j - x_i \leq b_k$ 。可以发现每个约束条件都形如最短路中的三角不等式 $d_u - d_v \leq w_{u,v}$ 。因此连一条边 (i, j, b_k) 建图。

若要判断解的存在性, 使用 `spfa` 判断是否存在负环, 有则无解。

若要使得所有量两两的值最接近, 源点到各点的距离初始成 0, 跑最远路。

若要使得某一变量与其他变量的差尽可能大，则源点到各点距离初始化成 ∞ ，跑最短路。

最近公共祖先

```
1  const LL N = 5e5+10, SP = log2(N)+1;
2  vector<int> G[N];
3  int pa[N][SP], dep[N];
4
5  void dfs(int u, int fa) {
6      pa[u][0] = fa; dep[u] = dep[fa] + 1;
7      FOR (i, 1, SP) pa[u][i] = pa[pa[u][i-1]][i-1];
8      for (int& v: G[u]) {
9          if (v == fa) continue;
10         dfs(v, u);
11     }
12 }
13
14 int lca(int u, int v) {
15     if (dep[u] < dep[v]) swap(u, v);
16     int t = dep[u] - dep[v];
17     FOR (i, 0, SP) if (t & (1 << i)) u = pa[u][i];
18     FORD (i, SP-1, -1) {
19         int uu = pa[u][i], vv = pa[v][i];
20         if (uu != vv) { u = uu; v = vv; }
21     }
22     return u == v ? u : pa[u][0];
23 }
```

树链剖分

- 将树上操作转化为区间操作，套用区间数据结构
- 别忘了选一种方法（取消注释）
- fa[N]: 存储每个节点的父节点
- dep[N]: 存储每个节点的深度
- idx[N]: 存储每个节点在线段树中的索引（DFS 序）
- out[N]: 存储每个节点子树在 DFS 序中的结束位置
- ridx[N]: 存储 DFS 序到节点的反向映射
- sz[N]: 存储每个节点的子树大小
- son[N]: 存储每个节点的重儿子（子树最大的儿子）
- top[N]: 存储每个节点所在重链的顶端节点
- clk: DFS 序计数器
- init(): 初始化（先建图再调用）
- go(u, v, f): f 是一个形如 f(int l, int r) 的函数。对树上节点 u 到节点 v 的简单路径，分解为 dfs 序中的区间 $[l, r]$ ，调用函数 f
- 子树操作: u 的子树的 dfs 序区间为 $[idx[u], out[u]]$

```
1  const int N = 3e4+10;
2
3  vector<int> G[N];
4  int fa[N], dep[N], idx[N], out[N], ridx[N];
5  namespace hld {
6      int sz[N], son[N], top[N], clk;
7      void predfs(int u, int d) {
8          dep[u] = d; sz[u] = 1;
9          int& maxs = son[u] = -1;
10         for (int& v: G[u]) {
11             if (v == fa[u]) continue;
12             fa[v] = u;
13             predfs(v, d+1);
14             sz[u] += sz[v];
15             if (maxs == -1 || sz[v] > sz[maxs]) maxs = v;
16         }
17     }
18     void dfs(int u, int tp) {
19         top[u] = tp; idx[u] = ++clk; ridx[clk] = u;
20         if (son[u] != -1) dfs(son[u], tp);
21     }
```

```

21     for (int& v: G[u])
22         if (v != fa[u] && v != son[u]) dfs(v, v);
23     out[u] = clk;
24 }
25 void init(){
26     clk = 0;
27     predfs(1, 1);
28     dfs(1, 1);
29 }
30 template<typename T>
31 int go(int u, int v, T&& f = [] (int, int) {} ) {
32     int uu = top[u], vv = top[v];
33     while (uu != vv) {
34         if (dep[uu] < dep[vv]) { swap(uu, vv); swap(u, v); }
35         f(idx[uu], idx[u]);
36         u = fa[uu]; uu = top[u];
37     }
38     if (dep[u] < dep[v]) swap(u, v);
39     // 下面两行代码选择一个
40     // f(idx[v], idx[u]); // 包含 lca(u, v)
41     // if (u != v) f(idx[v] + 1, idx[u]); // 不包含 lca(u, v)
42     return v;
43 }
44 int up(int u, int d) { // 查询 u 节点向上走 d 步的节点编号
45     while (d) {
46         if (dep[u] - dep[top[u]] < d) {
47             d -= dep[u] - dep[top[u]];
48             u = top[u];
49         } else return ridx[idx[u] - d];
50         u = fa[u]; --d;
51     }
52     return u;
53 }
54 int finds(int u, int rt) { // 找 u 在 rt 的哪个儿子的子树中
55     while (top[u] != top[rt]) {
56         u = top[u];
57         if (fa[u] == rt) return u;
58         u = fa[u];
59     }
60     return ridx[idx[rt] + 1];
61 }
62 }

```

网络流

• 最大流

```

1  const LL INF = LONG_LONG_MAX;
2
3  struct E {
4      LL to, cp;
5      E(LL to, LL cp): to(to), cp(cp) {}
6  };
7
8  struct Dinic {
9      static const LL M = 1E5 * 5;
10     LL m, s, t;
11     vector<E> edges;
12     vector<LL> G[M];
13     LL d[M];
14     LL cur[M];
15
16     void init(LL n, LL s, LL t) {
17         this->s = s; this->t = t;
18         for (LL i = 0; i <= n; i++) G[i].clear();
19         edges.clear(); m = 0;
20     }
21
22     void addedge(LL u, LL v, LL cap) {
23         edges.emplace_back(v, cap);
24         edges.emplace_back(u, 0);

```

```

25     G[u].push_back(m++);
26     G[v].push_back(m++);
27 }
28
29 bool BFS() {
30     memset(d, 0, sizeof d);
31     queue<LL> Q;
32     Q.push(s); d[s] = 1;
33     while (!Q.empty()) {
34         LL x = Q.front(); Q.pop();
35         for (LL& i: G[x]) {
36             E &e = edges[i];
37             if (!d[e.to] && e.cp > 0) {
38                 d[e.to] = d[x] + 1;
39                 Q.push(e.to);
40             }
41         }
42     }
43     return d[t];
44 }
45
46 LL DFS(LL u, LL cp) {
47     if (u == t || !cp) return cp;
48     LL tmp = cp, f;
49     for (LL& i = cur[u]; i < G[u].size(); i++) {
50         E& e = edges[G[u][i]];
51         if (d[u] + 1 == d[e.to]) {
52             f = DFS(e.to, min(cp, e.cp));
53             e.cp -= f;
54             edges[G[u][i] ^ 1].cp += f;
55             cp -= f;
56             if (!cp) break;
57         }
58     }
59     return tmp - cp;
60 }
61
62 LL go() {
63     LL flow = 0;
64     while (BFS()) {
65         memset(cur, 0, sizeof cur);
66         flow += DFS(s, INF);
67     }
68     return flow;
69 }
70 } DC;

```

● 最小费用最大流

```

1  const LL M = 5e4+10;
2  const int INF = INT_MAX;
3
4  struct E {
5      int from, to, cp, v;
6      E() {}
7      E(int f, int t, int cp, int v) : from(f), to(t), cp(cp), v(v) {}
8  };
9
10 struct MCMF {
11     int n, m, s, t;
12     vector<E> edges;
13     vector<int> G[M];
14     bool inq[M];
15     int d[M], p[M], a[M];
16
17     void init(int _n, int _s, int _t) {
18         n = _n; s = _s; t = _t;
19         FOR (i, 0, n + 1) G[i].clear();
20         edges.clear(); m = 0;
21     }
22
23     void addedge(int from, int to, int cap, int cost) {

```

```

24     edges.emplace_back(from, to, cap, cost);
25     edges.emplace_back(to, from, 0, -cost);
26     G[from].push_back(m++);
27     G[to].push_back(m++);
28 }
29
30 bool BellmanFord(int &flow, int &cost) {
31     FOR (i, 0, n + 1) d[i] = INF;
32     memset(inq, 0, sizeof inq);
33     d[s] = 0, a[s] = INF, inq[s] = true;
34     queue<int> Q; Q.push(s);
35     while (!Q.empty()) {
36         int u = Q.front(); Q.pop();
37         inq[u] = false;
38         for (int& idx: G[u]) {
39             E &e = edges[idx];
40             if (e.cp && d[e.to] > d[u] + e.v) {
41                 d[e.to] = d[u] + e.v;
42                 p[e.to] = idx;
43                 a[e.to] = min(a[u], e.cp);
44                 if (!inq[e.to]) {
45                     Q.push(e.to);
46                     inq[e.to] = true;
47                 }
48             }
49         }
50     }
51     if (d[t] == INF) return false;
52     flow += a[t];
53     cost += a[t] * d[t];
54     int u = t;
55     while (u != s) {
56         edges[p[u]].cp -= a[t];
57         edges[p[u] ^ 1].cp += a[t];
58         u = edges[p[u]].from;
59     }
60     return true;
61 }
62
63 pair<int, int> go() {
64     int flow = 0, cost = 0;
65     while (BellmanFord(flow, cost));
66     return {flow, cost};
67 }
68 } MM;

