$H P \underline{s} = D \underline{s}$

$H \underline{x} = D \underline{s} \iff H\underline{x} = S \underline{d} \iff \arg\min \|\underline{e}\|^2$

$\underline{d} = S^{-1} H \underline{x}$

$\underline{r} = H \underline{x} \qquad \underline{x} = P \underline{s}$

amplification $\alpha\, d \otimes s \qquad \alpha \in \mathbb{C} \qquad d_i$ réel, ≥ 0

$$P = \underset{P, d, \alpha}{\arg\min} \| H P \underline{s} - \alpha D s \|^2$$

$$H P \underline{s} = \alpha D \underline{s} \implies HP = \alpha D$$

$$H \underline{x} = \alpha D \underline{s} \qquad \alpha^* = H P D^{-1} \quad \text{bof}$$

$$D^{-1} H \underline{x} = \alpha \underline{s}$$

$$\underline{s}^H D^{-1} H \underline{x} = \alpha \underline{s}^H \underline{s}$$

$$\alpha^* = \frac{\underline{s}^H D^{-1} H x}{\|\underline{s}\|^2} = \frac{\underline{s}^H D^{-1} H P \underline{s}}{\|\underline{s}\|^2} = \frac{s^H D^{-1} \underline{r}}{\|\underline{s}\|^2}$$

↳ on a ce qu'on veut pour $\underline{r} = D \underline{s}$ la sortie opt

amplification $e^{i\varphi}\, \underline{d} \otimes \underline{s} \qquad d_i \geq 0$, réel

$$P = \underset{P, d, \varphi}{\arg\min} \| H P \underline{s} - e^{i\varphi} \underline{d} \otimes \underline{s} \|^2$$

$$H P \underline{s} = (\cos(\varphi) + i \sin(\varphi))\, D \underline{s}$$

$$\frac{\underline{s}^H D^{-1} H \underline{x}}{\|\underline{s}\|^2} = e^{i\varphi} \implies \varphi = \arctan\left(\frac{Im(e^{i\varphi})}{Re(e^{i\varphi})}\right) \begin{cases} +0 \text{ si } Re(e^{i\varphi}) > 0 \\ +\pi \text{ si } Re(e^{i\varphi}) < 0 \\ sign(b)\frac{\pi}{2} \text{ si } = 0 \end{cases}$$

Pr garder l'info à l'encodage (sans décodage itératif)

$$(a+ib)(c+id) = ac - bd + i(ad + bc)$$

↳ $sign(ac - bd) = sign(a)$

$sign(ad + be) = sign(b)$ $\implies$ * si $a > 0$ et $b > 0$

$$\begin{cases} ac - bd \geq 0 \\ ad + bc \geq 0 \end{cases} \quad \begin{cases} ae \geq bd \\ ad \geq -bc \end{cases}$$

$$\begin{cases} c \geq \frac{bd}{a} \quad \text{car } a > 0 \\ \frac{ad}{b} \geq -c \implies c \geq \frac{ad}{b} \end{cases}$$

$r e^{i\theta} \in QPSK$

$\hookrightarrow r > 0$

$\theta \in \frac{\pi}{4} \times (2K+1)$

$K \in \mathbb{Z}$

$r e^{i\theta} \times r' e^{i\varphi} = r r' e^{i(\theta+\varphi)}$

pour rester dans le même quadrant du plan complexe (divisé en 4)

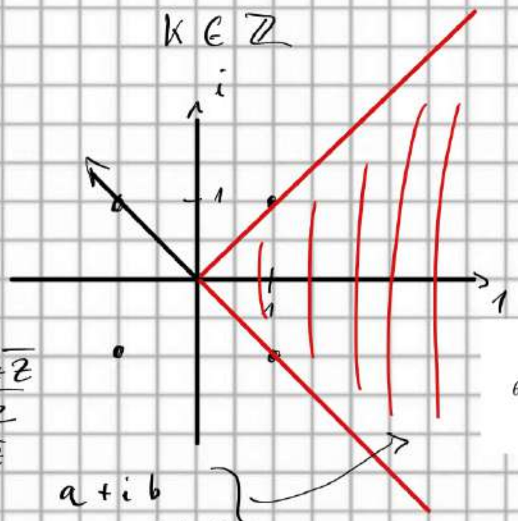
il faut $r' > 0$ et $\varphi \in]-\frac{\pi}{4}, \frac{\pi}{4}[$

i.e pour $r' e^{i\varphi} = a + ib$

$|a+ib| = \sqrt{a^2+b^2} > 0$

implique

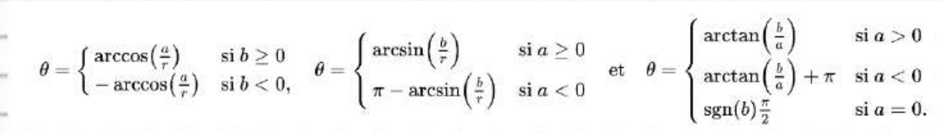
$a > 0$



$a = \frac{z+\bar{z}}{2}$

$b = \frac{z-\bar{z}}{2i}$

$\frac{b}{a} = \frac{z-\bar{z}}{(z+\bar{z})i}$



$$\theta = \begin{cases} \arccos\left(\frac{a}{r}\right) & \text{si } b \geq 0 \\ -\arccos\left(\frac{a}{r}\right) & \text{si } b < 0, \end{cases} \quad \theta = \begin{cases} \arcsin\left(\frac{b}{r}\right) & \text{si } a \geq 0 \\ \pi - \arcsin\left(\frac{b}{r}\right) & \text{si } a < 0 \end{cases} \quad \text{et} \quad \theta = \begin{cases} \arctan\left(\frac{b}{a}\right) & \text{si } a > 0 \\ \arctan\left(\frac{b}{a}\right) + \pi & \text{si } a < 0 \\ \operatorname{sgn}(b)\frac{\pi}{2} & \text{si } a = 0. \end{cases}$$

fct angle matlab.

$$a + ib \quad \left.\begin{array}{l} a > 0 \\ \text{et } b < a \\ \text{et } -b < a \end{array}\right\} \; |b| < a$$

$\overline{(a+ib)\, i} = \overline{-b+ia} = -b - ia$

C2PO

avec seulement α

$\Rightarrow$ pas pris en compte, mais l'initialisation MRT semble empêcher ce problème

$x = H^H s$?

→ initialisation en ZF semble également empêcher ce pb, mais donne de moins bons résultats pour un grand SNR

à vérifier

→ contrainte de positivité peut-être non nécessaire.

$$\alpha^* = \frac{\underline{s}^H D^{-1} H \underline{x}}{\|\underline{s}\|^2}$$

$$= \frac{\underline{s}^H D^{-1} \underline{R}}{\|\underline{s}\|^2}$$

$$= \frac{1}{\underline{s}^H \underline{s}} \overline{\begin{pmatrix} s_1 & \cdots & s_K \end{pmatrix}} \begin{pmatrix} \frac{1}{d_1} & & \\ & \ddots & \\ & & \frac{1}{d_K} \end{pmatrix} \begin{pmatrix} R_1 \\ \vdots \\ R_K \end{pmatrix}$$

$$= \frac{1}{\underline{s}^H \underline{s}} \sum_{i=1}^{K} \frac{s_i}{d_i} r_i = \frac{\sum_{j=1}^{K} \frac{s_j r_j}{d_j}}{\sum_{l=1}^{K} |s_l|^2}$$

? Contraindre α à $|\alpha| > 0$

et $\arctan\left(\frac{\mathrm{Im}(\alpha)}{\mathrm{Re}(\alpha)}\right) \in \left]-\frac{\pi}{4} ; \frac{\pi}{4}\right[$

On détermine $\underline{d} = \operatorname{argmin} \|\alpha^* D \underline{s} - \underline{R}\|^2$

s.t. $d_i > 0$, réel <— plutôt car $\frac{1}{d_i}$

ou $d_i \geq 0$

$$\|\alpha^* D \underline{s} - \underline{R}\|^2 = \left\| \frac{\underline{s}^H D^{-1} \underline{R}}{\|\underline{s}\|^2} \underline{d} \otimes \underline{s} - \underline{R} \right\|^2$$

$$= \sum_{i=1}^{K} \left|\mathrm{Re}\left(\alpha^* d_i s_i - R_i\right)\right|^2 + \left|\mathrm{Im}\left(\alpha^* d_i s_i - R_i\right)\right|^2$$

$$= \sum_{i=1}^{K} \mathrm{Re}\left(\alpha^* d_i s_i - R_i\right)^2 + \mathrm{Im}\left(\alpha^* d_i s_i - R_i\right)^2$$

$$= \sum_{i=1}^{K} \left[d_i \mathrm{Re}\left(\alpha^* s_i\right) - \mathrm{Re}(R_i)\right]^2 + \left[d_i \mathrm{Im}(\alpha^* s_i) - \mathrm{Im}(R_i)\right]^2$$

$$\left(\begin{array}{l} \mathrm{Re}(z_1 z_2) = \mathrm{Re}(z_1)\mathrm{Re}(z_2) - \mathrm{Im}(z_1)\mathrm{Im}(z_2) \\ \mathrm{Im}(z_1 z_2) = \mathrm{Re}(z_1)\mathrm{Im}(z_2) + \mathrm{Re}(z_2)\mathrm{Im}(z_1) \end{array}\right)$$

$$\mathrm{Re}(z_1 + z_2) = \mathrm{Re}(z_1) + \mathrm{Re}(z_2)$$

$$(a + ib)(c + id) = ac - bd + i(ad + bc)$$

$$\mathrm{Im}(z_1 + z_2) = \mathrm{Im}(z_1) + \mathrm{Im}(z_2)$$

$$= \sum_{i=1}^{K} \left[d_i \, \mathrm{Re}\left(\frac{\sum_{j=1}^{K} \frac{s_j r_j}{d_j}}{\sum_{l=1}^{K} |s_l|^2} s_i \right) - \mathrm{Re}(r_i) \right]^2 + \left[d_i \, \mathrm{Im}\left(\frac{\sum_{j=1}^{K} \frac{s_j r_j}{d_j}}{\sum_{l=1}^{K} |s_l|^2} s_i \right) - \mathrm{Im}(r_i) \right]^2$$

$$= \sum_{i=1}^{K} \left[\frac{d_i}{\sum_{l=1}^{K} |s_l|^2} \mathrm{Re}\left(\sum_{j=1}^{K} \frac{s_j r_j s_i}{d_j} \right) - \mathrm{Re}(r_i) \right]^2 + \left[\frac{d_i}{\sum_{l=1}^{K} |s_l|^2} \mathrm{Im}\left(\sum_{j=1}^{K} \frac{s_j r_j s_i}{d_j} \right) - \mathrm{Im}(r_i) \right]^2$$

$$= \sum_{i=1}^{K} \left[\frac{d_i}{\sum_{l=1}^{K} |s_l|^2} \sum_{j=1}^{K} \mathrm{Re}\left(\frac{s_j r_j s_i}{d_j} \right) - \mathrm{Re}(r_i) \right]^2 + \left[\frac{d_i}{\sum_{l=1}^{K} |s_l|^2} \sum_{j=1}^{K} \mathrm{Im}\left(\frac{s_j r_j s_i}{d_j} \right) - \mathrm{Im}(r_i) \right]^2$$

$d_i \in \mathbb{R}$

$$= \sum_{i=1}^{K} \left[\frac{d_i}{\sum_{l=1}^{K} |s_l|^2} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_i) - \mathrm{Re}(r_i) \right]^2 + \left[\frac{d_i}{\sum_{l=1}^{K} |s_l|^2} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Im}(s_j r_j s_i) - \mathrm{Im}(r_i) \right]^2$$

s.t. $-d_i \leq 0$

$$\sum_{l=1}^{K} |s_l|^2 = \sum_{l=1}^{K} \mathrm{Re}(s_l)^2 + \mathrm{Im}(s_l)^2 = \sum_{l=1}^{K} 1 = K$$ si QPSK ou M-PSK sur le cercle unité

On minimise donc $f(d_1, \dots, d_K) = \sum_{i=1}^{K} \left[\frac{d_i}{K} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_i) - \mathrm{Re}(r_i) \right]^2 + \left[\frac{d_i}{K} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Im}(s_j r_j s_i) - \mathrm{Im}(r_i) \right]^2$

