Non-negative vector optimization

Application to precoding for massive antenna

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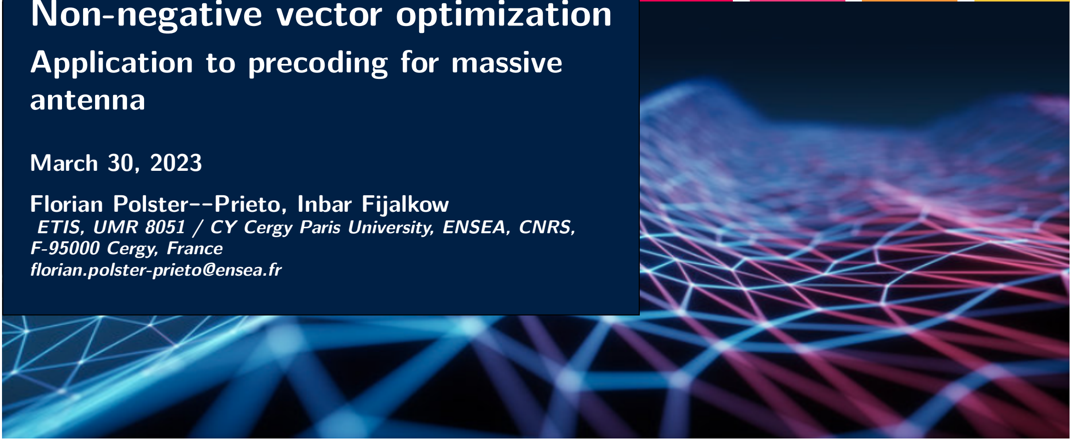

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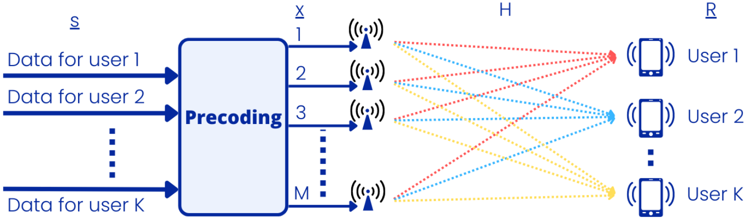
Plan

Quantized massive MIMO precoding

This project focuses on the transmission of data in massive MIMO systems with 1-bit signals (i.e Q-PSK constellation: $\forall z,\ \mathbb{Q}(z) = \pm\frac{1}{\sqrt{2}} \pm j\frac{1}{\sqrt{2}}$).



The channel is thus modeled as:

$$\underset{K\times 1}{\mathbf{R}} = \underset{K\times M}{\mathbf{H}} \times \mathbb{Q}(\underset{M\times 1}{\mathbf{x}}) = \underset{K\times M}{\mathbf{H}} \times \mathbb{Q}(\underset{M\times K}{\mathbf{P}} \times \underset{K\times 1}{\mathbf{s}})$$

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Quantized Zero-Forcing

Quantized Zero-Forcing

The precoding matrix $\mathbf{P}$ is defined as the pseudo-inverse of $\mathbf{H}$

$$\mathbf{P} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}$$

The precoded vector is then defined as

$$\mathbf{x} = \mathbb{Q}(\mathbf{P}\mathbf{s})$$

[Saxena 2017] shows that it achieves asymptotically in K and M (for $\gamma = \frac{K}{M} > 10$) the best quantized symbol error rate.
Our goal is to improve the precoding step for small values of γ.

