Shannon-Nyquist sampling theorem

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September 2022

Theorem:

According to Shannon:

If a function $x(t)$ contains no frequencies higher than B hertz, it is completely determined by giving its ordinates at a series of points spaced $1/(2B)$ seconds apart.

Theoretical Proof:

We know there's a relationship between sinusoids and circles. So for this experiment, let's use a unit circle.

Let's consider a circle of unknown radius. We look at the circle one point at a time through blinking and that blinking rate will be our sampling frequency.

Now when we see a point in space, if we open our eyes at the frequency of the signal, then we'll only see one point. However, if we open our eyes at twice the frequency of the signal, we'll see two points at the ends of the circle. Hence, our system becomes a two-point system with a probability of 0.5 of seeing the point at either end of the circle. And according to Shannon, this system gives us 1 bit of information.

Now, using this information, we can construct a circle of the required radius (amplitude) and we know the frequency (our blinking rate). Hence, that information is enough to construct a signal. And this process can continue for other signals. If we have higher frequency signals, then we can use the Nyquist rate to re-construct all the signals and get overall information of > 1 bit. So Nyquist sampling rate has to have at least 1 bit of information to construct the original signal.