```

树上路径交

- 前置模板：最近公共祖先

```

1 int intersection(int x1, int y1, int x2, int y2) {
2     int t[4] = {lca(x1, x2), lca(x1, y2), lca(y1, x2), lca(y1, y2)};
3     int p1 = 0, p2 = 0;
4     FOR(j, 0, 4)
5         if (dep[t[j]] > dep[p1]) p2 = p1, p1 = t[j];
6     else if (dep[t[j]] > dep[p2]) p2 = t[j];
7     int h1 = lca(x1, y1), h2 = lca(x2, y2);
8     if (p1 == p2) {
9         if (dep[p1] < dep[h1] || dep[p1] < dep[h2]) return 0;
10        else return 1;
11    }
12    else {
13        int ans = dep[p1] + dep[p2] - 2 * dep[lca(p1, p2)] + 1;
14        return ans;
15    }
16 }

```

树上点分治（树的重心）

```
1  const LL N = 2e4+10, N2 = N * 2;
2
3  int h[N], e[N2], ne[N2], idx;
4
5  void add(int a, int b){
6      e[idx] = b, ne[idx] = h[a], h[a] = idx++;
7  }
8
9  vector<bool> vis;
10
11 // 获取子树的重心（自动处理父子关系）（如果有两个重心，输出编号小的那个）
12 // 若重心为 u，则 mx[u] 为以 u 为重心子树大小的最大值
13 int q[N], fa[N], sz[N], mx[N];
14 int get_rt(int u) {
15     int p = 0, cur = -1;
16     q[p++] = u; fa[u] = -1;
17     while (++cur < p) {
18         u = q[cur]; mx[u] = 0; sz[u] = 1;
19         for (int i = h[u]; i != -1; i = ne[i]){
20             int j = e[i];
21             if(vis[j] or j == fa[u]) continue;
22             fa[q[p++]] = j; = u;
23         }
24     }
25     FORD (i, p - 1, -1) {
26         u = q[i];
27         mx[u] = max(mx[u], p - sz[u]);
28         if (mx[u] * 2 <= p) return u;
29         sz[fa[u]] += sz[u];
30         mx[fa[u]] = max(mx[fa[u]], sz[u]);
31     }
32     // assert(0);
33 }
34
35 // 分治 dfs（起点任意）
36 void dfs(int u) {
37     cout << "u: " << u;
38     u = get_rt(u);
39     vis[u] = true;
40     // 处理子树逻辑
41     cout << " centroid: " << u << '\n';
42     // 如果在此处 DFS，会遍历整棵子树 (if(vis[u]) return)
43     // ...
44
45     for(int i=h[u]; i!=-1; i=ne[i]){
46         int j = e[i];
47         if(vis[j]) continue;
48         dfs(j);
49     }
50 }
```

二分图

最大匹配

- 最小覆盖数 = 最大匹配数
- 最大独立集 = 顶点数 - 二分图匹配数
- DAG 最小路径覆盖数 = 结点数 - 拆点后二分图最大匹配数

```
1  const int N = 500+10;
2
3  struct MaxMatch {
4      int n;
5      vector<int> G[N];
6      int vis[N], left[N], clk;
7
8      void init(int n) {
9          this->n = n;
10         FOR (i, 0, n + 1) G[i].clear();
11     }
```

```

11     memset(left, -1, sizeof left);
12     memset(vis, -1, sizeof vis);
13 }
14
15 bool dfs(int u) {
16     for (int v: G[u])
17         if (vis[v] != clk) {
18             vis[v] = clk;
19             if (left[v] == -1 || dfs(left[v])) {
20                 left[v] = u;
21                 return true;
22             }
23         }
24     return false;
25 }
26
27 int match() {
28     int ret = 0;
29     for (clk = 0; clk <= n; ++clk)
30         if (dfs(clk)) ++ret;
31     return ret;
32 }
33 } MM;
34 // !!!!!!!!!!!!!!!!!!!!!!!!!!!!! Tester Start !!!!!!!!!!!!!!!!!!!!!!!!!!!!!
35 void solve(){
36     LL n1, n2, m, n, i, t1, t2;
37     cin >> n1 >> n2 >> m;
38     n = n1 + n2;
39     MM.init(n);
40     for(i=0; i<m; i++){
41         cin >> t1 >> t2;
42         MM.G[t1].push_back(n1+t2);
43     }
44     cout << MM.match() << '\n';
45 }

```

最大权匹配

- $py[j] = i$ 表示右侧顶点 j 与左侧顶点 i 匹配

```

1 namespace R {
2     const int M = 400 + 5;
3     const int INF = 2E9;
4     int n;
5     int w[M][M], kx[M], ky[M], py[M], vy[M], slk[M], pre[M];
6
7     LL KM() {
8         FOR (i, 1, n + 1)
9             FOR (j, 1, n + 1)
10                 kx[i] = max(kx[i], w[i][j]);
11         FOR (i, 1, n + 1) {
12             fill(vy, vy + n + 1, 0);
13             fill(slk, slk + n + 1, INF);
14             fill(pre, pre + n + 1, 0);
15             int k = 0, p = -1;
16             for (py[k = 0] = i; py[k]; k = p) {
17                 int d = INF;
18                 vy[k] = 1;
19                 int x = py[k];
20                 FOR (j, 1, n + 1)
21                     if (!vy[j]) {
22                         int t = kx[x] + ky[j] - w[x][j];
23                         if (t < slk[j]) { slk[j] = t; pre[j] = k; }
24                         if (slk[j] < d) { d = slk[j]; p = j; }
25                     }
26                 FOR (j, 0, n + 1)
27                     if (vy[j]) { kx[py[j]] -= d; ky[j] += d; }
28                     else slk[j] -= d;
29             }
30             for (; k; k = pre[k]) py[k] = py[pre[k]];
31         }
32     }
33 }

```

```

32         LL ans = 0;
33         FOR (i, 1, n + 1) ans += kx[i] + ky[i];
34         return ans;
35     }
36 }
37 // !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!! Tester Start !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
38 void solve(){
39     LL n1, n2, i, t1, t2, t3, m, n, j;
40     cin >> n1 >> n2 >> m;
41     // 初始化
42     n = max(n1, n2);
43     R::n = n;
44     for(i=0; i<=n; i++){
45         for(j=0; j<=n; j++){
46             R::w[i][j] = 0;
47         }
48     }
49     // 读数据
50     for(i=0; i<m; i++){
51         cin >> t1 >> t2 >> t3;
52         R::w[t1][t2] = t3;
53     }
54     // 计算
55     LL maxx = R::KM();
56     cout << maxx << '\n';
57     // 结果转换
58     vector<pair<LL, LL>> anss;
59     for(i=1; i<=n; i++){ // 注意遍历最大范围
60         if(R::w[R::py[i]][i]){
61             anss.push_back({R::py[i], i});
62         }else{
63             // 未匹配
64             anss.push_back({R::py[i], 0});
65         }
66     }
67     sort(anss.begin(), anss.end());
68     for(i=0; i<n1; i++){
69         cout << anss[i].second << ' ';
70     }
71 }

```

Tarjan

割点

- 判断割点（无向图）
- 注意原图可能不连通

```

1  int dfn[N], low[N], clk;
2  void init() { clk = 0; memset(dfn, 0, sizeof dfn); }
3  void tarjan(int u, int fa) {
4      low[u] = dfn[u] = ++clk;
5      int cc = fa != -1;
6      for (int& v: G[u]) {
7          if (v == fa) continue;
8          if (!dfn[v]) {
9              tarjan(v, u);
10             low[u] = min(low[u], low[v]);
11             cc += low[v] >= dfn[u];
12         } else low[u] = min(low[u], dfn[v]);
13     }
14     if (cc > 1) // u 是割点
15 }

```

桥

- 无向图
- 注意原图不连通和重边


```

1  int dfn[N], low[N], clk;
2  void init() { memset(dfn, 0, sizeof dfn); clk = 0; }
3  void tarjan(int u, int fa) {
4      low[u] = dfn[u] = ++clk;
5      int _fst = 0;
6      for (E& e: G[u]) {
7          int v = e.to; if (v == fa && ++_fst == 1) continue;
8          if (!dfn[v]) {
9              tarjan(v, u);
10             if (low[v] > dfn[u]) // (u, v) 是桥
11                 low[u] = min(low[u], low[v]);
12             } else low[u] = min(low[u], dfn[v]);
13     }
14 }

```

强连通分量缩点

- 有向图
- B: 强连通分量的数量计数器
- bl[N]: 记录每个顶点所属的强连通分量编号
- bcc[N]: 存储每个强连通分量包含的顶点列表

```

1  int low[N], dfn[N], clk, B, bl[N];
2  vector<int> bcc[N];
3  void init() { B = clk = 0; memset(dfn, 0, sizeof dfn); }
4  void tarjan(int u) {
5      static int st[N], p;
6      static bool in[N];
7      dfn[u] = low[u] = ++clk;
8      st[p++] = u; in[u] = true;
9      for (int& v: G[u]) {
10         if (!dfn[v]) {
11             tarjan(v);
12             low[u] = min(low[u], low[v]);
13         } else if (in[v]) low[u] = min(low[u], dfn[v]);
14     }
15     if (dfn[u] == low[u]) {
16         while (1) {
17             int x = st[--p]; in[x] = false;
18             bl[x] = B; bcc[B].push_back(x);
19             if (x == u) break;
20         }
21         ++B;
22     }
23 }

