

Pendulum

- a) For a pendulum with given physical parameters m, l, g , write down the equations of motion and an expression for the exergy E . Explain the normalization procedure to get $\dot{\varphi} = p, \dot{p} = -\sin \varphi$. What are the “units” of time, energy and angular momentum in this normalized form?
- b) In `pendulum.py`, the normalized equations are integrated using the forward-Euler method. Compare the behavior for different time steps Δt .
- c) Implement alternative methods and investigate the behavior for different Δt again:
 - i) the 4th order Runge-Kutta method.
 - ii) the trapezoidal rule.
Hint: To get an easy implementation, introduce $\varphi^* := \varphi_n + \frac{\Delta t}{2} p_n$ and $p^* := p_n - \frac{\Delta t}{2} \sin \varphi_n$. Plug this into the discretized equations of motion to get a single transcendent equation for p_{n+1} . Then, solve that in a separate function `solvePendel` by means of Newton-Raphson iteration.
 - iii) the drift-kick / Leapfrog / Verlet method