Analyse de l'amplification sans contrainte de d

$$f(d) = \| d \circ s - R \|_2^2 = \sum_{i=1}^{4} \mathrm{Re}(d_i s_i - R_i)^2 + \mathrm{Im}(d_i s_i - R_i)^2$$

$d_i \in \mathbb{C}$

$$= \sum_{i=1}^{4} \overline{(d_i s_i - R_i)}\,(d_i s_i - R_i)$$

$$\hookrightarrow \overline{d_i s_i}\, d_i s_i - \overline{d_i s_i} R_i - \overline{R_i} d_i s_i + R_i^2$$

$$\frac{\partial f(d)}{\partial d_i} = \frac{\partial}{\partial d_i} \mathrm{Re}(d_i s_i - R_i)^2 + \mathrm{Im}(d_i s_i - R_i)^2$$

$$= \mathrm{Re}(d_i s_i - R_i) \frac{\partial}{\partial d_i} \underbrace{\mathrm{Re}(d_i s_i)}_{\mathrm{Re}(d_i)\mathrm{Re}(s_i) - \mathrm{Im}(d_i)\mathrm{Im}(s_i)} + \mathrm{Im}(d_i s_i - R_i) \frac{\partial}{\partial d_i} \underbrace{\mathrm{Im}(d_i s_i)}_{\mathrm{Im}(d_i)\mathrm{Re}(s_i) + \mathrm{Re}(d_i)\mathrm{Im}(s_i)}$$

$$\hookrightarrow \frac{\partial}{\partial \mathrm{Re}(d_i)} = \mathrm{Re}(d_i s_i - R_i)\,\mathrm{Re}(s_i) + \mathrm{Im}(d_i s_i - R_i)\,\mathrm{Im}(s_i) = 0 \quad (1)$$

$$\frac{\partial}{\partial \mathrm{Im}(d_i)} = \mathrm{Re}(d_i s_i - R_i)\,(-\mathrm{Im}(s_i)) + \mathrm{Im}(d_i s_i - R_i)\,\mathrm{Re}(s_i) = 0 \quad (2)$$

(1) $$\left(\mathrm{Re}(d_i s_i) - \mathrm{Re}(R_i)\right)\mathrm{Re}(s_i) = -\left(\mathrm{Im}(d_i s_i) - \mathrm{Im}(R_i)\right)\mathrm{Im}(s_i)$$

$$\mathrm{Re}(d_i s_i) + \mathrm{Im}(d_i s_i) = \mathrm{Re}(R_i)\,\mathrm{Re}(s_i) + \mathrm{Im}(R_i)\,\mathrm{Im}(s_i)$$

$$\mathrm{Re}(d_i)\mathrm{Re}(s_i) - \mathrm{Im}(d_i)\mathrm{Im}(s_i) + \mathrm{Re}(d_i)\mathrm{Im}(s_i) + \mathrm{Im}(d_i)\mathrm{Re}(s_i) = \mathrm{Re}(R_i)\,\mathrm{Re}(s_i) + \mathrm{Im}(R_i)\,\mathrm{Im}(s_i)$$

$$\mathrm{Re}(d_i)\left(\mathrm{Re}(s_i) + \mathrm{Im}(s_i)\right) + \mathrm{Im}(d_i)\left(\mathrm{Re}(s_i) - \mathrm{Im}(s_i)\right) = \mathrm{Re}(R_i)\,\mathrm{Re}(s_i) + \mathrm{Im}(R_i)\,\mathrm{Im}(s_i)$$

(2) $$-\left(\mathrm{Re}(d_i s_i) - \mathrm{Re}(R_i)\right)\mathrm{Im}(s_i) + \left(\mathrm{Im}(d_i s_i) - \mathrm{Im}(R_i)\right)\mathrm{Re}(s_i) = 0$$

$$-\mathrm{Re}(d_i s_i) + \mathrm{Im}(d_i s_i) = \mathrm{Re}(s_i)\,\mathrm{Im}(R_i) - \mathrm{Re}(R_i)\,\mathrm{Im}(s_i)$$

$$-\mathrm{Re}(d_i)\mathrm{Re}(s_i) + \mathrm{Im}(d_i)\mathrm{Im}(s_i) + \mathrm{Re}(d_i)\mathrm{Im}(s_i) + \mathrm{Im}(d_i)\mathrm{Re}(s_i) = \mathrm{Re}(s_i)\,\mathrm{Im}(R_i) - \mathrm{Re}(R_i)\,\mathrm{Im}(s_i)$$

$$\mathrm{Im}(d_i) = \frac{\underbrace{\boxed{\mathrm{Re}(s_i)\,\mathrm{Im}(R_i) - \mathrm{Re}(R_i)\,\mathrm{Im}(s_i)}}_{b_i} + \mathrm{Re}(d_i)\left(\mathrm{Re}(s_i) - \mathrm{Im}(s_i)\right)}{\mathrm{Re}(s_i) + \mathrm{Im}(s_i)}$$

