$$\underline{r} = H\underline{x} \qquad \underline{x} = P\underline{s}$$

$$H P \underline{s} = D \underline{s}$$
$$H \underline{x} = D \underline{s} \Leftrightarrow H\underline{x} = S\underline{d} \Leftrightarrow \operatorname{argmin} \|\underline{e}\|^2$$
$$\underline{d} = S^{-1} H \underline{x}$$

amplification $\alpha\, \underline{d} \otimes \underline{s}$ $\qquad \alpha \in \mathbb{C} \qquad d_i$ réel, ≥ 0

$$P = \underset{P, d, \alpha}{\operatorname{argmin}} \| H P \underline{s} - \alpha D \underline{s} \|^2$$

$$H P \underline{s} = \alpha D \underline{s} \Rightarrow \qquad HP = \alpha D$$

$$H\underline{x} = \alpha D \underline{s} \qquad \alpha^* = H P D^{-1} \quad \text{bof}$$

$$D^{-1} H \underline{x} = \alpha \underline{s}$$

$$\underline{s}^H D^{-1} H \underline{x} = \alpha\, \underline{s}^H \underline{s}$$

$$\alpha^* = \frac{\underline{s}^H D^{-1} H \underline{x}}{\|\underline{s}\|^2} = \frac{\underline{s}^H D^{-1} H P \underline{s}}{\|\underline{s}\|^2} = \frac{\underline{s}^H D^{-1} \underline{r}}{\|\underline{s}\|^2}$$

↳ on a ce qu'on veut pour $\underline{r} = D\underline{s}$ la sortie opt

amplification $e^{i\varphi}\, \underline{d} \otimes \underline{s} \qquad d_i \geq 0$, réel

$$P = \underset{P, d, \varphi}{\operatorname{argmin}} \| H P \underline{s} - e^{i\varphi}\, \underline{d} \otimes \underline{s} \|^2$$

$$H P \underline{s} = \left(\cos(\varphi) + i \sin(\varphi) \right) D \underline{s}$$

$$\frac{\underline{s}^H D^{-1} H \underline{x}}{\|\underline{s}\|^2} = e^{i\varphi} \Rightarrow \varphi = \arctan\left(\frac{\operatorname{Im}(e^{i\varphi})}{\operatorname{Re}(e^{i\varphi})} \right) \begin{cases} +0 \text{ si } \operatorname{Re}(e^{i\varphi}) > 0 \\ +\pi \text{ si } \operatorname{Re}(e^{i\varphi}) < 0 \\ \operatorname{sign}(b)\frac{\pi}{2} \text{ si } = 0 \end{cases}$$

Pr garder l'info à l'encodage (sans décodage itératif)

$$(a + ib)(c + id) = ac - bd + i(ad + bc)$$

↳ $\operatorname{sign}(ac - bd) = \operatorname{sign}(a)$
$\operatorname{sign}(ad + bc) = \operatorname{sign}(b)$ $\Rightarrow$

* si $a > 0$ et $b > 0$

$$\begin{cases} ac - bd \geq 0 \\ ad + bc \geq 0 \end{cases} \qquad \begin{cases} ac \geq bd \\ ad \geq -bc \end{cases}$$

$$\begin{cases} c \geq \dfrac{bd}{a} & \text{car } a > 0 \\ \dfrac{ad}{b} \geq -c \Rightarrow & c \geq \dfrac{ad}{b} \end{cases}$$

$r e^{i\theta} \in QPSK$

$1 > r > 0$

$\theta \in \frac{\pi}{4} \times (2K+1)$

$K \in \mathbb{Z}$

$r e^{i\theta} \times r' e^{i\varphi} = r r' e^{i(\theta+\varphi)}$

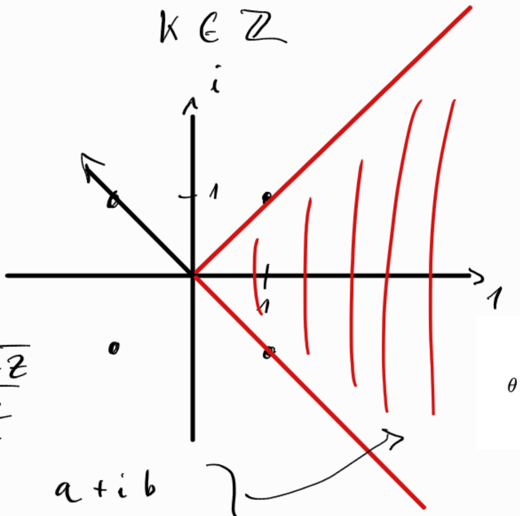
pour rester dans le même quadrant du plan complexe (divisé en 4)

il faut $r' > 0$ et $\varphi \in \left]-\frac{\pi}{4} ; \frac{\pi}{4}\right[$

i.e pour $r' e^{i\varphi} = a + ib$

$|a+ib| = \sqrt{a^2+b^2} > 0$

implique $a > 0$



$a = \frac{z+\bar{z}}{2}$

$b = \frac{z-\bar{z}}{2i}$

$\frac{b}{a} = \frac{z-\bar{z}}{(z+\bar{z})i}$

$a+ib$

$a > 0$

et $b < a$

et $-b < a$

$|b| < a$

$$\theta = \begin{cases} \arccos\left(\frac{a}{r}\right) & \text{si } b \geq 0 \\ -\arccos\left(\frac{a}{r}\right) & \text{si } b < 0, \end{cases} \quad \theta = \begin{cases} \arcsin\left(\frac{b}{r}\right) & \text{si } a \geq 0 \\ \pi - \arcsin\left(\frac{b}{r}\right) & \text{si } a < 0 \end{cases} \quad \text{et} \quad \theta = \begin{cases} \arctan\left(\frac{b}{a}\right) & \text{si } a > 0 \\ \arctan\left(\frac{b}{a}\right) + \pi & \text{si } a < 0 \\ \operatorname{sgn}(b)\frac{\pi}{2} & \text{si } a = 0. \end{cases}$$

fct angle matlab.

$\overline{(a+ib)\,i} = \overline{-b+ia} = -b-ia$

C2PO

avec seulement α

=> pas pris en compte, mais l'initialisation MRT semble empêcher ce problème

$x = H^H s$?

→ initialisation en ZF semble également empêcher ce pb, mais donne de moins bons résultats pour un grand SNR

à vérifier

→ contrainte de positivité peut-être non nécessaire.

$$\alpha^* = \frac{\underline{s}^H D^{-1} H \underline{x}}{\|\underline{s}\|^2}$$

$$= \frac{\underline{s}^H D^{-1} \underline{R}}{\|\underline{s}\|^2}$$

$$= \frac{1}{\underline{s}^H \underline{s}} \overline{\begin{pmatrix} s_1 & \cdots & s_K \end{pmatrix}} \begin{pmatrix} \frac{1}{d_1} & & \\ & \ddots & \\ & & \frac{1}{d_K} \end{pmatrix} \begin{pmatrix} R_1 \\ \vdots \\ R_K \end{pmatrix}$$

$$= \frac{1}{\underline{s}^H \underline{s}} \sum_{i=1}^{K} \frac{\overline{s_i}}{d_i} r_i = \frac{\sum_{j=1}^{K} \frac{\overline{s_j} r_j}{d_j}}{\sum_{l=1}^{K} |s_l|^2}$$

? Contraindre α à $|\alpha| > 0$

et $\arctan\left(\frac{\mathrm{Im}(\alpha)}{\mathrm{Re}(\alpha)}\right) \in \left]-\frac{\pi}{4} ; \frac{\pi}{4}\right[$

On détermine $\underline{d} = \operatorname{argmin} \|\alpha^* D \underline{s} - \underline{R}\|^2$

s.t. $d_i > 0$, réel ← plutôt car $\frac{1}{d_i}$

ou $d_i \geq 0$

$$\|\alpha^* D \underline{s} - \underline{R}\|^2 = \left\| \frac{\underline{s}^H D^{-1} \underline{R}}{\|\underline{s}\|^2} \underline{d} \otimes \underline{s} - \underline{R} \right\|^2$$

$$= \sum_{i=1}^{K} \left|\mathrm{Re}\left(\alpha^* d_i s_i - R_i\right)\right|^2 + \left|\mathrm{Im}\left(\alpha^* d_i s_i - R_i\right)\right|^2$$

$$= \sum_{i=1}^{K} \mathrm{Re}\left(\alpha^* d_i s_i - R_i\right)^2 + \mathrm{Im}\left(\alpha^* d_i s_i - R_i\right)^2$$

$$= \sum_{i=1}^{K} \left[d_i \mathrm{Re}\left(\alpha^* s_i\right) - \mathrm{Re}(R_i)\right]^2 + \left[d_i \mathrm{Im}(\alpha^* s_i) - \mathrm{Im}(R_i)\right]^2$$

$$\left(\begin{aligned} \mathrm{Re}(z_1 z_2) &= \mathrm{Re}(z_1)\mathrm{Re}(z_2) - \mathrm{Im}(z_1)\mathrm{Im}(z_2) \\ \mathrm{Im}(z_1 z_2) &= \mathrm{Re}(z_1)\mathrm{Im}(z_2) + \mathrm{Re}(z_2)\mathrm{Im}(z_1) \end{aligned}\right)$$

