Worksheet 4

Contagion Dynamics

Networks and Complex Systems

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Schülerakademie 5.2 (Roßleben 2016)

Exercise 4.1 A Network of the Monastery School Roßleben

The aim of this task is the creation of a network that represents the building of the Monastery School Roßleben. The network's rooms are nodes and a pair of nodes is connected with an edge if there is a *direct* connection between them. In the end the network should be available in a file as an adjacency matrix or edge list such that it can be loaded into python and further analysed.

Subexercise 4.1.1 Dividing the Problem up

Discuss in a large group how the problem could be divided such that many of you can work in parallel on it. In the end there should be one file with the whole network. Create also a second file that gives the name of the rooms, such as:

Node number	floor	room description
1	0	Administration Office
2	1	Hall
⋮		

Subexercise 4.1.2 Degree distribution

Extract the degree distribution $P(k)$ of the network. Illustrate it as a histogram.

Exercise 4.2 Pandemic in the Monastery School

The Course 5.3 'Germs and Diseases' is experimenting with infectious germs. Simulate the spread of a disease that starts in their laboratory. Use the SI model dynamics on the network created in Exercise 4.1.

Subexercise 4.2.1 Complete Infection

Simulate the contagion process for varying infection rates $\beta \in [0,1]$. After how many time steps t_{end} is the whole network infected for the different rates β? Illustrate the results for $t_{\text{end}}(\beta)$ in a figure and interpret your findings.

Optional: Average $t_{\text{end}}(\beta)$ for $s = 20$ simulations. How does this change the curve?

Subexercise 4.2.2 Vulnerable Courses

The contagion again starts from laboratory of course 5.3. We want to determine which of the other five courses is most vulnerable to the contagion. Simulate the SI-Process for $\beta = 0.01$ and investigate at which point in time the rooms of the other courses and the Administration office are infected.

Which rooms are most vulnerable? How can you explain these results?

Exercise 4.3 *Betweenness* Centrality

In addition to the degree k_i of a node there exist also other measures of centrality of a node, for example, the *betweenness* centrality g_i. It measures in how many shortest paths between all pairs of nodes a certain node i exists. To be specific it is defined as

$$g_i = \sum_{s \neq i \neq t} \frac{\sigma_{st}(i)}{\sigma_{st}}, \tag{1}$$

where σ_{st} is the number of shortest paths between nodes s and t and $\sigma_{st}(i)$ the number of these paths that go through node i. For finding all shortest paths between nodes s and t in a network G you can use the function `networkx.all_shortest_paths(G,s,t)`.

Subexercise 4.3.1 Betweenness Centrality of Graph Models

- Sketch a path graph P_n and find $g_i \, \forall \, i \in V$ in dependence of the number n of nodes.
- Calculate the betweenness g_i for the cycle graph C_n.
- Which two graphs with n nodes have $g_i = 0 \, \forall \, i \in V$? Show that both are not isomorph to each other if $n > 1$.

Subexercise 4.3.2 Betweenness Centrality of a Complete Graph after Deletion of an Edge

Be K_n the complete graph with n nodes and $e = \{u, v\}$ one of its edges. Show that the betweenness centrality of the graph $K_n - e$ is then given by

$$g_i = \begin{cases} \frac{1}{n-2} & \text{if } i \in \{u, v\} \\ 0 & \text{else} \end{cases} \tag{2}$$

beträgt.

Subexercise 4.3.3 Betweenness Centrality of two connected n-Cliques

The graph G consists of two n-cliques that are connected via a single node that connects to all other nodes. Find formulas for the betweenness g_i of this connection node i.

Optional: Generalise this for c of such n-cliques that are connected through that single central node and find the function $g_i(c, n)$.

Exercise 4.4 Contagion Dynamic – Vaccination

We want to investigate how the contagion process is slowed down if we vaccinate certain rooms. We achieve this by removing these rooms from the network.

Subexercise 4.4.1 Random Vaccination

At first we want to vaccinate the rooms randomly. Vaccinate a single room and investigate whether the time t_{end} to the total infection is changed. Repeat this experiment a couple of times. Does the behaviour change under this repetitions? Interpret the result.

Subexercise 4.4.2 Random Next Neighbour Vaccination

Now we will repeat the random vaccination but use our insights from the *friendship paradox*. We do not vaccinate a random room but a random neighbour of a random room. Repeat the experiments and investigate whether the behaviour is changed notably.

Subexercise 4.4.3 Systematic Investigation of the Vaccination Success

After these initial observations we will analyse the impact of vaccinations systematically. For this we vaccinate not only a single room but iteratively rooms, at first one, then a second, and so on, until we removed half of all rooms. For each of these iterative steps we calculate:

- The number N_1 of nodes that are infected after a single step.
- The number N_{10} of nodes that are infected after ten steps.
- The number N_{100} of nodes that are infected after a hundred steps.
- The number N_{1000} of nodes that are infected after a thousand steps.

Vary the infection rate $\beta \in \{0.01, 0.1, 0.5, 1\}$.

Note that you can stop the dynamic when all nodes are infected. For this you can use the python commands `if` and `break`.

Repeat the experiments with the next neighbour vaccination strategy.

Interpret your results and discuss which strategy is more efficient. Why is this the case? Under which conditions will the whole network be infected?

Subexercise 4.4.4 Optional: Vaccination by *Betweenness* Centrality

Repeat the experiments for a vaccination strategy such that you vaccinate nodes in decreasing order of their betweenness centrality g_i. How does the contagion dynamic change under this targeted vaccination strategy?