Machete Cálculo lambda

17 de diciembre de 2024

Machete: Tipos y Términos

Las expresiones de tipos (o simplemente tipos) son:

$$\sigma ::= \operatorname{Bool} \mid \operatorname{Nat} \mid \sigma \to \sigma$$

Los términos están dados por:

$$M ::= x \mid \lambda x : \sigma M \mid MM \mid \text{true} \mid \text{false} \mid \text{if } M \text{ then } M \text{ else } M \mid \text{zero} \mid \text{succ}(M) \mid \text{pred}(M) \mid \text{isZero}(M)$$

Machete: Axiomas y Reglas de Tipado

$$\Gamma \vdash \text{true} : \text{Bool} \quad (\text{axtrue})$$

$$\Gamma \vdash \text{false} : \text{Bool} \quad (\text{axfalse})$$

$$\Gamma, x : \sigma \vdash x : \sigma \qquad (\text{axv})$$

$$\frac{\Gamma \vdash M : \text{Bool} \quad \Gamma \vdash P : \sigma \quad \Gamma \vdash Q : \sigma}{\Gamma \vdash \text{if } M \text{ then } P \text{ else } Q : \sigma} \quad (\text{if}$$

$$\frac{\Gamma, x : \sigma \vdash M : \tau}{\Gamma \vdash \lambda x : \sigma . M : \sigma \to \tau} \quad (\to \text{i})$$

$$\frac{\Gamma \vdash M : \sigma \to \tau \quad \Gamma \vdash N : \sigma}{\Gamma \vdash M : \tau} \quad (\to \text{e})$$

$$\frac{\Gamma \vdash M : \text{Nat}}{\Gamma \vdash \text{succ}(M) : \text{Nat}} \quad (\text{succ})$$

$$\frac{\Gamma \vdash M : \text{Nat}}{\Gamma \vdash \text{pred}(M) : \text{Nat}} \quad (\text{pred})$$

$$\frac{\Gamma \vdash M : \text{Nat}}{\Gamma \vdash \text{isZero}(M) : \text{Bool}} \quad (\text{isZero})$$

Semántica Operacional

Valores

$$V ::= \text{true} \mid \text{false} \mid \lambda x : \sigma.M \mid \text{zero} \mid \text{succ}(V)$$

(Los valores de tipo Nat pueden escribirse como n, lo cual abrevia $\operatorname{succ}^n(\operatorname{zero})$).

Reglas de Evaluación en un Paso

$$\frac{M_1 \to M_1'}{M_1 M_2 \to M_1' M_2} \quad (app_l \circ \mu)$$

$$\frac{M_2 \to M_2'}{V M_2 \to V M_2'} \quad (app_r \circ \nu)$$

$$(\lambda x : \sigma.M)V \to M\{x := V\} \quad (\beta)$$

if true then M_2 else $M_3 \to M_2$ (if_t)

if false then M_2 else $M_3 \to M_3$ (if_f)

$$\frac{M_1 \to M_1'}{\text{if } M_1 \text{ then } M_2 \text{ else } M_3 \to \text{if } M_1' \text{ then } M_2 \text{ else } M_3} \quad (if_c)$$

Reglas Adicionales

$$\operatorname{pred}(\operatorname{succ}(n)) \to n \pmod{pred}$$

$$pred(zero) \rightarrow zero \quad (pred_0, opcional)$$

$$isZero(zero) \rightarrow true \quad (isZero_0)$$

$$isZero(succ(n)) \rightarrow false \quad (isZero_n)$$

$$\frac{M \to N}{\operatorname{succ}(M) \to \operatorname{succ}(N)} \quad (succ_c)$$

$$\frac{M \to N}{\operatorname{pred}(M) \to \operatorname{pred}(N)} \quad (pred_c)$$

$$\frac{M \to N}{\mathrm{isZero}(M) \to \mathrm{isZero}(N)} \quad (isZero_c)$$

Deducción natural

Reglas básicas

Deducción natural

Reglas derivadas

Cuantificación universal

Regla de eliminación

$$\frac{\Gamma \vdash \forall X. \, \sigma}{\Gamma \vdash \sigma \{X := t\}} \forall E$$