```

点双连通分量 / 广义圆方树

- 数组开两倍
- 一条边也被计入点双了（适合拿来建圆方树），可以用点数 \leq 边数过滤
- B: 双连通分量的数量（编号从 0 开始）。
- bc[B]: 存储第 B 个双连通分量包含的节点。
- be[B]: 存储第 B 个双连通分量包含的边（索引）。
- bno[x]: 标记节点 x 属于哪个双连通分量（用于去重）。

```

1  struct E { int to, nxt; } e[N];
2  int hd[N], ecnt;
3  void addedge(int u, int v) {
4      e[ecnt] = {v, hd[u]};
5      hd[u] = ecnt++;
6  }
7  int low[N], dfn[N], clk, B, bno[N];
8  vector<int> bc[N], be[N];
9  bool vise[N];
10 void init() {
11     memset(vise, 0, sizeof vise);
12     memset(hd, -1, sizeof hd);
13     memset(dfn, 0, sizeof dfn);
14     memset(bno, -1, sizeof bno);

```

```

15     B = clk = ecnt = 0;
16 }
17
18 void tarjan(int u, int feid) {
19     static int st[N], p;
20     static auto add = [&](int x) {
21         if (bno[x] != B) { bno[x] = B; bc[B].push_back(x); }
22     };
23     low[u] = dfn[u] = ++clk;
24     for (int i = hd[u]; ~i; i = e[i].nxt) {
25         if ((feid ^ i) == 1) continue;
26         if (!vise[i]) { st[p++] = i; vise[i] = vise[i ^ 1] = true; }
27         int v = e[i].to;
28         if (!dfn[v]) {
29             tarjan(v, i);
30             low[u] = min(low[u], low[v]);
31             if (low[v] >= dfn[u]) {
32                 bc[B].clear(); be[B].clear();
33                 while (1) {
34                     int eid = st[--p];
35                     add(e[eid].to); add(e[eid ^ 1].to);
36                     be[B].push_back(eid);
37                     if ((eid ^ i) <= 1) break;
38                 }
39                 ++B;
40             }
41         } else low[u] = min(low[u], dfn[v]);
42     }
43 }

```

计算几何

二维几何：点与向量

```

1  #define y1 yy1
2  #define nxt(i) ((i + 1) % s.size())
3  typedef double LD;
4  const LD PI = 3.14159265358979323846;
5  const LD eps = 1E-10;
6  int sgn(LD x) { return fabs(x) < eps ? 0 : (x > 0 ? 1 : -1); }
7  struct L;
8  struct P;
9  typedef P V;
10 struct P {
11     LD x, y;
12     explicit P(LD x = 0, LD y = 0): x(x), y(y) {}
13     explicit P(const L& l);
14 };
15 struct L {
16     P s, t;
17     L() {}
18     L(P s, P t): s(s), t(t) {}
19 };
20
21 P operator + (const P& a, const P& b) { return P(a.x + b.x, a.y + b.y); }
22 P operator - (const P& a, const P& b) { return P(a.x - b.x, a.y - b.y); }
23 P operator * (const P& a, LD k) { return P(a.x * k, a.y * k); }
24 P operator / (const P& a, LD k) { return P(a.x / k, a.y / k); }
25 inline bool operator < (const P& a, const P& b) {
26     return sgn(a.x - b.x) < 0 || (sgn(a.x - b.x) == 0 && sgn(a.y - b.y) < 0);
27 }
28 bool operator == (const P& a, const P& b) { return !sgn(a.x - b.x) && !sgn(a.y - b.y); }
29 P::P(const L& l) { *this = l.t - l.s; }
30 ostream &operator << (ostream &os, const P &p) {
31     return (os << "(" << p.x << ", " << p.y << ")");
32 }
33 istream &operator >> (istream &is, P &p) {
34     return (is >> p.x >> p.y);
35 }
36

```

```

37 LD dist(const P& p) { return sqrt(p.x * p.x + p.y * p.y); }
38 LD dot(const V& a, const V& b) { return a.x * b.x + a.y * b.y; }
39 LD det(const V& a, const V& b) { return a.x * b.y - a.y * b.x; }
40 LD cross(const P& s, const P& t, const P& o = P()) { return det(s - o, t - o); }
41 // -----

```

象限

```

1 // 象限
2 int quad(P p) {
3     int x = sgn(p.x), y = sgn(p.y);
4     if (x > 0 && y >= 0) return 1;
5     if (x <= 0 && y > 0) return 2;
6     if (x < 0 && y <= 0) return 3;
7     if (x >= 0 && y < 0) return 4;
8     assert(0);
9 }
10
11 // 仅适用于参照点在所有点一侧的情况
12 struct cmp_angle {
13     P p;
14     bool operator () (const P& a, const P& b) {
15         // int qa = quad(a - p), qb = quad(b - p);
16         // if (qa != qb) return qa < qb;
17         int d = sgn(cross(a, b, p));
18         if (d) return d > 0;
19         return dist(a - p) < dist(b - p);
20     }
21 };

```

线

```

1 // 是否平行
2 bool parallel(const L& a, const L& b) {
3     return !sgn(det(P(a), P(b)));
4 }
5 // 直线是否相等
6 bool l_eq(const L& a, const L& b) {
7     return parallel(a, b) && parallel(L(a.s, b.t), L(b.s, a.t));
8 }
9 // 逆时针旋转 r 弧度
10 P rotation(const P& p, const LD& r) { return P(p.x * cos(r) - p.y * sin(r), p.x * sin(r) + p.y * cos(r)); }
11 P RotateCCW90(const P& p) { return P(-p.y, p.x); }
12 P RotateCW90(const P& p) { return P(p.y, -p.x); }
13 // 单位法向量
14 V normal(const V& v) { return V(-v.y, v.x) / dist(v); }

```

点与线

```

1 // 点在线段上 <= 0 包含端点 < 0 则不包含
2 bool p_on_seg(const P& p, const L& seg) {
3     P a = seg.s, b = seg.t;
4     return !sgn(det(p - a, b - a)) && sgn(dot(p - a, p - b)) <= 0;
5 }
6 // 点到直线距离
7 LD dist_to_line(const P& p, const L& l) {
8     return fabs(cross(l.s, l.t, p)) / dist(l);
9 }
10 // 点到线段距离 (考虑端点和垂线)
11 LD dist_to_seg(const P& p, const L& l) {
12     if (l.s == l.t) return dist(p - l);
13     V vs = p - l.s, vt = p - l.t;
14     if (sgn(dot(l, vs)) < 0) return dist(vs);
15     else if (sgn(dot(l, vt)) > 0) return dist(vt);
16     else return dist_to_line(p, l);
17 }

```

线与线

```
1 // 求直线交 需要事先保证有界 (需保证不平行)
2 P l_intersection(const L& a, const L& b) {
3     LD s1 = det(P(a), b.s - a.s), s2 = det(P(a), b.t - a.s);
4     return (b.s * s2 - b.t * s1) / (s2 - s1);
5 }
6 // 向量夹角的弧度
7 LD angle(const V& a, const V& b) {
8     LD r = asin(fabs(det(a, b)) / dist(a) / dist(b));
9     if (sgn(dot(a, b)) < 0) r = PI - r;
10    return r;
11 }
12 // 线段和直线是否有交 1 = 规范 (+), 2 = 不规范 (L, T, I)
13 int s_l_cross(const L& seg, const L& line) {
14     int d1 = sgn(cross(line.s, line.t, seg.s));
15     int d2 = sgn(cross(line.s, line.t, seg.t));
16     if ((d1 ^ d2) == -2) return 1; // proper
17     if (d1 == 0 || d2 == 0) return 2;
18     return 0;
19 }
20 // 线段的交 1 = 规范, 2 = 不规范
21 int s_cross(const L& a, const L& b, P& p) {
22     int d1 = sgn(cross(a.t, b.s, a.s)), d2 = sgn(cross(a.t, b.t, a.s));
23     int d3 = sgn(cross(b.t, a.s, b.s)), d4 = sgn(cross(b.t, a.t, b.s));
24     if ((d1 ^ d2) == -2 && (d3 ^ d4) == -2) { p = l_intersection(a, b); return 1; }
25     if (!d1 && p_on_seg(b.s, a)) { p = b.s; return 2; }
26     if (!d2 && p_on_seg(b.t, a)) { p = b.t; return 2; }
27     if (!d3 && p_on_seg(a.s, b)) { p = a.s; return 2; }
28     if (!d4 && p_on_seg(a.t, b)) { p = a.t; return 2; }
29     return 0;
30 }
```