$- \lambda_i d_i$

$n \in [\![1 ; K]\!]$

$$\frac{\partial}{\partial d_n} \sum_{i=1}^{K} \left[\frac{d_i}{K} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_i) - \mathrm{Re}(r_i) \right]^2 = \sum_{i=1}^{K} \frac{\partial}{\partial d_n} \Big[\underbrace{\frac{d_i}{K} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_i) - \mathrm{Re}(r_i)}_{g_i} \Big]^2$$

$$= \sum_{i=1}^{K} 2 g_i \times \frac{\partial g_i}{\partial d_n} = \sum_{i=1}^{K} 2 g_i \left[\frac{d_i}{K} \frac{\partial}{\partial d_n} \left[\sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_i) \right] + \frac{\partial}{\partial d_n} \left[\frac{d_i}{K} \right] \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_i) \right]$$

$$\left(\left(\frac{1}{x}\right)' = -\frac{1}{x^2} \right) \quad = \sum_{i=1}^{K} 2 g_i \frac{d_i}{K} \frac{\partial}{\partial d_n} \left[\frac{1}{d_n} \mathrm{Re}(s_n r_n s_i) \right] + \sum_{i=1}^{K} 2 g_i \frac{\partial}{\partial d_n} \left[\frac{d_i}{K} \right] \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_i)$$

$$= \sum_{i=1}^{K} 2 g_i \frac{d_i}{K} \mathrm{Re}(s_n r_n s_i) \times \left(-\frac{1}{d_n^2} \right) + 2 g_n \frac{1}{K} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_n)$$

Donc $$\frac{\partial f(d)}{\partial d_n} = \sum_{i=1}^{K} 2 g_i \frac{d_i}{K} \mathrm{Re}(s_n r_n s_i) \times \left(-\frac{1}{d_n^2} \right) + 2 g_n \frac{1}{K} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_n)$$
$$+ \sum_{i=1}^{K} 2 g_i \frac{d_i}{K} \mathrm{Im}(s_n r_n s_i) \times \left(-\frac{1}{d_n^2} \right) + 2 g_n \frac{1}{K} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Im}(s_j r_j s_n) - \lambda_n$$

On calcule $\frac{\partial f(d)}{\partial d_n} = 0$

$$-\frac{1}{d_n^2}\sum_{i=1}^{K} 2 g_i \frac{d_i}{K} \mathrm{Re}(s_n r_n s_i) + 2 g_n \frac{1}{K}\sum_{j=1}^{K}\frac{1}{d_j}\mathrm{Re}(s_j r_j s_n)$$

$$= -\frac{1}{d_n^2}\sum_{i=1}^{K} 2\left[\frac{d_i}{K}\sum_{j=1}^{K}\frac{1}{d_j}\mathrm{Re}(s_j r_j s_i) - \mathrm{Re}(r_i)\right]\frac{d_i}{K}\mathrm{Re}(s_n r_n s_i)$$

$$+ 2\left[\frac{d_n}{K}\sum_{j=1}^{K}\frac{1}{d_j}\mathrm{Re}(s_j r_j s_n) - \mathrm{Re}(r_n)\right]\frac{1}{K}\sum_{j=1}^{K}\frac{1}{d_j}\mathrm{Re}(s_j r_j s_n)$$

$$= -\frac{1}{d_n^2}\sum_{i=1}^{K}\left[2\frac{d_i^2}{K^2}\mathrm{Re}(s_n r_n s_i)\sum_{j=1}^{K}\frac{1}{d_j}\mathrm{Re}(s_j r_j s_i) - 2\,\mathrm{Re}(r_i)\frac{d_i}{K}\mathrm{Re}(s_n r_n s_i)\right]$$

$$+ 2\frac{d_n}{K^2}\sum_{j=1}^{K}\frac{1}{d_j}\mathrm{Re}(s_j r_j s_n)\sum_{j=1}^{K}\frac{1}{d_j}\mathrm{Re}(s_j r_j s_n) - \mathrm{Re}(r_n)\frac{1}{K}\sum_{j=1}^{K}\frac{1}{d_j}\mathrm{Re}(s_j r_j s_n)$$

? amplification $D \in \mathcal{M}_{K,1}(\mathbb{C})$, $d_j = r_j e^{i\varphi_j}$

se limiter à $r_j \geq 0$ et $\varphi_j \in \left]-\frac{\pi}{4} ; \frac{\pi}{4}\right[$

pour éviter une solution en $\underset{x}{\operatorname{argmin}} \|0\|^2$

$$d_j = r_j e^{i\varphi_j} = a_j + i b_j$$

$(\Leftrightarrow)$ se limiter à $a_j \geq 0$ et $b_j \leq a_j$ $\quad -b_j \leq a_j$

signal pas envoyé $=0$

↳ ? $a_j = b_j$ perte d'info
on pourrait ajouter un angle d'incertitude $\lambda < 1$
$b_j \leq \lambda a_j$

Convexité des fcts :

$$a_j \geq 0 \Leftrightarrow -a_j \leq 0$$

soit $z = a + ib \in \mathbb{C}$
et $z_2 = c + id$

$$f(z) = -a = -\left(\frac{z+\bar{z}}{2}\right)$$

$$f\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = -x_1 = -\left(\frac{x_1+x_2+x_1-x_2}{2}\right) = -x_1$$

$\alpha \in [0,1]$

$$f(\alpha z + (1-\alpha) z_2) = -\alpha a - (1-\alpha) c$$

$$\alpha f(z) + (1-\alpha) f(z_2) = -\alpha a - (1-\alpha) c$$

f convexe

$$b \leq a \iff \frac{b}{a} - 1 \leq 0 \quad \text{si } a \neq 0 \text{, sinon } a = b = 0$$

$$f(z) = \frac{b}{a} - 1 = \frac{z - \overline{z}}{(z + \overline{z})i} - 1$$

$$f\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{x_2}{x_1} - 1 = \frac{x_1 + x_2 - (x_1 - x_2)}{(x_1 + x_2 + x_1 - x_2)} - 1$$

$$\alpha \in [0;1] \quad f(\alpha z + (1-\alpha) z_2) = f(\alpha a + i\alpha b + (1-\alpha)c + i(1-\alpha)d)$$

$$= f(\alpha a + (1-\alpha)c + i(\alpha b + (1-\alpha)d))$$

$$= \frac{\alpha b + (1-\alpha)d}{\alpha a + (1-\alpha)c} - 1 = \frac{\alpha b + (1-\alpha)d - \alpha a - (1-\alpha)c}{\alpha a + (1-\alpha)c}$$

$$= \frac{\alpha(b-a) + (1-\alpha)(d-c)}{\alpha a + (1-\alpha)c} = \frac{\alpha b - \alpha a - \alpha d + \alpha c + d - c}{\alpha a + c - \alpha c}$$

$$= \frac{\alpha(b - a + c - d) + d - c}{\alpha(a-c) + c}$$

espace ds lequel est f $\longrightarrow$ $z = a + ib \in \mathbb{C}$

$$\begin{cases} a > 0 \\ b \leq a \\ -b \leq a \end{cases}$$

$$\left.\begin{array}{l} \alpha a + (1-\alpha)c > 0 \\ \quad \text{car } a > 0,\ c > 0 \\ \quad \text{et } \alpha \in [0;1] \\ \underbrace{\alpha}_{\geqslant 0}\underbrace{(b-a)}_{\leq 0} + \underbrace{(1-\alpha)}_{\geqslant 0}\underbrace{(d-c)}_{\leq 0} \leq 0 \end{array}\right\} \leq 0$$

$$\alpha f(z) + (1-\alpha) f(z_2) = \alpha\left(\frac{b}{a} - 1\right) + (1-\alpha)\left(\frac{d}{c} - 1\right)$$

$$= \alpha \frac{b}{a} - \alpha + (1-\alpha)\frac{d}{c} - (1-\alpha)$$

$$= \alpha \frac{b}{a} + (1-\alpha)\frac{d}{c} - 1$$

$$= \frac{\alpha bc + (1-\alpha)da - ac}{ac} = \frac{\alpha bc + da - \alpha da - ac}{ac}$$

$$f\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{x_2}{x_1} - 1 \qquad \frac{\partial f}{\partial x_1} = -\frac{x_2}{x_1^2}$$

$z_1 = a + ib$

$z_2 = c + id$

$$\frac{\partial^2 f}{\partial x_1^2} = \frac{u'v - uv'}{v^2} = \frac{x_2 \times 2x_1}{x_1^4} = \frac{2x_2}{x_1^3} \qquad \frac{\partial f}{\partial x_2} = \frac{1}{x_1}$$

$$\frac{d}{c} - 1 \geqslant \frac{b}{a} - 1 + \begin{pmatrix} -\frac{b}{a^2} \\ \frac{1}{a} \end{pmatrix} \cdot \begin{pmatrix} c - a \\ d - b \end{pmatrix}$$

$$\frac{\partial^2 f}{\partial x_2^2} = 0$$

$$= \frac{b}{a} - 1 - \frac{bc}{a^2} + \cancel{\frac{b}{a}} + \frac{d}{a} - \cancel{\frac{b}{a}}$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = -\frac{1}{x_1^2}$$

$$\frac{d}{c} - \frac{b}{a} + \frac{bc}{a^2} - \frac{d}{a} = \frac{da^2 - bac + bc^2 - dac}{\underbrace{a^2c}_{} > 0} \ (1)$$

$|b| \leq a \Leftrightarrow \frac{|b|}{a} - 1 \leq 0$ si $a \neq 0$, sinon $a = b = 0$

$$f(z) = \frac{|b|}{a} - 1 = \left|\frac{z-\bar{z}}{2i}\right| \times \frac{2}{z+\bar{z}} - 1$$

$$= \frac{|z-\bar{z}|}{|2i|} \times \frac{2}{z+\bar{z}} - 1 = \frac{|z-\bar{z}|}{z+\bar{z}} - 1 = \frac{|z-\bar{z}|(\overline{z+\bar{z}})}{|z+\bar{z}|^2} - 1$$

$\alpha \in [0;1]$:

$z_1 = a + ib$

$z_2 = c + id$

$$f(\alpha z_1 + (1-\alpha) z_2) = f(\alpha a + (1-\alpha)c + i(\alpha b + (1-\alpha) d))$$

$$(1) = \frac{|\alpha b + (1-\alpha) d|}{\alpha a + (1-\alpha) c} - 1 = \frac{|\alpha b + (1-\alpha) d| - \alpha a - (1-\alpha) c}{\alpha a + (1-\alpha) c}$$

$$\alpha f(z_1) + (1-\alpha) f(z_2) = \alpha\left(\frac{|b|}{a} - 1\right) + (1-\alpha)\left(\frac{|d|}{c} - 1\right)$$

$$= \frac{\alpha|b|}{a} - \alpha + \frac{(1-\alpha)|d|}{c} - 1 + \alpha$$

$(1) \leq (2)$

$(1)-(2) \leq 0$

$$(2) = \frac{\alpha|b|}{a} + \frac{(1-\alpha)|d|}{c} - 1$$

$$(1)-(2) = \frac{|\alpha b + (1-\alpha) d|}{\alpha a + (1-\alpha) c} - \frac{\alpha|b|}{a} - \frac{(1-\alpha)|d|}{c}$$

$$= \frac{ac|\alpha b + (1-\alpha) d| - \alpha|b|(\alpha a + (1-\alpha)c)c - (1-\alpha)|d|(\alpha a + (1-\alpha)c)a}{\underbrace{(\alpha a + (1-\alpha)c)ac}_{\geqslant 0}}$$

inégalité triangulaire

$$ac|\alpha b + (1-\alpha) d| - \alpha|b|(\alpha a + (1-\alpha)c)c - (1-\alpha)|d|(\alpha a + (1-\alpha)c)a \leq ac\alpha|b| + ac(1-\alpha)|d|$$
$$- \alpha|b|(\alpha a + (1-\alpha)c)c - (1-\alpha)|d|(\alpha a + (1-\alpha)c)a$$

$$= a|b|c\left[a - (\alpha a + (1-\alpha)c)\right] + a(1-\alpha)|d|\left[c - (\alpha a + (1-\alpha)c)\right]$$

$$= ac\alpha|b| + ac(1-\alpha)|d| - \alpha^2|b|ac - \alpha|b|(1-\alpha)c^2 - (1-\alpha)a^2|d|\alpha - (1-\alpha)^2|d|ac$$

$$f\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{|x_2|}{x_1} - 1 \qquad \frac{\partial f}{\partial x_1} = -\frac{|x_2|}{x_1^2} \qquad \frac{\partial f}{\partial x_2} = \frac{\text{sign}(x_2)}{x_1}$$

$$\frac{\partial^2 f}{\partial x_1^2} = \frac{2|x_2|}{x_1^3} \qquad \frac{\partial^2 f}{\partial x_2^2} = \frac{\delta(x_2)}{x_1}$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = -\frac{\text{sign}(x_2)}{x_1^2} \qquad \frac{\partial^2 f}{\partial x_2 \partial x_1} = -\frac{\text{sign}(x_2)}{x_1^2}$$

$\delta(x)$ fonction dirac $\begin{cases} \to +\infty \text{ en } 0 \\ 0 \text{ sinon} \end{cases}$

$$\text{Hess} : \begin{pmatrix} \frac{2|x_2|}{x_1^3} & -\frac{\text{sign}(x_2)}{x_1^2} \\ -\frac{\text{sign}(x_2)}{x_1^2} & \frac{\delta(x_2)}{x_1} \end{pmatrix}$$

$$\text{trace}(\text{Hess}) = \sum \lambda_i = \frac{2|x_2|}{x_1^3} + \frac{\delta(x_2)}{x_1}$$