Regla de introducción

$$\frac{\Gamma \vdash \sigma \quad X \notin \mathsf{fv}(\Gamma)}{\Gamma \vdash \forall X.\,\sigma} \forall I$$

Cuantificación existencial

Regla de introducción

$$\frac{\Gamma \vdash \sigma\{X := t\}}{\Gamma \vdash \exists X. \sigma} \exists I$$

Regla de eliminación

$$\frac{\Gamma \vdash \exists X. \sigma \quad \Gamma, \sigma \vdash \tau \quad X \notin \mathsf{fv}(\Gamma, \tau)}{\Gamma \vdash \tau} \exists \mathsf{E}$$

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Inferencia de Tipos Machete

Paradigmas (de Lenguajes) de Programación

1. Algoritmo de inferencia

- $\mathbb{W}(x) \leadsto \{x : \mathbf{X}_k\} \vdash x : \mathbf{X}_k, \quad \mathbf{X}_k \text{ incógnita fresca}$
- $\mathbb{W}(\theta) \leadsto \emptyset \vdash \theta : Nat$
- $\mathbb{W}(true) \leadsto \emptyset \vdash true : Bool$
- $\mathbb{W}(false) \leadsto \emptyset \vdash false : Bool$
- $\mathbb{W}(succ(U)) \leadsto S(\Gamma) \vdash S(succ(M)) : Nat \text{ donde}$
 - $\mathbb{W}(U) = \Gamma \vdash M : \tau$
 - $S = MGU\{\tau \stackrel{?}{=} Nat\}$
- $\mathbb{W}(pred(U)) \leadsto S(\Gamma) \vdash S(pred(M)) : Nat \text{ donde}$
 - $\mathbb{W}(U) = \Gamma \vdash M : \tau$
 - $S = MGU\{\tau \stackrel{?}{=} Nat\}$
- $\mathbb{W}(iszero(U)) \leadsto S(\Gamma) \vdash S(iszero(M)) : Bool \text{ donde}$
 - $\mathbb{W}(U) = \Gamma \vdash M : \tau$
 - $S = MGU\{\tau \stackrel{?}{=} Nat\}$
- $\mathbb{W}(if\ U\ then\ V\ else\ W) \leadsto S(\Gamma_1) \cup S(\Gamma_2) \cup S(\Gamma_3) \vdash S(if\ M\ then\ P\ else\ Q): S(\sigma)\ donde$
 - $\mathbb{W}(U) = \Gamma_1 \vdash M : \rho$
 - $\mathbb{W}(V) = \Gamma_2 \vdash P : \sigma$
 - $\mathbb{W}(W) = \Gamma_3 \vdash Q : \tau$
 - $S = MGU\{\sigma \stackrel{?}{=} \tau, \rho \stackrel{?}{=} Bool\} \cup \{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_i, x : \sigma_2 \in \Gamma_j, i, j \in \{1, 2, 3\}\}$
- $\mathbb{W}(\lambda x.U) \leadsto \Gamma' \vdash \lambda x : \tau'.M : \tau' \to \rho \text{ donde}$
 - $\mathbb{W}(U) = \Gamma \vdash M : \rho$
 - $\tau' = \left\{ \begin{array}{l} \alpha \text{ si } x : \alpha \in \Gamma \\ \mathbf{X}_k \text{ con } \mathbf{X}_k \text{ variable fresca en otro caso} \end{array} \right.$
 - $\Gamma' = \Gamma \ominus \{x\}$
- $\mathbb{W}(UV) \leadsto S(\Gamma_1) \cup S(\Gamma_2) \vdash S(MN) : S(\mathbf{X}_k)$ donde
 - $\mathbb{W}(U) = \Gamma_1 \vdash M : \tau$
 - $\mathbb{W}(V) = \Gamma_2 \vdash N : \rho$
 - X_k variable fresca
 - $S = MGU\{\tau \stackrel{?}{=} \rho \rightarrow \mathbf{X}_k\} \cup \{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_1, x : \sigma_2 \in \Gamma_2\}$

