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On the search for gravitational-wave progenitors: Reducing the computational costs of binary population synthesis using adaptive importance sampling

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PREFACE

Preface

ABSTRACT

Abstract

Popular Summary

CONTENTS

I RESEARCH 1

1 INTRODUCTION 3
- 1.1 Gravitational-waves 3
- 1.2 Binary stellar evolution models 3
- 1.3 Common methods 3
- 1.4 rare events simulations 3
- 1.5 Motivation 4
- 1.6 Outline 5

2 STATISTICAL METHOD 7
- 2.1 Simulating populations of binaries 7
- 2.2 Adptive Importance Sampling 9
- 2.3 Fiducial method 9
- 2.4 Advanced Adaptive Importance Sampling 9
- 2.5 Other implementations 9
- 2.6 Population Synthesis 9
- 2.7 Fiducial Model 9

3 RESULTS 11
- 3.1 Adaptive Importance Sampling Toy Model 11
- 3.2 Binary population Blck hole - Neutron star mergers 11
- 3.3 Exploring Adaptive importance Sampling 11

4 DISCUSSION 13

5 CONCLUSION 15

II APPENDIX 17

A A BRIEF GUIDE TO ADAPTIVE IMPORTANCE SAMPLING 19

B CODE 21

LIST OF FIGURES

LIST OF TABLES

Table 1 Table of variables used 9

ACRONYMS

Part I

RESEARCH

1 INTRODUCTION

GRAVITATIONAL-WAVES

Currently, one of the leading ideas in the field is that binary mergers originate from two massive stars that are born in a binary system, evolve over millions of years and at the end of their lives die in energetic explosions leaving behind a neutron star or black hole that spiral in and eventually merge to produce the gravitational-waves that we observe today. By comparing statistical studies of gravitational-wave observations with simulated compact object merger populations from theoretical models, we can learn about the evolution of stars in binary systems and the underlying physical processes involved. However (i) the progenitor systems of gravitational waves are extremely rare (ii) the stellar evolution simulations include many uncertainties that need to be explored and (iii) computational resources are scarce - making large inference studies of compact object mergers computationally intractable. This is currently a main challenge in this field limiting more detailed simulations.

Gravitational-wave progenitors

BINARY STELLAR EVOLUTION MODELS

detailed stellar models

binary population synthesis

COMMON METHODS

RARE EVENTS SIMULATIONS

Naive simulation becomes inefficient as the rare event probabilities gets smaller. Days or weeks are required to obtain an accurate estimate. For that reason, we need special techniques to speed up the estimation process. These special techniques for the simulation of rare events can be collected under two main categories: importance sampling and splitting . Both categories modify the simulation so that the rare event of interest occurs more frequently than in Monte Carlo simulation. It is important to note that the mathematical influence of these modifications has to be compensated to obtain the true probability. However, these two categories differ in the type of modifications.

In importance sampling the underlying probability measure is transformed to push the sample paths towards the rare event. In splitting, the underlying probability measure stays the same. Instead, a selection mechanism is used to pick the sample paths that are likely to reach rare event. Then, the chosen sample paths are split or cloned into multiple copies. This results in an artificial drift towards the rare event. A general discussion on both methods is provided below.

Importance sampling (IS) is a powerful Monte Carlo simulation variance-reduction technique that has achieved success in simulating many types of rare event problems. The generic idea of IS is to modify the probability law of the underlying system to sample the important events more frequently. This new probability measure is called the change of measure or IS distribution. Simulating the system under the IS distribution would naturally result in a biased estimator unless a correction is applied. This translation of the outcome from IS distribution to the original probability distribution is done by the likelihood ratio. The likelihood ratio is associated either with a single outcome or a sample path – sequence of outcomes. The likelihood ratio of a sample path can simply be defined as the probability of the sample path under the original measure over the probability of the same sample path under the IS measure. If the original probability laws are known, we can trace the sample path through time to calculate the probability of the sample path under the original probability measure. In this thesis, we are interested in the problems where the original probability laws are known. For more details on IS , see the recent surveys by Bucklew [21], Juneja and Shahahbuddin [68] or Rubino and Tuffin [96]. A crucial problem in IS simulation is to identify a proper choice of the change of measure. The IS distribution should be chosen such that the target event is no longer rare, and thus will be observed more frequently. However, one should be also careful about "too much occurrence of the rare event". Although the IS estimator is proved to be unbiased, it may overestimate with a high probability. Hence, if chosen incorrectly, the resulting estimator may have a greater variance than the one from Monte Carlo simulation. The variance can even become infinite. In that case, the estimator may also give biased results, even when it is unbiased in a theoretical sense. Mathematically, it is possible to pinpoint the optimal change of measure. To obtain zero-variance, every sample path generated under the IS measure should hit the rare event. This is only possible if the new distribution is chosen as the original distribution conditioned on the occurrence of the rare event. Although theoretically the optimal change of measure is known, it is practically useless since it involves a-priori knowledge of the probability of interest.