$\geqslant 0$ car $x_1 > 0$ par déf de l'espace d'entrée de f

$$\det(H_{es}) = \prod \lambda_i = \frac{2|x_2|}{x_1^{4}}\delta(x_2) - \left[\frac{\text{sign}(x_2)^2}{(x_1)^{4}}\right] \leq 0 \quad \text{si } x_2 \neq 0$$

amplification $\alpha^* D \Leftrightarrow \quad d_j \in \mathbb{C}$ avec $d_j = r_j e^{i\varphi} \; \forall j \in [\![1;K]\!]$

$\alpha^* \in \mathbb{C}, d_j \in \mathbb{R}$ $\quad r_j \in \mathbb{R}$ $\to$ peut induire des rotations de π si $r<0$ amplitude différente par utilisateurs

ou $\mathbb{R}^+$ $\quad$ 1 seule rotation globale

$$\|D\underline{s} - \underline{R}\|^2 = \sum_{j=1}^{K} \text{Re}(d_j s_j - R_j)^2 + \text{Im}(d_j s_j - R_j)^2$$

$$= \sum_{j=1}^{K} \left[\text{Re}(d_j s_j) - \text{Re}(R_j)\right]^2 + \left[\text{Im}(d_j s_j) - \text{Im}(R_j)\right]^2$$

$$f(r_1, \dots, r_K, \varphi) = \sum_{j=1}^{K} \left[r_j \text{Re}(e^{i\varphi} s_j) - \text{Re}(R_j)\right]^2 + \left[r_j \text{Im}(e^{i\varphi} s_j) - \text{Im}(R_j)\right]^2$$

$$= \sum_{j=1}^{K} \left[r_j\left(\text{Re}(e^{i\varphi})\text{Re}(s_j) - \text{Im}(e^{i\varphi})\text{Im}(s_j)\right) - \text{Re}(R_j)\right]^2$$
$$+ \left[r_j\left(\text{Re}(e^{i\varphi})\text{Im}(s_j) + \text{Re}(s_j)\text{Im}(e^{i\varphi})\right) - \text{Im}(R_j)\right]^2$$

$$\left(\begin{array}{l} \text{Re}(z_1 z_2) = \text{Re}(z_1)\text{Re}(z_2) - \text{Im}(z_1)\text{Im}(z_2) \\ \text{Im}(z_1 z_2) = \text{Re}(z_1)\text{Im}(z_2) + \text{Re}(z_2)\text{Im}(z_1) \end{array}\right)$$

$e^{i\varphi} = \cos\varphi + i\sin\varphi$

$\text{Re}(e^{i\varphi}) = \cos\varphi$

$\text{Im}(e^{i\varphi}) = \sin\varphi$

$$f(r_1, \dots, r_K, \varphi) = \sum_{j=1}^{K} \left[r_j\left(\cos(\varphi)\text{Re}(s_j) - \sin(\varphi)\text{Im}(s_j)\right) - \text{Re}(R_j)\right]^2$$
$$+ \left[r_j\left(\cos(\varphi)\text{Im}(s_j) + \sin(\varphi)\text{Re}(s_j)\right) - \text{Im}(R_j)\right]^2$$

On calcule grad f : $\quad \forall n \in [\![1;K]\!]$

$$\frac{\partial f}{\partial r_n} = 2\,\text{Re}(e^{i\varphi} s_n)\left[r_n \text{Re}(e^{i\varphi} s_n) - \text{Re}(R_n)\right]$$
$$+ 2\,\text{Im}(e^{i\varphi} s_n)\left[r_n \text{Im}(e^{i\varphi} s_n) - \text{Im}(R_n)\right]$$

$$\frac{\partial f}{\partial \varphi} : \frac{\partial}{\partial \varphi}\left(\left[\overbrace{r_j\left(\cos(\varphi)\mathrm{Re}(s_j)-\sin(\varphi)\mathrm{Im}(s_j)\right)-\mathrm{Re}(R_j)}^{u}\right]^2\right)$$

$\sin(x)' = \cos(x)$
$\cos(x)' = -\sin(x)$

$$= 2\frac{\partial}{\partial \varphi}\left(r_j\left[\cos(\varphi)\mathrm{Re}(s_j)-\sin(\varphi)\mathrm{Im}(s_j)\right]\right) \times u$$

$$= 2\left(-r_j\,\mathrm{Re}(s_j)\sin(\varphi) - r_j\,\mathrm{Im}(s_j)\cos(\varphi)\right) \times u$$

$$= -2r_j\left(\mathrm{Re}(s_j)\sin(\varphi)+\mathrm{Im}(s_j)\cos(\varphi)\right)\times\left[r_j\left(\cos(\varphi)\mathrm{Re}(s_j)-\sin(\varphi)\mathrm{Im}(s_j)\right)-\mathrm{Re}(R_j)\right]$$

$$= -2r_j\,\mathrm{Im}(e^{i\varphi}s_j)\times\left[r_j\,\mathrm{Re}(e^{i\varphi}s_j)-\mathrm{Re}(R_j)\right]$$

$$\frac{\partial}{\partial \varphi}\left(\left[\overbrace{r_j\left(\cos(\varphi)\mathrm{Im}(s_j)+\sin(\varphi)\mathrm{Re}(s_j)\right)-\mathrm{Im}(R_j)}^{u'}\right]^2\right)$$

$$= 2\left(-r_j\sin(\varphi)\mathrm{Im}(s_j)+\cos(\varphi)\mathrm{Re}(s_j)\,r_j\right)\times u'$$

$$= 2r_j\left(\cos(\varphi)\mathrm{Re}(s_j)-\sin(\varphi)\mathrm{Im}(s_j)\right)\left[r_j\left(\cos(\varphi)\mathrm{Im}(s_j)+\sin(\varphi)\mathrm{Re}(s_j)\right)-\mathrm{Im}(R_j)\right]$$

$$= 2r_j\,\mathrm{Re}(e^{i\varphi}s_j)\left[r_j\,\mathrm{Im}(e^{i\varphi}s_j)-\mathrm{Im}(R_j)\right]$$

$$\left(\frac{z+\bar z}{2}\right)^2 = \frac{z^2+2z\bar z+\bar z^2}{4} \qquad \left(\frac{z+\bar z}{2}\right)\left(\frac{z-\bar z}{2i}\right) = \frac{z^2-z\bar z+z\bar z-\overline{zz}}{4i} = \frac{zz-\overline{zz}}{4i}$$

$$= \frac{1}{2}\mathrm{Im}(zz)$$

On a donc $\displaystyle\frac{\partial f}{\partial \varphi} = \sum_{j=1}^{K} -2r_j\,\mathrm{Im}(e^{i\varphi}s_j)\left[r_j\,\mathrm{Re}(e^{i\varphi}s_j)-\mathrm{Re}(R_j)\right] + 2r_j\,\mathrm{Re}(e^{i\varphi}s_j)\left[r_j\,\mathrm{Im}(e^{i\varphi}s_j)-\mathrm{Im}(R_j)\right]$

On annule grad f

$$\frac{\partial f}{\partial r_n} = 0 \iff 2\,\mathrm{Re}(e^{i\varphi}s_n)\left[r_n\,\mathrm{Re}(e^{i\varphi}s_n)-\mathrm{Re}(R_n)\right] + 2\,\mathrm{Im}(e^{i\varphi}s_n)\left[r_n\,\mathrm{Im}(e^{i\varphi}s_n)\,\mathrm{Im}(R_n)\right]=0$$

$$\iff r_n\,\mathrm{Re}(e^{i\varphi}s_n)^2-\mathrm{Re}(e^{i\varphi}s_n)\mathrm{Re}(R_n)+r_n\,\mathrm{Im}(e^{i\varphi}s_n)^2-\mathrm{Im}(e^{i\varphi}s_n)\mathrm{Im}(R_n)=0$$

lorsque $\vec{R_n} \perp \overrightarrow{e^{i\varphi}s_n}$
$r_n = 0$

$$r_n = \frac{\mathrm{Re}(e^{i\varphi}s_n)\mathrm{Re}(R_n)+\mathrm{Im}(e^{i\varphi}s_n)\mathrm{Im}(R_n)}{\mathrm{Re}(e^{i\varphi}s_n)^2+\mathrm{Im}(e^{i\varphi}s_n)^2}$$

$\leftarrow$ 1 si $s_n \in M$-PSK

$$\frac{\partial f}{\partial \varphi} = 0 \iff \sum_{j=1}^{K} -2 r_j \mathrm{Im}(e^{i\varphi}s_j)\left[r_j \mathrm{Re}(e^{i\varphi}s_j) - \mathrm{Re}(R_j)\right] + 2 r_j \mathrm{Re}(e^{i\varphi}s_j)\left[r_j \mathrm{Im}(e^{i\varphi}s_j) - \mathrm{Im}(R_j)\right] = 0$$

$$\iff \sum_{j=1}^{K} \left[r_j^2 \mathrm{Re}(e^{i\varphi}s_j)\mathrm{Im}(e^{i\varphi}s_j) - r_j \mathrm{Re}(e^{i\varphi}s_j)\mathrm{Im}(R_j)\right]$$

$$= \sum_{j=1}^{K} \left[r_j^2 \mathrm{Im}(e^{i\varphi}s_j)\mathrm{Re}(e^{i\varphi}s_j) - r_j \mathrm{Im}(e^{i\varphi}s_j)\mathrm{Re}(R_j)\right]$$

? égalité si $\forall j \in [\![1;K]\!]$

suffisant mais pas nécessaire ?

$$\cancel{r_j^2 \mathrm{Re}(e^{i\varphi}s_j)\mathrm{Im}(e^{i\varphi}s_j)} - r_j \mathrm{Re}(e^{i\varphi}s_j)\mathrm{Im}(R_j) = \cancel{r_j^2 \mathrm{Im}(e^{i\varphi}s_j)\mathrm{Re}(e^{i\varphi}s_j)} - r_j \mathrm{Im}(e^{i\varphi}s_j)\mathrm{Re}(R_j)$$

$$\Rightarrow \quad r_j \mathrm{Re}(e^{i\varphi}s_j)\mathrm{Im}(R_j) = r_j \mathrm{Im}(e^{i\varphi}s_j)\mathrm{Re}(R_j)$$

$*$ si $r_j \neq 0$ $\quad \mathrm{Re}(e^{i\varphi}s_j)\mathrm{Im}(R_j) = \mathrm{Im}(e^{i\varphi}s_j)\mathrm{Re}(R_j)$

$$\iff \left[\mathrm{Re}(e^{i\varphi})\mathrm{Re}(s_j) - \mathrm{Im}(e^{i\varphi})\mathrm{Im}(s_j)\right]\mathrm{Im}(R_j) = \left[\mathrm{Im}(e^{i\varphi})\mathrm{Re}(s_j) + \mathrm{Im}(s_j)\mathrm{Re}(e^{i\varphi})\right]\mathrm{Re}(R_j)$$

$$\cos(\varphi)\mathrm{Re}(s_j)\mathrm{Im}(R_j) - \sin(\varphi)\mathrm{Im}(s_j)\mathrm{Im}(R_j) = \sin(\varphi)\mathrm{Re}(s_j)\mathrm{Re}(R_j) + \cos(\varphi)\mathrm{Im}(s_j)\mathrm{Re}(R_j)$$

$$\cos(\varphi)\left[\mathrm{Re}(s_j)\mathrm{Im}(R_j) - \mathrm{Im}(s_j)\mathrm{Re}(R_j)\right] = \sin(\varphi)\left[\underbrace{\mathrm{Re}(s_j)\mathrm{Re}(R_j) + \mathrm{Im}(s_j)\mathrm{Im}(R_j)}_{\delta_j}\right]$$

$*$ si $\delta_j \neq 0$ et $\cos\varphi \neq 0$

$$\frac{\sin(\varphi)}{\cos(\varphi)} = \tan(\varphi) = \frac{\mathrm{Re}(s_j)\mathrm{Im}(R_j) - \mathrm{Im}(s_j)\mathrm{Re}(R_j)}{\mathrm{Re}(s_j)\mathrm{Re}(R_j) + \mathrm{Im}(s_j)\mathrm{Im}(R_j)}$$

$$\varphi = \arctan\left[\frac{\mathrm{Re}(s_j)\mathrm{Im}(R_j) - \mathrm{Im}(s_j)\mathrm{Re}(R_j)}{\mathrm{Re}(s_j)\mathrm{Re}(R_j) + \mathrm{Im}(s_j)\mathrm{Im}(R_j)}\right]$$

$*$ si $\delta_j = 0$ ou $\cos(\varphi) = 0$

$*$ $\delta_j = 0$ et $\cos(\varphi) \neq 0$

$$\varphi = \mathrm{sign}\left[\mathrm{Re}(s_j)\mathrm{Im}(R_j) - \mathrm{Im}(s_j)\mathrm{Re}(R_j)\right] \times \frac{\pi}{2}$$

$\cos\varphi = 0$:

$\varphi = \frac{\pi}{2}$ ou $-\frac{\pi}{2}$

* $s_j \neq 0$ et $\cos\varphi = 0$

$\varphi =$

* $s_j = 0$ et $\cos\varphi = 0$

$\varphi =$

* si $r_j = 0$ $\quad r_j \operatorname{Re}(e^{i\varphi}s_j)\operatorname{Im}(R_j) = r_j \operatorname{Im}(e^{i\varphi}s_j)\operatorname{Re}(R_j)$

or on a $r_j = \operatorname{Re}(e^{i\varphi}s_j)\operatorname{Re}(R_j) + \operatorname{Im}(e^{i\varphi}s_j)\operatorname{Im}(R_j) = 0$ $\Leftrightarrow$

$$\left[\operatorname{Re}(e^{i\varphi})\operatorname{Re}(s_j) - \operatorname{Im}(e^{i\varphi})\operatorname{Im}(s_j)\right]\operatorname{Re}(R_j) + \left[\operatorname{Im}(e^{i\varphi})\operatorname{Re}(s_j) + \operatorname{Re}(e^{i\varphi})\operatorname{Im}(s_j)\right]\operatorname{Im}(R_j) = 0$$

$$\cos(\varphi)\operatorname{Re}(s_j)\operatorname{Re}(R_j) - \sin(\varphi)\operatorname{Im}(s_j)\operatorname{Re}(R_j) + \sin(\varphi)\operatorname{Re}(s_j)\operatorname{Im}(R_j) + \cos(\varphi)\operatorname{Im}(s_j)\operatorname{Im}(R_j) = 0$$

$$\cos(\varphi)\left[\operatorname{Re}(s_j)\operatorname{Re}(R_j) + \operatorname{Im}(s_j)\operatorname{Im}(R_j)\right] = \sin(\varphi)\left[\operatorname{Im}(s_j)\operatorname{Re}(R_j) - \operatorname{Re}(s_j)\operatorname{Im}(R_j)\right]$$

$$r_j = \operatorname{Re}(e^{i\varphi}s_j)\operatorname{Re}(R_j) + \operatorname{Im}(e^{i\varphi}s_j)\operatorname{Im}(R_j)$$

$$\Leftrightarrow \sum_{j=1}^{K}\left[r_j^2 \operatorname{Re}(e^{i\varphi}s_j)\operatorname{Im}(e^{i\varphi}s_j) - r_j \operatorname{Re}(e^{i\varphi}s_j)\operatorname{Im}(R_j)\right]$$
$$= \sum_{j=1}^{K}\left[r_j^2 \operatorname{Im}(e^{i\varphi}s_j)\operatorname{Re}(e^{i\varphi}s_j) - r_j \operatorname{Im}(e^{i\varphi}s_j)\operatorname{Re}(R_j)\right]$$

$$\Leftrightarrow \sum_{j=1}^{K} \cancel{+}\, r_j \operatorname{Re}(e^{i\varphi}s_j)\operatorname{Im}(R_j) = \sum_{j=1}^{K} \cancel{+}\, r_j \operatorname{Im}(e^{i\varphi}s_j)\operatorname{Re}(R_j)$$

$$\Leftrightarrow \sum_{j=1}^{K} \operatorname{Re}(e^{i\varphi}s_j)^2 \operatorname{Im}(R_j)\operatorname{Re}(R_j) + \operatorname{Re}(e^{i\varphi}s_j)\operatorname{Im}(e^{i\varphi}s_j)\operatorname{Im}(R_j)^2$$
$$= \sum_{j=1}^{K} \operatorname{Re}(e^{i\varphi}s_j)\operatorname{Re}(R_j)^2\operatorname{Im}(e^{i\varphi}s_j) + \operatorname{Im}(e^{i\varphi}s_j)^2\operatorname{Re}(R_j)\operatorname{Im}(R_j)$$

$$\Leftrightarrow \sum_{j=1}^{K}\left[\operatorname{Re}(e^{i\varphi})\operatorname{Re}(s_j) - \operatorname{Im}(e^{i\varphi})\operatorname{Im}(s_j)\right]^2\operatorname{Im}(R_j)\operatorname{Re}(R_j) +$$
$$\left[\operatorname{Re}(e^{i\varphi})\operatorname{Re}(s_j) - \operatorname{Im}(e^{i\varphi})\operatorname{Im}(s_j)\right]\left[\operatorname{Re}(e^{i\varphi})\operatorname{Im}(s_j) + \operatorname{Im}(e^{i\varphi})\operatorname{Re}(s_j)\right]\operatorname{Im}(R_j)^2$$

$$= \sum_{j=1}^{K} \left[\mathrm{Re}(e^{i\varphi})\mathrm{Re}(s_j) - \mathrm{Im}(e^{i\varphi})\mathrm{Im}(s_j)\right] \mathrm{Re}(R_j)^2 \left[\mathrm{Re}(e^{i\varphi})\mathrm{Im}(s_j) + \mathrm{Im}(e^{i\varphi})\mathrm{Re}(s_j)\right]$$
$$+ \left[\mathrm{Re}(e^{i\varphi})\mathrm{Im}(s_j) + \mathrm{Im}(e^{i\varphi})\mathrm{Re}(s_j)\right]^2 \mathrm{Re}(R_j)\mathrm{Im}(R_j)$$

$$\Leftrightarrow \sum_{j=1}^{K} \mathrm{Re}(e^{i\varphi})^2\mathrm{Re}(s_j)^2\mathrm{Im}(R_j)\mathrm{Re}(R_j) - 2\,\mathrm{Re}(e^{i\varphi})\,\mathrm{Im}(e^{i\varphi})\,\mathrm{Re}(s_j)\mathrm{Im}(s_j)\mathrm{Im}(R_j)\mathrm{Re}(R_j)$$
$$+ \mathrm{Im}(e^{i\varphi})^2\mathrm{Im}(s_j)^2\,\mathrm{Im}(R_j)\,\mathrm{Re}(R_j) + \mathrm{Re}(e^{i\varphi})^2\mathrm{Re}(s_j)\mathrm{Im}(s_j)\mathrm{Im}(R_j)^2$$
$$+ \mathrm{Re}(e^{i\varphi})\mathrm{Im}(e^{i\varphi})\,\mathrm{Re}(s_j)^2\,\mathrm{Im}(R_j)^2 - \mathrm{Im}(e^{i\varphi})\,\mathrm{Re}(e^{i\varphi})\,\mathrm{Im}(s_j)^2\,\mathrm{Im}(R_j)^2 - \mathrm{Im}(e^{i\varphi})^2\,\mathrm{Im}(s_j)\mathrm{Re}(s_j)\,\mathrm{Im}(R_j)^2$$

$$= \sum_{j=1}^{4} \mathrm{Re}(e^{i\varphi})^2\mathrm{Re}(s_j)\,\mathrm{Im}(s_j)\,\mathrm{Re}(R_j)^2 + \mathrm{Re}(e^{i\varphi})\mathrm{Im}(e^{i\varphi})\,\mathrm{Re}(s_j)^2\,\mathrm{Re}(R_j)^2$$
$$- \mathrm{Re}(e^{i\varphi})\mathrm{Im}(e^{i\varphi})\,\mathrm{Im}(s_j)^2\,\mathrm{Re}(R_j)^2 - \mathrm{Im}(e^{i\varphi})^2\mathrm{Im}(s_j)\mathrm{Re}(s_j)\mathrm{Re}(R_j)^2$$
$$+ \mathrm{Re}(e^{i\varphi})^2\mathrm{Im}(s_j)^2\,\mathrm{Re}(R_j)\,\mathrm{Im}(R_j) + 2\,\mathrm{Re}(e^{i\varphi})\,\mathrm{Im}(s_j)\mathrm{Im}(e^{i\varphi})\mathrm{Re}(s_j)\,\mathrm{Re}(R_j)\mathrm{Im}(R_j)$$
$$+ \mathrm{Im}(e^{i\varphi})^2\,\mathrm{Re}(s_j)^2\,\mathrm{Re}(R_j)\,\mathrm{Im}(R_j)$$

et $\cos\left(\frac{\pi}{2} - a\right) = \sin a$

$\sin\left(\frac{\pi}{2} - a\right) = \cos a$

Plutôt $f(r_1, \dots, r_n, \varphi) = f(r_1, \dots, r_n, \cos\varphi, \sin\varphi)$

$$= \sum_{j=1}^{4} \left[r_j\left(\cos(\varphi)\mathrm{Re}(s_j) - \sin(\varphi)\mathrm{Im}(s_j)\right) - \mathrm{Re}(R_j)\right]^2$$
$$+ \left[r_j\left(\cos(\varphi)\mathrm{Im}(s_j) + \sin(\varphi)\mathrm{Re}(s_j)\right) - \mathrm{Im}(R_j)\right]^2$$

$\frac{\partial f}{\partial r_n}$ et r_n pareil qu'au dessus

$\cos^2 x + \sin^2 x = 1$

$$\frac{\partial}{\partial \cos x}(\sin x) = \frac{\partial}{\partial a}\left(\sqrt{1-a^2}\right) = \frac{-2a}{2\sqrt{1-a^2}} = \frac{-\cos x}{\sin x} = \frac{-1}{\tan x}$$
$$\overrightarrow{a = \cos x}$$

$$\frac{\partial}{\partial \sin x}(\cos x) = \frac{\partial}{\partial b}\left(\sqrt{1-b^2}\right) = \frac{-2b}{2\sqrt{1-b^2}} = \frac{-\sin x}{\cos x} = -\tan x$$
$$\overrightarrow{b = \sin x}$$

$\frac{\partial f}{\partial \cos\varphi}$: $\frac{\partial}{\partial \cos\varphi}\left[r_j\left(\cos(\varphi)\,\mathrm{Re}(s_j) - \sin\varphi\,\mathrm{Im}(s_j)\right) - \mathrm{Re}(R_j)\right]^2$

$$= 2\,u_j \frac{\partial}{\partial \cos\varphi}\left[r_j \cos\varphi\,\mathrm{Re}(s_j) - r_j\sin(\varphi)\,\mathrm{Im}(s_j)\right]$$

$$= 2\,u_j\left[r_j\mathrm{Re}(s_j) + r_j\frac{\mathrm{Im}(s_j)}{\tan(\varphi)}\right]$$

$\tan = \frac{\sin}{\cos}$

$$= 2\left[r_j\,\mathrm{Re}(e^{i\varphi}s_j) - \mathrm{Re}(R_j)\right]\left[r_j\,\mathrm{Re}(s_j) + r_j\frac{\mathrm{Im}(s_j)}{\tan(\varphi)}\right)$$

$$= 2\; r_j^2 \, Re(e^{i\varphi} s_j) Re(s_j) + r_j^2 \frac{Re(e^{i\varphi} s_j) Im(s_j)}{\tan\varphi} - r_j Re(R_j) Re(s_j) + Re(R_j)\, r_j \frac{Im(s_j)}{\tan\varphi}$$

$$\frac{d}{d\cos\varphi}\Big[r_j \big(\cos(\varphi) Im(s_j) + \sin(\varphi) Re(s_j) \big) - Im(R_j) \Big]^2$$

$$= 2u' \frac{d}{d\cos\varphi}\Big(r_j \cos(\varphi) Im(s_j) + r_j \sin(\varphi) Re(s_j) \Big]$$

$$= 2u_j' \Big[r_j Im(s_j) - r_j \frac{Re(s_j)}{\tan(\varphi)} \Big]$$

$$= 2\Big[r_j Im(e^{i\varphi} s_j) - Im(R_j) \Big] \Big[r_j Im(s_j) - r_j \frac{Re(s_j)}{\tan(\varphi)}$$

$$= 2 r_j^2 Im(e^{i\varphi} s_j) Im(s_j) - r_j^2 \frac{Im(e^{i\varphi} s_j)}{\tan\varphi} - r_j Im(R_j) Im(s_j) + r_j \frac{Im(R_j) Re(s_j)}{\tan\varphi}$$

$$r_j = Re(e^{i\varphi} s_j) Re(R_j) + Im(e^{i\varphi} s_j) Im(R_j)$$

$$= \big[\cos\varphi\, Re(s_j) - \sin\varphi\, Im(s_j)\big] Re(R_j) + \big[\cos\varphi\, Im(s_j) + \sin\varphi\, Re(s_j)\big] Im(R_j)$$

$$= \cos(\varphi) \Big[\underbrace{Re(s_j) Re(R_j) + Im(s_j) Im(R_j)}_{a_j} \Big] + \sin\varphi \Big[\underbrace{Re(s_j) Im(R_j) - Im(s_j) Re(R_j)}_{b_j} \Big]$$

$$\Leftrightarrow \sum_{j=1}^{K} \cancel{2}\, r_j Re(e^{i\varphi} s_j) Im(R_j) = \sum_{j=1}^{K} \cancel{2}\, r_j Im(e^{i\varphi} s_j) Re(R_j)$$