$$\mathrm{Re}(z_1 + z_2) = \mathrm{Re}(z_1) + \mathrm{Re}(z_2)$$

$$\mathrm{Im}(z_1 + z_2) = \mathrm{Im}(z_1) + \mathrm{Im}(z_2)$$

$$(a + ib)(c + id) = ac - bd + i(ad + bc)$$

$$= \sum_{i=1}^{K} \left[d_i \, \mathrm{Re}\left(\frac{\sum_{j=1}^{K} \frac{s_j r_j}{d_j}}{\sum_{l=1}^{K} |s_l|^2} s_i \right) - \mathrm{Re}(r_i) \right]^2 + \left[d_i \, \mathrm{Im}\left(\frac{\sum_{j=1}^{K} \frac{s_j r_j}{d_j}}{\sum_{l=1}^{K} |s_l|^2} s_i \right) - \mathrm{Im}(r_i) \right]^2$$

$$= \sum_{i=1}^{K} \left[\frac{d_i}{\sum_{l=1}^{K} |s_l|^2} \mathrm{Re}\left(\sum_{j=1}^{K} \frac{s_j r_j s_i}{d_j} \right) - \mathrm{Re}(r_i) \right]^2 + \left[\frac{d_i}{\sum_{l=1}^{K} |s_l|^2} \mathrm{Im}\left(\sum_{j=1}^{K} \frac{s_j r_j s_i}{d_j} \right) - \mathrm{Im}(r_i) \right]^2$$

$$= \sum_{i=1}^{K} \left[\frac{d_i}{\sum_{l=1}^{K} |s_l|^2} \sum_{j=1}^{K} \mathrm{Re}\left(\frac{s_j r_j s_i}{d_j} \right) - \mathrm{Re}(r_i) \right]^2 + \left[\frac{d_i}{\sum_{l=1}^{K} |s_l|^2} \sum_{j=1}^{K} \mathrm{Im}\left(\frac{s_j r_j s_i}{d_j} \right) - \mathrm{Im}(r_i) \right]^2$$

$d_i \in \mathbb{R}$

$$= \sum_{i=1}^{K} \left[\frac{d_i}{\sum_{l=1}^{K} |s_l|^2} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_i) - \mathrm{Re}(r_i) \right]^2 + \left[\frac{d_i}{\sum_{l=1}^{K} |s_l|^2} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Im}(s_j r_j s_i) - \mathrm{Im}(r_i) \right]^2$$

$$\sum_{l=1}^{K} |s_l|^2 = \sum_{l=1}^{K} \mathrm{Re}(s_l)^2 + \mathrm{Im}(s_l)^2 = \sum_{l=1}^{K} 1 = K$$ si QPSK ou M-PSK sur le cercle unité

On minimise donc $f(d_1, \dots, d_K) = \sum_{i=1}^{K} \left[\frac{d_i}{K} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_i) - \mathrm{Re}(r_i) \right]^2 + \left[\frac{d_i}{K} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Im}(s_j r_j s_i) - \mathrm{Im}(r_i) \right]^2 - \lambda_i d_i$ s.t. $-d_i \leq 0$

$n \in [\![1; K]\!]$

$$\frac{\partial}{\partial_n} \sum_{i=1}^{K} \left[\frac{d_i}{K} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_i) - \mathrm{Re}(r_i) \right]^2 = \sum_{i=1}^{K} \frac{\partial}{\partial_n} \Big[\underbrace{\frac{d_i}{K} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_i) - \mathrm{Re}(r_i)}_{g_i} \Big]^2$$

$$= \sum_{i=1}^{K} 2 g_i \times \frac{\partial g_i}{\partial d_n} = \sum_{i=1}^{K} 2 g_i \left[\frac{d_i}{K} \frac{\partial}{\partial d_n} \left[\sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_i) \right] + \frac{\partial}{\partial d_n}\left[\frac{d_i}{K}\right] \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_i) \right]$$

$\left(\left(\frac{1}{x}\right)' = -\frac{1}{x^2} \right)$

$$= \sum_{i=1}^{K} 2 g_i \frac{d_i}{K} \frac{\partial}{\partial d_n} \left[\frac{1}{d_n} \mathrm{Re}(s_n r_n s_i) \right] + \sum_{i=1}^{K} 2 g_i \frac{\partial}{\partial d_n} \left[\frac{d_i}{K} \right] \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_i)$$

$$= \sum_{i=1}^{K} 2 g_i \frac{d_i}{K} \mathrm{Re}(s_n r_n s_i) \times \left(-\frac{1}{d_n^2} \right) + 2 g_n \frac{1}{K} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_n)$$

Donc
$$\frac{\partial f(d)}{\partial d_n} = \sum_{i=1}^{K} 2 g_i \frac{d_i}{K} \mathrm{Re}(s_n r_n s_i) \times \left(-\frac{1}{d_n^2} \right) + 2 g_n \frac{1}{K} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Re}(s_j r_j s_n)$$
$$+ \sum_{i=1}^{K} 2 g_i \frac{d_i}{K} \mathrm{Im}(s_n r_n s_i) \times \left(-\frac{1}{d_n^2} \right) + 2 g_n \frac{1}{K} \sum_{j=1}^{K} \frac{1}{d_j} \mathrm{Im}(s_j r_j s_n) - \lambda_n$$

On calcule $\frac{\partial f(d)}{\partial d_n} = 0$

$$-\frac{1}{d_n^2}\sum_{i=1}^{K} 2 g_i \frac{d_i}{K} \mathrm{Re}\left(s_n r_n s_i\right) + 2 g_n \frac{1}{K}\sum_{j=1}^{K}\frac{1}{d_j} \mathrm{Re}\left(s_j r_j s_n\right)$$

$$= -\frac{1}{d_n^2}\sum_{i=1}^{K} 2\left[\frac{d_i}{K}\sum_{j=1}^{K}\frac{1}{d_j}\mathrm{Re}\left(s_j r_j s_i\right) - \mathrm{Re}\left(r_i\right)\right]\frac{d_i}{K}\mathrm{Re}\left(s_n r_n s_i\right)$$

$$+2\left[\frac{d_n}{K}\sum_{j=1}^{K}\frac{1}{d_j}\mathrm{Re}\left(s_j r_j s_n\right) - \mathrm{Re}\left(r_n\right)\right]\frac{1}{K}\sum_{j=1}^{K}\frac{1}{d_j}\mathrm{Re}\left(s_j r_j s_n\right)$$

$$= -\frac{1}{d_n^2}\sum_{i=1}^{K}\left[2\frac{d_i^2}{K^2}\mathrm{Re}\left(s_n r_n s_i\right)\sum_{j=1}^{K}\frac{1}{d_j}\mathrm{Re}\left(s_j r_j s_i\right) - 2\,\mathrm{Re}\left(r_i\right)\frac{d_i}{K}\mathrm{Re}\left(s_n r_n s_i\right)\right]$$

$$+2\frac{d_n}{K^2}\sum_{j=1}^{K}\frac{1}{d_j}\mathrm{Re}\left(s_j r_j s_n\right)\sum_{j=1}^{K}\frac{1}{d_j}\mathrm{Re}\left(s_j r_j s_n\right) - \mathrm{Re}\left(r_n\right)\frac{1}{K}\sum_{j=1}^{K}\frac{1}{d_j}\mathrm{Re}\left(s_j r_j s_n\right)$$

semble trop compliqué

amplification $\alpha^* D \iff \partial_j \in \mathbb{C}$ avec $\partial_j = r_j e^{i\varphi} \; \forall j \in [\![1;K]\!]$

$\alpha^* \in \mathbb{C}, \partial_j \in \mathbb{R}$

$r_j \in \mathbb{R} \rightarrow$ peut induire des rotations de π si $r<0$ amplitude différente par utilisateurs

ou $\mathbb{R}^+$

1 seule rotation globale

$$\| D \underline{s} - \underline{R} \|^2 = \sum_{j=1}^{K} \operatorname{Re}(\partial_j s_j - R_j)^2 + \operatorname{Im}(\partial_j s_j - R_j)^2$$

$$= \sum_{j=1}^{K} \left[\operatorname{Re}(\partial_j s_j) - \operatorname{Re}(R_j) \right]^2 + \left[\operatorname{Im}(\partial_j s_j) - \operatorname{Im}(R_j) \right]^2$$

$$f(r_1, \dots, r_K, \varphi) = \sum_{j=1}^{K} \left[r_j \operatorname{Re}(e^{i\varphi} s_j) - \operatorname{Re}(R_j) \right]^2 + \left[r_j \operatorname{Im}(e^{i\varphi} s_j) - \operatorname{Im}(R_j) \right]^2$$

$$= \sum_{j=1}^{K} \left[r_j \left(\operatorname{Re}(e^{i\varphi}) \operatorname{Re}(s_j) - \operatorname{Im}(e^{i\varphi}) \operatorname{Im}(s_j) \right) - \operatorname{Re}(R_j) \right]^2$$
$$+ \left[r_j \left(\operatorname{Re}(e^{i\varphi}) \operatorname{Im}(s_j) + \operatorname{Re}(s_j) \operatorname{Im}(e^{i\varphi}) \right) - \operatorname{Im}(R_j) \right]^2$$

$$\left(\begin{aligned} \operatorname{Re}(z_1 z_2) &= \operatorname{Re}(z_1)\operatorname{Re}(z_2) - \operatorname{Im}(z_1)\operatorname{Im}(z_2) \\ \operatorname{Im}(z_1 z_2) &= \operatorname{Re}(z_1)\operatorname{Im}(z_2) + \operatorname{Re}(z_2)\operatorname{Im}(z_1) \end{aligned} \right)$$

$e^{i\varphi} = \cos\varphi + i \sin\varphi$

$\operatorname{Re}(e^{i\varphi}) = \cos\varphi$

$\operatorname{Im}(e^{i\varphi}) = \sin\varphi$

$$f(r_1, \dots, r_K, \varphi) = \sum_{j=1}^{K} \left[r_j \left(\cos(\varphi) \operatorname{Re}(s_j) - \sin(\varphi) \operatorname{Im}(s_j) \right) - \operatorname{Re}(R_j) \right]^2$$
$$+ \left[r_j \left(\cos(\varphi) \operatorname{Im}(s_j) + \sin(\varphi) \operatorname{Re}(s_j) \right) - \operatorname{Im}(R_j) \right]^2$$