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Precoding optimization with C2PO

In the case of small values of γ, C2PO [Balatsoukas 2019] outperforms Quantized Zero-Forcing.
It introduces an amplification of the input vector $\underline{s}$ by a complex coefficient α:

C2PO Optimization Problem

$$\{\mathbf{x}^*, \alpha^*\} = \arg\min_{\mathbf{x},\alpha} \|\alpha\mathbf{s} - \mathbf{H}\mathbf{x}\|_2^2$$

For a given $\mathbf{x}$:

- The optimal α^* value is $\alpha^* = \mathbf{s}^H\mathbf{H}\mathbf{x}/\|\mathbf{s}\|_2^2$
- Leading to $\mathbf{x}^* = \arg\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x}\|_2^2$ with $\mathbf{A} = (\mathbf{I}_K - \mathbf{s}\mathbf{s}^H/\|\mathbf{s}\|_2^2)\mathbf{H}$
- Add a regularizer $-\frac{\delta}{2}\|\mathbf{x}\|_2^2$ to avoid the full zero solution

This problem is then solved with a forward-backward splitting (FBS) [Goldstein 2015] algorithm called C2PO.

C2PO algorithm

Algorithm 1: C2PO

Input: $\mathbf{s}, \mathbf{H}, P, \tau^{(t)}, \delta$

Initialize $\mathbf{x}^{(0)} = \mathbf{H}^H\mathbf{s}$

Compute $\rho^{(t)}$

for $t \in [1, t_{max}]$ **do**

$$\mathbf{z}^{(t)} = \mathbf{x}^{(t-1)} - \tau^{(t)} \frac{d\,\|\mathbf{Ax}\|_2^2}{d\mathbf{x}}\bigg|_{\mathbf{x}^{(t-1)}} = \mathbf{x}^{(t-1)} - \tau^{(t)}\mathbf{A}^H\mathbf{A}\mathbf{x}^{(t-1)}$$

$$\mathbf{x}^{(t)} = \mathrm{prox}_g(\mathbf{z}^{(t)}; \rho^{(t)}, \xi)$$

end

Quantize the output $\mathbf{x}^{(t_{max})}$ to the used alphabet

Output: $\mathbf{x}^{(\mathbf{tmax})}$

Non-negative vector optimization

To obtain better results ⇒ relax the degree of freedom of the problem

$$\alpha \mathbf{s} \rightsquigarrow \begin{pmatrix} d_1 \\ \vdots \\ d_k \end{pmatrix} \otimes \mathbf{s} \quad \text{with } d_i \geq 0$$

If non-constrained: $d_i s_i = (\mathbf{Hx})_i = \mathbf{R}_i$ makes no sense

Non negative C2PO Optimization Problem

$$\{\mathbf{x}^*, \mathbf{d}^*\} = \text{arg} \min_{\mathbf{x} \in \mathcal{S}^M, d_i \geq 0} \|\mathbf{d} \otimes \underline{s} - \mathbf{Hx}\|_2^2$$

$\otimes$: element-wise product

$$\mathbf{d} \otimes \mathbf{s} = \mathbf{Ds} = \mathbf{Sd} \quad \text{where} \quad \mathbf{D} = \text{Diag}(\mathbf{d}),\ \mathbf{S} = \text{Diag}(\mathbf{s})$$

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Optimization of the amplification

We first solve the optmization over $\mathbf{d}$ using the Karush-Kuhn-Tucker conditions:

Optimization of $\mathbf{d}$

$$\mathbf{d}^* = \arg\min_{d_i \geq 0} \|\mathbf{D}\mathbf{s} - \mathbf{R}\|_2^2$$

gives $\forall k \in [\![1; K]\!]$

$$d_k = \Big(\Re(s_k)\Re(R_k) + \Im(s_k)\Im(R_k)\Big)^+$$

with $(x)^+ = \max(x, 0)$

Rewriting of the problem

We find

$$\mathbf{d} = \left(\underline{\underline{\mathbf{S}}}\mathbf{A}\underline{\underline{\mathbf{x}}}\right)^{+} = \left(\mathbf{M}\underline{\underline{\mathbf{x}}}\right)^{+}$$

and $\quad \mathbf{R} = \mathcal{I}_K \mathbf{A}\underline{\underline{\mathbf{x}}}$

