Diffusion Equation II

Again, we want to address the diffusion equation

$$\partial_t f = \nabla \cdot \kappa \nabla f$$

as discussed before.

a) Consider the one-dimensional setup with a diffusion coefficient that depends on position, i.e., $\kappa(x)$ and modify the forward Euler implementation accordingly. Use a conservative discretization based on the flux density $\vec{q} = -\kappa \nabla f$ at "staggered" positions $x_{j+1/2}$. Do the results match your "physical" expectations?

b) You find a python implementation of the backward Euler method for the case of constant κ in `heatBackward.py`. Improve its accuracy to $\mathcal{O}(\Delta t^2)$ by using the trapezoidal rule for integration. Can you estimate and verify the stability property for this case? Can you implement Dirichlet type boundary conditions?

c) Generalize the implicit method from b) to $\kappa(x)$ as in a): What does the discretization matrix look like?

juergen.dreher@rub.de kevin.schoeffler@rub.de