On calcule grad f : $\quad \forall n \in [\![1;K]\!]$

$$\frac{\partial f}{\partial r_n} = 2 \operatorname{Re}(e^{i\varphi} s_n) \left[r_n \operatorname{Re}(e^{i\varphi} s_n) - \operatorname{Re}(R_n) \right]$$
$$+ 2 \operatorname{Im}(e^{i\varphi} s_n) \left[r_n \operatorname{Im}(e^{i\varphi} s_n) - \operatorname{Im}(R_n) \right]$$

$$\frac{\partial f}{\partial \varphi} : \quad \frac{d}{d\varphi}\Big(\big(\underbrace{\big[r_j\big(\cos(\varphi)\,Re(s_j) - \sin(\varphi)\,Im(s_j)\big) - Re(R_j)\big]}_{u}\big)^2\Big)$$

$\sin(x)' = \cos(x)$
$\cos(x)' = -\sin(x)$

$$= 2\,\frac{d}{d\varphi}\Big(r_j\big[\cos(\varphi)Re(s_j) - \sin(\varphi)Im(s_j)\big]\Big) \times u$$

$$= 2\big(-r_j\,Re(s_j)\sin(\varphi) - r_j\,Im(s_j)\cos(\varphi)\big) \times u$$

$$= -2\,r_j\big(Re(s_j)\sin(\varphi) + Im(s_j)\cos(\varphi)\big) \times \big[r_j\big(\cos(\varphi)Re(s_j) - \sin(\varphi)Im(s_j)\big) - Re(R_j)\big]$$

$$= -2\,r_j\,Im(e^{i\varphi}s_j) \times \big[r_j\,Re(e^{i\varphi}s_j) - Re(R_j)\big]$$

$$\frac{d}{d\varphi}\Big(\big(\underbrace{\big[r_j\big(\cos(\varphi)\,Im(s_j) + \sin(\varphi)\,Re(s_j)\big) - Im(R_j)\big]}_{u'}\big)^2\Big)$$

$$= 2\big(-r_j\sin(\varphi)Im(s_j) + \cos(\varphi)Re(s_j)\,r_j\big) \times u'$$

$$= 2\,r_j\big(\cos(\varphi)Re(s_j) - \sin(\varphi)Im(s_j)\big)\big[r_j\big(\cos(\varphi)Im(s_j) + \sin(\varphi)Re(s_j)\big) - Im(R_j)\big]$$

$$= 2\,r_j\,Re(e^{i\varphi}s_j)\,\big[r_j\,Im(e^{i\varphi}s_j) - Im(R_j)\big]$$

$$\left(\frac{z+\bar z}{2}\right)^2 = \frac{z^2 + 2z\bar z + \bar z^2}{4} \qquad \left(\frac{z+\bar z}{2}\right)\left(\frac{z-\bar z}{2i}\right) = \frac{z^2 - z\bar z + z\bar z - \bar z\bar z}{4i} = \frac{zz - \overline{zz}}{4i}$$

$$= \frac{1}{2}\,Im(zz)$$

On a donc $$\frac{\partial f}{\partial \varphi} = \sum_{j=1}^{K} -2\,r_j\,Im(e^{i\varphi}s_j)\big[r_j\,Re(e^{i\varphi}s_j) - Re(R_j)\big] + 2\,r_j\,Re(e^{i\varphi}s_j)\big[r_j\,Im(e^{i\varphi}s_j) - Im(R_j)\big]$$

On annule grad f

$$\frac{\partial f}{\partial r_n} = 0 \iff 2\,Re(e^{i\varphi}s_n)\big[r_n\,Re(e^{i\varphi}s_n) - Re(R_n)\big] + 2\,Im(e^{i\varphi}s_n)\big[r_n\,Im(e^{i\varphi}s_n) - Im(R_n)\big] = 0$$

$$\iff r_n\,Re(e^{i\varphi}s_n)^2 - Re(e^{i\varphi}s_n)Re(R_n) + r_n\,Im(e^{i\varphi}s_n)^2 - Im(e^{i\varphi}s_n)Im(R_n) = 0$$

lorsque $\vec{R_n} \perp \overrightarrow{e^{i\varphi}s_n}$
$r_n = 0$

$$r_n = \frac{Re(e^{i\varphi}s_n)Re(R_n) + Im(e^{i\varphi}s_n)Im(R_n)}{Re(e^{i\varphi}s_n)^2 + Im(e^{i\varphi}s_n)^2}$$

1 si $s_n \in$ M-PSK

$$\frac{\partial f}{\partial \varphi} = 0 \iff \sum_{j=1}^{K} -2 r_j \operatorname{Im}(e^{i\varphi} s_j)\left[r_j \operatorname{Re}(e^{i\varphi} s_j) - \operatorname{Re}(R_j)\right] + 2 r_j \operatorname{Re}(e^{i\varphi} s_j)\left[r_j \operatorname{Im}(e^{i\varphi} s_j) - \operatorname{Im}(R_j)\right] = 0$$

$$\iff \sum_{j=1}^{K} \left[r_j^2 \operatorname{Re}(e^{i\varphi} s_j) \operatorname{Im}(e^{i\varphi} s_j) - r_j \operatorname{Re}(e^{i\varphi} s_j) \operatorname{Im}(R_j)\right]$$

$$= \sum_{j=1}^{K} \left[r_j^2 \operatorname{Im}(e^{i\varphi} s_j) \operatorname{Re}(e^{i\varphi} s_j) - r_j \operatorname{Im}(e^{i\varphi} s_j) \operatorname{Re}(R_j)\right]$$

Plutot $f(r_1, \dots, r_n, \varphi) = f(r_1, \dots, r_n, \cos\varphi, \sin\varphi)$ et $\cos\left(\frac{\pi}{2} - a\right) = \sin a$

$\sin\left(\frac{\pi}{2} - a\right) = \cos a$

$$= \sum_{j=1}^{4} \left[r_j \left(\cos(\varphi) Re(s_j) - \sin(\varphi) Im(s_j) \right) - Re(R_j) \right]^2$$

$$+ \left[r_j \left(\cos(\varphi) Im(s_j) + \sin(\varphi) Re(s_j) \right) - Im(R_j) \right]^2$$

$\frac{\partial f}{\partial r_n}$ et r_n pareil qu'au dessus

$$\cos^2 x + \sin^2 x = 1$$

$$\frac{\partial}{\partial \cos x}(\sin x) = \frac{\partial}{\partial a}\left(\sqrt{1-a^2}\right) = \frac{-2a}{2\sqrt{1-a^2}} = \frac{-\cos x}{\sin x} = \frac{-1}{\tan x}$$

$a = \cos x$

$$\frac{\partial}{\partial \sin x}(\cos x) = \frac{\partial}{\partial b}\left(\sqrt{1-b^2}\right) = \frac{-2b}{2\sqrt{1-b^2}} = \frac{-\sin x}{\cos x} = -\tan x$$

$b = \sin x$

$\frac{\partial f}{\partial \cos\varphi}$: $\frac{\partial}{\partial \cos\varphi} \left[r_j \left(\cos(\varphi)\, Re(s_j) - \sin\varphi\, Im(s_j) \right) - Re(R_j) \right]^2$

$$= 2 u_j \frac{\partial}{\partial \cos\varphi} \left[r_j \cos\varphi\, Re(s_j) - r_j \sin(\varphi)\, Im(s_j) \right]$$

$$= 2 u_j \left[r_j Re(s_j) + r_j \frac{Im(s_j)}{\tan(\varphi)} \right]$$

$\tan = \frac{\sin}{\cos}$

$$= 2 \left[r_j\, Re(e^{i\varphi} s_j) - Re(R_j) \right] \left[r_j\, Re(s_j) + r_j \frac{Im(s_j)}{\tan(\varphi)} \right]$$

$$= 2 \quad r_j^2 Re(e^{i\varphi} s_j) Re(s_j) + r_j^2 \frac{Re(e^{i\varphi} s_j) Im(s_j)}{\tan\varphi} - r_j Re(R_j) Re(s_j) + Re(R_j) r_j \frac{Im(s_j)}{\tan\varphi}$$

$$\frac{d}{d\cos\varphi} \left[r_j \left(\cos(\varphi) Im(s_j) + \sin(\varphi) Re(s_j) \right) - Im(R_j) \right]^2$$

$$= 2 u' \frac{d}{d\cos\varphi} \left(r_j \cos(\varphi) Im(s_j) + r_j \sin(\varphi) Re(s_j) \right]$$

$$= 2 u_j' \left[r_j Im(s_j) - r_j \frac{Re(s_j)}{\tan(\varphi)} \right]$$

$$= 2 \left[r_j Im(e^{i\varphi} s_j) - Im(R_j) \right] \left[r_j Im(s_j) - r_j \frac{Re(s_j)}{\tan(\varphi)} \right.$$

$$= 2 r_j^2 Im(e^{i\varphi} s_j) Im(s_j) - r_j^2 \frac{Im(e^{i\varphi} s_j)}{\tan\varphi} - r_j Im(R_j) Im(s_j) + r_j \frac{Im(R_j) Re(s_j)}{\tan\varphi}$$