2. Algoritmo de unificación (Martelli-Montanari)

2.1. Reglas

Se enuncian las reglas para constructores de tipo C en general de cualquier aridad, y en particular para los constructores de tipo de λ^b

$$\sigma, \tau ::= Nat \mid Bool \mid \sigma \rightarrow \tau$$

1. Descomposición

$$\{\sigma_1 \to \sigma_2 \stackrel{?}{=} \tau_1 \to \tau_2\} \cup G \mapsto \{\sigma_1 \stackrel{?}{=} \tau_1, \sigma_2 \stackrel{?}{=} \tau_2\} \cup G$$

$$\{Bool \stackrel{?}{=} Bool\} \cup G \mapsto G$$

$$\{Nat \stackrel{?}{=} Nat\} \cup G \mapsto G$$

Caso general

$$\{C(\sigma_1,\ldots,\sigma_n)\stackrel{?}{=}C(\tau_1,\ldots,\tau_n)\}\cup G\mapsto \{\sigma_1\stackrel{?}{=}\tau_1,\ldots,\sigma_n\stackrel{?}{=}\tau_n\}\cup G$$

2. Eliminación de par trivial

$$\{\mathbf{X}_k \stackrel{?}{=} \mathbf{X}_k\} \cup G \mapsto G$$

3. Swap: si σ no es una variable

$$\{\sigma \stackrel{?}{=} \mathbf{X}_k\} \cup G \mapsto \{\mathbf{X}_k \stackrel{?}{=} \sigma\} \cup G$$

4. Eliminación de variable: si $X_k \notin FV(\sigma)$

$$\{\mathbf{X}_k \stackrel{?}{=} \sigma\} \cup G \mapsto_{\{\mathbf{X}_k := \sigma\}} G\{\mathbf{X}_k := \sigma\}$$

5. Colisión

$$\{\sigma \stackrel{?}{=} \tau\} \cup G \mapsto \mathbf{falla}, \operatorname{con}(\sigma, \tau) \in T \cup T^{-1} \text{ donde}$$

 $T = \{(Bool, Nat), (Nat, \sigma_1 \to \sigma_2), (Bool, \sigma_1 \to \sigma_2)\} \text{ y } T^{-1} \text{ representa invertir cada par}$
 $Caso\ general: \operatorname{si} C \neq C' \text{ son constructores de tipo diferentes}$
 $\{C(\dots) \stackrel{?}{=} C'(\dots)\} \cup G \mapsto \mathbf{falla}$

6. Occur check: si $X_k \neq \sigma$ y $X_k \in FV(\sigma)$

$$\{\mathbf{X}_k \stackrel{?}{=} \sigma\} \cup G \mapsto \mathbf{falla}$$

2.2. Ejemplos

2.2.1. Secuencia exitosa

$$\{(Nat \to X_1) \to (X_1 \to X_3) \stackrel{?}{=} X_2 \to (X_4 \to X_4) \to X_2\}$$

$$\mapsto^1 \qquad \{Nat \to X_1 \stackrel{?}{=} X_2, X_1 \to X_3 \stackrel{?}{=} (X_4 \to X_4) \to X_2\}$$

$$\mapsto^3 \qquad \{X_2 \stackrel{?}{=} Nat \to X_1, X_1 \to X_3 \stackrel{?}{=} (X_4 \to X_4) \to X_2\}$$

$$\mapsto^4_{\{X_2 := Nat \to X_1\}} \qquad \{X_1 \to X_3 \stackrel{?}{=} (X_4 \to X_4) \to (Nat \to X_1)\}$$

$$\mapsto^1 \qquad \{X_1 \stackrel{?}{=} X_4 \to X_4, X_3 \stackrel{?}{=} Nat \to X_1\}$$