A common way to launch IS schemes for state-dependent changes of measures is to use adaptive IS techniques that attempt to learn the zero-variance change of measure by an iterative procedure. In every iteration a number of sample paths is generated, and based on the 'scores' of these paths the current change of measure is updated. For more information on adaptive IS techniques, we refer to Desai [34] and Kollman [74]. One of these adaptive methods is called the cross-entropy method which will be discussed briefly in the following section

MOTIVATION

OUTLINE

2 STATISTICAL METHOD

In this chapter we will discuss the statistical method that we developed forbinary Population synthesis. Focusing on the difference between our method and traditional sampling methods in BS.

SIMULATING POPULATIONS OF BINARIES

In binary population synthesis, one can simulate the evolution of a synthetic population of binary systems. For such simulations, bianries are randomly drawn from the distributions of initial binary parameters and evaluating it through binary evolution prescriptions. By doing so, they present a rapid code that can compute the evolution of many stars within a simulation. Since it is now known a priori which initial conditions will produce an event of interest, the full initial parameter space needs to be explored. Traditional methods in BPS tackle this problem by randomly drawing binaries $\mathbf{x_i} \sim P(\mathbf{x_i})$ from their prior distributions which are based on observations, and evaluating them with the BPS model, u, into their final state $\mathbf{x_f}$,

$$\mathbf{x_f} = u(\mathbf{x_i}). \tag{1}$$

Many parameters are used in BPS but most binaries can be described uniquely by a few important initial vari- ables: the initial mass of the primary star (i.e., the most massive star) $M_{1,i}$, the mass ratio, $q_i = M_{1,i}/M_{2,i}$, between the two stars and the initial separation, a_i, eccentricity e_i. Depending on the binary evolution we can also add the kick velocity received when the primary or secondary collapses to a compact object $\mathbf{v}_{\text{kick, SN}}$. Each initial binary sample can thus be represented by

$$\mathbf{x_i} = (M_{1,i}, a_i, q_i, e_i, \mathbf{v}_{\text{kick, SN1}}, \mathbf{v}_{\text{kick, SN2}}) \tag{2}$$

Often, BPS is used to study binaries that evolve to a certain subtype $\mathbf{X_t}$,

So in such a study the goal is to model the distribution:

$$\psi(\mathbf{x_f}) = \begin{cases} 1 & \text{if } \mathbf{x_f} \in \mathbf{X_t} \\ 0 & \text{else} \end{cases} \tag{3}$$

that equals unity if $\mathbf{x_i}$ evaluated to the target binary system $\mathbf{X_t}$ and zero if not. We will use this function throughout the paper as it is the main objective to perform the inference on. Say something about this function.

However, in cases when the target population is a rare event in the simulation, e.g. when simulating Compact object binaries since most systems will disrupt during the supernova kick, and thus $\psi(\mathbf{x_f}) = 0$ for most systems. Therefore simulating populations of such rare events becomes extremely computational expensive.

Therefore, instead, present the variance reduction method , adaptive importance sampling [1], that generates samples from a distribution function which is automatically adapted to the scientific target by focusing on areas of the parameter space found to produce events of interest. Instead of drawing random binaries from the prior distribution P we draw them from a distribution that focuses around the target distribution of interest, thereby minimizing the computational costs spend on areas that don't produce events of interest, whilst maximizing the computational costs spend on binaries that become the rare event.

The method consists of three main steps that are also shown in Figure

1. The parameter space is explored to find a small population of events of interest.

2. The set of known events is used to build an instrumental distribution, to adaptively guide future sampling of the parameter space.

3. Later simulations of population members are then drawn from this instrumental distribution. The information from these simulations can in turn be used to further improve the instrumental distribution. By doing so, the method minimizes the computatio

Table 1: Table of variables used

variable	description
u	BPS model
$\mathbf{x_i}$	initial state of a binary system
$\mathbf{x_f}$	final state of a binary system
$\mathbf{X_t}$	target subpopulation of binaries of interest
M_1	
a	
q	
e	
ν	
ν_k	
θ_k	
ϕ_k	

ADPTIVE IMPORTANCE SAMPLING

FIDUCIAL METHOD

Exploratory phase

Improve distribution

Run simulation (step 3)

ADVANCED ADAPTIVE IMPORTANCE SAMPLING

OTHER IMPLEMENTATIONS

POPULATION SYNTHESIS

For the population synthesis model we use COMPAS (Alejandro et al in prep and Stevenson et al. 2017)

based on (...)

FIDUCIAL MODEL

Our fiducial model

SN

mass transfer and stabillity

CE treatment

fall back

kicks

3 RESULTS

ADAPTIVE IMPORTANCE SAMPLING TOY MODEL

BINARY POPULATION BLCK HOLE - NEUTRON STAR MERGERS

EXPLORING ADAPTIVE IMPORTANCE SAMPLING

4 DISCUSSION

5 CONCLUSION

Part II

APPENDIX

a

A BRIEF GUIDE TO ADAPTIVE IMPORTANCE SAMPLING

b

CODE