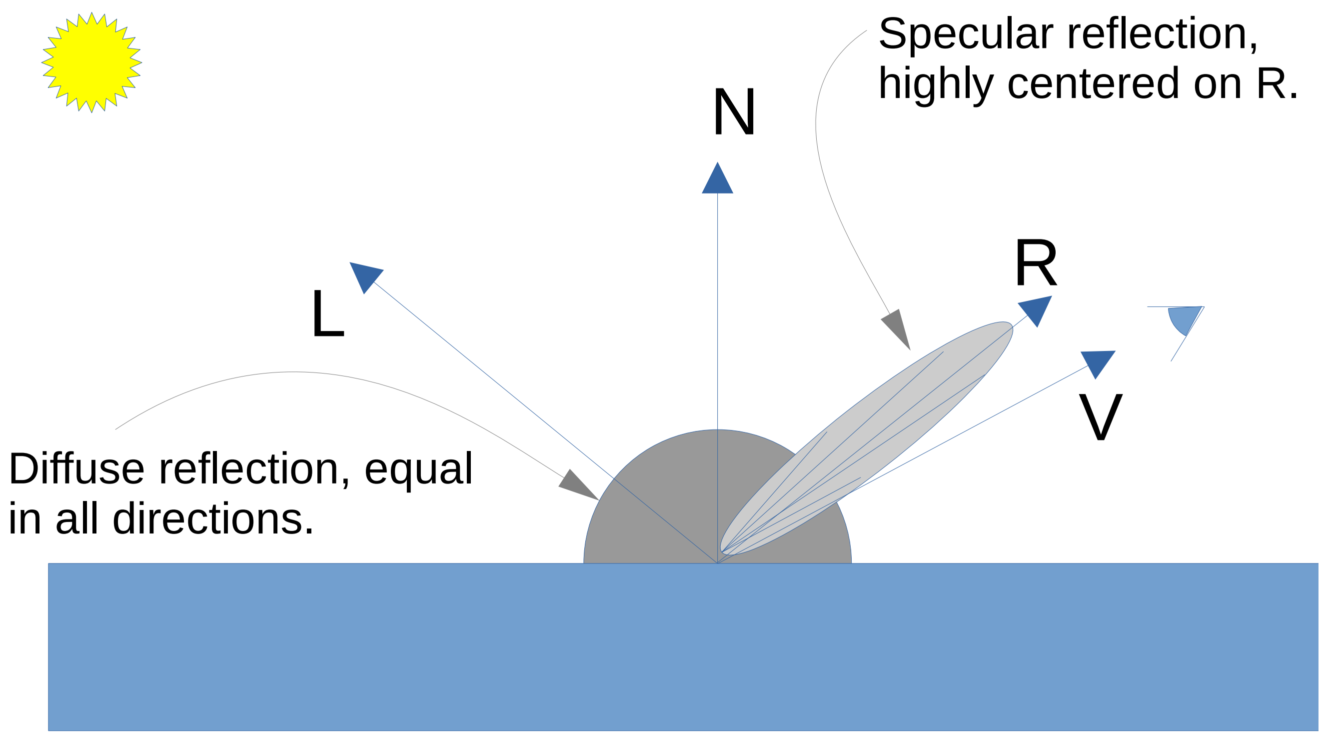
# BRDF
# Bidirectional Reflectance Distribution Function



# Micro-facet BRDF Lighting

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The general BRDF lighting equation is

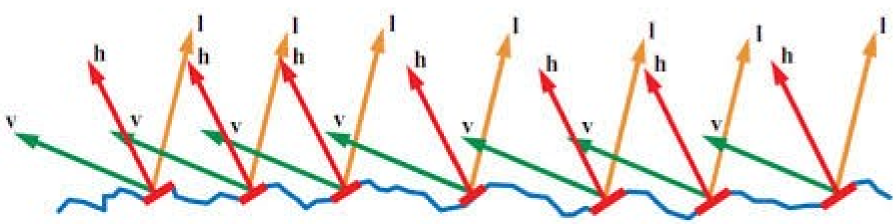
$$I_o = I_i \, (N \cdot L)_+ \; BRDF$$

where the BRDF portion is:

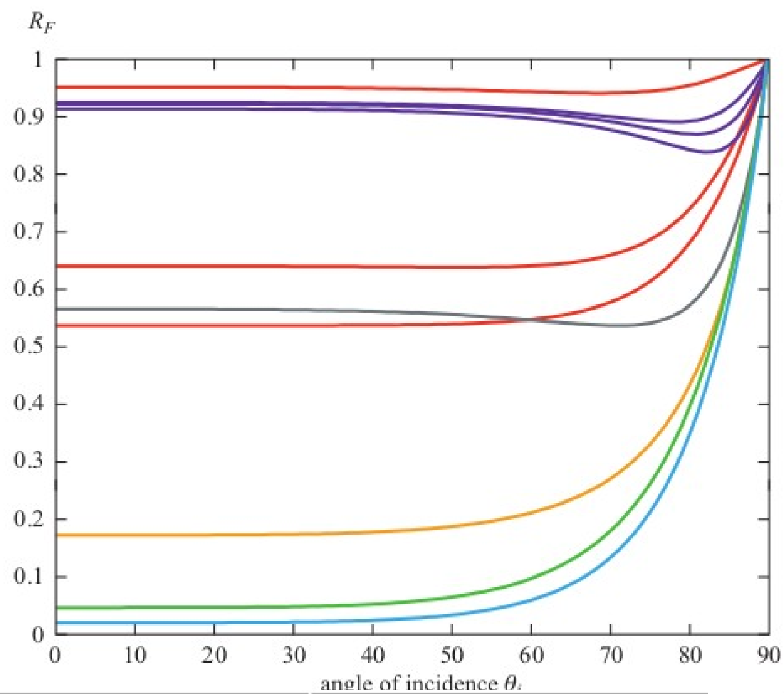
$$BRDF = \frac{K_d}{\pi} \; + \; \frac{D(H) \;\; F(L, H) \;\; G(L, V, H)}{4\,(L \cdot N)\,(V \cdot N)}$$

and where $H$ is half way between $L$ and $V$.

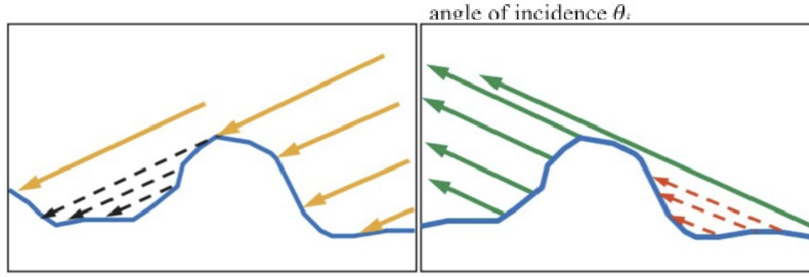
- **D term (distribution):** The fraction of a surface aligned with $H$, so it reflect from $L$ to $V$. This is a probability distributions, and so must integrate to 1.




- **F term (Fresnel):** The fraction of light reflected from (not absorbed by) a micro-facet. Calculated theoretically. Varies greatly with angle of incidence.




- **G term:** Accounts for self-shadowing and self-occlusion between micro-facets. Function is mostly 1, but falls toward 0 at extreme angles.



## Functions for D, F, and G

A nice **starter set** for $D$, $G$, and $F$ (but see the next page for more):

$$BRDF = \frac{K_d}{\pi} + \frac{D(H)\ F(L,H)\ G(L,V,H)}{4\,(L\cdot N)\,(V\cdot N)}$$

- Phong-BRDF with roughness parameter $\alpha$ : 0(rough) ... $\infty$ (mirror)

$$D(H) = \frac{\alpha+2}{2\pi}(N\cdot H)^{\alpha}$$

- Schlick's approximation of the Fresnel equation:

$$F(L,H) = K_s + (1-K_s)(1-L\cdot H)^5$$

- A well known approximation to more carefully derived shadow/occlusion term:

$$\frac{G(L,V,H)}{(L\cdot N)\,(V\cdot N)} \approx \frac{1}{(L\cdot H)^2}$$

# MicroFacet BRDFs (Phong, Beckman, GGX)

All microfacet BRDFs have this general form:

$$I_o = I_i \, (N \cdot L)_+ \; BRDF$$

$$\text{BRDF} = \frac{K_d}{\pi} + \frac{F(L,H)\;\; G(L,V,H)\;\; D(H)}{4\,(L \cdot N)\,(V \cdot N)}$$

where $N$, $L$, and $V$ are unit length vectors for surface orientation and light and eye directions.

