$$\frac{\partial}{\partial Re(x_l)} RMSE_i = Re(s_i)\Big[\underbrace{a_i\, 2\cos\varphi(-\sin\varphi)\frac{\partial}{\partial Re(x_l)}\text{[redacted]}\,\varphi + \cos^2(\varphi)\frac{\partial}{\partial Re(x_l)}\text{[redacted]}\,a_i}_{A_1}$$

$$\underbrace{+\cos\varphi\sin\varphi\frac{\partial}{\partial Re(x_l)}\text{[redacted]}\,b_i + \cos^2(\varphi)\, b_i\frac{\partial}{\partial Re(x_l)}\text{[redacted]}\,\varphi - \sin^2(\varphi)\, b_i\frac{\partial}{\partial Re(x_l)}\text{[redacted]}\,\varphi}_{A_2}\Big]$$

$$-Im(s_i)\Big[\underbrace{\cos\varphi\sin\varphi\frac{\partial}{\partial Re(x_l)}\text{[redacted]}\,a_i + \cos^2(\varphi)\, a_i\frac{\partial}{\partial Re(x_l)}\text{[redacted]}\,\varphi - \sin^2(\varphi)\, a_i\frac{\partial}{\partial Re(x_l)}\text{[redacted]}\,\varphi}_{B_1}$$

$$\underbrace{+\,2\, b_i\sin(\varphi)\cos(\varphi)\frac{\partial}{\partial Re(x_l)}\text{[redacted]}\,\varphi + \sin^2(\varphi)\frac{\partial}{\partial Re(x_l)}\text{[redacted]}\,b_i}_{B_2}\Big] - \underbrace{Re(H_{i,l})}_{C}$$

$$\frac{\partial}{\partial Re(x_l)} IMSE_i = Im(s_i)\big[A_1 + A_2\big] + Re(s_i)\big[B_1 + B_2\big] - Im(H_{i,l})$$

$$\frac{\partial a_i}{\partial Re(x_l)} = Re(s_i)Re(H_{i,l}) + Im(s_i)Im(H_{i,l})$$

$$\frac{\partial b_i}{\partial Re(x_l)} = Re(s_i)Im(H_{i,l}) - Im(s_i)Re(H_{i,l})$$

$$\frac{\partial\varphi}{\partial Re(x_l)} = \cancel{\frac{1}{2}} \times \frac{1}{1+(\text{système})^2} \qquad \text{système} = \tan(2\varphi)$$

$$\times \sum_{j=1}^{K} Im(s_j)Re(s_j)\Bigg[\frac{\cancel{2}\, Im(H_{j,l})Im(R_j)\,\text{dénom} - Im(R_j)^2\,\cancel{2}\sum_{j=1}^{K} Im(s_j)Re(s_j)\big[Re(R_j)Im(H_{j,l}) + Im(R_j)Re(H_{j,l})\big]}{(\text{dénom})^2}$$

$$-\frac{\cancel{2}\, Re(H_{j,l})Re(R_j)\,\text{dénom} - Re(R_j)^2\,\cancel{2}\sum_{j=1}^{K} Im(s_j)Re(s_j)\big[Re(R_j)Im(H_{j,l}) + Im(R_j)Re(H_{j,l})\big]}{(\text{dénom})^2}\Bigg]$$

$$\text{dénom} = 2\sum_{j=1}^{K} Im(s_j)Re(s_j)\, Re(R_j)\, Im(R_j)$$

$$f(\underline{d}) = \|\underline{d} \times \underline{s} - \underline{R}\|_2^2 = \sum_{i=1}^{4} Re(d_i s_i - R_i)^2 + Im(d_i s_i - R_i)^2$$

$d_i \in \mathbb{C}$

$$= \sum_{i=1}^{4} \overline{(d_i s_i - R_i)}\,(d_i s_i - R_i)$$

$$\hookrightarrow \overline{d_i s_i}\, d_i s_i - \overline{d_i s_i} R_i - \overline{R_i}\, d_i s_i + R_i^2$$

$$\frac{\partial f(\underline{d})}{\partial d_i} = \frac{\partial}{\partial d_i} Re(d_i s_i - R_i)^2 + Im(d_i s_i - R_i)^2$$

$$= Re(d_i s_i - R_i)\frac{\partial}{\partial d_i} \underbrace{Re(d_i s_i)}_{Re(d_i)Re(s_i) - Im(d_i)Im(s_i)} + Im(d_i s_i - R_i)\frac{\partial}{\partial d_i} \underbrace{Im(d_i s_i)}_{Im(d_i)Re(s_i) + Re(d_i)Im(s_i)}$$

$$\frac{\partial}{\partial Re(d_i)} = Re(d_i s_i - R_i)\,Re(s_i) + Im(d_i s_i - R_i)\,Im(s_i) = 0 \quad (1)$$

$$\frac{\partial}{\partial Im(d_i)} = Re(d_i s_i - R_i)\,(-Im(s_i)) + Im(d_i s_i - R_i)\,Re(s_i) = 0 \quad (2)$$

(1) $$\big(Re(d_i s_i) - Re(R_i)\big)Re(s_i) = -\big(Im(d_i s_i) - Im(R_i)\big)Im(s_i)$$

$$Re(d_i s_i) + Im(d_i s_i) = Re(R_i)Re(s_i) + Im(R_i)Im(s_i)$$

$$Re(d_i)Re(s_i) - Im(d_i)Im(s_i) + Re(d_i)Im(s_i) + Im(d_i)Re(s_i) = Re(R_i)Re(s_i) + Im(R_i)Im(s_i)$$

$$Re(d_i)\big(Re(s_i) + Im(s_i)\big) + Im(d_i)\big(Re(s_i) - Im(s_i)\big) = Re(R_i)Re(s_i) + Im(R_i)Im(s_i)$$

(2) $$-\big(Re(d_i s_i) - Re(R_i)\big)Im(s_i) + \big(Im(d_i s_i) - Im(R_i)\big)Re(s_i) = 0$$

$$-Re(d_i s_i) + Im(d_i s_i) = Re(s_i)Im(R_i) - Re(R_i)Im(s_i)$$

$$-Re(d_i)Re(s_i) + Im(d_i)Im(s_i) + Re(d_i)Im(s_i) + Im(d_i)Re(s_i) = Re(s_i)Im(R_i) - Re(R_i)Im(s_i)$$

$$Im(d_i) = \frac{\underbrace{Re(s_i)Im(R_i) - Re(R_i)Im(s_i)}_{b_i} + Re(d_i)\big(Re(s_i) - Im(s_i)\big)}{Re(s_i) + Im(s_i)}$$

