

**Pendulum**

- a) For a pendulum with given physical parameters  $m$ ,  $l$ ,  $g$ , write down the equations of motion and an expression for the exergy  $E$ . Explain the normalization procedure to get  $\dot{\varphi} = p$ ,  $\dot{p} = -\sin \varphi$ . What are the “units” of time, energy and angular momentum in this normalized form?
- b) In `pendulum.py`, the normalized equations are integrated using the forward-Euler method. Compare the behavior for different time steps  $\Delta t$ .
- c) Implement alternative methods and investigate the behavior for different  $\Delta t$  again:
  - i) the 4th order Runge-Kutta method.
  - ii) the trapezoidal rule.  
Hint: To get an easy implementation, introduce  $\varphi^* := \varphi_n + \frac{\Delta t}{2} p_n$  and  $p^* := p_n - \frac{\Delta t}{2} \sin \varphi_n$ . Plug this into the discretized equations of motion to get a single transcendent equation for  $p_{n+1}$ . Then, solve that in a separate function `solvePendel` by means of Newton-Raphson iteration.
  - iii) the drift-kick / Leapfrog / Verlet method