We use the following notations to write the optimization in a matrix form with exclusively real values:

$$\underline{\underline{\mathbf{x}}} = \begin{pmatrix} \Re(\mathbf{x}) \\ \Im(\mathbf{x}) \end{pmatrix}, \quad \underline{\underline{\mathbf{S}}} = \left(\mathrm{Diag}\left(\Re(\mathbf{s})\right),\ \mathrm{Diag}\left(\Im(\mathbf{s})\right)\right)$$

$$\mathbf{A} = \begin{pmatrix} \Re(\mathbf{H}) & -\Im(\mathbf{H}) \\ \Im(\mathbf{H}) & \Re(\mathbf{H}) \end{pmatrix}, \quad \mathcal{I}_K = \begin{pmatrix} \mathbf{I}_K & j\mathbf{I}_K \end{pmatrix}$$

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Derivation of real vector C2PO

The previous notations lead us to the following derivative

Derivative of the MSE

$$\frac{d\|\mathbf{S}(\mathbf{M}\underline{\underline{\mathbf{x}}})^{+} - \mathcal{I}_K \mathbf{A}\underline{\underline{\mathbf{x}}}\|_2^2}{d\underline{\underline{\mathbf{x}}}} = 2\Re\left[\left(\mathbf{S}\left[\mathbf{M}\otimes\mathcal{U}\right] - \mathcal{I}_K\mathbf{A}\right)^T \overline{\left(\mathbf{S}(\mathbf{M}\underline{\underline{\mathbf{x}}})^{+} - \mathcal{I}_K\mathbf{A}\underline{\underline{\mathbf{x}}}\right)}\right]$$

with

$$\mathcal{U} = \begin{pmatrix} U\left(\sum_{i=1}^{2M}\mathbf{M}_{1,i}\underline{\underline{\mathbf{x}}}_i\right) & \dots & U\left(\sum_{i=1}^{2M}\mathbf{M}_{1,i}\underline{\underline{\mathbf{x}}}_i\right) \\ \vdots & \ddots & \vdots \\ U\left(\sum_{i=1}^{2M}\mathbf{M}_{K,i}\underline{\underline{\mathbf{x}}}_i\right) & \dots & U\left(\sum_{i=1}^{2M}\mathbf{M}_{K,i}\underline{\underline{\mathbf{x}}}_i\right) \end{pmatrix} \text{ and } U(x) = \begin{cases} 0 \text{ if } x \leq 0 \\ 1 \text{ if } x > 0 \end{cases}$$

We implemented a C2PO version with our method: Real vector C2PO.

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Simulation settings

Dataset:

- Pairs of $(\mathbf{H}, \mathbf{s})$ where the Quantized ZF fails (using the SER metric)
- 6 different databsets for different pairs of (K, M)
- 1000 different pairs in each dataset

Table: Number of iterations for the dataset construction to obtain 1000 settings

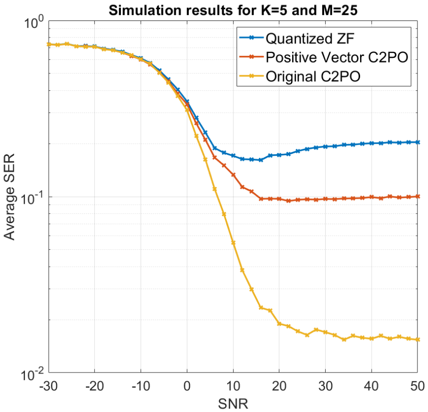
γ	K	M	Realisations
4	5	20	9617
4	25	100	2566
5	5	25	26042
5	20	100	6634
10	5	50	2 293 382
10	10	100	1 203 298