semble être une mauvaise idée à cause de $\frac{1}{\tan\varphi}$

$$r_j = Re(e^{i\varphi} s_j) Re(R_j) + Im(e^{i\varphi} s_j) Im(R_j)$$

$$= \left[\cos\varphi\, Re(s_j) - \sin\varphi\, Im(s_j) \right] Re(R_j) + \left[\cos\varphi\, Im(s_j) + \sin\varphi\, Re(s_j) \right] Im(R_j)$$

$$= \cos(\varphi) \left[\underbrace{Re(s_j) Re(R_j) + Im(s_j) Im(R_j)}_{a_j} \right] + \sin\varphi \left[\underbrace{Re(s_j) Im(R_j) - Im(s_j) Re(R_j)}_{b_j} \right]$$

$$\Leftrightarrow \sum_{j=1}^{K} \cancel{t}\, r_j Re(e^{i\varphi} s_j) Im(R_j) = \sum_{j=1}^{K} \cancel{t}\, r_j Im(e^{i\varphi} s_j) Re(R_j)$$

$$\Leftrightarrow \sum_{j=1}^{K} \left(\cos(\varphi) a_j + \sin(\varphi) b_j \right) \left[\cos(\varphi) Re(s_j) - \sin(\varphi) Im(s_j) \right] Im(R_j)$$

$$= \sum_{j=1}^{K} \left(\cos(\varphi) a_j + \sin(\varphi) b_j \right) \left[\cos\varphi\, Im(s_j) + \sin\varphi\, Re(s_j) \right] Re(R_j)$$

$$\Leftrightarrow \sum_{j=1}^{K} \cos(\varphi)^2 a_j Re(s_j) Im(R_j) - \cos\varphi \sin\varphi\, a_j Im(s_j) Im(R_j) + \sin(\varphi)\cos(\varphi) b_j Re(s_j) Im(R_j)$$
$$- \sin(\varphi)^2 b_j Im(s_j) Im(R_j)$$

$$= \sum_{j=1}^{K} \cos(\varphi)^2 a_j Im(s_j) Re(R_j) + \cos\varphi \sin\varphi\, a_j Re(s_j) Re(R_j) + \cos\varphi \sin\varphi\, b_j Im(s_j) Re(R_j)$$
$$+ \sin(\varphi)^2 b_j Re(s_j) Re(R_j)$$

$$\Leftrightarrow \sum_{j=1}^{K} \cos^2(\varphi) a_j \left[Re(s_j) Im(R_j) - Im(s_j) Re(R_j) \right] - \sin^2(\varphi) b_j \left[Im(s_j) Im(R_j) + Re(s_j) Re(R_j) \right]$$

$$- \cos(\varphi)\sin(\varphi)\, a_j \left[Im(s_j) Im(R_j) + Re(s_j) Re(R_j) \right]$$

$$+ \sin(\varphi)\cos(\varphi)\, b_j \left[Re(s_j) Im(R_j) - Im(s_j) Re(R_j) \right] = 0$$

$$\Leftrightarrow \sum_{j=1}^{K} \cos^2(\varphi) a_j b_j - \sin^2(\varphi) b_j a_j - \cos(\varphi)\sin(\varphi) a_j^2 + \sin(\varphi)\cos(\varphi) b_j^2 = 0$$

$$\cos^2 + \sin^2 = 1 \qquad \sin^2 = 1 - \cos^2 \qquad \cos(2a) = \cos^2 a - \sin^2 a$$

$$\cos(a)\sin(a) = \frac{1}{2}\sin(2a)$$

$$\Leftrightarrow \sum_{j=1}^{K} a_j b_j \left(\cos(\varphi)^2 - \sin(\varphi)^2 \right) + \cos\varphi \sin\varphi \left[b_j^2 - a_j^2 \right] = 0$$

$$\Leftrightarrow \cos(2\varphi) \sum_{j=1}^{K} a_j b_j = \frac{1}{2}\sin(2\varphi) \sum_{j=1}^{K} (a_j^2 - b_j^2)$$

$$\Rightarrow \frac{\sin(2\varphi)}{\cos(2\varphi)} = \tan(2\varphi) = \frac{2\sum_{j=1}^{K} a_j b_j}{\sum_{j=1}^{K} (a_j^2 - b_j^2)}$$

$$\varphi = \frac{1}{2} \arctan\left(\frac{2\sum_{j=1}^{K} a_j b_j}{\sum_{j=1}^{K} (a_j^2 - b_j^2)} \right) \quad \text{si } \sum_{j=1}^{K} (a_j^2 - b_j^2) \neq 0$$

$$\text{sinon} \quad \varphi = \frac{\pi}{4} \times \text{sign}\left(\sum_{j=1}^{K} a_j b_j \right)$$

$$a_j = Re(s_j) Re(R_j) + Im(s_j) Im(R_j)$$
$$b_j = Re(s_j) Im(R_j) - Im(s_j) Re(R_j)$$

$$a_j b_j = Re(s_j)^2 Re(R_j) Im(R_j) - Re(s_j) Im(s_j) Re(R_j)^2$$
$$+ Im(s_j) Re(s_j) Im(R_j)^2 - Im(s_j)^2 Im(R_j) Re(R_j)$$

$$= Re(R_j) Im(R_j) \left[Re(s_j)^2 - Im(s_j)^2 \right] + Im(s_j) Re(s_j) \left[Im(R_j)^2 - Re(R_j)^2 \right]$$

$$a_j^2 = \mathrm{Re}(s_j)^2 \mathrm{Re}(R_j)^2 + \mathrm{Im}(s_j)^2 \mathrm{Im}(R_j)^2 + 2\,\mathrm{Re}(s_j)\,\mathrm{Im}(s_j)\,\mathrm{Re}(R_j)\,\mathrm{Im}(R_j)$$

$$b_j^2 = \mathrm{Re}(s_j)^2 \mathrm{Im}(R_j)^2 + \mathrm{Im}(s_j)^2 \mathrm{Re}(R_j)^2 - 2\,\mathrm{Re}(s_j)\,\mathrm{Im}(R_j)\,\mathrm{Im}(s_j)\,\mathrm{Re}(R_j)$$

$$a_j^2 - b_j^2 = \mathrm{Re}(R_j)^2\left[\mathrm{Re}(s_j)^2 - \mathrm{Im}(s_j)^2\right] + \mathrm{Im}(R_j)^2\left[\mathrm{Im}(s_j)^2 - \mathrm{Re}(s_j)^2\right]$$
$$+ 4\,\mathrm{Re}(R_j)\mathrm{Im}(R_j)\,\mathrm{Re}(s_j)\,\mathrm{Im}(s_j)$$
$$= \left[\mathrm{Re}(R_j)^2 - \mathrm{Im}(R_j)^2\right]\left[\mathrm{Re}(s_j)^2 - \mathrm{Im}(s_j)^2\right]$$
$$+ 4\,\mathrm{Re}(R_j)\mathrm{Im}(R_j)\,\mathrm{Re}(s_j)\,\mathrm{Im}(s_j)$$

Soit $z = a + ib$ tq $a^2 + b^2 = 1$ ($z \in M\text{-PSK}$)
$\Leftrightarrow a^2 = 1 - b^2$

On a donc $a^2 - b^2 = 1 - b^2 - b^2 = 1 - 2b^2$

Si $|a| = |b|$ (i.e $z \in Q\text{-PSK}$)
$a^2 - b^2 = 0$

Cas de s_j

Pas forcément R_j car non quantifié ds les calculs

de $\sum_{j=1}^{K} a_j b_j = \sum_{j=1}^{K} \mathrm{Im}(s_j)\,\mathrm{Re}(s_j)\left[\mathrm{Im}(R_j)^2 - \mathrm{Re}(R_j)^2\right]$

et $\sum_{j=1}^{K} a_j^2 - b_j^2 = \sum_{j=1}^{K} 4\,\mathrm{Re}(R_j)\,\mathrm{Im}(R_j)\,\mathrm{Re}(s_j)\,\mathrm{Im}(s_j)$

$$\forall n \in [\![1; K]\!] \quad d_n = s_n e^{i\varphi}$$
$$= \left[\mathrm{Re}(e^{i\varphi}s_n)\,\mathrm{Re}(R_n) + \mathrm{Im}(e^{i\varphi}s_n)\,\mathrm{Im}(S_n)\right] e^{i\varphi}$$
$$= \Big(\cos(\varphi)\left[\mathrm{Re}(s_n)\,\mathrm{Re}(R_n) + \mathrm{Im}(s_n)\mathrm{Im}(R_n)\right] + \sin(\varphi)\left[\mathrm{Re}(s_j)\mathrm{Im}(R_j) - \mathrm{Im}(s_j)\,\mathrm{Re}(R_j)\right]\Big) e^{i\varphi}$$

avec $e^{i\varphi} = \cos(\varphi) + i\sin(\varphi)$

et $\varphi = \frac{1}{2}\arctan\left(\dfrac{2\sum_{j=1}^{K} \mathrm{Im}(s_j)\,\mathrm{Re}(s_j)\left[\mathrm{Im}(R_j)^2 - \mathrm{Re}(R_j)^2\right]}{4\sum_{j=1}^{K} \mathrm{Re}(R_j)\,\mathrm{Im}(R_j)\,\mathrm{Re}(s_j)\,\mathrm{Im}(s_j)}\right)$

$$= \left(\cos\varphi\; a_n + \sin\varphi\; b_n\right)\left(\cos\varphi + i\sin\varphi\right)$$

$$= \cos^2(\varphi)\, a_n + i\, a_n \cos\varphi\sin\varphi + \cos\varphi\sin\varphi\; b_n + i\sin^2(\varphi)\, b_n$$