(1) $\mathrm{Re}(d_i)\left(\mathrm{Re}(s_i)+\mathrm{Im}(s_i)\right)+\mathrm{Im}(d_i)\left(\mathrm{Re}(s_i)-\mathrm{Im}(s_i)\right)=\mathrm{Re}(R_i)\,\mathrm{Re}(s_i)+\mathrm{Im}(R_i)\,\mathrm{Im}(s_i)$

$$\mathrm{Re}(d_i) = \frac{\overbrace{\mathrm{Re}(R_i)\mathrm{Re}(s_i)+\mathrm{Im}(R_i)\,\mathrm{Im}(s_i)}^{a_i} - \mathrm{Im}(d_i)\left(\mathrm{Re}(s_i)-\mathrm{Im}(s_i)\right)}{\mathrm{Re}(s_i)+\mathrm{Im}(s_i)}$$

$$\mathrm{Re}(d_i)\left(\mathrm{Re}(s_i)+\mathrm{Im}(s_i)\right) = a_i - \frac{b_i + \mathrm{Re}(d_i)\left(\mathrm{Re}(s_i)-\mathrm{Im}(s_i)\right)}{\mathrm{Re}(s_i)+\mathrm{Im}(s_i)}$$

$$\mathrm{Re}(d_i)\left(\mathrm{Re}(s_i)+\mathrm{Im}(s_i)\right)^2 = a_i\left(\mathrm{Re}(s_i)+\mathrm{Im}(s_i)\right) - b_i + \mathrm{Re}(d_i)\left(\mathrm{Re}(s_i)-\mathrm{Im}(s_i)\right)$$

$$\mathrm{Re}(d_i)\left[\mathrm{Re}(s_i)^2+\mathrm{Im}(s_i)^2+2\,\mathrm{Re}(s_i)\,\mathrm{Im}(s_i) - \mathrm{Re}(s_i)+\mathrm{Im}(s_i)\right]$$

$=$

$$\frac{\partial}{\partial d_i}\left(\overline{d_i s_i}\; d_i s_i - \overline{d_i s_i}\,R_i - \overline{R_i}\,d_i s_i + \cancel{R_i^2}\right)$$

$$= \overline{d_i}\,|s_i|^2 - \overline{R_i}\,s_i = 0$$

$\frac{\partial \bar z}{\partial z} = 0 \rightarrow$ pour une fct holomorphe

$$\hookrightarrow \overline{d_i}\,|s_i|^2 = \overline{R_i}\,s_i$$

$$\overline{d_i} = \frac{\overline{R_i}\,s_i}{|s_i|^2} = \frac{\overline{R_i}\,\cancel{s_i}}{\overline{s_i}\,\cancel{s_i}} \Leftrightarrow d_i = \frac{R_i}{s_i}$$

$$\|\underline{d} \times \underline{s} - R\|_2^2 \Rightarrow \sum_{i=1}^{K}\overline{(d_i s_i - R_i)}\,(d_i s_i - R_i)$$

$$= \sum_{i=1}^{K}\overline{\left(\frac{R_i}{s_i}s_i - R_i\right)}\left(\frac{R_i}{s_i}s_i - R_i\right)$$

$$= \sum_{i=1}^{K} 0$$

? amplification $D \in \mathcal{M}_{K,1}(\mathbb{C})$, $d_j = r_j e^{i\varphi_j}$

se limiter à $r_j \geq 0$ et $\varphi_j \in \left]-\frac{\pi}{4};\frac{\pi}{4}\right[$

pour éviter une solution en $\arg\min_{\underline{x}} \|\underline{0}\|^2$

$$d_j = r_j e^{i\varphi_j} = a_j + i\,b_j$$

$$|b| < a$$

$$|b| - a < 0$$

$$f\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = |x_1| - x_2 \qquad \frac{\partial f}{\partial x_1} = \operatorname{sign}(x_1)$$

$$\frac{\partial f}{\partial x_2} = -1$$

$$\frac{\partial^2 f}{\partial x_1^2} = \delta(x_1) \qquad \frac{\partial^2 f}{\partial x_1 \partial x_2} = 0$$

$$\frac{\partial^2 f}{\partial x_2^2} = 0$$

$$\text{Hess} = \begin{pmatrix} \delta(x_1) & 0 \\ 0 & 0 \end{pmatrix}$$

$$\text{trace} = \delta(x_1) = \sum \lambda_i = \begin{cases} 1 \text{ si } x_1 = 0 \\ 0 \text{ sinon} \end{cases}$$

$$\det(\text{Hess}) = \delta(x_1) = \prod \lambda_i = 0 \quad \forall x_1$$

$$\hookrightarrow 1 \text{ des } \lambda_i = 0$$

$$\underline{d}^* = \operatorname{argmin} \| \underline{d} \otimes \underline{s} - \underline{R} \|_2^2$$

$$d_j = r_j e^{i\varphi_j} \qquad \forall j \in [\![1;k]\!]$$

$$r_j > 0$$

$$\varphi_j \in \left]-\frac{\pi}{4}; \frac{\pi}{4}\right[$$

$$\| \underline{d} \otimes \underline{s} - \underline{R} \|_2^2 = \sum_{j=1}^{K} \left[\operatorname{Re}(d_j s_j - R_j)\right]^2 + \left[\operatorname{Im}(d_j s_j - R_j)\right]^2$$

$$= \sum_{j=1}^{k} \left[r_j \operatorname{Re}(e^{i\varphi_j} s_j) - \operatorname{Re}(R_j) \right]^2 + \left[r_j \operatorname{Im}(e^{i\varphi_j} s_j) - \operatorname{Im}(R_j) \right]^2$$

$$\text{s.t.} \quad r_j > 0, \quad \varphi_j > -\frac{\pi}{4}, \quad \varphi_j < \frac{\pi}{4}$$

$$-r_j < 0, \quad -\varphi_j - \frac{\pi}{4} < 0, \quad \varphi_j - \frac{\pi}{4} < 0$$

$$= \sum_{j=1}^{K} \left[r_j \left(\cos\varphi_j \operatorname{Re}(s_j) - \sin\varphi_j \operatorname{Im}(s_j) \right) - \operatorname{Re}(R_j) \right]^2 + \left[r_j \left(\cos\varphi_j \operatorname{Im}(s_j) + \sin\varphi_j \operatorname{Re}(s_j) \right) - \operatorname{Im}(R_j) \right]^2$$

$$f(r_1, \dots, r_K, \varphi_1, \dots, \varphi_K) = \| \underline{d} \otimes \underline{s} - \underline{R} \|_2^2 - \sum_{j=1}^{K} \lambda_j r_j$$
$$- \sum_{j=1}^{K} \left(\varphi_j + \frac{\pi}{4}\right) \alpha_j + \sum_{j=1}^{K} \left(\varphi_j - \frac{\pi}{4}\right) \beta_j$$

$$\frac{\partial f(\underline{r}, \underline{\varphi})}{\partial r_\ell} = 2\operatorname{Re}(e^{i\varphi_j} s_j) \left[r_j \operatorname{Re}(e^{i\varphi_j} s_j) - \operatorname{Re}(R_j) \right]$$
$$+ 2 \operatorname{Im}(e^{i\varphi_j} s_j) \left[r_j \operatorname{Im}(e^{i\varphi_j} s_j) - \operatorname{Im}(R_j) \right] - \lambda_\ell$$

$$\frac{\partial}{\partial \varphi_j} \operatorname{Re}(d_j s_j - R_j) = \frac{\partial}{\partial \varphi_j} r_j \cos\varphi_j \operatorname{Re}(s_j) - r_j \sin\varphi_j \operatorname{Im}(s_j)$$
$$= - r_j \operatorname{Re}(s_j) \sin\varphi_j - r_j \operatorname{Im}(s_j) \cos\varphi_j$$
$$= - r_j \operatorname{Im}(e^{i\varphi_j})$$