$$\Leftrightarrow \sum_{j=1}^{K} \big(\cos(\varphi) a_j + \sin(\varphi) b_j \big) \Big[\cos(\varphi) Re(s_j) - \sin(\varphi) Im(s_j) \Big] Im(R_j)$$

$$= \sum_{j=1}^{K} \big(\cos(\varphi) a_j + \sin(\varphi) b_j \big) \Big[\cos\varphi\, Im(s_j) + \sin\varphi\, Re(s_j) \Big] Re(R_j)$$

$$\Leftrightarrow \sum_{j=1}^{K} \cos(\varphi)^2 a_j Re(s_j) Im(R_j) - \cos\varphi \sin\varphi\, a_j Im(s_j) Im(R_j) + \sin(\varphi)\cos(\varphi)\, b_j Re(s_j) Im(R_j)$$
$$- \sin(\varphi)^2 b_j Im(s_j) Im(R_j)$$

$$= \sum_{j=1}^{K} \cos(\varphi)^2 a_j Im(s_j) Re(R_j) + \cos\varphi \sin\varphi\, a_j Re(s_j) Re(R_j) + \cos\varphi \sin\varphi\, b_j Im(s_j) Re(R_j)$$
$$+ \sin(\varphi)^2 b_j Re(s_j) Re(R_j)$$

$$\Leftrightarrow \sum_{j=1}^{K} \cos^2(\varphi) a_j \Big[Re(s_j) Im(R_j) - Im(s_j) Re(R_j) \Big] - \sin^2(\varphi) b_j \Big[Im(s_j) Im(R_j) + Re(s_j) Re(R_j) \Big]$$

$$- \cos(\varphi)\sin(\varphi)\, a_j \left[\mathrm{Im}(s_j)\,\mathrm{Im}(R_j) + \mathrm{Re}(s_j)\,\mathrm{Re}(R_j) \right]$$

$$+ \sin(\varphi)\cos(\varphi)\, b_j \left[\mathrm{Re}(s_j)\,\mathrm{Im}(R_j) - \mathrm{Im}(s_j)\,\mathrm{Re}(R_j) \right] = 0$$

$$\iff \sum_{j=1}^{K} \cos^2(\varphi)\, a_j b_j - \sin^2(\varphi)\, b_j a_j - \cos(\varphi)\sin(\varphi)\, a_j^2 + \sin(\varphi)\cos(\varphi)\, b_j^2 = 0$$

$\cos^2 + \sin^2 = 1$ $\quad \sin^2 = 1 - \cos^2$ $\quad \cos(2a) = \cos^2 a - \sin^2 a$

$$\cos(a)\sin(a) = \frac{1}{2}\sin(2a)$$

$$\iff \sum_{j=1}^{K} a_j b_j \left(\cos(\varphi)^2 - \sin(\varphi)^2 \right) + \cos\varphi \sin\varphi \left[b_j^2 - a_j^2 \right] = 0$$

$$\iff \cos(2\varphi) \sum_{j=1}^{K} a_j b_j = \frac{1}{2}\sin(2\varphi) \sum_{j=1}^{K} \left(a_j^2 - b_j^2 \right)$$

$$\Rightarrow \frac{\sin(2\varphi)}{\cos(2\varphi)} = \tan(2\varphi) = \frac{2\sum_{j=1}^{K} a_j b_j}{\sum_{j=1}^{K} \left(a_j^2 - b_j^2 \right)}$$

$$\varphi = \frac{1}{2}\arctan\left(\frac{2\sum_{j=1}^{K} a_j b_j}{\sum_{j=1}^{K} \left(a_j^2 - b_j^2 \right)} \right) \quad \text{si } \sum_{j=1}^{K} \left(a_j^2 - b_j^2 \right) \neq 0$$

$$\text{sinon} \quad \varphi = \frac{\pi}{4} \times \mathrm{sign}\left(\sum_{j=1}^{K} a_j b_j \right)$$

$$a_j = \mathrm{Re}(s_j)\,\mathrm{Re}(R_j) + \mathrm{Im}(s_j)\,\mathrm{Im}(R_j)$$

$$b_j = \mathrm{Re}(s_j)\,\mathrm{Im}(R_j) - \mathrm{Im}(s_j)\,\mathrm{Re}(R_j)$$

$$a_j b_j = \mathrm{Re}(s_j)^2\,\mathrm{Re}(R_j)\,\mathrm{Im}(R_j) - \mathrm{Re}(s_j)\,\mathrm{Im}(s_j)\,\mathrm{Re}(R_j)^2$$

$$+ \mathrm{Im}(s_j)\,\mathrm{Re}(s_j)\,\mathrm{Im}(R_j)^2 - \mathrm{Im}(s_j)^2\,\mathrm{Im}(R_j)\,\mathrm{Re}(R_j)$$

$$= \mathrm{Re}(R_j)\,\mathrm{Im}(R_j)\left[\mathrm{Re}(s_j)^2 - \mathrm{Im}(s_j)^2 \right] + \mathrm{Im}(s_j)\,\mathrm{Re}(s_j)\left[\mathrm{Im}(R_j)^2 - \mathrm{Re}(R_j)^2 \right]$$

$$a_j^2 = \mathrm{Re}(s_j)^2\,\mathrm{Re}(R_j)^2 + \mathrm{Im}(s_j)^2\,\mathrm{Im}(R_j)^2 + 2\,\mathrm{Re}(s_j)\,\mathrm{Im}(s_j)\,\mathrm{Re}(R_j)\,\mathrm{Im}(R_j)$$

$$b_j^2 = \mathrm{Re}(s_j)^2\,\mathrm{Im}(R_j)^2 + \mathrm{Im}(s_j)^2\,\mathrm{Re}(R_j)^2 - 2\,\mathrm{Re}(s_j)\,\mathrm{Im}(R_j)\,\mathrm{Im}(s_j)\,\mathrm{Re}(R_j)$$

$$a_j^2 - b_j^2 = \mathrm{Re}(R_j)^2\left[\mathrm{Re}(s_j)^2 - \mathrm{Im}(s_j)^2\right] + \mathrm{Im}(R_j)^2\left[\mathrm{Im}(s_j)^2 - \mathrm{Re}(s_j)^2\right]$$

$$+ 4\,\mathrm{Re}(R_j)\,\mathrm{Im}(R_j)\,\mathrm{Re}(s_j)\,\mathrm{Im}(s_j)$$

$$= \left[\mathrm{Re}(R_j)^2 - \mathrm{Im}(R_j)^2\right]\left[\mathrm{Re}(s_j)^2 - \mathrm{Im}(s_j)^2\right]$$

$$+ 4\,\mathrm{Re}(R_j)\,\mathrm{Im}(R_j)\,\mathrm{Re}(s_j)\,\mathrm{Im}(s_j)$$

Soit $z = a + ib$ tq $a^2 + b^2 = 1$ ($z \in$ M-PSK)

$\Leftrightarrow a^2 = 1 - b^2$

On a donc $a^2 - b^2 = 1 - b^2 - b^2 = 1 - 2b^2$

Si $|a| = |b|$ (i.e $z \in$ Q-PSK)

$a^2 - b^2 = 0$

Cas de s_j

pas forcément R_j car non quantifié ds les calculs

de $\displaystyle\sum_{j=1}^{K} a_j b_j = \sum_{j=1}^{K} \mathrm{Im}(s_j)\,\mathrm{Re}(s_j)\left[\mathrm{Im}(R_j)^2 - \mathrm{Re}(R_j)^2\right]$

et $\displaystyle\sum_{j=1}^{K} a_j^2 - b_j^2 = \sum_{j=1}^{K} 4\,\mathrm{Re}(R_j)\,\mathrm{Im}(R_j)\,\mathrm{Re}(s_j)\,\mathrm{Im}(s_j)$

$$\forall n \in [\![1; K]\!] \quad d_n = s_n e^{i\varphi}$$

$$= \left[\mathrm{Re}(e^{i\varphi} s_n)\,\mathrm{Re}(R_n) + \mathrm{Im}(e^{i\varphi} s_n)\,\mathrm{Im}(S_n)\right] e^{i\varphi}$$

$$= \Big(\cos(\varphi)\left[\mathrm{Re}(s_n)\,\mathrm{Re}(R_n) + \mathrm{Im}(s_n)\,\mathrm{Im}(R_n)\right] + \sin(\varphi)\left[\mathrm{Re}(s_j)\,\mathrm{Im}(R_j) - \mathrm{Im}(s_j)\,\mathrm{Re}(R_j)\right]\Big) e^{i\varphi}$$

$$\left(\begin{array}{l} \text{avec } e^{i\varphi} = \cos(\varphi) + i\sin(\varphi) \\ \text{et } \varphi = \dfrac{1}{2}\arctan\left(\dfrac{2\sum_{j=1}^{K}\mathrm{Im}(s_j)\,\mathrm{Re}(s_j)\left[\mathrm{Im}(R_j)^2 - \mathrm{Re}(R_j)^2\right]}{4\sum_{j=1}^{K}\mathrm{Re}(R_j)\,\mathrm{Im}(R_j)\,\mathrm{Re}(s_j)\,\mathrm{Im}(s_j)}\right. \end{array}\right)$$

$$= \left(\cos\varphi\, a_n + \sin\varphi\, b_n\right)\left(\cos\varphi + i\sin\varphi\right)$$

$$= \cos^2(\varphi)\, a_n + i\, a_n \cos\varphi\sin\varphi + \cos\varphi\sin\varphi\, b_n + i\sin^2(\varphi)\, b_n$$

On a $\quad K\updownarrow R = \underset{K}{\updownarrow}\overset{\overset{M}{\longleftrightarrow}}{H}\, x \updownarrow M$

$$\forall n \in [\![1;K]\!],\quad R_n = \sum_{i=1}^{M} H_{n,i}\, x_i$$

On cherche $\quad \underset{\underline{x}}{\arg\min} \left\| \underline{d} \otimes \underline{s} - \underline{R} \right\|_2^2$

on calcule donc $\quad \forall l \in [\![1;M]\!]$

$$\frac{\partial}{\partial x_l}\left(\left\| \underline{d} \otimes \underline{s} - \underline{R} \right\|_2^2 \right) = \frac{\partial}{\partial x_l}\left(\sum_{i=1}^{K} \left| \underline{d} \otimes \underline{s} - \underline{R} \right|_i^2 \right)$$

$$= \frac{\partial}{\partial x_l}\left(\sum_{i=1}^{K} \mathrm{Re}\left(d_i \otimes s_i - R_i\right)^2 + \mathrm{Im}\left(d_i \otimes s_i - R_i\right)^2 \right)$$

$$= \sum_{i=1}^{K} \frac{\partial}{\partial x_l}\left(\mathrm{Re}\left(d_i \otimes s_i - R_i\right)^2\right) + \sum_{i=1}^{K} \frac{\partial}{\partial x_l}\left(\mathrm{Im}\left(d_i \otimes s_i - R_i\right)^2\right)$$

$$\frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{Re}\left(d_i \otimes s_i - R_i\right)^2 = \frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{Re}\left(s_i\left(\cos(\varphi)^2 a_i + \cos(\varphi)\sin(\varphi)\left(b_i + i\, a_i\right) + i\sin(\varphi)^2 b_i\right) - \sum_{j=1}^{M} H_{i,j}\, x_j \right)^2$$

$$= \frac{\partial}{\partial \mathrm{Re}(x_l)} \left[\mathrm{Re}\left(s_i\left[\cos(\varphi)^2 a_i + \cos\varphi\sin\varphi\, b_i\right]\right) + \mathrm{Re}\left(i \times s_i\left[\cos\varphi\sin\varphi\, a_i + \sin^2(\varphi)\, b_i\right]\right) - \sum_{j=1}^{M} \mathrm{Re}\left(H_{i,j}\, x_j\right) \right]^2$$

$$= \frac{\partial}{\partial \mathrm{Re}(x_l)} \Bigg[\underbrace{\underbrace{\mathrm{Re}(s_i)\left(\cos^2(\varphi)\, a_i + \cos\varphi\sin\varphi\, b_i\right)}_{(A)} + \overbrace{\underbrace{\mathrm{Re}(i\, s_i)}_{= -\mathrm{Im}(s_i)}\left(\cos\varphi\sin\varphi\, a_i + \sin^2(\varphi)\, b_i\right)}^{(B)} - \underbrace{\sum_{j=1}^{M} \mathrm{Re}\left(H_{i,j}\, x_j\right)}_{(C)}}_{\mathrm{RMSE}_i} \Bigg]^2$$

$$= 2\, \mathrm{RMSE}_i \frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{RMSE}_i$$

$$\left(\begin{array}{l} \mathrm{Re}\left(H_{i,j}\, x_j\right) = \mathrm{Re}\left(H_{i,j}\right)\mathrm{Re}\left(x_j\right) - \mathrm{Im}\left(H_{i,j}\right)\mathrm{Im}\left(x_j\right) \\ \\ a_i = \mathrm{Re}\left(s_i\right)\mathrm{Re}\left(R_i\right) + \mathrm{Im}\left(s_i\right)\mathrm{Im}\left(R_i\right) \\ b_i = \mathrm{Re}\left(s_i\right)\mathrm{Im}\left(R_i\right) - \mathrm{Im}\left(s_i\right)\mathrm{Re}\left(R_i\right) \end{array} \right)$$