$$\mapsto^4_{\{X_1 := X_4 \to X_4\}} \qquad \{X_3 \stackrel{?}{=} Nat \to (X_4 \to X_4)\}$$

$$\mapsto^4_{\{X_3 := Nat \to (X_4 \to X_4)\}} \emptyset$$

El MGU es

$$\begin{aligned} & \{ \mathbf{X}_3 := Nat \to (\mathbf{X}_4 \to \mathbf{X}_4) \} \circ \{ \mathbf{X}_1 := \mathbf{X}_4 \to \mathbf{X}_4 \} \circ \{ \mathbf{X}_2 := Nat \to \mathbf{X}_1 \} \\ & = \{ \mathbf{X}_2 := Nat \to (\mathbf{X}_4 \to \mathbf{X}_4), \ \mathbf{X}_1 := \mathbf{X}_4 \to \mathbf{X}_4, \ \mathbf{X}_3 := Nat \to (\mathbf{X}_4 \to \mathbf{X}_4) \} \end{aligned}$$

2.2.2. Secuencia fallida

$$\{X_{1} \to (X_{2} \to X_{1}) \stackrel{?}{=} X_{2} \to ((X_{1} \to Nat) \to X_{1})\}$$

$$\mapsto^{1} \{X_{1} \stackrel{?}{=} X_{2}, X_{2} \to X_{1} \stackrel{?}{=} (X_{1} \to Nat) \to X_{1}\}$$

$$\mapsto^{4}_{\{X_{1} := X_{2}\}} \{X_{2} \to X_{2} \stackrel{?}{=} (X_{2} \to Nat) \to X_{2}\}$$

$$\mapsto^{1} \{X_{2} \stackrel{?}{=} X_{2} \to Nat, X_{2} \stackrel{?}{=} X_{2}\}$$

$$\mapsto^{6}$$
 falla

2.2.3. Constructores en general

El MGU es

Se usan los constructores de tipos de listas,

$$\sigma ::= \ldots \mid [\sigma]$$

$$\{(X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4 \stackrel{?}{=} ((X_1 \to X_2) \to [X_1] \to [X_2]) \to X_5\}$$

$$\mapsto^1 \qquad \{X_3 \to X_4 \to X_4 \stackrel{?}{=} (X_1 \to X_2) \to [X_1] \to [X_2], X_4 \to [X_3] \to X_4 \stackrel{?}{=} X_5\}$$

$$\mapsto^3 \qquad \{X_3 \to X_4 \to X_4 \stackrel{?}{=} (X_1 \to X_2) \to [X_1] \to [X_2], X_5 \stackrel{?}{=} X_4 \to [X_3] \to X_4\}$$

$$\mapsto^4_{\{X_5 := X_4 \to [X_3] \to X_4\}} \{X_3 \to X_4 \to X_4 \stackrel{?}{=} (X_1 \to X_2) \to [X_1] \to [X_2]\}$$

$$\mapsto^1 \qquad \{X_3 \stackrel{?}{=} X_1 \to X_2, X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2]\}$$

$$\mapsto^4_{\{X_3 := X_1 \to X_2\}} \qquad \{X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2]\}$$

