Finite Differences and Convergence Tests II

We continue with the derivation and test of finite difference formulae:

- Verify the standard 3-point stencil for the *second* derivative f'' as given in the lecture notes.
- The discretizations addressed so far were based on equal spacings h between function values, employing function values $f(x), f(x-h), f(x+h)$ etc.
  What about a formula that use values like $f(x), f(x-h^-), f(x+h^+)$, i.e. different spacings on either side? Can you derive, using Taylor expansion, stencils for f' and f'' in this setting? Which order of accuracy do you get?
  Hint: When making convergence tests, use a fixed ratio h^+/h^- for the limit $h^+, h^- \to 0$.
- Derive a one-sided, second order accurate approximation for $f'(x)$ that employs $f(x)$, $f(x-h)$, and $f(x-2h)$. Verify !
  Hint: Use your previous results.
- Richardson extrapolation is a simple way to derive higher-order finite difference formulae from lower order stencils (see the lecture notes). Use it to improve the $\mathcal{O}(h^2)$ central difference formulae for f' and f'', respectively, and verify!

Planetary Motion I

Here, we want to address the motion of a "point mass" under the influence of a central force

$$\vec{F}(\vec{r}) = -4\pi^2 m \frac{\vec{r}}{|\vec{r}|^3}$$

Closed, elliptical trajectories are approximated in file `planet.py` as positions of the mass at times t_i with $i = 0, 1, 2, ...(N-1)$. They are plotted together with velocity arrows, which in turn are approximated by central finite differences.

a) Compute the particle's acceleration $\vec{a} = \ddot{\vec{r}}$ with finite differences and plot the respective vector arrows.

b) Compute $|\vec{F}(\vec{r}_i)/m - \vec{a}_i|$ for each t_i and analyze the way it behaves with varying time spacings Δt and eccentricities ε.
   Which error sources come into play when computing $\vec{r}$, $\dot{\vec{r}}$ and $\ddot{\vec{r}}$? How can they be quantified or alleviated ? Could root-finding methods help here ?

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