$$\text{et } \varphi = \frac{1}{2} \arctan\left(\frac{\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j)\left[\mathrm{Im}(R_j)^2 - \mathrm{Re}(R_j)^2\right]}{2 \sum_{j=1}^{K} \mathrm{Re}(R_j)\mathrm{Im}(R_j)\mathrm{Re}(s_j)\mathrm{Im}(s_j)} \right)$$

$$(A) \quad \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \mathrm{Re}(s_i)\left(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i\right) = \mathrm{Re}(s_i) \frac{\partial}{\partial \mathrm{Re}(x_\ell)}\Big[\underbrace{\cos^2(\varphi) a_i}_{A_1} + \underbrace{\cos\varphi \sin\varphi\, b_i}_{A_2}\Big]$$

$$(A_1) : \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \cos^2(\varphi) a_i = a_i \underbrace{\frac{\partial}{\partial \mathrm{Re}(x_\ell)} \cos^2(\varphi)}_{A_1'} + \cos^2(\varphi) \underbrace{\frac{\partial}{\partial \mathrm{Re}(x_\ell)} a_i}_{A_1''}$$

$$(A_1') \quad \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \cos^2(\varphi) = 2\cos(\varphi) \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \cos(\varphi)$$

$$= 2\cos(\varphi)\left(-\sin(\varphi)\right) \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \varphi \qquad \left(f(g)\right)' = g' f'(g)$$

$$\frac{\partial}{\partial \mathrm{Re}(x_\ell)} \varphi = \frac{1}{2} \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \left[\arctan\left(\frac{\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j)\left[\mathrm{Im}(R_j)^2 - \mathrm{Re}(R_j)^2\right]}{2\sum_{j=1}^{K} \mathrm{Re}(R_j)\mathrm{Im}(R_j)\mathrm{Re}(s_j)\mathrm{Im}(s_j)} \right) \right]$$

$$= \arctan(\text{système})$$

$$= \frac{1}{2}\left(\frac{1}{1+(\text{système})^2}\right) \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \text{système}$$

$$= \frac{1}{2}\left(\frac{1}{1+(\text{système})^2}\right) \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \left(\frac{\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j)\left[\mathrm{Im}\left(\sum_{n=1}^{M} H_{j,n} x_n\right)^2 - \mathrm{Re}\left(\sum_{n=1}^{M} H_{j,n} x_n\right)^2\right]}{2\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j)\,\mathrm{Im}\left(\sum_{n=1}^{M} H_{j,n} x_n\right)\mathrm{Re}\left(\sum_{n=1}^{M} H_{j,n} x_n\right)} \right)$$

$$= \frac{1}{2+2\,\text{système}^2} \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \left(\frac{\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j)\left[\left(\sum_{n=1}^{M} \mathrm{Re}(H_{j,n})\mathrm{Im}(x_n) + \mathrm{Im}(H_{j,n})\mathrm{Re}(x_n)\right)^2 - \left(\sum_{n=1}^{M} \mathrm{Re}(H_{j,n})\mathrm{Re}(x_n) - \mathrm{Im}(H_{j,n})\mathrm{Im}(x_n)\right)^2\right]}{\text{dénom} \rightarrow 2\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j)\left(\sum_{n=1}^{M} \mathrm{Re}(H_{j,n})\mathrm{Im}(x_n) + \mathrm{Im}(H_{j,n})\mathrm{Re}(x_n)\right)\left(\sum_{n=1}^{M} \mathrm{Re}(H_{j,n})\mathrm{Re}(x_n) - \mathrm{Im}(H_{j,n})\mathrm{Im}(x_n)\right)} \right)$$

$$\frac{\partial}{\partial \mathrm{Re}(x_\ell)} \text{syst} = \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \left(\frac{\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j)\left[\sum_{n=1}^{M} \mathrm{Re}(H_{j,n})\mathrm{Im}(x_n) + \mathrm{Im}(H_{j,n})\mathrm{Re}(x_n)\right]^2}{\text{dénom}} \right)$$

$$- \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \left(\frac{\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j)\left[\sum_{n=1}^{M} \mathrm{Re}(H_{j,n})\mathrm{Re}(x_n) - \mathrm{Im}(H_{j,n})\mathrm{Im}(x_n)\right]^2}{\text{dénom}} \right)$$

$$= \sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j) \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \frac{\left[\sum_{n=1}^{M} \mathrm{Re}(H_{j,n})\mathrm{Im}(x_n) + \mathrm{Im}(H_{j,n})\mathrm{Re}(x_n)\right]^2}{\text{dénom}} \Big\} \text{syst}_1 \qquad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$- \sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j) \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \frac{\left[\sum_{n=1}^{M} \mathrm{Re}(H_{j,n})\mathrm{Re}(x_n) - \mathrm{Im}(H_{j,n})\mathrm{Im}(x_n)\right]^2}{\text{dénom}} \Big\} \text{syst}_2$$

syst$_1$ $$\frac{\partial}{\partial \mathrm{Re}(x_l)} \frac{\left[\sum_{m=1}^{M} \mathrm{Re}(H_{j,m})\mathrm{Im}(x_m)+\mathrm{Im}(H_{j,m})\mathrm{Re}(x_m)\right]^2}{\text{dénum}} = \frac{u_1' \times \text{dénum} - u_1(\text{dénum})'}{(\text{dénum})^2} \qquad u_1 = \mathrm{Im}(R_j)^2$$

$$u_1' = 2 \times \mathrm{Im}(H_{j,l}) \left[\sum_{m=1}^{M} \mathrm{Re}(H_{j,m})\mathrm{Im}(x_m)+\mathrm{Im}(H_{j,m})\mathrm{Re}(x_m)\right]$$

$$v = 2\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j) \overbrace{\left(\sum_{m=1}^{K} \mathrm{Re}(H_{j,m})\mathrm{Re}(x_m)-\mathrm{Im}(H_{j,m})\mathrm{Im}(x_m)\right)}^{\mathrm{Re}(R_j)} \overbrace{\left(\sum_{m=1}^{K} \mathrm{Im}(H_{j,m})\mathrm{Re}(x_m)+\mathrm{Re}(H_{j,m})\mathrm{Im}(x_m)\right)}^{\mathrm{Im}(R_j)}$$

$$\frac{\partial v}{\partial \mathrm{Re}(x_l)} = 2\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j) \left[\mathrm{Re}(R_j)\frac{\partial}{\partial \mathrm{Re}(x_l)}\mathrm{Im}(R_j) + \mathrm{Im}(R_j)\frac{\partial}{\partial \mathrm{Re}(x_l)}\mathrm{Re}(R_j)\right]$$

$$= 2\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j)\left[\mathrm{Re}(R_j)\,\mathrm{Im}(H_{j,l}) + \mathrm{Im}(R_j)\,\mathrm{Re}(H_{j,l})\right] = (\text{dénum})'$$

syst$_2$ $$\frac{\partial}{\partial \mathrm{Re}(x_l)} \frac{\left[\sum_{m=1}^{M} \mathrm{Re}(H_{j,m})\mathrm{Re}(x_m)-\mathrm{Im}(H_{j,m})\mathrm{Im}(x_m)\right]^2}{\text{dénum}} = \frac{u_2' \times \text{dénum} - u_2(\text{dénum})'}{(\text{dénum})^2} \qquad u_2 = \mathrm{Re}(R_j)^2$$

$$u_2' = 2\,\mathrm{Re}(H_{j,l})\left[\sum_{m=1}^{M} \mathrm{Re}(H_{j,m})\mathrm{Re}(x_m)-\mathrm{Im}(H_{j,m})\mathrm{Im}(x_m)\right]$$

$$= 2\,\mathrm{Re}(H_{j,l})\,\mathrm{Re}(R_j)$$

$$\frac{\partial \varphi}{\partial \mathrm{Re}(x_l)} = \frac{1}{2} \times \frac{1}{1+(\text{système})^2} \times \underbrace{\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j)\left[\frac{u_1'(\text{dénum}) - u_1(\text{dénum})'}{(\text{dénum})^2} - \frac{u_2'(\text{dénum}) - u_2(\text{dénum})'}{(\text{dénum})^2}\right]_j}$$

$$\text{système} = \tan(2\varphi)$$

$$\frac{\partial}{\partial \mathrm{Re}(x_l)} \cos^2(\varphi) = 2\cos(\varphi)(-\sin(\varphi)) \times \frac{1}{2} \times \frac{1}{1+(\text{système})^2} \times$$

(A_1'') $$\frac{\partial}{\partial \mathrm{Re}(x_l)} a_i = \frac{\partial}{\partial \mathrm{Re}(x_l)}\left[\mathrm{Re}(s_i)\mathrm{Re}(R_i) + \mathrm{Im}(s_i)\mathrm{Im}(R_i)\right]$$

$$= \mathrm{Re}(s_i)\frac{\partial}{\partial \mathrm{Re}(x_l)}\mathrm{Re}\left(\sum_{m=1}^{M} H_{i,m}x_m\right) + \mathrm{Im}(s_i)\frac{\partial}{\partial \mathrm{Re}(x_l)}\mathrm{Im}\left(\sum_{m=1}^{M} H_{i,m}x_m\right)$$

$$= \mathrm{Re}(s_i)\frac{\partial}{\partial \mathrm{Re}(x_l)}\sum_{m=1}^{M} \mathrm{Re}(H_{i,m})\mathrm{Re}(x_m) - \mathrm{Im}(H_{i,m})\mathrm{Im}(x_m) + \mathrm{Im}(s_i)\frac{\partial}{\partial \mathrm{Re}(x_l)}\sum_{m=1}^{M} \mathrm{Re}(H_{i,m})\mathrm{Im}(x_m) + \mathrm{Im}(H_{i,m})\mathrm{Re}(x_m)$$

$$= \mathrm{Re}(s_i)\,\mathrm{Re}(H_{i,l}) + \mathrm{Im}(s_i)\,\mathrm{Im}(H_{i,l})$$

(A_1) ✓

$$(A_2) \quad \frac{\partial}{\partial \mathrm{Re}(x_l)} \left[\cos\varphi \sin\varphi \, b_i \right] = \cos\varphi \sin\varphi \underbrace{\frac{\partial}{\partial \mathrm{Re}(x_l)} b_i}_{A_2'} + \cos\varphi \, b_i \underbrace{\frac{\partial}{\partial \mathrm{Re}(x_l)} \sin\varphi}_{A_2''} + \sin\varphi \, b_i \underbrace{\frac{\partial}{\partial \mathrm{Re}(x_l)} \cos\varphi}_{A_2'''}$$

$$(A_2') \quad \frac{\partial}{\partial \mathrm{Re}(x_l)} b_i = \frac{\partial}{\partial \mathrm{Re}(x_l)} \left[\mathrm{Re}(s_i)\, \mathrm{Im}(R_i) - \mathrm{Im}(s_i)\, \mathrm{Re}(R_i) \right]$$

$$= \mathrm{Re}(s_i) \frac{\partial}{\mathrm{Re}(x_l)} \mathrm{Im}\left(\sum_{n=1}^{M} H_{i,n} x_n \right) - \mathrm{Im}(s_i) \frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{Re}\left(\sum_{n=1}^{M} H_{i,n} x_n \right)$$

$$= \mathrm{Re}(s_i) \frac{\partial}{\partial \mathrm{Re}(x_l)} \sum_{n=1}^{M} \mathrm{Im}(H_{i,n}) \mathrm{Re}(x_n) + \mathrm{Re}(H_{i,n}) \mathrm{Im}(x_n) - \mathrm{Im}(s_i) \frac{\partial}{\partial \mathrm{Re}(x_l)} \sum_{n=1}^{M} \mathrm{Re}(H_{i,n}) \mathrm{Re}(x_n) - \mathrm{Im}(H_{i,n}) \mathrm{Im}(x_n)$$

$$= \mathrm{Re}(s_i)\, \mathrm{Im}(H_{i,l}) - \mathrm{Im}(s_i)\, \mathrm{Re}(H_{i,l})$$

$$(A_2'') \quad \frac{\partial}{\partial \mathrm{Re}(x_l)} \sin\varphi = \cos(\varphi) \frac{\partial}{\partial \mathrm{Re}(x_l)} \varphi$$

$$(A_2''') \quad \frac{\partial}{\partial \mathrm{Re}(x_l)} \cos\varphi = -\sin(\varphi) \frac{\partial}{\partial \mathrm{Re}(x_l)} \varphi$$