多边形

面积、凸包

```
1 typedef vector<P> S;
2
3 // 点是否在多边形中 0 = 在外部 1 = 在内部 -1 = 在边界上
4 int inside(const S& s, const P& p) {
5     int cnt = 0;
6     FOR (i, 0, s.size()) {
7         P a = s[i], b = s[nxt(i)];
8         if (p_on_seg(p, L(a, b))) return -1;
9         if (sgn(a.y - b.y) <= 0) swap(a, b);
10        if (sgn(p.y - a.y) > 0) continue;
11        if (sgn(p.y - b.y) <= 0) continue;
12        cnt += sgn(cross(b, a, p)) > 0;
13    }
14    return bool(cnt & 1);
15 }
16 // 多边形面积, 有向面积可能为负
17 LD polygon_area(const S& s) {
18     LD ret = 0;
19     FOR (i, 1, (LL)s.size() - 1)
20         ret += cross(s[i], s[i + 1], s[0]);
21     return ret / 2;
22 }
23 // 构建凸包 点不可以重复 < 0 边上可以有点, <= 0 则不能
24 // 会改变输入点的顺序
25 const int MAX_N = 1000;
26 S convex_hull(S& s) {
27     // assert(s.size() >= 3);
28     sort(s.begin(), s.end());
29     S ret(MAX_N * 2);
30     int sz = 0;
31     FOR (i, 0, s.size()) {
32         while (sz > 1 && sgn(cross(ret[sz - 1], s[i], ret[sz - 2])) < 0) --sz;
33         ret[sz++] = s[i];
34     }
```

```

35     int k = sz;
36     FORD (i, (LL)s.size() - 2, -1) {
37         while (sz > k && sgn(cross(ret[sz - 1], s[i], ret[sz - 2])) < 0) --sz;
38         ret[sz++] = s[i];
39     }
40     ret.resize(sz - (s.size() > 1));
41     return ret;
42 }
43
44 P ComputeCentroid(const vector<P> &p) {
45     P c(0, 0);
46     LD scale = 6.0 * polygon_area(p);
47     for (unsigned i = 0; i < p.size(); i++) {
48         unsigned j = (i + 1) % p.size();
49         c = c + (p[i] + p[j]) * (p[i].x * p[j].y - p[j].x * p[i].y);
50     }
51     return c / scale;
52 }

```

旋转卡壳

若干点中点对距离最大值

```

1 LD rotatingCalipers(vector<P>& qs) {
2     int n = qs.size();
3     if (n == 2)
4         return dist(qs[0] - qs[1]);
5     int i = 0, j = 0;
6     FOR (k, 0, n) {
7         if (!(qs[i] < qs[k])) i = k;
8         if (qs[j] < qs[k]) j = k;
9     }
10    LD res = 0;
11    int si = i, sj = j;
12    while (i != sj || j != si) {
13        res = max(res, dist(qs[i] - qs[j]));
14        if (sgn(cross(qs[(i+1)%n] - qs[i], qs[(j+1)%n] - qs[j])) < 0)
15            i = (i + 1) % n;
16        else j = (j + 1) % n;
17    }
18    return res;
19 }
20
21 int main() {
22     int n;
23     while (cin >> n) {
24         S v(n);
25         FOR (i, 0, n) cin >> v[i].x >> v[i].y;
26         convex_hull(v);
27         printf("%.0f\n", rotatingCalipers(v));
28     }
29 }

```

半平面交

左半平面交

```

1 struct LV {
2     P p, v; LD ang;
3     LV() {}
4     LV(P s, P t): p(s), v(t - s) { ang = atan2(v.y, v.x); }
5 }; // 另一种向量表示
6
7 bool operator < (const LV &a, const LV& b) { return a.ang < b.ang; }
8 bool on_left(const LV& l, const P& p) { return sgn(cross(l.v, p - l.p)) >= 0; }
9 P l_intersection(const LV& a, const LV& b) {
10     P u = a.p - b.p; LD t = cross(b.v, u) / cross(a.v, b.v);
11     return a.p + a.v * t;
12 }
13
14 S half_plane_intersection(vector<LV>& L) {

```

```

15     int n = L.size(), fi, la;
16     sort(L.begin(), L.end());
17     vector<P> p(n); vector<LV> q(n);
18     q[fi = la = 0] = L[0];
19     FOR (i, 1, n) {
20         while (fi < la && !on_left(L[i], p[la - 1])) la--;
21         while (fi < la && !on_left(L[i], p[fi])) fi++;
22         q[++la] = L[i];
23         if (sgn(cross(q[la].v, q[la - 1].v)) == 0) {
24             la--;
25             if (on_left(q[la], L[i].p)) q[la] = L[i];
26         }
27         if (fi < la) p[la - 1] = l_intersection(q[la - 1], q[la]);
28     }
29     while (fi < la && !on_left(q[fi], p[la - 1])) la--;
30     if (la - fi <= 1) return vector<P>();
31     p[la] = l_intersection(q[la], q[fi]);
32     return vector<P>(p.begin() + fi, p.begin() + la + 1);
33 }
34
35 S convex_intersection(const vector<P> &v1, const vector<P> &v2) {
36     vector<LV> h; int n = v1.size(), m = v2.size();
37     FOR (i, 0, n) h.push_back(LV(v1[i], v1[(i + 1) % n]));
38     FOR (i, 0, m) h.push_back(LV(v2[i], v2[(i + 1) % m]));
39     return half_plane_intersection(h);
40 }

```

圓

```

1 struct C {
2     P p; LD r;
3     C(LD x = 0, LD y = 0, LD r = 0): p(x, y), r(r) {}
4     C(P p, LD r): p(p), r(r) {}
5 };

```

三点求圓心

```

1 P compute_circle_center(P a, P b, P c) {
2     b = (a + b) / 2;
3     c = (a + c) / 2;
4     return l_intersection({b, b + RotateCW90(a - b)}, {c, c + RotateCW90(a - c)});
5 }

```

圓线交点、圓圓交点

- 圆和线的交点关于圆心是顺时针的

```

1 vector<P> c_l_intersection(const L& l, const C& c) {
2     vector<P> ret;
3     P b(l), a = l.s - c.p;
4     LD x = dot(b, b), y = dot(a, b), z = dot(a, a) - c.r * c.r;
5     LD D = y * y - x * z;
6     if (sgn(D) < 0) return ret;
7     ret.push_back(c.p + a + b * (-y + sqrt(D + eps)) / x);
8     if (sgn(D) > 0) ret.push_back(c.p + a + b * (-y - sqrt(D)) / x);
9     return ret;
10 }
11
12 vector<P> c_c_intersection(C a, C b) {
13     vector<P> ret;
14     LD d = dist(a.p - b.p);
15     if (sgn(d) == 0 || sgn(d - (a.r + b.r)) > 0 || sgn(d + min(a.r, b.r) - max(a.r, b.r)) < 0)
16         return ret;
17     LD x = (d * d - b.r * b.r + a.r * a.r) / (2 * d);
18     LD y = sqrt(a.r * a.r - x * x);
19     P v = (b.p - a.p) / d;
20     ret.push_back(a.p + v * x + RotateCCW90(v) * y);
21     if (sgn(y) > 0) ret.push_back(a.p + v * x - RotateCCW90(v) * y);
22     return ret;
23 }

```

圖圓位置关系

```
1 // 1: 内含 2: 内切 3: 相交 4: 外切 5: 相离
2 int c_c_relation(const C& a, const C& v) {
3     LD d = dist(a.p - v.p);
4     if (sgn(d - a.r - v.r) > 0) return 5;
5     if (sgn(d - a.r - v.r) == 0) return 4;
6     LD l = fabs(a.r - v.r);
7     if (sgn(d - l) > 0) return 3;
8     if (sgn(d - l) == 0) return 2;
9     if (sgn(d - l) < 0) return 1;
10 }
```

圖与多边形交

- HDU 5130
- 注意顺时针逆时针（可能要取绝对值）

```
1 LD sector_area(const P& a, const P& b, LD r) {
2     LD th = atan2(a.y, a.x) - atan2(b.y, b.x);
3     while (th <= 0) th += 2 * PI;
4     while (th > 2 * PI) th -= 2 * PI;
5     th = min(th, 2 * PI - th);
6     return r * r * th / 2;
7 }
8
9 LD c_tri_area(P a, P b, P center, LD r) {
10     a = a - center; b = b - center;
11     int ina = sgn(dist(a) - r) < 0, inb = sgn(dist(b) - r) < 0;
12     // dbg(a, b, ina, inb);
13     if (ina && inb) {
14         return fabs(cross(a, b)) / 2;
15     } else {
16         auto p = c_l_intersection(L(a, b), C(0, 0, r));
17         if (ina ^ inb) {
18             auto cr = p_on_seg(p[0], L(a, b)) ? p[0] : p[1];
19             if (ina) return sector_area(b, cr, r) + fabs(cross(a, cr)) / 2;
20             else return sector_area(a, cr, r) + fabs(cross(b, cr)) / 2;
21         } else {
22             if ((int) p.size() == 2 && p_on_seg(p[0], L(a, b))) {
23                 if (dist(p[0] - a) > dist(p[1] - a)) swap(p[0], p[1]);
24                 return sector_area(a, p[0], r) + sector_area(p[1], b, r)
25                     + fabs(cross(p[0], p[1])) / 2;
26             } else return sector_area(a, b, r);
27         }
28     }
29 }
30
31 typedef vector<P> S;
32 LD c_poly_area(S poly, const C& c) {
33     LD ret = 0; int n = poly.size();
34     FOR (i, 0, n) {
35         int t = sgn(cross(poly[i] - c.p, poly[(i + 1) % n] - c.p));
36         if (t) ret += t * c_tri_area(poly[i], poly[(i + 1) % n], c.p, c.r);
37     }
38     return ret;
39 }
```