On a $\quad K\updownarrow R = \underset{K}{\updownarrow}\overset{M}{\overleftrightarrow{H}}\, x \updownarrow M$

$$\forall n \in [\![1;K]\!], \quad R_n = \sum_{i=1}^{M} H_{n,i}\, x_i$$

On cherche $\quad \underset{x}{\arg\min} \; \| \underline{d} \otimes \underline{s} - \underline{R} \|_2^2$

on calcule donc $\forall \ell \in [\![1;M]\!]$

$$\frac{\partial}{\partial x_\ell}\left(\| \underline{d} \otimes \underline{s} - \underline{R} \|_2^2 \right) = \frac{\partial}{\partial x_\ell}\left(\sum_{i=1}^{K} | \underline{d} \otimes \underline{s} - \underline{R} |_i^2 \right)$$

$$= \frac{\partial}{\partial x_\ell}\left(\sum_{i=1}^{K} Re(d_i \otimes s_i - R_i)^2 + Im(d_i \otimes s_i - R_i)^2 \right)$$

$$= \sum_{i=1}^{K} \frac{\partial}{\partial x_\ell}\left(Re(d_i \otimes s_i - R_i)^2 \right) + \sum_{i=1}^{K} \frac{\partial}{\partial x_\ell}\left(Im(d_i \otimes s_i - R_i)^2 \right)$$

$$\frac{\partial}{\partial Re(x_\ell)} Re(d_i \otimes s_i - R_i)^2 = \frac{\partial}{\partial Re(x_\ell)} Re\left(s_i\left(\cos(\varphi)^2 a_i + \cos(\varphi)\sin(\varphi)(b_i + i\, a_i) + i\sin(\varphi)^2 b_i \right) - \sum_{j=1}^{M} H_{i,j}\, x_j \right)^2$$

$$= \frac{\partial}{\partial Re(x_\ell)}\left[Re\left(s_i\left[\cos(\varphi)^2 a_i + \cos\varphi\sin\varphi\, b_i \right] \right) + Re\left(i \times s_i\left[\cos\varphi\sin\varphi\, a_i + \sin^2(\varphi)\, b_i \right] \right) - \sum_{j=1}^{M} Re(H_{i,j}\, x_j) \right]^2$$

$$= \frac{\partial}{\partial Re(x_\ell)}\Big[\underbrace{\underbrace{Re(s_i)\left(\cos^2(\varphi)\, a_i + \cos\varphi\sin\varphi\, b_i\right)}_{(A)} + \overbrace{\underbrace{Re(i\, s_i)}_{= -Im(s_i)}\left(\cos\varphi\sin\varphi\, a_i + \sin^2(\varphi)\, b_i\right)}^{(B)} - \underbrace{\sum_{j=1}^{M} Re(H_{i,j}\, x_j)}_{(C)}}_{RMSE_i} \Big]^2$$

$$= 2\, RMSE_i \frac{\partial}{\partial Re(x_\ell)} RMSE_i$$

$$\left(\begin{aligned} Re(H_{i,j}\, x_j) &= Re(H_{i,j})\, Re(x_j) - Im(H_{i,j})\, Im(x_j) \\ a_i &= Re(s_i)\, Re(R_i) + Im(s_i)\, Im(R_i) \\ b_i &= Re(s_i)\, Im(R_i) - Im(s_i)\, Re(R_i) \end{aligned} \right)$$

et $\varphi = \frac{1}{2} \arctan\left(\frac{\sum_{j=1}^{K} Im(s_j) Re(s_j) \left[Im(R_j)^2 - Re(R_j)^2 \right]}{2 \sum_{j=1}^{K} Re(R_j) Im(R_j) Re(s_j) Im(s_j)} \right)$

(A) $\frac{\partial}{\partial Re(x_l)} Re(s_i) \left(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i \right) = Re(s_i) \frac{\partial}{\partial Re(x_l)} \left[\underbrace{\cos^2(\varphi) a_i}_{A_1} + \underbrace{\cos\varphi \sin\varphi\, b_i}_{A_2} \right]$

(A_1) : $\frac{\partial}{\partial Re(x_l)} \cos^2(\varphi) a_i = a_i \underbrace{\frac{\partial}{\partial Re(x_l)} \cos^2(\varphi)}_{A_1'} + \cos^2(\varphi) \underbrace{\frac{\partial}{\partial Re(x_l)} a_i}_{A_1''}$

(A_1') $\frac{\partial}{\partial Re(x_l)} \cos^2(\varphi) = 2 \cos(\varphi) \frac{\partial}{\partial Re(x_l)} \cos(\varphi)$

$= 2 \cos(\varphi) \left(-\sin(\varphi) \right) \frac{\partial}{\partial Re(x_l)} \varphi$ $\qquad (f(g))' = g' f'(g)$

$$\frac{\partial}{\partial Re(x_l)} \varphi = \frac{1}{2} \frac{\partial}{\partial Re(x_l)} \left[\arctan\left(\frac{\sum_{j=1}^{K} Im(s_j) Re(s_j) \left[Im(R_j)^2 - Re(R_j)^2 \right]}{2 \sum_{j=1}^{K} Re(R_j) Im(R_j) Re(s_j) Im(s_j)} \right) \right]$$

$= \arctan(\text{système})$

$$= \frac{1}{2} \left(\frac{1}{1 + (\text{système})^2} \right) \frac{\partial}{\partial Re(x_l)} \text{système}$$

$$= \frac{1}{2} \left(\frac{1}{1 + (\text{système})^2} \right) \frac{\partial}{\partial Re(x_l)} \left(\frac{\sum_{j=1}^{K} Im(s_j) Re(s_j) \left[Im\left(\sum_{m=1}^{M} H_{j,m} x_m \right)^2 - Re\left(\sum_{m=1}^{M} H_{j,m} x_m \right)^2 \right]}{2 \sum_{j=1}^{K} Im(s_j) Re(s_j)\, Im\left(\sum_{m=1}^{M} H_{j,m} x_m \right) Re\left(\sum_{m=1}^{M} H_{j,m} x_m \right)} \right)$$

$$= \frac{1}{2 + 2\,\text{système}^2} \frac{\partial}{\partial Re(x_l)} \left(\frac{\sum_{j=1}^{K} Im(s_j) Re(s_j) \left[\left(\sum_{m=1}^{M} Re(H_{j,m}) Im(x_m) + Im(H_{j,m}) Re(x_m) \right)^2 - \left(\sum_{m=1}^{M} Re(H_{j,m}) Re(x_m) - Im(H_{j,m}) Im(x_m) \right)^2 \right]}{2 \sum_{j=1}^{K} Im(s_j) Re(s_j) \left(\sum_{m=1}^{M} Re(H_{j,m}) Im(x_m) + Im(H_{j,m}) Re(x_m) \right) \left(\sum_{m=1}^{M} Re(H_{j,m}) Re(x_m) - Im(H_{j,m}) Im(x_m) \right)} \right)$$

dénum $\rightarrow$

$$\frac{\partial}{\partial Re(x_l)} \text{syst} = \frac{\partial}{\partial Re(x_l)} \left(\frac{\sum_{j=1}^{K} Im(s_j) Re(s_j) \left[\sum_{m=1}^{M} Re(H_{j,m}) Im(x_m) + Im(H_{j,m}) Re(x_m) \right]^2}{\text{dénum}} \right)$$

$$- \frac{\partial}{\partial Re(x_l)} \left(\frac{\sum_{j=1}^{K} Im(s_j) Re(s_j) \left[\sum_{m=1}^{M} Re(H_{j,m}) Re(x_m) - Im(H_{j,m}) Im(x_m) \right]^2}{\text{dénum}} \right)$$

$$= \left.\sum_{j=1}^{K} Im(s_j) Re(s_j) \frac{\partial}{\partial Re(x_l)} \frac{\left[\sum_{m=1}^{M} Re(H_{j,m}) Im(x_m) + Im(H_{j,m}) Re(x_m) \right]^2}{\text{dénum}} \right\} \text{syst}_1$$

$$\left.- \sum_{j=1}^{K} Im(s_j) Re(s_j) \frac{\partial}{\partial Re(x_l)} \frac{\left[\sum_{m=1}^{M} Re(H_{j,m}) Re(x_m) - Im(H_{j,m}) Im(x_m) \right]^2}{\text{dénum}} \right\} \text{syst}_2$$

$\left(\frac{u}{v} \right)' = \frac{u'v - uv'}{v^2}$

$$\text{syst}_1 \quad \frac{\partial}{\partial \mathrm{Re}(x_l)} \frac{\left[\sum_{m=1}^{M} \mathrm{Re}(H_{j,m})\mathrm{Im}(x_m) + \mathrm{Im}(H_{j,m})\mathrm{Re}(x_m)\right]^2}{\text{dénum}} = \frac{u_1' \times \text{dénum} - u_1(\text{dénum})'}{(\text{dénum})^2} \qquad u_1 = \mathrm{Im}(R_j)^2$$

$$u_1' = 2 \times \mathrm{Im}(H_{j,l}) \left[\sum_{m=1}^{M} \mathrm{Re}(H_{j,m})\mathrm{Im}(x_m) + \mathrm{Im}(H_{j,m})\mathrm{Re}(x_m)\right]$$

$$v = 2 \sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j) \overbrace{\left(\sum_{m=1}^{K} \mathrm{Re}(H_{j,m})\mathrm{Re}(x_m) - \mathrm{Im}(H_{j,m})\mathrm{Im}(x_m)\right)}^{\mathrm{Re}(R_j)} \overbrace{\left(\sum_{m=1}^{K} \mathrm{Im}(H_{j,m})\mathrm{Re}(x_m) + \mathrm{Re}(H_{j,m})\mathrm{Im}(x_m)\right)}^{\mathrm{Im}(R_j)}$$

