Please remember to register for the final examination in eCampus no later than January 23, 2026.

Sound Waves, 1-dimensional

The equations of ideal, adiabatic gas dynamics are the *continuity equation* $\frac{\partial \rho}{\partial t} + \vec{v} \cdot \nabla \rho = -\rho \nabla \cdot \vec{v}$ for *mass density* $\rho(\vec{r}, t)$, the *momentum equation* $\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p$ for the gas *velocity* $\vec{v}(\vec{r}, t)$, and an *energy equation* like $\frac{\partial p}{\partial t} + \vec{v} \cdot \nabla p = -\gamma p \nabla \cdot \vec{v}$ for the gas *pressure* $p(\vec{r}, t)$.
Here, $\gamma =$ const. is the adiabatic index of the gas, and the equations are written in *advection form*.

If all quantities depend only on one space coordinate, x, and time, and if we address only the longitudinal component of $\vec{v}$ as $v_x(x, t)$, the system becomes considerably simpler. Linearizing the system further for small perturbations around a constant background density ρ_0, constant background pressure p_0, and for a gas at rest, $\vec{v}_0 = 0$, we get two coupled equations for the perturbations $p_1(x, t) := p(x, t) - p_0$ and $v_x(x, t)$ as

$$\frac{\partial v_x}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p_1}{\partial x}, \quad \frac{\partial p_1}{\partial t} = -\gamma p_0 \frac{\partial v_x}{\partial x}$$

a) Show that this system admits *traveling-wave* solutions of the form $v_x(x, t) = V(x - ct)$, $p_1(x, t) = P(x - ct)$. What is the relationship between the envelope functions V and P, what is the sound velocity c?

b) Solve the system numerically with periodic boundary conditions in x. Use an isolated pressure pulse and $v_x(x, t = 0) = 0$ as initial conditions (see `sound.py`).
You can do Leap-Frogging between v_x and p_1, use a Lax-Wendroff or Runge-Kutta scheme, or ...

c) Can you change the initial conditions for an isolated perturbation to move only to the left or only to the right?

juergen.dreher@rub.de kevin.schoeffler@rub.de