圖的离散化、面积并

SPOJ: CIRU, EOJ: 284

- 版本 1: 复杂度 $O(n^3 \log n)$ 。虽然常数小，但还是难以接受。
- 优点？想不出来。
- 原理上是用竖线进行切分，然后对每一个切片分别计算。
- 扫描线部分可以魔改，求各种东西。

```
1 inline LD rt(LD x) { return sgn(x) == 0 ? 0 : sqrt(x); }
2 inline LD sq(LD x) { return x * x; }
3
```

```

4 // 圆弧
5 // 如果按照 x 离散化, 圆弧是 " 横着的 "
6 // 记录圆弧的左端点、右端点、中点的坐标, 和圆弧所在的圆
7 // 调用构造要保证  $c.x - x.r \leq x_l < x_r \leq c.y + x.r$ 
8 //  $t = 1$  下圆弧  $t = -1$  上圆弧
9 struct CV {
10     LD yl, yr, ym; C o; int type;
11     CV() {}
12     CV(LD yl, LD yr, LD ym, C c, int t)
13         : yl(yl), yr(yr), ym(ym), type(t), o(c) {}
14 };
15
16 // 辅助函数 求圆上纵坐标
17 pair<LD, LD> c_point_eval(const C& c, LD x) {
18     LD d = fabs(c.p.x - x), h = rt(sq(c.r) - sq(d));
19     return {c.p.y - h, c.p.y + h};
20 }
21 // 构造上下圆弧
22 pair<CV, CV> pairwise_curves(const C& c, LD xl, LD xr) {
23     LD yl1, yl2, yr1, yr2, ym1, ym2;
24     tie(yl1, yl2) = c_point_eval(c, xl);
25     tie(ym1, ym2) = c_point_eval(c, (xl + xr) / 2);
26     tie(yr1, yr2) = c_point_eval(c, xr);
27     return {CV(yl1, yr1, ym1, c, 1), CV(yl2, yr2, ym2, c, -1)};
28 }
29
30 // 离散化之后同一切片内的圆弧应该是不相交的
31 bool operator < (const CV& a, const CV& b) { return a.ym < b.ym; }
32 // 计算圆弧和连接圆弧端点的线段构成的封闭图形的面积
33 LD cv_area(const CV& v, LD xl, LD xr) {
34     LD l = rt(sq(xr - xl) + sq(v.yr - v.yl));
35     LD d = rt(sq(v.o.r) - sq(l / 2));
36     LD ang = atan(l / d / 2);
37     return ang * sq(v.o.r) - d * l / 2;
38 }
39
40 LD circle_union(const vector<C>& cs) {
41     int n = cs.size();
42     vector<LD> xs;
43     FOR (i, 0, n) {
44         xs.push_back(cs[i].p.x - cs[i].r);
45         xs.push_back(cs[i].p.x);
46         xs.push_back(cs[i].p.x + cs[i].r);
47         FOR (j, i + 1, n) {
48             auto pts = c_c_intersection(cs[i], cs[j]);
49             for (auto& p: pts) xs.push_back(p.x);
50         }
51     }
52     sort(xs.begin(), xs.end());
53     xs.erase(unique(xs.begin(), xs.end(), [](LD x, LD y) { return sgn(x - y) == 0; }), xs.end());
54     LD ans = 0;
55     FOR (i, 0, (int) xs.size() - 1) {
56         LD xl = xs[i], xr = xs[i + 1];
57         vector<CV> intv;
58         FOR (k, 0, n) {
59             auto& c = cs[k];
60             if (sgn(c.p.x - c.r - xl) <= 0 && sgn(c.p.x + c.r - xr) >= 0) {
61                 auto t = pairwise_curves(c, xl, xr);
62                 intv.push_back(t.first); intv.push_back(t.second);
63             }
64         }
65         sort(intv.begin(), intv.end());
66
67         vector<LD> areas(intv.size());
68         FOR (i, 0, intv.size()) areas[i] = cv_area(intv[i], xl, xr);
69
70         int cc = 0;
71         FOR (i, 0, intv.size()) {
72             if (cc > 0) {
73                 ans += (intv[i].yl - intv[i - 1].yl + intv[i].yr - intv[i - 1].yr) * (xr - xl) / 2;
74                 ans += intv[i - 1].type * areas[i - 1];

```



```

75         ans -= intv[i].type * areas[i];
76     }
77     cc += intv[i].type;
78 }
79 }
80 return ans;
81 }

```

- 版本 2: 复杂度 $O(n^2 \log n)$ 。
- 原理是: 认为所求部分是一个奇怪的多边形 + 若干弓形。然后对于每个圆分别求贡献的弓形, 并累加多边形有向面积。
- 同样可以魔改扫描线的部分, 用于求周长、至少覆盖 k 次等等。
- 内含、内切、同一个圆的情况, 通常需要特殊处理。
- 下面的代码是 k 圆覆盖。

```

1  inline LD angle(const P& p) { return atan2(p.y, p.x); }
2
3  // 圆弧上的点
4  // p 是相对于圆心的坐标
5  // a 是在圆上的 atan2 [-PI, PI]
6  struct CP {
7      P p; LD a; int t;
8      CP() {}
9      CP(P p, LD a, int t): p(p), a(a), t(t) {}
10 };
11 bool operator < (const CP& u, const CP& v) { return u.a < v.a; }
12 LD cv_area(LD r, const CP& q1, const CP& q2) {
13     return (r * r * (q2.a - q1.a) - cross(q1.p, q2.p)) / 2;
14 }
15
16 LD ans[N];
17 void circle_union(const vector<C>& cs) {
18     int n = cs.size();
19     FOR (i, 0, n) {
20         // 有相同的圆的话只考虑第一次出现
21         bool ok = true;
22         FOR (j, 0, i)
23             if (sgn(cs[i].r - cs[j].r) == 0 && cs[i].p == cs[j].p) {
24                 ok = false;
25                 break;
26             }
27         if (!ok) continue;
28         auto& c = cs[i];
29         vector<CP> ev;
30         int belong_to = 0;
31         P bound = c.p + P(-c.r, 0);
32         ev.emplace_back(bound, -PI, 0);
33         ev.emplace_back(bound, PI, 0);
34         FOR (j, 0, n) {
35             if (i == j) continue;
36             if (c_c_relation(c, cs[j]) <= 2) {
37                 if (sgn(cs[j].r - c.r) >= 0) // 完全被另一个圆包含, 等于说叠了一层
38                     belong_to++;
39                 continue;
40             }
41             auto its = c_c_intersection(c, cs[j]);
42             if (its.size() == 2) {
43                 P p = its[1] - c.p, q = its[0] - c.p;
44                 LD a = angle(p), b = angle(q);
45                 if (sgn(a - b) > 0) {
46                     ev.emplace_back(p, a, 1);
47                     ev.emplace_back(bound, PI, -1);
48                     ev.emplace_back(bound, -PI, 1);
49                     ev.emplace_back(q, b, -1);
50                 } else {
51                     ev.emplace_back(p, a, 1);
52                     ev.emplace_back(q, b, -1);
53                 }
54             }
55         }
56         sort(ev.begin(), ev.end());
57         int cc = ev[0].t;

```

```

58     FOR (j, 1, ev.size()) {
59         int t = cc + belong_to;
60         ans[t] += cross(ev[j - 1].p + c.p, ev[j].p + c.p) / 2;
61         ans[t] += cv_area(c.r, ev[j - 1], ev[j]);
62         cc += ev[j].t;
63     }
64 }
65 }

```

最小圆覆盖

- 随机增量。期望复杂度 $O(n)$ 。

```

1  P compute_circle_center(P a, P b) { return (a + b) / 2; }
2  bool p_in_circle(const P& p, const C& c) {
3      return sgn(dist(p - c.p) - c.r) <= 0;
4  }
5  C min_circle_cover(const vector<P> &in) {
6      vector<P> a(in.begin(), in.end());
7      dbg(a.size());
8      random_shuffle(a.begin(), a.end());
9      P c = a[0]; LD r = 0; int n = a.size();
10     FOR (i, 1, n) if (!p_in_circle(a[i], {c, r})) {
11         c = a[i]; r = 0;
12         FOR (j, 0, i) if (!p_in_circle(a[j], {c, r})) {
13             c = compute_circle_center(a[i], a[j]);
14             r = dist(a[j] - c);
15             FOR (k, 0, j) if (!p_in_circle(a[k], {c, r})) {
16                 c = compute_circle_center(a[i], a[j], a[k]);
17                 r = dist(a[k] - c);
18             }
19         }
20     }
21     return {c, r};
22 }

```

圆的反演

```

1  C inv(C c, const P& o) {
2      LD d = dist(c.p - o);
3      assert(sgn(d) != 0);
4      LD a = 1 / (d - c.r);
5      LD b = 1 / (d + c.r);
6      c.r = (a - b) / 2 * R2;
7      c.p = o + (c.p - o) * ((a + b) * R2 / 2 / d);
8      return c;
9  }

```

三维计算几何

```

1  struct P;
2  struct L;
3  typedef P V;
4
5  struct P {
6      LD x, y, z;
7      explicit P(LD x = 0, LD y = 0, LD z = 0): x(x), y(y), z(z) {}
8      explicit P(const L& l);
9  };
10
11 struct L {
12     P s, t;
13     L() {}
14     L(P s, P t): s(s), t(t) {}
15 };
16
17 struct F {
18     P a, b, c;
19     F() {}
20     F(P a, P b, P c): a(a), b(b), c(c) {}