$$\frac{\partial}{\partial \varphi_j} \operatorname{Im}(d_j s_j - R_j) = \frac{\partial}{\partial \varphi_j} r_j \cos\varphi_j \operatorname{Im}(s_j) + r_j \sin\varphi_j \operatorname{Re}(s_j)$$
$$= - r_j \operatorname{Im}(s_j) \sin\varphi + r_j \operatorname{Re}(s_j) \cos\varphi_j$$
$$= r_j \operatorname{Re}(e^{i\varphi_j} s_j)$$

$$\frac{\partial f(\underline{r}, \underline{\varphi})}{\partial \varphi_\ell} = - 2 r_\ell \operatorname{Im}(e^{i\varphi_\ell}) \left[r_\ell \operatorname{Re}(e^{i\varphi_\ell} s_\ell) - \operatorname{Re}(R_\ell) \right] +$$
$$2 r_\ell \operatorname{Re}(e^{i\varphi_\ell} s_\ell) \left[r_\ell \operatorname{Im}(e^{i\varphi_\ell} s_\ell) - \operatorname{Im}(R_\ell) \right] - \alpha_\ell + \beta_j$$

$$\frac{\partial f(\underline{r}, \underline{\varphi})}{\partial r_\ell} = 0 \Leftrightarrow \frac{\lambda_\ell}{2} = r_j \left(\operatorname{Re}(e^{i\varphi_\ell} s_\ell)^2 + \operatorname{Im}(e^{i\varphi_\ell} s_\ell)^2 \right) - \operatorname{Re}(e^{i\varphi_\ell} s_\ell) \operatorname{Re}(R_\ell)$$
$$- \operatorname{Im}(e^{i\varphi_\ell} s_\ell) \operatorname{Im}(R_\ell)$$

$$\Leftrightarrow r_j = \frac{\frac{\lambda_\ell}{2} + \operatorname{Re}(e^{i\varphi_\ell} s_\ell) \operatorname{Re}(R_\ell) + \operatorname{Im}(e^{i\varphi_\ell} s_\ell) \operatorname{Im}(R_\ell)}{\operatorname{Re}(e^{i\varphi_\ell} s_\ell)^2 + \operatorname{Im}(e^{i\varphi_\ell} s_\ell)^2} \quad \leftarrow = 1$$

$$\begin{matrix} -\lambda_\ell r_\ell = 0 \\ \lambda \geq 0 \end{matrix} \Rightarrow \begin{cases} \text{soit } r_\ell = 0 \text{ et } \lambda \geq 0 \\ \text{soit } \lambda = 0 \text{ et } -r_\ell \leq 0 \end{cases}$$

$$r_l = \max\left(0, \left[\mathrm{Re}(e^{i\varphi_l}s_l)\mathrm{Re}(R_l) + \mathrm{Im}(e^{i\varphi_l}s_l)\mathrm{Im}(R_l)\right]\right)$$

$$\frac{\partial f(r,\varphi)}{\partial \varphi_l} = 0 \iff -2 r_l\, \mathrm{Im}(e^{i\varphi_l}s_l)\left[r_l\, \mathrm{Re}(e^{i\varphi_l}s_l) - \mathrm{Re}(R_l)\right] +$$

$$2 r_l\, \mathrm{Re}(e^{i\varphi_l}s_l)\left[r_l\, \mathrm{Im}(e^{i\varphi_l}s_l) - \mathrm{Im}(R_l)\right] - \alpha_l + \beta_j = 0$$

$$\iff \cancel{-2r_l^2\, \mathrm{Im}(e^{i\varphi_l}s_l)\mathrm{Re}(e^{i\varphi_l}s_l)} + 2r_l\, \mathrm{Im}(e^{i\varphi_l}s_l)\mathrm{Re}(R_l)$$

$$\cancel{+2r_l^2\, \mathrm{Re}(e^{i\varphi_l}s_l)\mathrm{Im}(e^{i\varphi_l}s_l)} - 2r_l\, \mathrm{Re}(e^{i\varphi_l}s_l)\mathrm{Im}(R_l) - \alpha_l + \beta_j = 0$$

$$r_l\, \mathrm{Im}(e^{i\varphi_l}s_l)\mathrm{Re}(R_l) + \frac{\beta_j}{2} = r_l\, \mathrm{Re}(e^{i\varphi_l}s_l)\mathrm{Im}(R_l) + \frac{\alpha_l}{2}$$

Cf. sharedphase

* si $r_l = 0$, trouver φ_l n'a pas d'intérêt

  ↳ $\mathrm{Re}(e^{i\varphi_l}s_l)\mathrm{Re}(R_l) + \mathrm{Im}(e^{i\varphi_l}s_l)\mathrm{Im}(R_l) \leq 0$

* sinon $r_l = \mathrm{Re}(e^{i\varphi_l}s_l)\mathrm{Re}(R_l) + \mathrm{Im}(e^{i\varphi_l}s_l)\mathrm{Im}(R_l)$

d'où

$$= \left[\cos\varphi\, \mathrm{Re}(s_l) - \sin\varphi\, \mathrm{Im}(s_l)\right]\mathrm{Re}(R_l) + \left[\cos\varphi\, \mathrm{Im}(s_l) + \sin\varphi\, \mathrm{Re}(s_l)\right]\mathrm{Im}(R_l)$$

$$= \cos\varphi\left[\mathrm{Re}(s_l)\mathrm{Re}(R_l) + \mathrm{Im}(s_l)\mathrm{Im}(R_l)\right] + \sin\varphi\left[\mathrm{Re}(s_l)\mathrm{Im}(R_l) - \mathrm{Im}(s_l)\mathrm{Re}(R_l)\right]$$

$$= \cos\varphi\, a_l + \sin\varphi\, b_l$$

$$r_l\, \mathrm{Im}(e^{i\varphi_l}s_l)\mathrm{Re}(R_l) + \frac{\beta_j}{2} = r_l\, \mathrm{Re}(e^{i\varphi_l}s_l)\mathrm{Im}(R_l) + \frac{\alpha_l}{2}$$

$$\cos(2\varphi_l)\, a_l b_l + \frac{\beta_j}{2} = \frac{1}{2}\sin(2\varphi_l)\left(a_l^2 - b_l^2\right) + \frac{\alpha_l}{2}$$

* si $\beta_j = 0$ et $\alpha_l = 0$

$$\tan(2\varphi_l) = \frac{2a_lb_l}{a_l^2 - b_l^2} = \frac{\mathrm{Im}(s_l)\cancel{\mathrm{Re}(s_l)}\left[\mathrm{Im}(R_l)^2 - \mathrm{Re}(R_l)^2\right]}{2\,\mathrm{Re}(R_l)\mathrm{Im}(R_l)\cancel{\mathrm{Re}(s_l)}\mathrm{Im}(s_l)}$$

$$\varphi_l = \frac{1}{2}\arctan\left(\frac{2a_lb_l}{a_l^2 - b_l^2}\right)$$

?