$$\frac{\partial v}{\partial \mathrm{Re}(x_l)} = 2 \sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j) \left[\mathrm{Re}(R_j) \frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{Im}(R_j) + \mathrm{Im}(R_j) \frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{Re}(R_j)\right]$$

$$= 2 \sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j) \left[\mathrm{Re}(R_j)\,\mathrm{Im}(H_{j,l}) + \mathrm{Im}(R_j)\,\mathrm{Re}(H_{j,l})\right] = (\text{dénum})'$$

$$\text{syst}_2 \quad \frac{\partial}{\partial \mathrm{Re}(x_l)} \frac{\left[\sum_{m=1}^{M} \mathrm{Re}(H_{j,m})\mathrm{Re}(x_m) - \mathrm{Im}(H_{j,m})\mathrm{Im}(x_m)\right]^2}{\text{dénum}} = \frac{u_2' \times \text{dénum} - u_2(\text{dénum})'}{(\text{dénum})^2} \qquad u_2 = \mathrm{Re}(R_j)^2$$

$$u_2' = 2\,\mathrm{Re}(H_{j,l}) \left[\sum_{m=1}^{M} \mathrm{Re}(H_{j,m})\mathrm{Re}(x_m) - \mathrm{Im}(H_{j,m})\mathrm{Im}(x_m)\right]$$

$$= 2\,\mathrm{Re}(H_{j,l})\,\mathrm{Re}(R_j)$$

$$\frac{\partial \varphi}{\partial \mathrm{Re}(x_l)} = \frac{1}{2} \times \frac{1}{1+(\text{système})^2} \times \underbrace{\sum_{j=1}^{K} \mathrm{Im}(s_j)\mathrm{Re}(s_j) \left[\frac{u_1'(\text{dénum}) - u_1(\text{dénum})'}{(\text{dénum})^2} - \frac{u_2'(\text{dénum}) - u_2(\text{dénum})'}{(\text{dénum})^2}\right]_j}$$

$$\text{système} = \tan(2\varphi)$$

$$\frac{\partial}{\partial \mathrm{Re}(x_l)} \cos^2(\varphi) = 2\cos(\varphi)(-\sin(\varphi)) \times \frac{1}{2} \times \frac{1}{1+(\text{système})^2} \times$$

$$(A_1'') \qquad \frac{\partial}{\partial \mathrm{Re}(x_l)} a_i = \frac{\partial}{\partial \mathrm{Re}(x_l)} \left[\mathrm{Re}(s_i)\mathrm{Re}(R_i) + \mathrm{Im}(s_i)\mathrm{Im}(R_i)\right]$$

$$= \mathrm{Re}(s_i) \frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{Re}\left(\sum_{m=1}^{M} H_{i,m}x_m\right) + \mathrm{Im}(s_i) \frac{\partial}{\partial \mathrm{Re}(x_l)} \mathrm{Im}\left(\sum_{m=1}^{M} H_{i,m}x_m\right)$$

$$= \mathrm{Re}(s_i) \frac{\partial}{\partial \mathrm{Re}(x_l)} \sum_{m=1}^{M} \mathrm{Re}(H_{i,m})\mathrm{Re}(x_m) - \mathrm{Im}(H_{i,m})\mathrm{Im}(x_m) + \mathrm{Im}(s_i) \frac{\partial}{\partial \mathrm{Re}(x_l)} \sum_{m=1}^{M} \mathrm{Re}(H_{i,m})\mathrm{Im}(x_m) + \mathrm{Im}(H_{i,m})\mathrm{Re}(x_m)$$

$$= \mathrm{Re}(s_i)\,\mathrm{Re}(H_{i,l}) + \mathrm{Im}(s_i)\,\mathrm{Im}(H_{i,l})$$

(A_1) ✓

$$(A_2)\quad \frac{\partial}{\partial \mathrm{Re}(x_\ell)}\left[\cos\varphi\sin\varphi\, b_i\right] = \cos\varphi\sin\varphi \underbrace{\frac{\partial}{\partial \mathrm{Re}(x_\ell)} b_i}_{A_2'} + \cos\varphi\, b_i \underbrace{\frac{\partial}{\partial \mathrm{Re}(x_\ell)} \sin\varphi}_{A_2''} + \sin\varphi\, b_i \underbrace{\frac{\partial}{\partial \mathrm{Re}(x_\ell)} \cos\varphi}_{A_2'''}$$

$$(A_2')\quad \frac{\partial}{\partial \mathrm{Re}(x_\ell)} b_i = \frac{\partial}{\partial \mathrm{Re}(x_\ell)}\left[\mathrm{Re}(s_i)\,\mathrm{Im}(R_i) - \mathrm{Im}(s_i)\,\mathrm{Re}(R_i)\right]$$

$$= \mathrm{Re}(s_i)\frac{\partial}{\partial \mathrm{Re}(x_\ell)}\mathrm{Im}\left(\sum_{m=1}^{M} H_{i,m} x_m\right) - \mathrm{Im}(s_i)\frac{\partial}{\partial \mathrm{Re}(x_\ell)}\mathrm{Re}\left(\sum_{m=1}^{M} H_{i,m} x_m\right)$$

$$= \mathrm{Re}(s_i)\frac{\partial}{\partial \mathrm{Re}(x_\ell)}\sum_{m=1}^{M} \mathrm{Im}(H_{i,m})\mathrm{Re}(x_m) + \mathrm{Re}(H_{i,m})\mathrm{Im}(x_m) - \mathrm{Im}(s_i)\frac{\partial}{\partial \mathrm{Re}(x_\ell)}\sum_{m=1}^{M} \mathrm{Re}(H_{i,m})\mathrm{Re}(x_m) - \mathrm{Im}(H_{i,m})\mathrm{Im}(x_m)$$

$$= \mathrm{Re}(s_i)\,\mathrm{Im}(H_{i,\ell}) - \mathrm{Im}(s_i)\,\mathrm{Re}(H_{i,\ell})$$

$$(A_2'')\quad \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \sin\varphi = \cos(\varphi)\frac{\partial}{\partial \mathrm{Re}(x_\ell)}\varphi$$

$$(A_2''')\quad \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \cos\varphi = -\sin(\varphi)\frac{\partial}{\partial \mathrm{Re}(x_\ell)}\varphi$$

(A_2) ✓

$$(B)\quad \frac{\partial}{\partial \mathrm{Re}(x_\ell)}\left(\underbrace{\mathrm{Re}(i s_i)}_{=-\mathrm{Im}(s_i)}\left(\cos\varphi\sin\varphi\, a_i + \sin^2(\varphi) b_i\right)\right) = -\mathrm{Im}(s_i)\frac{\partial}{\partial \mathrm{Re}(x_\ell)}\left[\underbrace{\cos\varphi\sin\varphi\, a_i}_{B_1} + \underbrace{\sin^2(\varphi) b_i}_{B_2}\right]$$

$$(B_1)\quad \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \cos\varphi\sin\varphi\, a_i = \cos\varphi\sin\varphi\frac{\partial}{\partial \mathrm{Re}(x_\ell)} a_i + \cos\varphi\, a_i \frac{\partial}{\partial \mathrm{Re}(x_\ell)}\sin\varphi + \sin\varphi\, a_i \frac{\partial}{\partial \mathrm{Re}(x_\ell)}\cos\varphi$$

$$(B_2)\quad \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \sin^2(\varphi) b_i = b_i \underbrace{\frac{\partial}{\partial \mathrm{Re}(x_\ell)}\sin^2(\varphi)}_{B_2'} + \sin^2(\varphi)\underbrace{\frac{\partial}{\partial \mathrm{Re}(x_\ell)} b_i}_{✓}$$

$$(B_2')\quad \frac{\partial}{\partial \mathrm{Re}(x_\ell)} \sin^2(\varphi) = 2\sin(\varphi)\frac{\partial}{\partial \mathrm{Re}(x_\ell)}\sin(\varphi)$$

$$= 2\sin(\varphi)\cos(\varphi)\frac{\partial}{\partial \mathrm{Re}(x_\ell)}\varphi \quad ✓$$

(C) $$\frac{\partial}{\partial Re(x_l)} - \sum_{j=1}^{M} Re(H_{i,j} x_j) = - \sum_{j=1}^{M} \frac{\partial}{\partial Re(x_l)} Re(H_{i,j}) Re(x_j) - Im(H_{i,j}) Im(x_j)$$

$$= - Re(H_{i,l})$$

$$\frac{\partial}{\partial Re(x_l)} Im(d_i \otimes s_i - R_i)^2 = \frac{\partial}{\partial Re(x_l)} Im\left(s_i \left[\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i + i\left(a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i\right)\right] - R_i\right)^2$$

$$= \frac{\partial}{\partial Re(x_l)} \Big[\underbrace{\overbrace{Im(s_i)\left(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i\right)}^{A} + \overbrace{Im(i s_i)\left(a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i\right)}^{B} - \overbrace{Im\left(\sum_{n=1}^{M} H_{i,n} x_n\right)}^{C}}_{IMSE_i}\Big]^2$$

$$= \frac{1}{2} IMSE_i \frac{\partial}{\partial Re(x_l)} IMSE_i$$

S

$$Im(i \times s_i) = Re(s_i)$$

(A) $$\frac{\partial}{\partial Re(x_l)} Im(s_i) \left(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i\right)$$

$$= Im(s_i) \underline{\frac{\partial}{\partial Re(x_l)} \left(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i\right)}$$

(B) $$\frac{\partial}{\partial Re(x_l)} Re(s_i) \left(a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i\right)$$

$$= Re(s_i) \underline{\frac{\partial}{\partial Re(x_l)} \left(a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i\right)}$$