```

```

21 };
22
23 P operator + (const P& a, const P& b) { return P(a.x + b.x, a.y + b.y, a.z + b.z); }
24 P operator - (const P& a, const P& b) { return P(a.x - b.x, a.y - b.y, a.z - b.z); }
25 P operator * (const P& a, LD k) { return P(a.x * k, a.y * k, a.z * k); }
26 P operator / (const P& a, LD k) { return P(a.x / k, a.y / k, a.z / k); }
27 inline int operator < (const P& a, const P& b) {
28     return sgn(a.x - b.x) < 0 || (sgn(a.x - b.x) == 0 && (sgn(a.y - b.y) < 0 ||
29         (sgn(a.y - b.y) == 0 && sgn(a.z - b.z) < 0)));
30 }
31 bool operator == (const P& a, const P& b) { return !sgn(a.x - b.x) && !sgn(a.y - b.y) && !sgn(a.z - b.z); }
32 P::P(const L& l) { *this = l.t - l.s; }
33 ostream &operator << (ostream &os, const P &p) {
34     return (os << "(" << p.x << ", " << p.y << ", " << p.z << ")");
35 }
36 istream &operator >> (istream &is, P &p) {
37     return (is >> p.x >> p.y >> p.z);
38 }
39
40 // -----
41 LD dist2(const P& p) { return p.x * p.x + p.y * p.y + p.z * p.z; }
42 LD dist(const P& p) { return sqrt(dist2(p)); }
43 LD dot(const V& a, const V& b) { return a.x * b.x + a.y * b.y + a.z * b.z; }
44 P cross(const P& v, const P& w) {
45     return P(v.y * w.z - v.z * w.y, v.z * w.x - v.x * w.z, v.x * w.y - v.y * w.x);
46 }
47 LD mix(const V& a, const V& b, const V& c) { return dot(a, cross(b, c)); } // 混合积

```

旋转

```

1 // 逆时针旋转 r 弧度
2 // axis = 0 绕 x 轴
3 // axis = 1 绕 y 轴
4 // axis = 2 绕 z 轴
5 P rotation(const P& p, const LD& r, int axis = 0) {
6     if (axis == 0)
7         return P(p.x, p.y * cos(r) - p.z * sin(r), p.y * sin(r) + p.z * cos(r));
8     else if (axis == 1)
9         return P(p.z * cos(r) - p.x * sin(r), p.y, p.z * sin(r) + p.x * cos(r));
10    else if (axis == 2)
11        return P(p.x * cos(r) - p.y * sin(r), p.x * sin(r) + p.y * cos(r), p.z);
12 }
13 // n 是单位向量 表示旋转轴
14 // 模板是顺时针的
15 P rotation(const P& p, const LD& r, const P& n) {
16     LD c = cos(r), s = sin(r), x = n.x, y = n.y, z = n.z;
17     // dbg(c, s);
18     return P((x * x * (1 - c) + c) * p.x + (x * y * (1 - c) + z * s) * p.y + (x * z * (1 - c) - y * s) * p.z,
19         (x * y * (1 - c) - z * s) * p.x + (y * y * (1 - c) + c) * p.y + (y * z * (1 - c) + x * s) * p.z,
20         (x * z * (1 - c) + y * s) * p.x + (y * z * (1 - c) - x * s) * p.y + (z * z * (1 - c) + c) * p.z);
21 }

```

线、面

函数相互依赖，所以交织在一起了。

```

1 // 点在线段上 <= 0 包含端点 < 0 则不包含
2 bool p_on_seg(const P& p, const L& seg) {
3     P a = seg.s, b = seg.t;
4     return !sgn(dist2(cross(p - a, b - a))) && sgn(dot(p - a, p - b)) <= 0;
5 }
6 // 点到直线距离
7 LD dist_to_line(const P& p, const L& l) {
8     return dist(cross(l.s - p, l.t - p)) / dist(l);
9 }
10 // 点到线段距离
11 LD dist_to_seg(const P& p, const L& l) {
12     if (l.s == l.t) return dist(p - l.s);
13     V vs = p - l.s, vt = p - l.t;
14     if (sgn(dot(l, vs)) < 0) return dist(vs);
15     else if (sgn(dot(l, vt)) > 0) return dist(vt);

```

```

16     else return dist_to_line(p, l);
17 }
18
19 P norm(const F& f) { return cross(f.a - f.b, f.b - f.c); }
20 int p_on_plane(const F& f, const P& p) { return sgn(dot(norm(f), p - f.a)) == 0; }
21
22 // 判两点在线段异侧 点在线段上返回 0 不共面无意义
23 int opposite_side(const P& u, const P& v, const L& l) {
24     return sgn(dot(cross(P(l), u - l.s), cross(P(l), v - l.s))) < 0;
25 }
26
27 bool parallel(const L& a, const L& b) { return !sgn(dist2(cross(P(a), P(b)))); }
28 // 线段相交
29 int s_intersect(const L& u, const L& v) {
30     return p_on_plane(F(u.s, u.t, v.s), v.t) &&
31         opposite_side(u.s, u.t, v) &&
32         opposite_side(v.s, v.t, u);
33 }

```

凸包

增量法。先将所有的点打乱顺序，然后选择四个不共面的点组成一个四面体，如果找不到说明凸包不存在。然后遍历剩余的点，不断更新凸包。对遍历到的点做如下处理。

1. 如果点在凸包内，则不更新。
2. 如果点在凸包外，那么找到所有原凸包上所有分隔了这个点可见面和不可见面的边，以这样的边的两个点和新的点创建新的面加入凸包中。

```

1
2 struct FT {
3     int a, b, c;
4     FT() { }
5     FT(int a, int b, int c) : a(a), b(b), c(c) { }
6 };
7
8 bool p_on_line(const P& p, const L& l) {
9     return !sgn(dist2(cross(p - l.s, P(l))));
10 }
11
12 vector<F> convex_hull(vector<P> &p) {
13     sort(p.begin(), p.end());
14     p.erase(unique(p.begin(), p.end()), p.end());
15     random_shuffle(p.begin(), p.end());
16     vector<FT> face;
17     FOR (i, 2, p.size()) {
18         if (p_on_line(p[i], L(p[0], p[1]))) continue;
19         swap(p[i], p[2]);
20         FOR (j, i + 1, p.size())
21             if (sgn(mix(p[1] - p[0], p[2] - p[1], p[j] - p[0]))) {
22                 swap(p[j], p[3]);
23                 face.emplace_back(0, 1, 2);
24                 face.emplace_back(0, 2, 1);
25                 goto found;
26             }
27     }
28 found:
29     vector<vector<int>> mk(p.size(), vector<int>(p.size()));
30     FOR (v, 3, p.size()) {
31         vector<FT> tmp;
32         FOR (i, 0, face.size()) {
33             int a = face[i].a, b = face[i].b, c = face[i].c;
34             if (sgn(mix(p[a] - p[v], p[b] - p[v], p[c] - p[v])) < 0) {
35                 mk[a][b] = mk[b][a] = v;
36                 mk[b][c] = mk[c][b] = v;
37                 mk[c][a] = mk[a][c] = v;
38             } else tmp.push_back(face[i]);
39         }
40         face = tmp;
41         FOR (i, 0, tmp.size()) {
42             int a = face[i].a, b = face[i].b, c = face[i].c;

```

```

43         if (mk[a][b] == v) face.emplace_back(b, a, v);
44         if (mk[b][c] == v) face.emplace_back(c, b, v);
45         if (mk[c][a] == v) face.emplace_back(a, c, v);
46     }
47 }
48 vector<F> out;
49 FOR (i, 0, face.size())
50     out.emplace_back(p[face[i].a], p[face[i].b], p[face[i].c]);
51 return out;
52 }

```

距离

- 欧氏距离

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- 曼哈顿距离

$$d(A, B) = |x_1 - x_2| + |y_1 - y_2|$$

便于求一个点任意一点到其他所有点的距离之和。

- 切比雪夫距离

$$d(A, B) = \max(|x_1 - x_2|, |y_1 - y_2|)$$

便于求任意两点间距离的最值。

- 距离转化

假设 $A(x_1, y_1), B(x_2, y_2)$,

- A, B 两点的曼哈顿距离为 $(x_1 + y_1, x_1 - y_1), (x_2 + y_2, x_2 - y_2)$ 两点之间的切比雪夫距离。
- A, B 两点的切比雪夫距离为 $(\frac{x_1 + y_1}{2}, \frac{x_1 - y_1}{2}), (\frac{x_2 + y_2}{2}, \frac{x_2 - y_2}{2})$ 两点之间的曼哈顿距离。
- 距离之和

```

1  sumx[0] = 0;
2  sumy[0] = 0;
3  LL i, tx, ty;
4  cin >> n;
5  for(i=1; i<=n; i++){
6      cin >> tx >> ty;
7      // 求曼哈顿距离之和
8      x[i] = hx[i] = tx;
9      y[i] = hy[i] = ty;
10     // 求切比雪夫距离之和
11     x[i] = hx[i] = tx + ty;
12     y[i] = hy[i] = tx - ty;
13 }
14 sort(hx+1, hx+1+n);
15 sort(hy+1, hy+1+n);
16 for(i=1; i<=n; i++){
17     sumx[i] = sumx[i-1] + hx[i];
18     sumy[i] = sumy[i-1] + hy[i];
19 }
20
21 LL calc_sum(LL i){
22     LL xi = lower_bound(hx+1, hx+1+n, x[i]) - hx;
23     LL yi = lower_bound(hy+1, hy+1+n, y[i]) - hy;
24     return xi * x[i] - sumx[xi] + sumx[n] - sumx[xi] - (n-xi) * x[i]
25     + yi * y[i] - sumy[yi] + sumy[n] - sumy[yi] - (n-yi) * y[i];
26 }
27