Cf page 12

$$-\left(\varphi_l+\frac{\pi}{4}\right)\alpha_l=0 \Rightarrow \begin{cases} \alpha_l=0 \text{ et } -\left(\varphi_l+\frac{\pi}{4}\right)\leq 0 \\ \qquad\qquad -\frac{\pi}{4}\leq\varphi_l \\ \text{ou} \\ -\left(\varphi_l+\frac{\pi}{4}\right)=0 \text{ et } \alpha_l\geq 0 \\ \varphi_l=-\frac{\pi}{4} \end{cases}$$

$$\left(\varphi_l-\frac{\pi}{4}\right)\beta_l=0 \Rightarrow \begin{cases} \beta_l=0 \text{ et } \varphi_l-\frac{\pi}{4}\leq 0 \\ \qquad\qquad \varphi_l\leq\frac{\pi}{4} \\ \text{ou} \\ \varphi_l-\frac{\pi}{4}=0 \text{ et } \beta_l\geq 0 \\ \varphi_l=\frac{\pi}{4} \end{cases}$$

$$\varphi_l^*=\max\left(-\frac{\pi}{4},\min\left(\frac{\pi}{4},\varphi_l\right)\right)$$

$\times$

$$\underline{\partial}^*=\operatorname{argmin}\|\underline{\partial}\otimes\underline{s}-\underline{R}\|_2^2$$

$$\partial_j=a_j+i b_j \qquad |b_j|\leq a_j$$

$$\|\underline{\partial}\otimes\underline{s}-\underline{R}\|_2^2=\sum_{j=1}^{K}\mathrm{Re}(\partial_j s_j-R_j)^2+\mathrm{Im}(\partial_j s_j-R_j)^2$$

$\mathrm{Re}(i s_j)$
$=-\mathrm{Im}(s_j)$

$$=\sum_{j=1}^{K}\left[\mathrm{Re}(a_j s_j+b_j i s_j)-\mathrm{Re}(R_j)\right]^2+\left[\mathrm{Im}(a_j s_j+b_j i s_j)-\mathrm{Im}(R_j)\right]^2$$

$\mathrm{Im}(i s_j)$
$=\mathrm{Re}(s_j)$

$$=\sum_{j=1}^{K}\left[a_j\mathrm{Re}(s_j)+b_j\mathrm{Re}(i s_j)-\mathrm{Re}(R_j)\right]^2+\left[a_j\mathrm{Im}(s_j)+b_j\mathrm{Im}(i s_j)-\mathrm{Im}(R_j)\right]^2$$

$$\text{s.t.} \qquad |b_j|\leq a_j \quad \forall j\in[\![1;K]\!]$$

$$|b_j|-a_j\leq 0$$

$$\mathcal{L}(\underline{a},\underline{b},\underline{\lambda})=\sum_{j=1}^{K}\left[a_j\mathrm{Re}(s_j)-b_j\mathrm{Im}(s_j)-\mathrm{Re}(R_j)\right]^2+\left[a_j\mathrm{Im}(s_j)+b_j\mathrm{Re}(s_j)-\mathrm{Im}(R_j)\right]^2$$
$$+\lambda_j(|b_j|-a_j)$$

$$\frac{\partial \mathcal{L}(\underline{a},\underline{b},\underline{\lambda})}{\partial a_\ell} = 2\left[a_\ell \operatorname{Re}(s_\ell) - b_\ell \operatorname{Im}(s_\ell) - \operatorname{Re}(R_\ell)\right]\operatorname{Re}(s_\ell)$$
$$+ 2\left[a_\ell \operatorname{Im}(s_\ell) + b_\ell \operatorname{Re}(s_\ell) - \operatorname{Im}(R_\ell)\right]\operatorname{Im}(s_\ell)$$
$$- \lambda_\ell$$

$$\frac{\partial \mathcal{L}(\underline{a},\underline{b},\underline{\lambda})}{\partial b_\ell} = 2\left[a_\ell \operatorname{Re}(s_\ell) - b_\ell \operatorname{Im}(s_\ell) - \operatorname{Re}(R_\ell)\right]\left(-\operatorname{Im}(s_\ell)\right)$$
$$+ 2\left[a_\ell \operatorname{Im}(s_\ell) + b_\ell \operatorname{Re}(s_\ell) - \operatorname{Im}(R_\ell)\right]\operatorname{Re}(s_\ell)$$
$$+ \lambda_\ell \operatorname{sign}(b_\ell)$$

$$\lambda_\ell\left(|b_\ell| - a_\ell\right) = 0 \Rightarrow \begin{cases} \lambda_\ell = 0 \quad \text{et} \quad |b_\ell| - a_\ell \leq 0 \\ \qquad\qquad\qquad\quad |b_\ell| \leq a_\ell \\ \text{ou} \\ |b_\ell| - a_\ell = 0 \quad \text{et} \quad \lambda_\ell \geq 0 \\ |b_\ell| = a_\ell \end{cases}$$

* soit $\lambda_\ell = 0$

$$\frac{\partial \mathcal{L}(\underline{a},\underline{b},\lambda)}{\partial \underline{a}} = 0 \iff \cancel{2}\left[a_\ell \operatorname{Re}(s_\ell) - b_\ell \operatorname{Im}(s_\ell) - \operatorname{Re}(R_\ell)\right]\operatorname{Re}(s_\ell)$$
$$+ \cancel{2}\left[a_\ell \operatorname{Im}(s_\ell) + b_\ell \operatorname{Re}(s_\ell) - \operatorname{Im}(R_\ell)\right]\operatorname{Im}(s_\ell)$$
$$- \cancel{\lambda_\ell} = 0$$

$$\iff a_\ell \operatorname{Re}(s_\ell)^2 - \cancel{b_\ell \operatorname{Im}(s_\ell)\operatorname{Re}(s_\ell)} - \operatorname{Re}(R_\ell)\operatorname{Re}(s_\ell) + a_\ell \operatorname{Im}(s_\ell)^2 + \cancel{b_\ell \operatorname{Re}(s_\ell)\operatorname{Im}(s_\ell)}$$
$$- \operatorname{Im}(R_\ell)\operatorname{Im}(s_\ell) = 0$$

$$a_\ell \underbrace{\left(\operatorname{Re}(s_\ell)^2 + \operatorname{Im}(s_\ell)^2\right)}_{1 \text{ car Q-PSK sur cercle unité}} = \operatorname{Re}(R_\ell)\operatorname{Re}(s_\ell) + \operatorname{Re}(R_\ell)\operatorname{Re}(s_\ell)$$

$$\frac{\partial \mathcal{L}(\underline{a},\underline{b},\underline{\lambda})}{\partial b_\ell} = 0 \iff \cancel{2}\left[a_\ell \operatorname{Re}(s_\ell) - b_\ell \operatorname{Im}(s_\ell) - \operatorname{Re}(R_\ell)\right]\left(-\operatorname{Im}(s_\ell)\right)$$

$$+ \cancel{2}\left[a_\ell \operatorname{Im}(s_\ell) + b_\ell \operatorname{Re}(s_\ell) - \operatorname{Im}(R_\ell)\right]\operatorname{Re}(s_\ell)$$