(A_2) ✓

$$(B) \quad \frac{\partial}{\partial \mathrm{Re}(x_l)} \Big(\underbrace{\mathrm{Re}(i s_i)}_{= -\mathrm{Im}(s_i)} \left(\cos\varphi \sin\varphi \, a_i + \sin^2(\varphi) b_i \right) \Big) = -\mathrm{Im}(s_i) \frac{\partial}{\partial \mathrm{Re}(x_l)} \Big[\underbrace{\cos\varphi \sin\varphi \, a_i}_{B_1} + \underbrace{\sin^2(\varphi) b_i}_{B_2} \Big]$$

$$(B_1) \quad \frac{\partial}{\partial \mathrm{Re}(x_l)} \cos\varphi \sin\varphi \, a_i = \cos\varphi \sin\varphi \frac{\partial}{\partial \mathrm{Re}(x_l)} a_i + \cos\varphi \, a_i \frac{\partial}{\partial \mathrm{Re}(x_l)} \sin\varphi + \sin\varphi \, a_i \frac{\partial}{\partial \mathrm{Re}(x_l)} \cos\varphi$$

$$(B_2) \quad \frac{\partial}{\partial \mathrm{Re}(x_l)} \sin^2(\varphi) b_i = b_i \underbrace{\frac{\partial}{\partial \mathrm{Re}(x_l)} \sin^2(\varphi)}_{B_2'} + \sin^2(\varphi) \underbrace{\frac{\partial}{\partial \mathrm{Re}(x_l)} b_i}_{✓}$$

$$(B_2') \quad \frac{\partial}{\partial \mathrm{Re}(x_l)} \sin^2(\varphi) = 2 \sin(\varphi) \frac{\partial}{\partial \mathrm{Re}(x_l)} \sin(\varphi)$$

$$= 2 \sin(\varphi) \cos(\varphi) \frac{\partial}{\partial \mathrm{Re}(x_l)} \varphi \qquad ✓$$

(C) $$\frac{\partial}{\partial \mathrm{Re}(x_l)} - \sum_{j=1}^{M} \mathrm{Re}(H_{i,j} x_j) = - \sum_{j=1}^{M} \frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{Re}(H_{i,j}) \mathrm{Re}(x_j) - \mathrm{Im}(H_{i,j}) \mathrm{Im}(x_j)$$

$$= - \mathrm{Re}(H_{i,l})$$

$$\frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{Im}(\partial_i \otimes s_i - R_i)^2 = \frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{Im}\Big(s_i \big[\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i + i(a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i)\big] - R_i\Big)^2$$

$$= \frac{\partial}{\partial \mathrm{Re}(x_l)} \Big[\underbrace{\overbrace{\mathrm{Im}(s_i)\big(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i\big)}^{A} + \overbrace{\underline{\mathrm{Im}(i s_i)}\big(a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i\big)}^{B} - \overbrace{\mathrm{Im}\Big(\sum_{n=1}^{M} H_{i,n} x_n\Big)}^{C}}_{\mathrm{IMSE}_i} \Big]^2$$

$$\mathrm{Im}(i \times s_i) = \mathrm{Re}(s_i)$$

$$= \frac{1}{2} \mathrm{IMSE}_i \frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{IMSE}_i$$

(A) $$\frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{Im}(s_i) \big(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i\big)$$

$$= \mathrm{Im}(s_i) \underbrace{\frac{\partial}{\partial \mathrm{Re}(x_l)} \big(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i\big)}_{\checkmark}$$

(B) $$\frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{Re}(s_i) \big(a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i\big)$$

$$= \mathrm{Re}(s_i) \underbrace{\frac{\partial}{\partial \mathrm{Re}(x_l)} \big(a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i\big)}_{\checkmark}$$

(C) $$\frac{\partial}{\partial \mathrm{Re}(x_l)} - \mathrm{Im}\Big(\sum_{n=1}^{M} H_{i,n} x_n\Big)$$

$$= - \frac{\partial}{\partial \mathrm{Re}(x_l)} \sum_{n=1}^{M} \mathrm{Re}(H_{i,n}) \mathrm{Im}(x_n) + \mathrm{Im}(H_{i,n}) \mathrm{Re}(x_n)$$

$$= - \mathrm{Im}(H_{i,l})$$

Dérivée en fonction de $\mathrm{Im}(x_l)$ $\forall\, l \in [\![1; K]\!]$

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} \mathrm{Re}\left(\partial_i \otimes s_i - R_i\right)^2 = \frac{\partial}{\partial \mathrm{Im}(x_l)} \mathrm{Re}\left(s_i\left(\cos(\varphi)^2 a_i + \cos(\varphi)\sin(\varphi)\left(b_i + i\, a_i\right) + i\sin(\varphi)^2 b_i\right) - \sum_{j=1}^{M} H_{i,j}\, x_j\right)^2$$

$$= 2\, \mathrm{RMSE}_i \frac{\partial}{\partial \mathrm{Im}(x_l)} \mathrm{RMSE}_i$$

$$(A)\quad \frac{\partial}{\partial \mathrm{Im}(x_l)} \mathrm{Re}(s_i)\left(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i\right) = \mathrm{Re}(s_i) \frac{\partial}{\partial \mathrm{Im}(x_l)}\Big[\underbrace{\cos^2(\varphi) a_i}_{A_1} + \underbrace{\cos\varphi \sin\varphi\, b_i}_{A_2}\Big]$$

$$(A_1)\quad \frac{\partial}{\partial \mathrm{Im}(x_l)} \cos^2(\varphi)\, a_i = a_i \underbrace{\frac{\partial}{\partial \mathrm{Im}(x_l)} \cos^2(\varphi)}_{A_1'} + \cos^2(\varphi) \underbrace{\frac{\partial}{\partial \mathrm{Im}(x_l)} a_i}_{A_1''}$$

$$(A_1')\quad \frac{\partial}{\partial \mathrm{Im}(x_l)} \cos^2(\varphi) = 2\cos(\varphi) \frac{\partial}{\partial \mathrm{Im}(x_l)} \cos(\varphi)$$

$$= 2\cos(\varphi)\left(-\sin(\varphi)\right) \frac{\partial}{\partial \mathrm{Im}(x_l)} \varphi$$

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} \varphi = \frac{1}{2} \times \frac{1}{1 + (\text{système})^2} \times \frac{\partial}{\partial \mathrm{Im}(x_l)} (\text{système})$$

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} \text{système} = \sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j) \frac{\partial}{\partial \mathrm{Im}(x_l)} \frac{\left[\sum_{m=1}^{M} \mathrm{Re}(H_{j,m})\mathrm{Im}(x_m) + \mathrm{Im}(H_{j,m})\mathrm{Re}(x_m)\right]^2}{\text{dénom}} \Big\}\, \text{syst}_1$$

$$- \sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j) \frac{\partial}{\partial \mathrm{Im}(x_l)} \frac{\left[\sum_{m=1}^{M} \mathrm{Re}(H_{j,m})\mathrm{Re}(x_m) - \mathrm{Im}(H_{j,m})\mathrm{Im}(x_m)\right]^2}{\text{dénom}} \Big\}\, \text{syst}_2$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

syst$_1$

$$u_1' \quad \frac{\partial}{\partial \mathrm{Im}(x_l)}\left[\sum_{m=1}^{M} \mathrm{Re}(H_{j,m})\,\mathrm{Im}(x_m) + \mathrm{Im}(H_{j,m})\,\mathrm{Re}(x_m)\right]^2$$

$$= 2\,\mathrm{Im}(R_j) \frac{\partial}{\partial \mathrm{Im}(x_l)}\left[\sum_{m=1}^{M} \mathrm{Re}(H_{j,m})\,\mathrm{Im}(x_m) + \mathrm{Im}(H_{j,m})\,\mathrm{Re}(x_m)\right]$$

$$= 2\,\mathrm{Im}(R_j)\,\mathrm{Re}(H_{j,l})$$

$$u_1 = \mathrm{Im}(R_j)^2$$

$$v = 2\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j)\Big(\overbrace{\sum_{n=1}^{K} \mathrm{Re}(H_{j,n})\mathrm{Re}(x_n) - \mathrm{Im}(H_{j,n})\mathrm{Im}(x_n)}^{\mathrm{Re}(R_j)}\Big)\Big(\overbrace{\sum_{n=1}^{K} \mathrm{Im}(H_{j,n})\mathrm{Re}(x_n) + \mathrm{Re}(H_{j,n})\mathrm{Im}(x_n)}^{\mathrm{Im}(R_j)}\Big)$$

$$\frac{\partial v}{\partial \mathrm{Im}(x_l)} = 2\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j)\left[\mathrm{Re}(R_j)\frac{\partial}{\partial \mathrm{Im}(x_l)}\mathrm{Im}(R_j) + \mathrm{Im}(R_j)\frac{\partial}{\partial \mathrm{Im}(x_l)}\mathrm{Re}(R_j)\right)$$

$$= 2\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j)\left[\mathrm{Re}(R_j)\mathrm{Re}(H_{j,l}) - \mathrm{Im}(R_j)\mathrm{Im}(H_{j,l})\right] = (\text{dénom})'$$

$$\frac{\partial}{\partial \mathrm{Im}(x_l)}\text{syst}_1 = \frac{u_1'\,\text{dénom} - u_1(\text{dénom})'}{(\text{dénom})^2}$$

syst_2

$$u_2 = -\mathrm{Re}(R_j)^2$$

$$u_2' = -2\,\mathrm{Re}(R_j)\frac{\partial}{\partial \mathrm{Im}(x_l)}\sum_{n=1}^{M} \mathrm{Re}(H_{j,n})\mathrm{Re}(x_n) - \mathrm{Im}(H_{j,n})\mathrm{Im}(x_n)$$

$$= 2\,\mathrm{Re}(R_j)\,\mathrm{Im}(H_{j,l})$$

$$\frac{\partial}{\partial \mathrm{Im}(x_l)}\text{syst}_2 = \frac{u_2'\,\text{dénom} - u_2(\text{dénom})'}{(\text{dénom})^2}$$

$$\frac{\partial \varphi}{\partial \mathrm{Im}(x_l)} = \frac{1}{2} \times \frac{1}{1+(\text{système})^2} \times \left[\sum_{j=1}^{K} \mathrm{Re}(s_j)\mathrm{Im}(s_j)\left[\frac{u_1'\,\text{dénom} - u_1(\text{dénom})'}{(\text{dénom})^2} + \frac{u_2'\,\text{dénom} - u_2(\text{dénom})'}{(\text{dénom})^2}\right]\right]$$

$$\frac{\partial}{\partial \mathrm{Im}(x_l)}\cos^2(\varphi) = 2\cos(\varphi)(-\sin(\varphi))\frac{\partial}{\partial \mathrm{Im}(x_l)}\varphi \quad \checkmark$$

(A_1'') $$\frac{\partial}{\partial \mathrm{Im}(x_l)}a_i = \frac{\partial}{\partial \mathrm{Im}(x_l)}\left[\mathrm{Re}(s_i)\mathrm{Re}(R_i) + \mathrm{Im}(s_i)\mathrm{Im}(R_i)\right]$$

$$= \mathrm{Re}(s_i)\frac{\partial}{\partial \mathrm{Im}(x_l)}\mathrm{Re}\left(\sum_{n=1}^{M} H_{i,n}x_n\right) + \mathrm{Im}(s_i)\frac{\partial}{\partial \mathrm{Im}(x_l)}\mathrm{Im}\left(\sum_{n=1}^{M} H_{i,n}x_n\right)$$

$$= \mathrm{Re}(s_i)\frac{\partial}{\partial \mathrm{Im}(x_l)}\sum_{n=1}^{M} \mathrm{Re}(H_{i,n})\mathrm{Re}(x_n) - \mathrm{Im}(H_{i,n})\mathrm{Im}(x_n) + \mathrm{Im}(s_i)\frac{\partial}{\partial \mathrm{Im}(x_l)}\sum_{n=1}^{M} \mathrm{Re}(H_{i,n})\mathrm{Im}(x_n) + \mathrm{Im}(H_{i,n})\mathrm{Re}(x_n)$$

$$= \mathrm{Re}(s_i)\left(-\mathrm{Im}(H_{i,l})\right) + \mathrm{Im}(s_i)\,\mathrm{Re}(H_{i,l})$$

(A_1) ✓

$(A_2)\quad \cos\varphi \sin\varphi\, b_i$

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} \cos\varphi \sin\varphi\, b_i = \cos\varphi \sin\varphi \underbrace{\frac{\partial}{\partial \mathrm{Im}(x_l)} b_i}_{A_2'} + \cos\varphi\, b_i \underbrace{\frac{\partial}{\partial \mathrm{Im}(x_l)} \sin\varphi}_{A_2''} + \sin\varphi\, b_i \underbrace{\frac{\partial}{\partial \mathrm{Im}(x_l)} \cos\varphi}_{A_2'''}$$

(A_2')