$$\mapsto^4_{\{X_4 := [X_1]\}} \qquad \{[X_1] \stackrel{?}{=} [X_2]\}$$

$$\mapsto^4_{\{X_4 := [X_1]\}} \qquad \{X_1 \stackrel{?}{=} X_2\}$$

$$\mapsto^4_{\{X_1 := X_2\}} \qquad \emptyset$$

$$\begin{split} \{X_1 := X_2\} \circ \{X_4 := [X_1]\} \circ \{X_3 := X_1 \to X_2\} \circ \{X_5 := X_4 \to [X_3] \to X_4\} \\ = \{X_5 := X_{[X_2]} \to [X_2 \to X_2] \to X_{[X_2]}, \ X_3 := X_2 \to X_2, \ X_4 := [X_2], \ X_1 := X_2\} \end{split}$$

```
\#(1\ 2\ 3\ 4) collect: [:each | each *\ 2] \longrightarrow \#(2\ 4\ 6\ 8)
#(1 2 3 4)
   inject: 0
   into: [:each:result | each + result ] ----
" testing "
#( 2 4 ) anySatisfy: [ :each | each odd ] \longrightarrow
                                                 false
#(24) allSatisfy: [:each | each even]
                                                 true
" findina "
'abcdef' includes: $e → true
'abcdef' contains: [:each | each isUppercase ] ----- false
'abcdef'
   detect: [:each | each isVowel]
   ifNone: [\$u] \longrightarrow \$a
" String – a collection of characters "
string := 'abc'.
                               'abcDEF'
string := string , 'DEF' ----
string beginsWith: 'abc'
string endsWith: 'abc' ----
string includesSubString: 'cD'
                                       true
string asLowercase --> 'abcdef
string asUppercase --- 'ABCDEF'
" OrderedCollection - an ordered collection of objects "
ordered := OrderedCollection new.
ordered addLast: 'world'.
ordered addFirst: 'hello'.
ordered size \longrightarrow 2
ordered at: 2 \longrightarrow
                      'world'
ordered removeLast ---- 'world'
ordered removeFirst ----
                              'hello'
ordered is Empty ---- true
" Set - an unordered collection of objects without duplicates "
set := Set new.
set add 'hello'; add: 'hello'.
set size --- 1
" Bag - an unordered collection of objects with duplicates "
bag := Bag new.
bag add: 'this'; add: 'that'; add: 'that'.
bag occurrencesOf: 'that' --> 2
bag remove: 'that'.
bag occurrencesOf: 'that' --> 1
" Dictionary - associates unique keys with objects "
dictionary := Dictionary new.
dictionary at: 'smalltalk' put: 80.
dictionary at: 'smalltalk' --> 80
dictionary at: 'squeak' ifAbsent: [82] ----
dictionary removeKey: 'smalltalk'.
dictionary is Empty --> true
```

Streams

File Streams

fileStream := FileDirectory default newFileNamed: 'tmp.txt'. fileStream nextPutAll: 'my cool stuff'. fileStream close.

fileStream := FileDirectory default oldFileNamed: 'tmp.txt'. fileStream contents —— 'my cool stuff'

Method Definition

messageSelectorAndArgumentNames
"comment stating purpose of message"
| temporary variable names |
statements

Class Definition

Object subclass: #NameOfSubclass instanceVariableNames: 'instVar1 instVar2' classVariableNames: " poolDictionaries: " category: 'Category-Name'

References

- 1. Andrew Black, Stéphane Ducasse, Oscar Nierstrasz and Damien Pollet, *Squeak by Example*, Square Bracket Associates, 2007, squeakbyexample.org.
- Chris Rathman, Terse guide to Squeak, wiki.squeak. org/squeak/5699.
- Smalltalk, Wikipedia, the free encyclopedia, en. wikipedia.org/wiki/Smalltalk.

Smalltalk Cheat Sheet

Software Composition Group University of Bern

May 21, 2008

1. The Environment

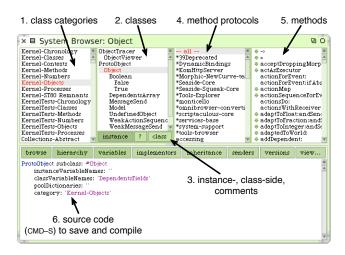


Figure 1: The Smalltalk Code Browser

- Do it (CMD-D): Evaluate selected code.
- Print it (CMD-P): Display the result of evaluating selected code.
- Debug it: Evaluate selected code step-by-step with the integrated debugger.
- Inspect it (CMD-I): Show an *object inspector* on the result of evaluating selected code.
- Explore it (CMD-SHIFT-I): Show an *object explorer* on the result of evaluating selected code.

2. The Language

- Everything is an object.
- Everything happens by sending messages.
- Single inheritance.
- Methods are public.
- Instance variables are private to objects.