$$+ \cancel{\lambda_\ell \operatorname{sign}(b_\ell)} = 0$$

$$\iff -a_\ell \cancel{\operatorname{Re}(s_\ell)\operatorname{Im}(s_\ell)} + b_\ell \operatorname{Im}(s_\ell)^2 + \operatorname{Re}(R_\ell)\operatorname{Im}(s_\ell)$$

$$+ a_\ell \cancel{\operatorname{Re}(s_\ell)\operatorname{Im}(s_\ell)} + b_\ell \operatorname{Re}(s_\ell)^2 - \operatorname{Im}(R_\ell)\operatorname{Re}(s_\ell) = 0$$

$$b_\ell\Big(\underbrace{\operatorname{Re}(s_\ell)^2 + \operatorname{Im}(s_\ell)^2}_{1 \text{ car cercle unité}}\Big) = \operatorname{Re}(s_\ell)\operatorname{Im}(R_\ell) - \operatorname{Re}(R_\ell)\operatorname{Im}(s_\ell)$$

$$a + ib = r e^{i\varphi}$$

$$= r\cos\varphi + i\, r\sin\varphi \implies \begin{array}{l} b = r\sin\varphi \\ a = r\cos\varphi \end{array}$$

$$r = \frac{a+ib}{e^{i\varphi}} = (a+ib)\,e^{-i\varphi} = (a+ib)\left(\cos(-\varphi) + i\sin(-\varphi)\right)$$

$$= a\cos(\varphi) - i\,a\sin(\varphi) + i\,b\cos(\varphi) + b\sin(\varphi)$$

$$= a\cos(\varphi) + b\sin(\varphi) \cancel{- i\, r\cos\varphi\sin\varphi + i\, r\sin\varphi\cos\varphi}$$

d'où nos résultats.

* soit $\lambda_\ell \geq 0$ $\quad |b_\ell| = a_\ell$

$$\frac{\partial \mathcal{L}(\underline{a},\underline{b},\lambda)}{\partial \underline{a}} = 0 \iff 2\left[a_\ell \operatorname{Re}(s_\ell) - b_\ell \operatorname{Im}(s_\ell) - \operatorname{Re}(R_\ell)\right]\operatorname{Re}(s_\ell)$$

$$+ 2\left[a_\ell \operatorname{Im}(s_\ell) + b_\ell \operatorname{Re}(s_\ell) - \operatorname{Im}(R_\ell)\right]\operatorname{Im}(s_\ell)$$

$$- \lambda_\ell = 0 \qquad (1)$$

$$a_\ell\Big[\underbrace{\operatorname{Re}(s_\ell)^2 + \operatorname{Im}(s_\ell)^2}_{1}\Big] = \frac{\lambda_\ell}{2} + \operatorname{Re}(R_\ell)\operatorname{Re}(s_\ell) + \operatorname{Im}(R_\ell)\operatorname{Im}(s_\ell)$$

$$\frac{\partial \mathcal{L}(\underline{a},\underline{b},\underline{\lambda})}{\partial b_\ell} = 0 \iff b_\ell\Big[\underbrace{\cancel{\operatorname{Im}(s_\ell)^2 + \operatorname{Re}(s_\ell)^2}}_{=1}\Big] + \frac{\lambda_\ell \operatorname{sign}(b_\ell)}{2}$$

$$= \operatorname{Re}(s_\ell)\operatorname{Im}(R_\ell) - \operatorname{Im}(s_\ell)\operatorname{Re}(R_\ell) \qquad (2)$$

$$\begin{cases} (1) & a_l = \dfrac{\lambda_l}{2} + Re(R_l)Re(s_l) + Im(R_l)Im(s_l) = \dfrac{\lambda_l}{2} + a_l^* \qquad |b_l| = sign(b_l) \times b_l \\ (2) & b_l + \dfrac{\lambda_l\, sign(b_l)}{2} = Re(s_l)Im(R_l) - Im(s_l)Re(R_l) = b_l^* \\ (3) & |b_l| = a_l \qquad\qquad sign(0) = 0 \end{cases}$$

$$(2) \quad b_l + (a_l - a_l^*)\, sign(b_l) = b_l^*$$

$$\Leftrightarrow b_l + (sign(b_l)\, b_l - a_l^*)\, sign(b_l) = b_l^*$$

$*$ si $b_l = 0$ $\quad sign(b_l) = 0$

$$a_l = 0 = \frac{\lambda_l}{2} + a_l^* \quad \text{et} \quad b_l^* = 0$$

$*$ si $b_l \neq 0$ $\qquad b_l + b_l \underbrace{sign(b_l)^2}_{=1} - a_l^* sign(b_l) = b_l^*$

$\hookrightarrow$ si $b_l > 0$ $\qquad sign(b_l) = 1$

$$2b_l - a_l^* = b_l^*$$

$$b_l = \frac{a_l^* + b_l^*}{2}$$

$$\frac{a_l^* + b_l^*}{2} + \frac{\lambda_l}{2} = b_l^* \Rightarrow \lambda_l = b_l^* - a_l^*$$

$$a_l = \frac{b_l^* - a_l^*}{2} + a_l^* = \frac{b_l^* + a_l^*}{2} \quad \checkmark$$

$\hookrightarrow$ si $b_l < 0$ $\qquad sign(b_l) = -1$

$$2b_l + a_l^* = b_l^*$$

$$b_l = \frac{b_l^* - a_l^*}{2}$$

$$\frac{b_l^* - a_l^*}{2} - \frac{\lambda_l}{2} = b_l^* \Rightarrow b_l^* - a_l^* = 2b_l^* + \lambda_l$$

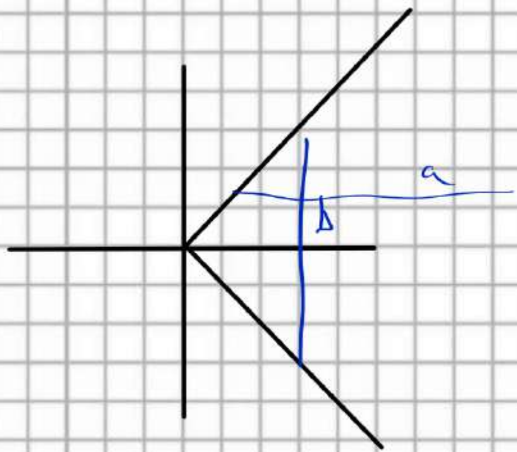
$$\lambda_l = -b_l^* - a_l^*$$

$$a_l = \frac{-b_l^* - a_l^*}{2} + a_l^* = \frac{-b_l^* + a_l^*}{2} \quad \checkmark$$

$$\begin{cases} a_l = Re(sl)Re(Rl) + Im(sl)Im(Rl) \\ \text{et } b_l = Re(sl)Im(Rl) - Re(Rl)Im(sl) & \text{si } |b_l| < a_l \\ pos = a_l = b_l = \dfrac{Re(sl)Re(Rl) + Im(sl)Im(Rl) + Re(sl)Im(Rl) - Re(Rl)Im(sl)}{2} & \text{si } b_l = a_l \\ -neg = a_l = -b_l = \dfrac{Re(sl)Re(Rl) + Im(sl)Im(Rl) - Re(sl)Im(Rl) + Re(Rl)Im(sl)}{2} & \text{si } b_l = -a_l \end{cases}$$

$$b_l = \min(pos, \max(neg, Re(sl)Im(Rl) - Re(Rl)Im(sl)))$$

$$a_l = \max(b_l, Re(sl)Re(Rl) + Im(sl)Re(Rl))$$



1) a) **Démontrer max(a;b)+min(a;b) = a+b**

b) Démontrer max(a;b)-min(a;b) = |a-b|

2) **Déduire min(a;b) =** $\dfrac{a+b-|a-b|}{2}$ **et max(a;b) =** $\dfrac{a+b+|a-b|}{2}$