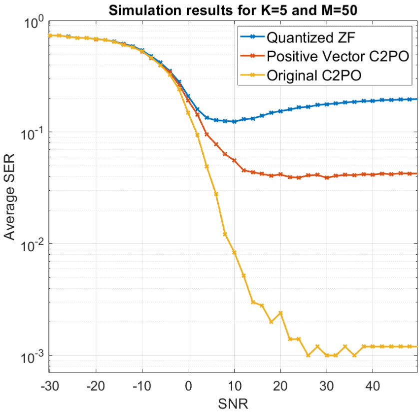
Symbol error rate (SER)

$$\text{SER}(\mathbf{s}, \mathbf{r}) = \frac{\|1_{\mathbf{s}-\mathbf{r}}\|_1}{K}$$

where $1_{\mathbf{s}-\mathbf{r}}$ is a vector whose i-th element is 0 if $s_i - r_i = 0$ and 1 else

Results



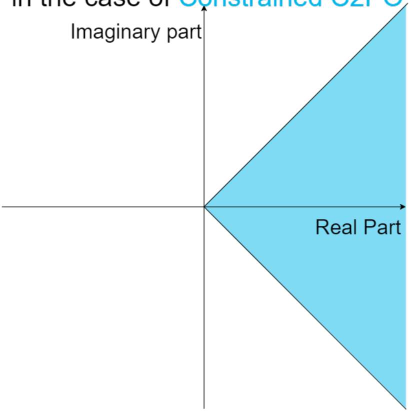


Bad performances compared to Original C2PO $\Rightarrow$ motivated us to improve our method

Proposed improvements

As the previous results were not conclusive compared to the Original C2PO, I proposed multiple ways to improve our method:

1. Model the $\max(0, x)$ function with a sigmoid to have a more precise derivative in 0 $\Rightarrow$ *Sigmoid C2PO*
2. Introduce a complex phase shared by the coefficients $d_i = r_i e^{j\varphi}$ $\Rightarrow$ *Shared Phase C2PO*
3. Constrain $d_i = r_i e^{j\varphi_i}$ to a certain area of the complex space ($\varphi_i \in [-\frac{\pi}{4}; \frac{\pi}{4}]$ and $r_i \geq 0$) to lose no information during a perfect transmission $\Rightarrow$ *Constrained C2PO*



The derivation of $\mathbf{d}$ and of the gradient are complicated (see report)

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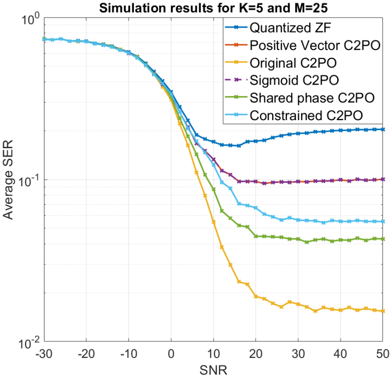
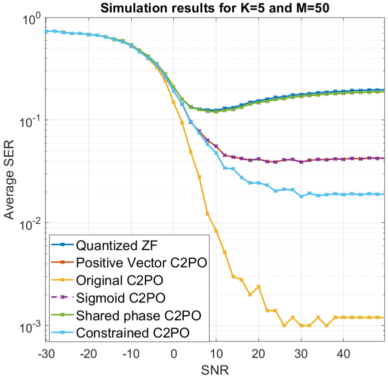
Results of the improved methods





Poor performances:

- FBS is not designed for non-convex optimisation
- Hyperparameter tuning may not be optimal

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Conclusion and perspectives

We studied the mathematical approaches of three methods for massive MIMO 1-bit precoding in the case of a constrained preamplification with a vector $\mathbf{d}$:

$$\{\mathbf{x}^*, \mathbf{d}^*\} = \arg\min_{\mathbf{x},\mathbf{d}} \|\mathbf{d} \otimes \mathbf{s} - \mathbf{H}\mathbf{x}\|_2^2$$

We showed that the different proposed methods perform worse than the Original C2PO. To further improve them and fairly compare all the presented methods:

1. Implement the methods using non-convex optimization tools such as SGD with momentum or other popular methods
2. Learn the hyperparameters (fixed or dynamically at each step) by training a Neural-Network as [Balatsoukas 2019], as a small nudge in a hyperparameter significantly changes the performances

Considered improvements can be used in practice as the Base Station has a high computationnal power