These sub expressions occur in several places:

$H = (L+V)/\|L+V\|$ is the so called **half** vector

$\tan\theta_v = \sqrt{(1.0 - (v \cdot N)^2)} \,/\, (v \cdot N)$ for an arbitrary vector v. (Which may be H, L, or V).

### F term

**F** is the Fresnel (reflection) is usually approximated by Schlick as

$$F(L,H) = K_s + (1-K_s)(1-L \cdot H)^5$$

where $K_s$ is the specular reflection color at $L=V=N=H$ .

The exact formulation (if you are interested) is

$$F(L,H) = \frac{1}{2}\;\frac{(g-c)^2}{(g+c)^2}\left(1 + \frac{(c(g+c)-1)^2}{(c(g-c)+1)^2}\right)$$

where

$$g = \sqrt{\eta_t^2/\eta_i^2 - 1 + c^2}\;,$$

$$c = |L \cdot H|$$

and $\eta_i$ and $\eta_t$ are indices of refraction of the two materials.

### D term

**D** is the micro-facet distribution, each controlled by it’s own roughness/shininess parameter Many exist:

**Phong**: $$D_p(H) = \frac{\alpha_p + 2}{2\pi}(N \cdot H)^{\alpha_p}$$

( $\alpha_p$ : $1..\infty$ ; increasing means smoother surface)

**Beckman**: $$D_b(H) = \frac{1}{\pi\alpha_b^2 (N \cdot H)^4}\; e^{\frac{-\tan^2\theta_H}{\alpha_b^2}}$$

( $\alpha_b$ : $0..1$ ; increasing means rougher surface)

similar to Phong for smooth surfaces using $\alpha_p = 2\alpha_b^{-2} - 2$

**GGX**: $$D_g(H) = \frac{\alpha_g^2}{\pi (N \cdot H)^4 (\alpha_g^2 + \tan^2\theta_H)^2}$$ or equivalently $$D_g(H) = \frac{\alpha_g^2}{\pi\,((N \cdot H)^2(\alpha_g^2 - 1) + 1)^2}$$

### Roughness/shininess parameters in D and conversions.

Phong’s shininess parameter $\alpha_p$ ranges over $[0,\infty]$ for rough to shiny surfaces

Beckman and GGX roughness parameters $\alpha_b$ and $\alpha_g$ range over $[0,1]$ for smooth to rough surfaces.

Conversions between $\alpha_p$ and $\alpha_g$ (or equivalently $\alpha_b$ ):

$$\alpha_g = \sqrt{\frac{2}{\alpha_p + 2}} \quad \text{and} \quad \alpha_p = -2 + 2/\alpha_g^2$$

**G is the self occluding and self-shadowing geometry term**

Many exist in the Smith form: $G(L, V, H) = G_1(L, H)\, G_1(V, H)$
where $G_1(v, H)$ is:

**Beckman** uses this very accurate rational approximation

$$G_1(v, H) = \begin{cases} \dfrac{3.535a + 2.181a^2}{1.0 + 2.276a + 2.577a^2} & \text{if } a < 1.6 \\ 1 & \text{otherwise} \end{cases}$$

with

$$a = 1/(\alpha_b \tan\theta_v)$$

**Phong**:

Same $G_1$ as Beckman, but with $a = (\sqrt{\alpha_p/2+1}) / \tan\theta_v$

***Beware roundoff errors in calculating the*** $G_1$ ***function:***

- The value of $(v \cdot N)$ may round up to greater than 1.0 (it shouldn't, but it does). If so, return $G_1(\ldots) = 1.0$ .
- The calculation of $\tan\theta_v$ may be zero. If so, don't divide by it, instead return $G_1(\ldots) = 1.0$ .

**GGX:**

$$G_1(v, m) = \frac{2}{1 + \sqrt{1+\alpha_g^2 \tan^2\theta_v}}$$

Sometimes $G$ is combined with most of the denominator and called the **visibility** term

$$V(L, V) = \frac{G(L, V, H)}{(L \cdot N)(V \cdot N)}$$

A simple approx is: $V(L, V) = 1$
is not too bad – darkens too fast and is independent of roughness.

Better approx is: $V(L, V) \approx 1/(L \cdot H)^2$
often considered good enough for real time graphics.

## Comparison Phong; Beckman; GGX