On utilise $\forall j \in [\![1;K]\!]$, $\delta_j^* = r_j e^{i\varphi_j} = r_j(\cos\varphi_j + i\sin\varphi_j)$

avec $r_j = \max\left(0; Re(e^{i\varphi_j} s_j) Re(R_j) + Im(e^{i\varphi_j} s_j) Im(R_j)\right)$

et $\varphi_j = \max\left(-\frac{\pi}{4}; \min\left(\frac{\pi}{4}, \arctan\left[\frac{Re(sl)Im(Rl) - Im(sl)Re(Rl)}{Re(sl)Re(Rl) + Im(sl)Im(Rl)}\right]\right)\right)$

$\times$ si $\beta_l = \alpha_l = 0$ et $al \neq 0$

$$rl\, Im(e^{i\varphi_l} s_l) Re(Rl) + \frac{\beta_j}{2} = rl\, Re(e^{i\varphi_l} s_l) Im(Rl) + \frac{\alpha_l}{2}$$

$$\cancel{rl}\, Im(e^{i\varphi_l} s_l) Re(Rl) = \cancel{rl}\, Re(e^{i\varphi_l} s_l) Im(Rl)$$

$$\left[\cos\varphi\, Im(sl) + \sin\varphi\, Re(sl)\right] Re(Rl) = \left[\cos\varphi\, Re(sl) - \sin\varphi\, Im(sl)\right] Im(Rl)$$

$$\sin\varphi\left[Re(sl)Re(Rl) + Im(sl)Im(Rl)\right] = \cos\varphi\left[Re(sl)Im(Rl) - Im(sl)Re(Rl)\right]$$

$$\tan(\varphi) = \frac{Re(sl)Im(Rl) - Im(sl)Re(Rl)}{Re(sl)Re(Rl) + Im(sl)Im(Rl)}$$

$$\varphi = \arctan\left[\frac{Re(sl)Im(Rl) - Im(sl)Re(Rl)}{Re(sl)Re(Rl) + Im(sl)Im(Rl)}\right]$$

avec $\min(x, y) = \frac{x + y - |x - y|}{2}$ $\qquad \max(0, x) = x \times U(x)$

$U = \begin{cases} 1 \text{ si } x > 0 \\ 0 \text{ sinon} \end{cases}$

et $\max(x, y) = \frac{x + y + |x - y|}{2}$

On cherche $\quad \underline{x}^* = \underset{\underline{x}}{\operatorname{argmin}} \|\underline{\delta} \otimes \underline{s} - \underline{R}\|_2^2 \qquad \underline{R} = H\underline{x}$

$R_j = \sum_{n=1}^{M} H_{j,n} x_n$

$$f(\underline{x}) = \sum_{i=1}^{K} \left(r_i\left[\cos\varphi_i Re(s_i) - \sin\varphi_i Im(s_i)\right] - Re(R_i)\right)^2$$
$$+ \left(r_i\left[\cos\varphi_i Im(s_i) + \sin\varphi_i Re(s_i)\right] - Im(R_i)\right)^2$$
$$= \sum_{i=1}^{K} RMSE_i^2 + IMSE_i^2$$

$$\frac{\partial f(x)}{\partial Re(x_l)} = 2\sum_{i=1}^{N} RMSE_i \frac{\partial}{\partial Re(x_l)} RMSE_i + 2\sum_{i=1}^{N} IMSE_i \frac{\partial}{\partial Re(x_l)} IMSE_i$$

$$Re(e^{i\varphi_i} s_i) Re(R_i) + Im(e^{i\varphi_i} s_i) Im(R_i)$$
$$= \cos(\varphi_i)\left[Re(s_i)Re(R_i) + Im(s_i)Im(R_i)\right] + \sin\varphi_i \left[Re(s_i)Im(R_i) - Re(R_i)Im(s_i)\right]$$
$$= \cos\varphi_i\, a_i + \sin\varphi_i\, a_i$$

$$RMSE_i = \max(0, \cos\varphi_i a_i + \sin\varphi_i a_i)\left[\cos\varphi_i Re(s_i) - \sin\varphi_i Im(s_i)\right] - Re(R_i)$$

(C) $*$ $$\frac{\partial}{\partial Re(x_l)} - \sum_{j=1}^{M} Re(H_{i,j} x_j) = -\sum_{j=1}^{M} \frac{\partial}{\partial Re(x_l)} Re(H_{i,j}) Re(x_j) - Im(H_{i,j}) Im(x_j)$$
$$= \boxed{-Re(H_{i,l})} = \frac{\partial}{\partial Re(x_l)} - Re(R_i)$$

$*$ $$\frac{\partial}{\partial Re(x_l)} - Im\left(\sum_{m=1}^{M} H_{i,m} x_m\right)$$
$$= -\frac{\partial}{\partial Re(x_l)} \sum_{m=1}^{M} Re(H_{i,m}) Im(x_m) + Im(H_{i,m}) Re(x_m)$$
$$= \boxed{-Im(H_{i,l})} = \frac{\partial}{\partial Re(x_l)} - Im(R_i)$$

$*$ $$\frac{\partial}{\partial Re(x_l)} a_i = \boxed{Re(s_i) R(H_{i,l}) + Im(s_i) Im(H_{i,l})}$$

$*$ $$\frac{\partial}{\partial Re(x_l)} b_i = \boxed{Re(s_i) Im(H_{i,l}) - Im(s_i) Re(H_{i,l})}$$