(C) $$\frac{\partial}{\partial Re(x_l)} - Im\left(\sum_{n=1}^{M} H_{i,n} x_n\right)$$

$$= -\frac{\partial}{\partial Re(x_l)} \sum_{n=1}^{M} Re(H_{i,n}) Im(x_n) + Im(H_{i,n}) Re(x_n)$$

$$= - Im(H_{i,l})$$

Dérivée en fonction de $Im(x_l)$ $\forall\, l \in [\![1; K]\!]$

$$\frac{\partial}{\partial Im(x_l)} Re(\partial_i \otimes s_i - R_i)^2 = \frac{\partial}{\partial Im(x_l)} Re\Big(s_i\big(\cos(\varphi)^2 a_i + \cos(\varphi)\sin(\varphi)(b_i + i\, a_i) + i \sin(\varphi)^2 b_i\big) - \sum_{j=1}^{M} H_{i,j} x_j\Big)^2$$

$$= 2\, RMSE_i \frac{\partial}{\partial Im(x_l)} RMSE_i$$

$$(A) \quad \frac{\partial}{\partial Im(x_l)} Re(s_i)\left(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i\right) = Re(s_i) \frac{\partial}{\partial Im(x_l)}\Big[\underbrace{\cos^2(\varphi) a_i}_{A_1} + \underbrace{\cos\varphi \sin\varphi\, b_i}_{A_2}\Big]$$

$$(A_1) : \frac{\partial}{\partial Im(x_l)} \cos^2(\varphi) a_i = a_i \underbrace{\frac{\partial}{\partial Im(x_l)} \cos^2(\varphi)}_{A_1'} + \cos^2(\varphi) \underbrace{\frac{\partial}{\partial Im(x_l)} a_i}_{A_1''}$$

$$(A_1') \quad \frac{\partial}{\partial Im(x_l)} \cos^2(\varphi) = 2\cos(\varphi) \frac{\partial}{\partial Im(x_l)} \cos(\varphi)$$

$$= 2\cos(\varphi)\,(-\sin(\varphi)) \frac{\partial}{\partial Im(x_l)} \varphi$$

$$\frac{\partial}{\partial Im(x_l)} \varphi = \frac{1}{2} \times \frac{1}{1 + (\text{système})^2} \times \frac{\partial}{\partial Im(x_l)} (\text{système})$$

$$\frac{\partial}{\partial Im(x_l)} \text{système} = \sum_{j=1}^{K} Im(s_j) Re(s_j) \frac{\partial}{\partial Im(x_l)} \frac{\left[\sum_{n=1}^{M} Re(H_{j,n}) Im(x_n) + Im(H_{j,n}) Re(x_n)\right]^2}{\text{dénom}} \Big\} \, syst_1$$

$$- \sum_{j=1}^{K} Im(s_j) Re(s_j) \frac{\partial}{\partial Im(x_l)} \frac{\left[\sum_{n=1}^{M} Re(H_{j,n}) Re(x_n) - Im(H_{j,n}) Im(x_n)\right]^2}{\text{dénom}} \Big\} \, syst_2$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$syst_1$ $\quad u_1'$

$$\frac{\partial}{\partial Im(x_l)} \left[\sum_{n=1}^{M} Re(H_{j,n}) Im(x_n) + Im(H_{j,n}) Re(x_n)\right]^2$$

$$= 2\, Im(R_j) \frac{\partial}{\partial Im(x_l)} \left[\sum_{n=1}^{M} Re(H_{j,n}) Im(x_n) + Im(H_{j,n}) Re(x_n)\right]$$

$$= 2\, Im(R_j)\, Re(H_{j,l})$$

$$u_1 = Im(R_j)^2$$

$$v = 2\sum_{j=1}^{K} Im(s_j)Re(s_j)\Big(\underbrace{\sum_{n=1}^{K} Re(H_{j,n})Re(x_n) - Im(H_{j,n})Im(x_n)}_{Re(R_j)}\Big)\Big(\underbrace{\sum_{n=1}^{K} Im(H_{j,n})Re(x_n) + Re(H_{j,n})Im(x_n)}_{Im(R_j)}\Big)$$

$$\frac{\partial v}{\partial Im(x_\ell)} = 2\sum_{j=1}^{K} Im(s_j)Re(s_j)\left[Re(R_j)\frac{\partial}{\partial Im(x_\ell)}Im(R_j) + Im(R_j)\frac{\partial}{\partial Im(x_\ell)}Re(R_j)\right]$$

$$= 2\sum_{j=1}^{K} Im(s_j)Re(s_j)\left[Re(R_j)Re(H_{j,\ell}) - Im(R_j)Im(H_{j,\ell})\right] = (\text{dénom})'$$

$$\frac{\partial}{\partial Im(x_\ell)}\text{syst}_1 = \frac{u_1'\,\text{dénom} - u_1(\text{dénom})'}{(\text{dénom})^2}$$

syst$_2$

$$u_2 = -Re(R_j)^2$$

$$u_2' = -2Re(R_j)\frac{\partial}{\partial Im(x_\ell)}\sum_{n=1}^{M} Re(H_{j,n})Re(x_n) - Im(H_{j,n})Im(x_n)$$

$$= 2Re(R_j)\,Im(H_{j,\ell})$$

$$\frac{\partial}{\partial Im(x_\ell)}\text{syst}_2 = \frac{u_2'\,\text{dénom} - u_2(\text{dénom})'}{(\text{dénom})^2}$$

$$\frac{\partial \varphi}{\partial Im(x_\ell)} = \frac{1}{2}\times\frac{1}{1+(\text{système})^2}\times\left[\sum_{j=1}^{K} Re(s_j)Im(s_j)\left[\frac{u_1'\,\text{dénom} - u_1(\text{dénom})'}{(\text{dénom})^2} + \frac{u_2'\,\text{dénom} - u_2(\text{dénom})'}{(\text{dénom})^2}\right]\right]$$

$$\frac{\partial}{\partial Im(x_\ell)}\cos^2(\varphi) = 2\cos(\varphi)(-\sin(\varphi))\frac{\partial}{\partial Im(x_\ell)}\varphi \quad ✓$$

(A_1'') $$\frac{\partial}{\partial Im(x_\ell)}a_i = \frac{\partial}{\partial Im(x_\ell)}\left[Re(s_i)Re(R_i) + Im(s_i)Im(R_i)\right]$$

$$= Re(s_i)\frac{\partial}{\partial Im(x_\ell)}Re\Big(\sum_{n=1}^{M} H_{i,n}x_n\Big) + Im(s_i)\frac{\partial}{\partial Im(x_\ell)}Im\Big(\sum_{n=1}^{M} H_{i,n}x_n\Big)$$

$$= Re(s_i)\frac{\partial}{\partial Im(x_\ell)}\sum_{n=1}^{M} Re(H_{i,n})Re(x_n) - Im(H_{i,n})Im(x_n) + Im(s_i)\frac{\partial}{\partial Im(x_\ell)}\sum_{n=1}^{M} Re(H_{i,n})Im(x_n) + Im(H_{i,n})Re(x_n)$$

$$= Re(s_i)\left(-Im(H_{i,\ell})\right) + Im(s_i)\,Re(H_{i,\ell})$$

(A_1) ✓

(A_2) $\cos\varphi \sin\varphi \, b_i$

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} \cos\varphi \sin\varphi \, b_i = \cos\varphi \sin\varphi \underbrace{\frac{\partial}{\partial \mathrm{Im}(x_l)} b_i}_{A_2'} + \cos\varphi \, b_i \underbrace{\frac{\partial}{\partial \mathrm{Im}(x_l)} \sin\varphi}_{A_2''} + \sin\varphi \, b_i \underbrace{\frac{\partial}{\partial \mathrm{Im}(x_l)} \cos\varphi}_{A_2'''}$$

(A_2')

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} b_i = \frac{\partial}{\partial \mathrm{Im}(x_l)} \left[\mathrm{Re}(s_i)\,\mathrm{Im}(R_i) - \mathrm{Im}(s_i)\,\mathrm{Re}(R_i) \right]$$

$$= \mathrm{Re}(s_i) \frac{\partial}{\partial \mathrm{Im}(x_l)} \sum_{n=1}^{M} \mathrm{Im}(H_{i,n} x_n) - \mathrm{Im}(s_i) \frac{\partial}{\partial \mathrm{Im}(x_l)} \sum_{n=1}^{M} \mathrm{Re}(H_{i,n} x_n)$$

$$= \mathrm{Re}(s_i) \frac{\partial}{\partial \mathrm{Im}(x_l)} \sum_{n=1}^{M} \mathrm{Re}(H_{i,n})\,\mathrm{Im}(x_n) + \mathrm{Im}(H_{i,n})\,\mathrm{Re}(x_n) - \mathrm{Im}(s_i) \frac{\partial}{\partial \mathrm{Im}(x_l)} \sum_{n=1}^{M} \mathrm{Re}(H_{i,n})\,\mathrm{Re}(x_n) - \mathrm{Im}(H_{i,n})\,\mathrm{Im}(x_n)$$

$$= \mathrm{Re}(s_i)\,\mathrm{Re}(H_{i,l}) - \mathrm{Im}(s_i)\left(-\mathrm{Im}(H_{i,l})\right)$$

$$= \mathrm{Re}(s_i)\,\mathrm{Re}(H_{i,l}) + \mathrm{Im}(s_i)\,\mathrm{Im}(H_{i,l})$$

(A_2'')

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} \sin\varphi = \cos(\varphi) \frac{\partial}{\partial \mathrm{Im}(x_l)} \varphi$$

(A_2''')

