Euler's Method for the Decay Equation

File `decay.py` approximates the solution to the simple decay equation

$$\frac{df(t)}{dt} = -\gamma f(t) =: G(f(t), t)$$

with $\gamma > 0$ using the forward Euler method, i.e.,

$$f_{n+1} = f_n + \Delta t G(f_n, t) = f_n - \gamma \Delta t \, f_n$$

(see lecture notes).

i) Study the solution behavior for different time steps Δt: Can you verify the different regimes of the iteration eigenvalue $\lambda = f_{n+1}/f_n = 1 - \gamma \Delta t$?
Which exact eigenvalue $\lambda_e = f(t + \Delta t)/f(t)$ does the differential equation give, and how is it related to the numerical λ ?

ii) Alternatively, implement the backward Euler scheme,

$$f_{n+1} = f_n - \gamma \Delta t \, f_{n+1}$$

and the trapezoidal scheme,

$$f_{n+1} = f_n - \gamma \, \Delta t \, \frac{f_n + f_{n+1}}{2}$$

and compare. What are the respective iteration eigenvalues here ?

Euler's Method for the Harmonic Oscillator

In a similar way, study the ODE system of the harmonic oscillator,

$$\dot{q}(t) = \omega p(t), \quad \dot{p}(t) = -\omega q(t)$$

as prepared in file `harmosc.py` (again, see the lecture notes).

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