```

```

28 // 求 i 点与其他所有点曼哈顿距离之和
29 calc_sum(i);
30 // 求 i 点与其他所有点切比雪夫距离之和
31 calc_sum(i) / 2;

```

字符串

最小表示法

- 寻找一个字符串的循环同构串中最小的那一个，输出偏移量

```

1 int min_string(string s){
2     int k = 0, i = 0, j = 1, n = s.length();
3     while (k < n && i < n && j < n) {
4         if (s[(i + k) % n] == s[(j + k) % n]) {
5             k++;
6         } else {
7             s[(i + k) % n] > s[(j + k) % n] ? i = i + k + 1 : j = j + k + 1;
8             if (i == j) i++;
9             k = 0;
10        }
11    }
12    return min(i, j);
13 }

```

字符串哈希

```

1 // 双值哈希开关
2 #define ENABLE_DOUBLE_HASH
3
4 typedef long long LL;
5 typedef unsigned long long ULL;
6
7 const int x = 135;
8 const int N = 4e5 + 10;
9 const int p1 = 1e9 + 7, p2 = 1e9 + 9;
10 ULL xp1[N], xp2[N], xp[N];
11
12 void init_xp() {
13     xp1[0] = xp2[0] = xp[0] = 1;
14     for (int i = 1; i < N; ++i) {
15         xp1[i] = xp1[i - 1] * x % p1;
16         xp2[i] = xp2[i - 1] * x % p2;
17         xp[i] = xp[i - 1] * x;
18     }
19 }
20
21 struct String {
22     string s;
23     int length, subsize;
24     bool sorted;
25     ULL h[N], hl[N];
26
27     // 预处理并返回全串哈希 O(n)
28     ULL hash() {
29         length = s.length();
30         ULL res1 = 0, res2 = 0;
31         h[length] = 0; // ATTENTION!
32         for (int j = length - 1; j >= 0; --j) {
33             #ifdef ENABLE_DOUBLE_HASH
34                 res1 = (res1 * x + s[j]) % p1;
35                 res2 = (res2 * x + s[j]) % p2;
36                 h[j] = (res1 << 32) | res2;
37             #else
38                 res1 = res1 * x + s[j];
39                 h[j] = res1;
40             #endif
41             // printf("%llu\n", h[j]);
42         }
43     }
44 }

```

```

43     return h[0];
44 }
45
46 // 获取子串哈希, 左闭右开区间  $O(1)$ 
47 ULL get_substring_hash(int left, int right) const {
48     int len = right - left;
49 #ifdef ENABLE_DOUBLE_HASH
50     // get hash of s[left...right-1]
51     unsigned int mask32 = ~(0u);
52     ULL left1 = h[left] >> 32, right1 = h[right] >> 32;
53     ULL left2 = h[left] & mask32, right2 = h[right] & mask32;
54     return (((left1 - right1 * xp1[len] % p1 + p1) % p1) << 32) |
55            (((left2 - right2 * xp2[len] % p2 + p2) % p2));
56 #else
57     return h[left] - h[right] * xp[len];
58 #endif
59 }
60
61 void get_all_subs_hash(int sublen) {
62     subsize = length - sublen + 1;
63     for (int i = 0; i < subsize; ++i)
64         hl[i] = get_substring_hash(i, i + sublen);
65     sorted = 0;
66 }
67
68 void sort_substring_hash() {
69     sort(hl, hl + subsize);
70     sorted = 1;
71 }
72
73 bool match(ULL key) const {
74     // if (!sorted) assert (0);
75     if (!subsize) return false;
76     return binary_search(hl, hl + subsize, key);
77 }
78
79 void init(string t) {
80     length = t.length();
81     s = t;
82 }
83 };
84
85 String S, T; // 栈溢出
86
87 // 验证 S 中长度为 ans 的子串是否都存在于 T 中 (是 0 否 1)
88 int check(String &S, String &T, int ans) {
89     if (T.length < ans) return 1;
90     T.get_all_subs_hash(ans); T.sort_substring_hash();
91     for (int i = 0; i < S.length - ans + 1; ++i)
92         if (!T.match(S.get_substring_hash(i, i + ans)))
93             return 1;
94     return 0;
95 }
96
97 // 返回是否匹配
98 bool match_once(String &S, String &T){
99     S.get_all_subs_hash(T.length);
100    S.sort_substring_hash();
101    return S.match(T.get_substring_hash(0, T.length));
102 }
103
104 // 返回匹配下标
105 vector<int> match_any(const String &text, const String &pattern) {
106     vector<int> positions;
107     int n = text.length;
108     int m = pattern.length;
109
110     if (m == 0 || m > n) return positions;
111
112     ULL pattern_hash = pattern.get_substring_hash(0, m);
113

```

```

114     for (int i = 0; i <= n - m; ++i) {
115         ULL text_sub_hash = text.get_substring_hash(i, i + m);
116         if (text_sub_hash == pattern_hash) {
117             positions.push_back(i);
118         }
119     }
120     return positions;
121 }
122
123 // 最长公共前缀 a[ai...] == b[bi...]
124 int LCP(const String &a, const String &b, int ai, int bi) {
125     int l = 0, r = min(a.length - ai, b.length - bi);
126     while (l < r) {
127         int mid = (l + r + 1) / 2;
128         if (a.get_substring_hash(ai, ai + mid) == b.get_substring_hash(bi, bi + mid))
129             l = mid;
130         else r = mid - 1;
131     }
132     return l;
133 }
134
135 // ----- Template End -----
136 // !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!! Tester Start !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
137
138 void solve(){
139     // cout << "AA\n";
140     init_xp(); // DON'T FORGET TO DO THIS!
141     // cout << "BB\n";
142     string s, t;
143     cin >> s >> t;
144     S.init(s), T.init(t);
145     S.hash(), T.hash();
146     cout << match_once(S, T) << '\n';
147
148     vector<int> v = match_any(S, T);
149     for(int ii: v) cout << ii << ' ';
150     cout << '\n';
151
152     cout << "LCP:" << LCP(S, T, 0, 0) << '\n';
153
154     // S 中所有长度为 l 的子串均在 T 中出现, 且 l 最大
155     LL l=0, r=S.length;
156     while (l < r){
157         int mid = l + r + 1 >> 1;
158         if (!check(S, T, mid)) l = mid;
159         else r = mid - 1;
160     }
161     cout << "check: " << l << '\n';
162 }
163

```

后缀自动机

- 状态: 后缀状态
- 计算每个状态出现的次数
- 统计不同子串的数量
- 查找与 s 的最长公共子串长度
- 查找子串 s 的出现次数

```

1  const LL N = 100005;
2
3  namespace sam {
4      const int M = N << 1;
5      int t[M][26], len[M] = {-1}, fa[M], cnt[M], sz = 2, last = 1;
6
7      void init() {
8          memset(t, 0, (sz + 10) * sizeof(t[0]));
9          memset(cnt, 0, sizeof(cnt));
10         sz = 2;
11         last = 1;

```



```

12     }
13
14     void ins(int ch) {
15         int p = last, np = last = sz++;
16         len[np] = len[p] + 1;
17         cnt[np] = 1; // 新创建的状态出现次数为 1
18         for (; p && !t[p][ch]; p = fa[p])
19             t[p][ch] = np;
20         if (!p) {
21             fa[np] = 1;
22             return;
23         }
24         int q = t[p][ch];
25         if (len[p] + 1 == len[q]) {
26             fa[np] = q;
27         } else {
28             int nq = sz++;
29             len[nq] = len[p] + 1;
30             memcpy(t[nq], t[q], sizeof(t[0]));
31             fa[nq] = fa[q];
32             fa[np] = fa[q] = nq;
33             for (; t[p][ch] == q; p = fa[p])
34                 t[p][ch] = nq;
35         }
36     }
37
38     int c[M] = {1}, a[M];
39     void rsort() {
40         FOR (i, 1, sz) c[i] = 0;
41         FOR (i, 1, sz) c[len[i]]++;
42         FOR (i, 1, sz) c[i] += c[i - 1];
43         FOR (i, 1, sz) a[--c[len[i]]] = i;
44     }
45
46     // 计算每个状态的出现次数
47     void calc_occurrences() {
48         rsort();
49         for (int i = sz - 1; i >= 0; --i) {
50             int u = a[i];
51             if (fa[u] > 1) {
52                 cnt[fa[u]] += cnt[u];
53             }
54         }
55     }
56 }
57
58 // 统计不同子串的数量
59 long long count_distinct_substrings() {
60     long long ans = 0;
61     for (int i = 2; i < sam.sz; ++i) {
62         ans += sam.len[i] - sam.len[sam.fa[i]];
63     }
64     return ans;
65 }
66
67 // 查找与 s 的最长公共子串长度
68 int longest_common_substring(const string &s) {
69     int now = 1, current_len = 0, max_len = 0;
70     for (char c : s) {
71         int ch = c - 'a';
72         while (now != 1 && !sam.t[now][ch]) {
73             now = sam.fa[now];
74             current_len = sam.len[now];
75         }
76         if (sam.t[now][ch]) {
77             now = sam.t[now][ch];
78             current_len++;
79         }
80         max_len = max(max_len, current_len);
81     }
82     return max_len;