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} \cos\varphi = -\sin(\varphi) \frac{\partial}{\partial \mathrm{Im}(x_l)} \varphi$$

(B)

$$\Big(\underbrace{\mathrm{Re}(i s_i)}_{= -\mathrm{Im}(s_i)}\Big)\left(\cos\varphi \sin\varphi \, a_i + \sin^2(\varphi)\, b_i\right)$$

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} B = -\mathrm{Im}(s_i) \frac{\partial}{\partial \mathrm{Im}(x_l)} \Big[\underbrace{\cos\varphi \sin\varphi \, a_i}_{B_1} + \underbrace{\sin^2(\varphi)\, b_i}_{B_2} \Big]$$

(B_1)

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} B_1 = \cos\varphi \sin\varphi \frac{\partial}{\partial \mathrm{Im}(x_l)} a_i + \cos\varphi \, a_i \frac{\partial}{\partial \mathrm{Im}(x_l)} \sin\varphi + \sin\varphi \, a_i \frac{\partial}{\partial \mathrm{Im}(x_l)} \cos\varphi$$

(B_2)

$$\frac{\partial}{\partial \mathrm{Im}(x_l)} B_2 = b_i \frac{\partial}{\partial \mathrm{Im}(x_l)} \sin^2(\varphi) + \sin^2(\varphi) \frac{\partial}{\partial \mathrm{Im}(x_l)} b_i$$

$$= b_i \, 2\sin(\varphi)\cos(\varphi) \frac{\partial}{\partial Im(x_l)} \varphi + \sin^2(\varphi) \frac{\partial}{\partial Im(x_l)} b_i$$

(C) $-\sum_{j=1}^{M} Re(H_{i,j} x_j)$

$$\frac{\partial}{\partial Im(x_l)} C = -\sum_{j=1}^{M} \frac{\partial}{\partial Im(x_l)} Re(H_{i,j}) Re(x_j) - Im(H_{i,j}) Im(x_j)$$

$$= -(-Im(H_{i,l})) = Im(H_{i,l})$$

$$* \; \frac{\partial}{\partial Im(x_l)} Im(d_i \otimes s_i - R_i)^2 = \frac{\partial}{\partial Im(x_l)} Im\left(s_i \left[\cos^2(\varphi) a_i + \cos\varphi \sin\varphi \, b_i + i(a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i)\right] - R_i\right)^2$$

$$= \frac{\partial}{\partial Im(x_l)} \Big[\underbrace{\overbrace{Im(s_i)(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi \, b_i)}^{A} + \overbrace{Im(i s_i)(a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i)}^{B} - \overbrace{Im\Big(\sum_{n=1}^{M} H_{i,n} x_n\Big)}^{C}}_{IMSE_i} \Big]^2$$

$$= \frac{1}{2} IMSE_i \frac{\partial}{\partial Im(x_l)} IMSE_i$$

$\hookrightarrow Im(i \times s_i) = Re(s_i)$

(A) $$\frac{\partial}{\partial Im(x_l)} A = Im(s_i) \frac{\partial}{\partial Im(x_l)} \left[\cos^2(\varphi) a_i + \cos\varphi \sin\varphi \, b_i\right]$$

(B) $$\frac{\partial}{\partial Im(x_l)} B = Re(s_i) \frac{\partial}{\partial Im(x_l)} \left[a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i\right]$$

(C) $$\frac{\partial}{\partial Im(x_l)} C = -\frac{\partial}{\partial Im(x_l)} \sum_{n=1}^{M} Im(H_{i,n} x_n)$$

$$= -\frac{\partial}{\partial Im(x_l)} \sum_{n=1}^{M} Re(H_{i,n}) Im(x_n) + Im(H_{i,n}) Re(x_n)$$

$$= -Re(H_{i,l})$$

On a donc : $\forall l \in [\![1;M]\!]$

$$\frac{\partial}{\partial Re(x_l)} \| d \otimes s - R \|_2^2 = \frac{\partial}{\partial Re(x_l)} \left(\sum_{i=1}^{K} | d \otimes s - R |_i^2 \right)$$

$$= \frac{\partial}{\partial Re(x_l)} \left(\sum_{i=1}^{K} Re(d_i \otimes s_i - R_i)^2 + Im(d_i \otimes s_i - R_i)^2 \right)$$

$$= \sum_{i=1}^{K} \frac{\partial}{\partial Re(x_l)} \left(\boxed{Re(d_i \otimes s_i - R_i)}^2 \right) + \sum_{i=1}^{K} \frac{\partial}{\partial Re(x_l)} \left(\boxed{Im(d_i \otimes s_i - R_i)}^2 \right)$$

$$= \sum_{i=1}^{K} 2\, RMSE_i \frac{\partial}{\partial Re(x_l)} RMSE_i + \sum_{i=1}^{K} 2\, IMSE_i \frac{\partial}{\partial Re(x_l)} IMSE_i$$

$$RMSE_i = Re(s_i)\left(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i\right) + \underbrace{Re(i s_i)}_{= -Im(s_i)} \left(\cos\varphi \sin\varphi\, a_i + \sin^2(\varphi) b_i\right) - \sum_{j=1}^{M} Re(H_{i,j} x_j)$$

$$IMSE_i = Im(s_i)\left(\cos^2(\varphi) a_i + \cos\varphi \sin\varphi\, b_i\right) + \underbrace{Im(i s_i)}_{= Re(s_i)} \left(a_i \cos\varphi \sin\varphi + \sin^2(\varphi) b_i\right) - Im\left(\sum_{n=1}^{M} H_{i,n} x_n\right)$$

avec $a_i = Re(s_i)\, Re(R_i) + Im(s_i)\, Im(R_i)$

$b_i = Re(s_i)\, Im(R_i) - Im(s_i)\, Re(R_i)$

$$R_i = \sum_{n=1}^{M} H_{i,n} x_n$$

et $$\varphi = \frac{1}{2} \arctan \left(\frac{\sum_{j=1}^{K} Im(s_j) Re(s_j) \left[Im(R_j)^2 - Re(R_j)^2 \right]}{2 \sum_{j=1}^{K} Re(R_j) Im(R_j) Re(s_j) Im(s_j)} \right)$$

$$\frac{\partial}{\partial Re(x_l)} RMSE_i = Re(s_i)\Big[\underbrace{a_i\, 2\cos\varphi(-\sin\varphi)\frac{\partial}{\partial Re(x_l)}\text{[redacted]} + \cos^2(\varphi)\frac{\partial}{\partial Re(x_l)}\text{[redacted]}}_{A_1}$$

$$\underbrace{+\cos\varphi\sin\varphi\frac{\partial}{\partial Re(x_l)}\text{[redacted]} + \cos^2(\varphi)\, b_i \frac{\partial}{\partial Re(x_l)}\text{[redacted]} - \sin^2(\varphi)\, b_i \frac{\partial}{\partial Re(x_l)}\text{[redacted]}}_{A_2}\Big]$$

$$-Im(s_i)\Big[\underbrace{\cos\varphi\sin\varphi\frac{\partial}{\partial Re(x_l)}\text{[redacted]} + \cos^2(\varphi)\, a_i\frac{\partial}{\partial Re(x_l)}\text{[redacted]} - \sin^2(\varphi)\, a_i\frac{\partial}{\partial Re(x_l)}\text{[redacted]}}_{B_1}$$

$$+\underbrace{2\, b_i \sin(\varphi)\cos(\varphi)\frac{\partial}{\partial Re(x_l)}\text{[redacted]} + \sin^2(\varphi)\frac{\partial}{\partial Re(x_l)}\text{[redacted]}}_{B_2}\Big] - \underbrace{Re(H_{i,l})}_{C}$$

$$\frac{\partial}{\partial Re(x_l)} IMSE_i = Im(s_i)\big[A_1 + A_2\big] + Re(s_i)\big[B_1 + B_2\big] - Im(H_{i,l})$$

$$\frac{\partial a_i}{\partial Re(x_l)} = Re(s_i)\,Re(H_{i,l}) + Im(s_i)\,Im(H_{i,l})$$

$$\frac{\partial b_i}{\partial Re(x_l)} = Re(s_i)\,Im(H_{i,l}) - Im(s_i)\,Re(H_{i,l})$$

$$\frac{\partial\varphi}{\partial Re(x_l)} = \cancel{\frac{1}{2}} \times \frac{1}{1+(\text{système})^2}$$

$$\Big(\text{système} = \tan(2\varphi)$$ mieux d'utiliser directement le résultat du quotient des sommes pour l'implémentation $\Big)$

$$\times \sum_{j=1}^{K} Im(s_j)Re(s_j)\Bigg[\frac{\cancel{2}\,Im(H_{j,l})\,Im(R_j)\,\text{dénom} - Im(R_j)^2\,\cancel{2}\sum_{j=1}^{K} Im(s_j)Re(s_j)\big[Re(R_j)Im(H_{j,l}) + Im(R_j)Re(H_{j,l})\big]}{(\text{dénom})^2}$$

$$-\frac{\cancel{2}\,Re(H_{j,l})\,Re(R_j)\,\text{dénom} - Re(R_j)^2\,\cancel{2}\sum_{j=1}^{K} Im(s_j)Re(s_j)\big[Re(R_j)Im(H_{j,l}) + Im(R_j)Re(H_{j,l})\big]}{(\text{dénom})^2}\Bigg]$$

$$\text{dénom} = 2\sum_{j=1}^{K} Im(s_j)Re(s_j)\,Re(R_j)\,Im(R_j)$$

$\frac{\partial}{\partial \mathrm{Im}(x_l)} RMSE_i$ et $\frac{\partial}{\partial \mathrm{Im}(x_l)} IMSE_i$ s'expriment de manière similaire