$*$ $$\frac{\partial}{\partial Re(x_l)} \arctan\left(\left[\frac{Re(s_i) Im(R_i) - Im(s_i) Re(R_i)}{Re(s_i) Re(R_i) + Im(s_i) Im(R_i)}\right]\right) = \frac{\partial}{\partial Re(x_l)} \arctan\left(\frac{b_i}{a_i}\right)$$

$$= \boxed{\frac{1}{1+\left(\frac{b_i}{a_i}\right)^2} \times \frac{\partial}{\partial Re(x_l)} \frac{b_i}{a_i} = \frac{1}{1+\left(\frac{b_i}{a_i}\right)^2} \times \left[\frac{(b_i)' a_i - b_i (a_i)'}{a_i^2}\right]}$$

$$\ast \quad \frac{\partial \varphi}{\partial \mathrm{Re}(x_l)} = \frac{\partial}{\partial \mathrm{Re}(x_l)} \frac{\min\left(\frac{\pi}{4}, \arctan\left(\frac{b_i}{a_i}\right)\right) - \frac{\pi}{4} - \left|\min\left(\frac{\pi}{4}, \arctan\left(\frac{b_i}{a_i}\right)\right) + \frac{\pi}{4}\right|}{2}$$

$$= \frac{\partial}{\partial \mathrm{Re}(x_l)} \frac{\dfrac{\arctan\left(\frac{b_i}{a_i}\right) + \frac{\pi}{4} - \left|\arctan\left(\frac{b_i}{a_i}\right) - \frac{\pi}{4}\right|}{2} - \frac{\pi}{4} - \left|\dfrac{\arctan\left(\frac{b_i}{a_i}\right) + \frac{\pi}{4} - \left|\arctan\left(\frac{b_i}{a_i}\right) - \frac{\pi}{4}\right|}{2} + \frac{\pi}{4}\right|}{2}$$

$$= \frac{\partial}{\partial \mathrm{Re}(x_l)} \frac{\arctan\left(\frac{b_i}{a_i}\right)}{4} + \cancel{\frac{\pi}{16}} \underbrace{- \frac{\left|\arctan\left(\frac{b_i}{a_i}\right) - \frac{\pi}{4}\right|}{2}}_{A} - \cancel{\frac{\pi}{8}} \underbrace{- \left|\frac{\arctan\left(\frac{b_i}{a_i}\right)}{2} + \frac{\pi}{8} - \frac{\left|\arctan\left(\frac{b_i}{a_i}\right) - \frac{\pi}{4}\right|}{2} + \frac{\pi}{8}\right|}_{B}$$

$f(g)' = g' f'(g)$

$$(A) \quad -\frac{1}{2} \frac{\partial}{\partial \mathrm{Re}(x_l)} \left|\arctan\left(\frac{b_i}{a_i}\right) - \frac{\pi}{4}\right| = -\frac{1}{2} \operatorname{sign}\left(\arctan\left(\frac{b_i}{a_i}\right) - \frac{\pi}{4}\right) \frac{\partial}{\partial \mathrm{Re}(x_l)} \left(\arctan\left(\frac{b_i}{a_i}\right) \cancel{- \frac{\pi}{4}}\right)$$

$$(B) \quad -\frac{\partial}{\partial \mathrm{Re}(x_l)} \left|\frac{\arctan\left(\frac{b_i}{a_i}\right) - \left|\arctan\left(\frac{b_i}{a_i}\right) - \frac{\pi}{4}\right|}{2} + \frac{\pi}{4}\right|$$

$$= -\operatorname{sign}\left(\frac{\arctan\left(\frac{b_i}{a_i}\right) - \left|\arctan\left(\frac{b_i}{a_i}\right) - \frac{\pi}{4}\right|}{2} + \frac{\pi}{4}\right) \frac{\partial}{\partial \mathrm{Re}(x_l)} \frac{\arctan\left(\frac{b_i}{a_i}\right) - \left|\arctan\left(\frac{b_i}{a_i}\right) - \frac{\pi}{4}\right|}{2} \cancel{+ \frac{\pi}{4}}$$

$$= -\operatorname{sign}\left(\frac{\arctan\left(\frac{b_i}{a_i}\right) - \left|\arctan\left(\frac{b_i}{a_i}\right) - \frac{\pi}{4}\right|}{2} + \frac{\pi}{4}\right) \left[\frac{1}{2} \frac{\partial}{\partial \mathrm{Re}(x_l)} \arctan\left(\frac{b_i}{a_i}\right) - \frac{1}{2} \operatorname{sign}\left(\arctan\left(\frac{b_i}{a_i}\right) - \frac{\pi}{4}\right) \frac{\partial}{\partial \mathrm{Re}(x_l)} \arctan\left(\frac{b_i}{a_i}\right)\right]$$

$$\ast \quad \frac{\partial}{\partial \mathrm{Re}(x_l)} n_i = \frac{\partial}{\partial \mathrm{Re}(x_l)} \max\left(0, \underbrace{\mathrm{Re}\left(e^{i\varphi_i} s_i\right) \mathrm{Re}(R_i) + \mathrm{Im}\left(e^{i\varphi_i} s_i\right) \mathrm{Im}(R_i)}_{= n_{i\,tot}}\right)$$

$$= \frac{\partial}{\partial \mathrm{Re}(x_l)} n_{i\,tot} \times U(n_{i\,tot})$$

$$= n_{i\,tot} \cancel{\frac{\partial}{\partial \mathrm{Re}(x_l)} U(n_{i\,tot})}_{=0} + U(n_{i\,tot}) \frac{\partial}{\partial \mathrm{Re}(x_l)} n_{i\,tot}$$

$$= U(n_{i\,tot}) \frac{\partial}{\partial \mathrm{Re}(x_l)} \left(\cos\varphi_i \, a_i + \sin\varphi_i \, b_i\right)$$

$$= U(n_{i\,tot}) \left[\cos\varphi_i \frac{\partial a_i}{\partial \mathrm{Re}(x_l)} - a_i \sin\varphi_i \frac{\partial \varphi_i}{\partial \mathrm{Re}(x_l)} + \sin\varphi_i \frac{\partial b_i}{\partial \mathrm{Re}(x_l)} + b_i \cos\varphi_i \frac{\partial \varphi_i}{\partial \mathrm{Re}(x_l)}\right]$$

✗ Donc $\frac{\partial}{\partial \mathrm{Re}(x_l)} RMSE_i = \frac{\partial}{\partial \mathrm{Re}(x_l)} r_i \cos\varphi_i \mathrm{Re}(s_i) - r_i \sin\varphi_i \mathrm{Im}(s_i)\Big] - \mathrm{Re}(R_i)$

$$= \mathrm{Re}(s_i)\Big[r_i(-\sin\varphi_i)\frac{\partial}{\partial \mathrm{Re}(x_l)}\varphi_i + \cos\varphi_i \frac{\partial}{\partial \mathrm{Re}(x_l)} r_i\Big] - \mathrm{Im}(s_i)\Big[r_i \cos\varphi_i \frac{\partial}{\partial \mathrm{Re}(x_l)}\varphi_i + \sin\varphi_i \frac{\partial}{\partial \mathrm{Re}(x_l)} r_i\Big] - \mathrm{Re}(H_{i,l})$$

$$IMSE_i = r_i\Big[\sin\varphi_i \mathrm{Re}(s_i) + \cos\varphi_i \mathrm{Im}(s_i)\Big] - \mathrm{Im}(R_i)$$

✗ Donc $\frac{\partial}{\partial \mathrm{Re}(x_l)} IMSE_i = \mathrm{Re}(s_i)\Big[r_i \cos\varphi_i \frac{\partial}{\partial \mathrm{Re}(x_l)}\varphi_i + \sin\varphi_i \frac{\partial}{\partial \mathrm{Re}(x_l)} r_i\Big]$

$$+ \mathrm{Im}(s_i)\Big[r_i(-\sin\varphi_i)\frac{\partial \varphi_i}{\partial \mathrm{Re}(x_l)} + \cos\varphi_i \frac{\partial r_i}{\partial \mathrm{Re}(x_l)}\Big] - \mathrm{Im}(H_{i,l})$$