Keywords

- self, the receiver.
- super, the receiver, method lookup starts in superclass.
- nil, the unique instance of the class UndefinedObject.
- true, the unique instance of the class True.
- false, the unique instance of the class False.
- thisContext, the current execution context.

Literals

- Integer 123 2r1111011 (123 in binary) 16r7B (123 in hexadecimal)
- Float 123.4 1.23e-4
- Character \$a
- String 'abc'
- Symbol #abc
- Array #(123 123.4 \$a 'abc' #abc)

Message Sends

- 1. *Unary messages* take no argument. 1 factorial sends the message factorial to the object 1.
- 2. *Binary messages* take exactly one argument. 3 + 4 sends message + with argument 4 to the object 3. #answer -> 42 sends -> with argument 42 to #answer. Binary selectors are built from one or more of the characters +, -, *, =, <, >, ...

3. Keyword messages take one or more arguments. 2 raisedTo: 6 modulo: 10 sends the message named raisedTo:modulo: with arguments 6 and 10 to the object 2.

Unary messages are sent first, then binary messages and finally keyword messages:

```
2 raisedTo: 1 + 3 factorial → 128
```

Messages are sent left to right. Use parentheses to change the order:

```
1 + 2 * 3 \longrightarrow 9
1 + (2 * 3) \longrightarrow 7
```

Syntax

- Comments "Comments are enclosed in double quotes"
- Temporary Variables l var l | var1 var2 |
- Assignment var := aStatement var1 := var2 := aStatement
- Statements aStatement1. aStatement2 aStatement1, aStatement2, aStatement3
- Messages receiver message (unary message) receiver + argument (binary message) receiver message: argument (keyword message) receiver message: argument1 with: argument2
- Cascade receiver message1; message2 receiver message1; message2: arg2; message3: arg3
- Blocks [aStatement1.aStatement2] [:argument1 | aStatement1. aStatement2] [:argument1:argument2||temp1 temp2|aStatement1]
- Return Statement ^ aStatement

3. Standard Classes

Logical expressions

```
true not \longrightarrow false
1 = 2 \text{ or: } [2 = 1] \longrightarrow \text{ false}
1 < 2 and: [2 > 1] \longrightarrow true
```

Conditionals

```
1 = 2 ifTrue: [Transcript show: '1 is equal to 2'].
1 = 2 ifFalse: [ Transcript show: '1 isn"t equal to 2' ].
100 factorial / 99 factorial = 100
   ifTrue: [ Transcript show: 'condition evaluated to true' ]
   ifFalse: [ Beeper beep ].
Loops
" conditional iteration "
[Sensor anyButtonPressed]
    whileFalse: [ "wait" ].
pen := Pen newOnForm: Display.
pen place: Sensor cursorPoint.
[Sensor anyButtonPressed]
    whileTrue: [ pen goto: Sensor cursorPoint ].
" fixed iteration "
180 timesRepeat: [
   pen turn: 88.
   pen go: 250 ].
1 to: 100 do: [:index |
    pen go: index * 4.
    bic turn: 89 ].
" infinite loop (press CMD+. to break) "
[pen goto: Sensor cursorPoint] repeat.
Blocks (anonymous functions)
" evaluation "
[1+2] value \longrightarrow 3
[:x \mid x + 2] value: 1 \longrightarrow 3
[:x:y|x+y] value: 1 value: 2 \longrightarrow 3
" processes "
[ (Delay forDuration: 5 seconds) wait.
 Transcript show: 'done' ] fork → aProcess
Collections
" iterating "
'abc' do: [:each | Transcript show: each ].
'abc'
    do: [:each | Transcript show: each ]
    separatedBy: [Transcript cr].
" transforming "
#(1 2 3 4) select: [:each | each even] \longrightarrow #(2 4)
\#(1\ 2\ 3\ 4)\ reject: [:each | each = 2] \longrightarrow \#(1\ 3\ 4)
```