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} b_i = \frac{\partial}{\partial \mathrm{Im}(x_l)} \left[\mathrm{Re}(s_i)\, \mathrm{Im}(R_i) - \mathrm{Im}(s_i)\, \mathrm{Re}(R_i)\right]$$

$$= \mathrm{Re}(s_i) \frac{\partial}{\partial \mathrm{Im}(x_l)} \sum_{n=1}^{M} \mathrm{Im}(H_{i,n} x_n) - \mathrm{Im}(s_i) \frac{\partial}{\partial \mathrm{Im}(x_l)} \sum_{n=1}^{M} \mathrm{Re}(H_{i,n} x_n)$$

$$= \mathrm{Re}(s_i) \frac{\partial}{\partial \mathrm{Im}(x_l)} \sum_{n=1}^{M} \mathrm{Re}(H_{i,n}) \mathrm{Im}(x_n) + \mathrm{Im}(H_{i,n}) \mathrm{Re}(x_n) - \mathrm{Im}(s_i) \frac{\partial}{\partial \mathrm{Im}(x_l)} \sum_{n=1}^{M} \mathrm{Re}(H_{i,n}) \mathrm{Re}(x_n) - \mathrm{Im}(H_{i,n}) \mathrm{Im}(x_n)$$

$$= \mathrm{Re}(s_i)\, \mathrm{Re}(H_{i,l}) - \mathrm{Im}(s_i) \left(-\mathrm{Im}(H_{i,l})\right)$$

$$= \mathrm{Re}(s_i)\, \mathrm{Re}(H_{i,l}) + \mathrm{Im}(s_i)\, \mathrm{Im}(H_{i,l})$$

(A_2'')

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} \sin\varphi = \cos(\varphi) \frac{\partial}{\partial \mathrm{Im}(x_l)} \varphi \quad ✓$$

(A_2''')

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} \cos\varphi = -\sin(\varphi) \frac{\partial}{\partial \mathrm{Im}(x_l)} \varphi \quad ✓$$

$(B)\quad \left(\underbrace{\mathrm{Re}(i s_i)}_{= -\mathrm{Im}(s_i)} \left(\cos\varphi \sin\varphi\, a_i + \sin^2(\varphi)\, b_i\right)\right)$

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} B = -\mathrm{Im}(s_i) \frac{\partial}{\partial \mathrm{Im}(x_l)} \Big[\underbrace{\cos\varphi \sin\varphi\, a_i}_{B_1} + \underbrace{\sin^2(\varphi)\, b_i}_{B_2}\Big]$$

(B_1)

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} B_1 = \cos\varphi \sin\varphi \underbrace{\frac{\partial}{\partial \mathrm{Im}(x_l)} a_i}_{✓} + \cos\varphi\, a_i \underbrace{\frac{\partial}{\partial \mathrm{Im}(x_l)} \sin\varphi}_{✓} + \sin\varphi\, a_i \underbrace{\frac{\partial}{\partial \mathrm{Im}(x_l)} \cos\varphi}_{✓}$$

(B_2)

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} B_2 = b_i \frac{\partial}{\partial \mathrm{Im}(x_l)} \sin^2(\varphi) + \sin^2(\varphi) \frac{\partial}{\partial \mathrm{Im}(x_l)} b_i$$

$$= b_i \, 2 \sin(\varphi) \cos(\varphi) \underbrace{\frac{\partial}{\partial \mathrm{Im}(x_l)} \varphi}_{\checkmark} + \sin^2(\varphi) \underbrace{\frac{\partial}{\partial \mathrm{Im}(x_l)} b_i}_{\checkmark}$$

(C) $-\sum_{j=1}^{M} \mathrm{Re}(H_{i,j} x_j)$

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} C = -\sum_{j=1}^{M} \frac{\partial}{\partial \mathrm{Im}(x_l)} \mathrm{Re}(H_{i,j}) \mathrm{Re}(x_j) - \mathrm{Im}(H_{i,j}) \mathrm{Im}(x_j)$$

$$= -(-\mathrm{Im}(H_{i,l})) = \mathrm{Im}(H_{i,l})$$

$$\times \; \frac{\partial}{\partial \mathrm{Im}(x_l)} \mathrm{Im}(d_i \otimes s_i - R_i)^2 = \frac{\partial}{\partial \mathrm{Im}(x_l)} \mathrm{Im}\left(s_i \left[\cos^2(\varphi) a_i + \cos\varphi \sin\varphi \, b_i + i\left(a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i\right)\right] - R_i\right)^2$$

$$= \frac{\partial}{\partial \mathrm{Im}(x_l)} \Bigg[\underbrace{\overbrace{\mathrm{Im}(s_i)\left(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi \, b_i\right)}^{A} + \overbrace{\underbrace{\mathrm{Im}(i s_i)}_{\mathrm{Im}(i \times s_i) = \mathrm{Re}(s_i)}\left(a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i\right)}^{B} - \overbrace{\mathrm{Im}\left(\sum_{n=1}^{M} H_{i,n} x_n\right)}^{C}}_{\mathrm{IMSE}_i}\Bigg]^2$$

$$= \frac{1}{2} \mathrm{IMSE}_i \frac{\partial}{\partial \mathrm{Im}(x_l)} \mathrm{IMSE}_i$$

(A) $$\frac{\partial}{\partial \mathrm{Im}(x_l)} A = \mathrm{Im}(s_i) \underbrace{\frac{\partial}{\partial \mathrm{Im}(x_l)} \left[\cos^2(\varphi) a_i + \cos\varphi \sin\varphi \, b_i\right]}_{\checkmark}$$

(B) $$\frac{\partial}{\partial \mathrm{Im}(x_l)} B = \mathrm{Re}(s_i) \frac{\partial}{\partial \mathrm{Im}(x_l)} \underbrace{\left[a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i\right]}_{\checkmark}$$

(C) $$\frac{\partial}{\partial \mathrm{Im}(x_l)} C = -\frac{\partial}{\partial \mathrm{Im}(x_l)} \sum_{n=1}^{M} \mathrm{Im}(H_{i,n} x_n)$$

$$= -\frac{\partial}{\partial \mathrm{Im}(x_l)} \sum_{n=1}^{M} \mathrm{Re}(H_{i,n}) \mathrm{Im}(x_n) + \mathrm{Im}(H_{i,n}) \mathrm{Re}(x_n)$$

$$= -\mathrm{Re}(H_{i,l})$$

On a donc : $\forall l \in [\![1;M]\!]$

$$\frac{\partial}{\partial \mathrm{Re}(x_l)} \| d \otimes s - R \|_2^2 = \frac{\partial}{\partial \mathrm{Re}(x_l)} \left(\sum_{i=1}^{K} | \underline{d} \otimes \underline{s} - \underline{R} |_i^2 \right)$$

$$= \frac{\partial}{\partial \mathrm{Re}(x_l)} \left(\sum_{i=1}^{K} \mathrm{Re}(d_i \otimes s_i - R_i)^2 + \mathrm{Im}(d_i \otimes s_i - R_i)^2 \right)$$

$$= \sum_{i=1}^{K} \frac{\partial}{\partial \mathrm{Re}(x_l)} \left(\boxed{\mathrm{Re}(d_i \otimes s_i - R_i)}^2 \right) + \sum_{i=1}^{K} \frac{\partial}{\partial \mathrm{Re}(x_l)} \left(\boxed{\mathrm{Im}(d_i \otimes s_i - R_i)}^2 \right)$$

$$= \sum_{i=1}^{K} 2\, \mathrm{RMSE}_i \frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{RMSE}_i + \sum_{i=1}^{K} 2\, \mathrm{IMSE}_i \frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{IMSE}_i$$

$$\mathrm{RMSE}_i = \mathrm{Re}(s_i) \left(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i \right) + \underbrace{\mathrm{Re}(i s_i)}_{= -\mathrm{Im}(s_i)} \left(\cos\varphi \sin\varphi\, a_i + \sin^2(\varphi) b_i \right) - \sum_{j=1}^{M} \mathrm{Re}(H_{i,j} x_j)$$

$$\mathrm{IMSE}_i = \mathrm{Im}(s_i) \left(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i \right) + \underbrace{\mathrm{Im}(i s_i)}_{= \mathrm{Re}(s_i)} \left(a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i \right) - \mathrm{Im}\left(\sum_{n=1}^{M} H_{i,n} x_n \right)$$

avec $a_i = \mathrm{Re}(s_i)\, \mathrm{Re}(R_i) + \mathrm{Im}(s_i)\, \mathrm{Im}(R_i)$

$b_i = \mathrm{Re}(s_i)\, \mathrm{Im}(R_i) - \mathrm{Im}(s_i)\, \mathrm{Re}(R_i)$

$R_i = \sum_{n=1}^{M} H_{i,n} x_n$

et $\varphi = \frac{1}{2} \arctan \left(\frac{\sum_{j=1}^{K} \mathrm{Im}(s_j) \mathrm{Re}(s_j) \left[\mathrm{Im}(R_j)^2 - \mathrm{Re}(R_j)^2 \right]}{2 \sum_{j=1}^{K} \mathrm{Re}(R_j) \mathrm{Im}(R_j) \mathrm{Re}(s_j) \mathrm{Im}(s_j)} \right)$

$$\frac{\partial}{\partial \mathrm{Re}(x_l)} RMSE_i = \mathrm{Re}(s_i)\Big[\underbrace{a_i\, 2\cos\varphi(-\sin\varphi)\frac{\partial}{\partial \mathrm{Re}(x_l)}\text{[redacted]}\,\varphi + \cos^2(\varphi)\frac{\partial}{\partial \mathrm{Re}(x_l)}\text{[redacted]}\,a_i}_{A_1}$$

$$\underbrace{+\cos\varphi\sin\varphi\frac{\partial}{\partial \mathrm{Re}(x_l)}\text{[redacted]}\,b_i + \cos^2(\varphi)\, b_i\frac{\partial}{\partial \mathrm{Re}(x_l)}\text{[redacted]}\,\varphi - \sin^2(\varphi)\, b_i\frac{\partial}{\partial \mathrm{Re}(x_l)}\text{[redacted]}\,\varphi}_{A_2}\Big]$$

$$-\mathrm{Im}(s_i)\Big[\underbrace{\cos\varphi\sin\varphi\frac{\partial}{\partial \mathrm{Re}(x_l)}\text{[redacted]}\,a_i + \cos^2(\varphi)\, a_i\frac{\partial}{\partial \mathrm{Re}(x_l)}\text{[redacted]}\,\varphi - \sin^2(\varphi)\, a_i\frac{\partial}{\partial \mathrm{Re}(x_l)}\text{[redacted]}\,\varphi}_{B_1}$$

$$\underbrace{+\,2\, b_i\sin(\varphi)\cos(\varphi)\frac{\partial}{\partial \mathrm{Re}(x_l)}\text{[redacted]}\,\varphi + \sin^2(\varphi)\frac{\partial}{\partial \mathrm{Re}(x_l)}\text{[redacted]}\,b_i}_{B_2}\Big] - \underbrace{\mathrm{Re}(H_{i,l})}_{C}$$

$$\frac{\partial}{\partial \mathrm{Re}(x_l)} IMSE_i = \mathrm{Im}(s_i)\big[A_1 + A_2\big] + \mathrm{Re}(s_i)\big[B_1 + B_2\big] - \mathrm{Im}(H_{i,l})$$

$$\frac{\partial a_i}{\partial \mathrm{Re}(x_l)} = \mathrm{Re}(s_i)\mathrm{Re}(H_{i,l}) + \mathrm{Im}(s_i)\mathrm{Im}(H_{i,l})$$

$$\frac{\partial b_i}{\partial \mathrm{Re}(x_l)} = \mathrm{Re}(s_i)\mathrm{Im}(H_{i,l}) - \mathrm{Im}(s_i)\mathrm{Re}(H_{i,l})$$

$$\frac{\partial\varphi}{\partial \mathrm{Re}(x_l)} = \cancel{\frac{1}{2}} \times \frac{1}{1 + (\text{système})^2}$$

$(\text{système} = \tan(2\varphi)$ mieux d'utiliser directement le résultat du quotient des sommes) pour l'implémentation

$$\times \sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j)\Bigg[\frac{\cancel{2}\,\mathrm{Im}(H_{j,l})\mathrm{Im}(R_j)\,\text{dénom} - \mathrm{Im}(R_j)^2\,\cancel{2}\sum_{j=1}^{K}\mathrm{Im}(s_j)\mathrm{Re}(s_j)\big[\mathrm{Re}(R_j)\mathrm{Im}(H_{j,l}) + \mathrm{Im}(R_j)\mathrm{Re}(H_{j}}{(\text{dénom})^2}$$

$$-\frac{\cancel{2}\,\mathrm{Re}(H_{j,l})\mathrm{Re}(R_j)\,\text{dénom} - \mathrm{Re}(R_j)^2\,\cancel{2}\sum_{j=1}^{K}\mathrm{Im}(s_j)\mathrm{Re}(s_j)\big[\mathrm{Re}(R_j)\mathrm{Im}(H_{j,l}) + \mathrm{Im}(R_j)\mathrm{Re}(H_{j,l})\big]}{(\text{dénom})^2}\Bigg]$$

$$\text{dénom} = 2\sum_{j=1}^{K}\mathrm{Im}(s_j)\mathrm{Re}(s_j)\,\mathrm{Re}(R_j)\mathrm{Im}(R_j)$$

Graph de ∂_i et α