✗ $\frac{\partial}{\partial \mathrm{Im}(x_l)} \|\underline{r} \otimes \underline{s} - \underline{R}\|_2^2 = \frac{\partial}{\partial \mathrm{Im}(x_l)} \sum_{i=1}^{K} (RMSE_i)^2 + (IMSE_i)^2$

$$= \sum_{i=1}^{K} 2\, RMSE_i \frac{\partial}{\partial \mathrm{Im}(x_l)} RMSE_i + 2\, IMSE_i \frac{\partial}{\partial \mathrm{Im}(x_l)} IMSE_i$$

$$RMSE_i = \max\left(0, \cos\varphi_i a_i + \sin\varphi_i a_i\right)\Big[\cos\varphi_i \mathrm{Re}(s_i) - \sin\varphi_i \mathrm{Im}(s_i)\Big] - \mathrm{Re}(R_i)$$

✗ $\frac{\partial}{\partial \mathrm{Im}(x_l)} a_i = \mathrm{Re}(s_i)\left(-\mathrm{Im}(H_{i,l})\right) + \mathrm{Im}(s_i)\,\mathrm{Re}(H_{i,l})$

✗ $\frac{\partial}{\partial \mathrm{Im}(x_l)} b_i = \mathrm{Re}(s_i)\,\mathrm{Re}(H_{i,l}) + \mathrm{Im}(s_i)\,\mathrm{Im}(H_{i,l})$

✗ $-\sum_{j=1}^{M} \mathrm{Re}(H_{i,j} x_j) = -\mathrm{Re}(R_i)$

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} C = -\sum_{j=1}^{M} \frac{\partial}{\partial \mathrm{Im}(x_l)} \mathrm{Re}(H_{i,j})\mathrm{Re}(x_j) - \mathrm{Im}(H_{i,j})\mathrm{Im}(x_j)$$

$$= -\left(-\mathrm{Im}(H_{i,l})\right) = \mathrm{Im}(H_{i,l})$$

$$* \quad \frac{\partial}{\partial \mathrm{Im}(x_l)} C = -\frac{\partial}{\partial \mathrm{Im}(x_l)} \sum_{n=1}^{M} \mathrm{Im}(H_{i,n} x_n) = -\frac{\partial}{\partial \mathrm{Im}(x_l)} \mathrm{Im}(R_i)$$

$$= -\frac{\partial}{\partial \mathrm{Im}(x_l)} \sum_{n=1}^{M} \mathrm{Re}(H_{i,n}) \mathrm{Im}(x_n) + \mathrm{Im}(H_{i,n}) \mathrm{Re}(x_n)$$

$$= -\mathrm{Re}(H_{i,l})$$

$$* \quad \frac{\partial}{\partial \mathrm{Im}(x_l)} \arctan\left(\frac{b_i}{a_i}\right)$$

$$= \frac{1}{1+\left(\frac{b_i}{a_i}\right)^2} \times \frac{\partial}{\partial \mathrm{Im}(x_l)} \frac{b_i}{a_i} = \frac{1}{1+\left(\frac{b_i}{a_i}\right)^2} \times \left[\frac{(b_i)' a_i - b_i (a_i)'}{a_i^2}\right]$$

$$* \quad \frac{\partial \varphi}{\partial \mathrm{Im}(x_l)} = \frac{\frac{\partial}{\partial \mathrm{Im}(x_l)} \arctan\left(\frac{b_i}{a_i}\right)}{4} - \frac{1}{2} \operatorname{sign}\left(\arctan\left(\frac{b_i}{a_i}\right) - \frac{\pi}{4}\right) \frac{\partial}{\partial \mathrm{Im}(x_l)} \left(\arctan\left(\frac{b_i}{a_i}\right) - \cancel{\frac{\pi}{4}}\right)$$

$$- \operatorname{sign}\left(\frac{\arctan\left(\frac{b_i}{a_i}\right) - \left|\arctan\left(\frac{b_i}{a_i}\right) - \frac{\pi}{4}\right|}{2} + \frac{\pi}{4}\right) \left[\frac{1}{2} \frac{\partial}{\partial \mathrm{Im}(x_l)} \arctan\left(\frac{b_i}{a_i}\right) - \frac{1}{2} \operatorname{sign}\left(\arctan\left(\frac{b_i}{a_i}\right) - \frac{\pi}{4}\right) \frac{\partial}{\partial \mathrm{Im}(x_l)} \arctan\left(\frac{b_i}{a_i}\right)\right]$$

$$* \quad \frac{\partial r_i}{\partial \mathrm{Im}(x_l)}$$

$$= U_{(r_i \, tot)} \left[\cos\varphi_i \frac{\partial a_i}{\partial \mathrm{Im}(x_l)} - a_i \sin\varphi_i \frac{\partial \varphi_i}{\partial \mathrm{Im}(x_l)} + \sin\varphi_i \frac{\partial b_i}{\partial \mathrm{Im}(x_l)} + b_i \cos\varphi_i \frac{\partial \varphi_i}{\partial \mathrm{Im}(x_l)}\right]$$

Donc $\frac{\partial}{\partial \mathrm{Im}(x_l)} RMSE_i =$

$$= \mathrm{Re}(s_i)\left[r_i(-\sin\varphi_i)\frac{\partial \varphi_i}{\partial \mathrm{Im}(x_l)} + \cos\varphi_i \frac{\partial r_i}{\partial \mathrm{Im}(x_l)}\right] - \mathrm{Im}(s_i)\left[r_i \cos\varphi_i \frac{\partial \varphi_i}{\partial \mathrm{Im}(x_l)} + \sin\varphi_i \frac{\partial r_i}{\partial \mathrm{Im}(x_l)}\right] + \mathrm{Im}(H_{i,l})$$

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} IMSE_i = \mathrm{Re}(s_i)\left[r_i \cos\varphi_i \frac{\partial \varphi_i}{\partial \mathrm{Re}(x_l)} + \sin\varphi_i \frac{\partial r_i}{\partial \mathrm{Re}(x_l)}\right]$$

$$+ \mathrm{Im}(s_i)\left[r_i(-\sin\varphi_i)\frac{\partial \varphi_i}{\partial \mathrm{Re}(x_l)} + \cos\varphi_i \frac{\partial r_i}{\partial \mathrm{Re}(x_l)}\right] - \mathrm{Re}(H_{i,l})$$

$$x^* = \arg\min \| \underline{s} - H\underline{x} \|_2^2$$

$$\ell_s \quad \overline{(\underline{s} - H\underline{x})}^T (s - H\underline{x})$$

$$= (\bar{s}^T - \bar{\underline{x}}^T \bar{H}^T)(s - H\underline{x})$$

$$= \bar{s}^T s - \bar{\underline{x}}^T \bar{H}^T s - \bar{s}^T H \underline{x} + x^T H^T H \underline{x}$$

$$\frac{\partial}{\partial \underline{x}} \ell = - \bar{s}^T H + H^T H \underline{x}$$