```

```

83 }
84
85 // 查找子串 s 的出现次数
86 int find_occurrences(const string &s) {
87     int now = 1;
88     for (char c : s) {
89         int ch = c - 'a';
90         if (!sam::t[now][ch]) return 0;
91         now = sam::t[now][ch];
92     }
93     return sam::cnt[now];
94 }

```

回文自动机

- 所有本质不同的回文串及其个数

```

1  const int N = 100010;
2
3  namespace pam {
4      int t[N][26], fa[N], len[N], rs[N], cnt[N], num[N];
5      int sz, n, last;
6      int _new(int l) {
7          len[sz] = l; cnt[sz] = num[sz] = 0;
8          return sz++;
9      }
10     void init() {
11         memset(t, 0, sz * sizeof t[0]);
12         rs[n = sz = 0] = -1;
13         last = _new(0);
14         fa[last] = _new(-1);
15     }
16     int get_fa(int x) {
17         while (rs[n - 1 - len[x]] != rs[n]) x = fa[x];
18         return x;
19     }
20     void ins(int ch) {
21         rs[++n] = ch;
22         int p = get_fa(last);
23         if (!t[p][ch]) {
24             int np = _new(len[p] + 2);
25             num[np] = num[fa[np]] = t[get_fa(fa[p])][ch] + 1;
26             t[p][ch] = np;
27         }
28         ++cnt[last = t[p][ch]];
29     }
30 }
31
32 // 计算每个回文子串的实际出现次数
33 void calc_count() {
34     // 按长度从大到小排序节点
35     int order[N];
36     for (int i = 0; i < pam::sz; ++i) order[i] = i;
37     sort(order, order + pam::sz, [&](int a, int b) {
38         return pam::len[a] > pam::len[b];
39     });
40
41     // 从长回文串向短回文串累加计数
42     for (int i = 0; i < pam::sz; ++i) {
43         int v = order[i];
44         if (pam::fa[v] != v) { // 不是辅助节点
45             pam::cnt[pam::fa[v]] += pam::cnt[v];
46         }
47     }
48 }
49
50 // 回溯获取所有回文串
51 void dfs(int u, string current, vector<pair<string, int>>& result) {
52     if (pam::len[u] <= 0) {
53         for (int c = 0; c < 26; ++c) {
54             if (pam::t[u][c]) {

```

```

55         char ch = 'a' + c;
56         string new_str;
57         if (pam::len[u] == -1) {
58             new_str = string(1, ch);
59         } else {
60             new_str = string(1, ch) + current + string(1, ch);
61         }
62         dfs(pam::t[u][c], new_str, result);
63     }
64 }
65 return;
66 }
67
68 result.emplace_back(current, pam::cnt[u]);
69
70 for (int c = 0; c < 26; ++c) {
71     if (pam::t[u][c]) {
72         char ch = 'a' + c;
73         string new_str = string(1, ch) + current + string(1, ch);
74         dfs(pam::t[u][c], new_str, result);
75     }
76 }
77 }
78
79 // ----- Template End -----
80 // !!!!!!!!!!!!!!!!!!!!!!!!!!!!! Tester Start !!!!!!!!!!!!!!!!!!!!!!!!!!!!!
81
82 void solve(){
83     string s;
84     cout << " 请输入字符串: ";
85     cin >> s;
86
87     // 初始化并构建回文自动机
88     pam::init();
89     for (char c : s) {
90         pam::ins(c - 'a'); // 转换为 0-25 的范围
91     }
92
93     // 计算每个回文子串的实际出现次数
94     calc_count();
95
96     // 回溯获取所有本质不同的回文串
97     vector<pair<string, int>> palindromes;
98     dfs(0, "", palindromes); // 从长度 0 的根节点开始
99     dfs(1, "", palindromes); // 从长度-1 的根节点开始
100
101     // 去重 (某些特殊情况可能产生重复)
102     sort(palindromes.begin(), palindromes.end());
103     auto last = unique(palindromes.begin(), palindromes.end());
104     palindromes.erase(last, palindromes.end());
105
106     // 输出结果
107     cout << " 字符串: " << s << endl;
108     cout << " 本质不同的回文子串总数: " << palindromes.size() << endl;
109     cout << " 所有回文子串及其出现次数: " << endl;
110     for (auto& [p, cnt] : palindromes) {
111         cout << " 回文串: \"" << p << "\", 出现次数: " << cnt << endl;
112     }
113 }

```

Manacher

- 查找最长回文子串

```

1  const LL N = 100005;
2
3  int RL[N];
4  void manacher(int* a, int n) { // "abc" => "#a#b#a#"
5      int r = 0, p = 0;
6      FOR (i, 0, n) {
7          if (i < r) RL[i] = min(RL[2 * p - i], r - i);

```

```

8         else RL[i] = 1;
9         while (i - RL[i] >= 0 && i + RL[i] < n && a[i - RL[i]] == a[i + RL[i]])
10             RL[i]++;
11         if (RL[i] + i - 1 > r) { r = RL[i] + i - 1; p = i; }
12     }
13     FOR (i, 0, n) --RL[i];
14 }
15
16 // 查找最长回文子串
17 string longestPalindrome(string s) {
18     int n = s.size();
19     if (n == 0) return "";
20
21     int len = 2 * n + 1;
22     int* arr = new int[len];
23     FOR(i, 0, len) {
24         if (i % 2 == 0) arr[i] = '#';
25         else arr[i] = s[i / 2];
26     }
27     manacher(arr, len);
28     int maxlen = 0, center = 0;
29     FOR(i, 0, len) {
30         if (RL[i] > maxlen) {
31             maxlen = RL[i];
32             center = i;
33         }
34     }
35     int start = (center - maxlen) / 2;
36     delete[] arr;
37     return s.substr(start, maxlen);
38 }

```

杂项

日期

```

1 string day_of_week[] = {"Mo", "Tu", "We", "Th", "Fr", "Sa", "Su"};
2
3 // 格里高利历 (yyyy-mm-dd) 转儒略历 (整型/天)
4 int date_to_int(int y, int m, int d){
5     return
6         1461 * (y + 4800 + (m - 14) / 12) / 4 +
7         367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
8         3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
9         d - 32075;
10 }
11
12 // 儒略历转格里高利历
13 void int_to_date(int jd, int &y, int &m, int &d){
14     int x, n, i, j;
15     x = jd + 68569;
16     n = 4 * x / 146097;
17     x -= (146097 * n + 3) / 4;
18     i = (4000 * (x + 1)) / 1461001;
19     x -= 1461 * i / 4 - 31;
20     j = 80 * x / 2447;
21     d = x - 2447 * j / 80;
22     x = j / 11;
23     m = j + 2 - 12 * x;
24     y = 100 * (n - 49) + i + x;
25 }

```

随机

随机素数表

NTT 素数表

$p = r2^k + 1$, 原根是 g 。

\$ (MOD, G, K, C) \$ 满足: MOD 是质数, G 是 MOD 的原根, $MOD - 1 = C \times 2^K$

挑选方法:

- MOD 大于系数最大值的平方乘以多项式长度
- $2^m \leq 2^K$, 其中 2^m 为多项式长度

3, 1, 1, 2; 5, 1, 2, 2; 17, 1, 4, 3; 97, 3, 5, 5; 193, 3, 6, 5; 257, 1, 8, 3; 7681, 15, 9, 17; 12289, 3, 12, 11; 40961, 5, 13, 3; 65537, 1, 16, 3; 786433, 3, 18, 10; 5767169, 11, 19, 3; 7340033, 7, 20, 3; 23068673, 11, 21, 3; 104857601, 25, 22, 3; 167772161, 5, 25, 3; 469762049, 7, 26, 3; 1004535809, 479, 21, 3; 2013265921, 15, 27, 31; 2281701377, 17, 27, 3; 3221225473, 3, 30, 5; 75161927681, 35, 31, 3; 77309411329, 9, 33, 7; 206158430209, 3, 36, 22; 2061584302081, 15, 37, 7; 2748779069441, 5, 39, 3; 6597069766657, 3, 41, 5; 39582418599937, 9, 42, 5; 79164837199873, 9, 43, 5; 263882790666241, 15, 44, 7; 1231453023109121, 35, 45, 3; 1337006139375617, 19, 46, 3; 3799912185593857, 27, 47, 5; 4222124650659841, 15, 48, 19; 7881299347898369, 7, 50, 6; 31525197391593473, 7, 52, 3; 180143985094819841, 5, 55, 6; 1945555039024054273, 27, 56, 5; 4179340454199820289, 29, 57, 3.

根号分治

二维可交换操作, 将时间复杂度较低的操作分配给规模较大的维度。

注意事项

- `1LL << k`
- `(LL)v.size()`
- 输入要读完
- 不要把 `while` 写成 `if`
- 树链剖分/dfs 序, 初始化或者询问不要忘记 `idx, ridx`
- 想清楚到底是要 `multiset` 还是 `set`
- 数据结构注意数组大小 (2 倍, 4 倍)
- 模意义下不要用除法
- 998244353
- 取模取全