(1) $\mathrm{Re}(d_i)\left(\mathrm{Re}(s_i)+\mathrm{Im}(s_i)\right)+\mathrm{Im}(d_i)\left(\mathrm{Re}(s_i)-\mathrm{Im}(s_i)\right)=\mathrm{Re}(R_i)\,\mathrm{Re}(s_i)+\mathrm{Im}(R_i)\,\mathrm{Im}(s_i)$

$$\mathrm{Re}(d_i)=\frac{\overbrace{\mathrm{Re}(R_i)\mathrm{Re}(s_i)+\mathrm{Im}(R_i)\,\mathrm{Im}(s_i)}^{a_i}-\mathrm{Im}(d_i)\left(\mathrm{Re}(s_i)-\mathrm{Im}(s_i)\right)}{\mathrm{Re}(s_i)+\mathrm{Im}(s_i)}$$

$$\mathrm{Re}(d_i)\left(\mathrm{Re}(s_i)+\mathrm{Im}(s_i)\right)=a_i-\frac{b_i+\mathrm{Re}(d_i)\left(\mathrm{Re}(s_i)-\mathrm{Im}(s_i)\right)}{\mathrm{Re}(s_i)+\mathrm{Im}(s_i)}$$

$$\mathrm{Re}(d_i)\left(\mathrm{Re}(s_i)+\mathrm{Im}(s_i)\right)^2=a_i\left(\mathrm{Re}(s_i)+\mathrm{Im}(s_i)\right)-b_i+\mathrm{Re}(d_i)\left(\mathrm{Re}(s_i)-\mathrm{Im}(s_i)\right)$$

$$\mathrm{Re}(d_i)\left[\mathrm{Re}(s_i)^2+\mathrm{Im}(s_i)^2+2\,\mathrm{Re}(s_i)\mathrm{Im}(s_i)-\mathrm{Re}(s_i)+\mathrm{Im}(s_i)\right]$$

$=$

$$\frac{\partial}{\partial d_i}\left(\overline{d_i s_i}\,d_i s_i-\overline{d_i s_i}R_i-\overline{R_i}d_i s_i+\cancel{R_i^2}\right)$$

$$=\overline{d_i}\,|s_i|^2-\overline{R_i}\,s_i=0$$

$\frac{\partial \bar z}{\partial z}=0$ → pour une fct holomorphe

$$\hookrightarrow\ \overline{d_i}\,|s_i|^2=\overline{R_i}\,s_i$$

$$\overline{d_i}=\frac{\overline{R_i}\,s_i}{|s_i|^2}=\frac{\overline{R_i}\,\cancel{s_i}}{\overline{s_i}\,\cancel{s_i}}\iff d_i=\frac{R_i}{s_i}$$

$$\|\underline{d}\times\underline{s}-\underline{R}\|_2^2\Rightarrow\sum_{i=1}^{K}\overline{(d_i s_i-R_i)}\,(d_i s_i-R_i)$$

$$=\sum_{i=1}^{K}\overline{\left(\frac{R_i}{s_i}s_i-R_i\right)}\left(\frac{R_i}{s_i}s_i-R_i\right)$$

$$=\sum_{i=1}^{K}0$$

aucun sens

De manière non analytique :

$$\underline{d}^*=\operatorname{argmin}\|\underline{d}\otimes\underline{s}-\underline{R}\|_2^2$$

$$\iff\underline{d}^*\otimes\underline{s}-\underline{R}=\underline{0}$$

$$\iff\underline{D}^*\underline{s}=\underline{S}\,\underline{d}^*=\underline{R}\Rightarrow\underline{d}^*=\underline{S}^{-1}\underline{R}$$

avec $\underline{S}^{-1}=\mathrm{Diag}\left(\frac{1}{s_1},\dots,\frac{1}{s_K}\right)$ d'où $d_i=\frac{R_i}{s_i}$ $\forall i\in[\![1;K]\!]$

On a donc $\underline{x}^* = \arg\min \| \underline{d}^* \otimes \underline{s} - \underline{R} \|_2^2$

et $\| \underline{d}^* \otimes \underline{s} - \underline{R} \|_2^2 = \sum_{i=1}^{K} \overline{\left(d_i^* s_i - R_i \right)} \left(d_i^* s_i - R_i \right)$

$$= \sum_{i=1}^{K} \overline{\left(\frac{R_i}{s_i} s_i - R_i \right)} \left(\frac{R_i}{s_i} s_i - R_i \right)$$

$$= \sum_{i=1}^{K} 0$$