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CALCULATING THE FLOWFIELD IN THE WAKE OF WIND TURBINES

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SUMMARY

The paper briefly outlines the known features of wind turbine wakes. A numerical model is described which can be used to calculate the wake flowfield. The effect of meteorological conditions on wake decay is examined in detail and the way in which this is included in the model is described. Comparison of experimental data with model calculations is used to draw conclusions about the wake flowfield.

It is concluded that the numerical wake model can be used with confidence to calculate the wake velocity field for wind turbines over a range of sizes and in a variety of meteorological conditions. It is considered that the model is sufficiently simple and quick to use that the technique will be of considerable use in the planning and design of windfarms and of turbines for windfarm operation.

NOTATION

U,V	axial and radial velocities
x,r	axial distance coordinate (downwind from wind turbine) and radial distance coordinate (from wake centreline)
$\overline{uv}$	Reynolds stress cross-correlation
U_o	free stream windspeed
U_H	free stream windspeed at hub height
U_c	wake centreline velocity
k	Von Karman constant (here taken to be 0.4)
k_1	dimensionless constant
L	Monin-Obukhov length
C_T	wind turbine thrust coefficient
d	velocity deficit
D_m	centreline velocity deficit at 2 diameters downstream
d_o	centreline velocity deficit
b	wake width
K_M	eddy diffusivity of momentum

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z height above ground

z_H wind turbine hub height

D wind turbine diameter

z_o roughness length

ϵ eddy viscosity

ϵ_a contribution to eddy viscosity due to ambient turbulence

INTRODUCTION

The grouping together of wind turbines into windfarms gives rise to interactive effects due to the operation of the turbines in each others wakes. The effects are both to reduce the power output of the windfarm and to increase the fatigue loads experienced by the wind turbines. These effects need to be taken into account during the design of both the wind turbines themselves and the overall windfarm.

The mathematical models of wind turbine wakes which have previously been subject to most development have been based on empirical descriptions of a wake (e.g. ref. 1). Although these models can be useful for estimating the effect of wake decay characteristics on windfarm efficiency, they do not provide any physical insight into the flow processes.

This paper describes the development of a wake model based on a numerical solution of the differential equations governing the flow. A relatively simple eddy viscosity turbulence model is used, which results in an algorithm which is both quick and easy to use. The eddy viscosity has two contributions - one describes the turbulent mixing due to turbulence generated within the shear layer of the wake, the other describes the effect of ambient turbulence in the atmosphere on wake mixing. The ambient turbulence term is a dominant term in many situations of practical interest and has considerable influence on the wake velocity field. The paper includes an examination of the influence of meteorological conditions on the wake decay.

CHARACTERISTICS OF WIND TURBINE WAKES

Studies of turbine wakes, both in the field and in wind tunnel experiments, indicate the existence of a complex near wake region which typically extends over about 2-4 diameters downstream. This region is dominated by the relaxation of axial and radial pressure gradients caused by the extraction of energy from the mean flow at the rotor disk. This causes the centreline velocity to drop and the wake width to increase as the air moves downstream from the rotor plane. The minimum centreline velocity is reached between 1 and 2 diameters downstream, beyond which the velocity begins to recover as fluid mixing takes over from pressure gradient effects as the dominant process in the flow. The tip vortices from a turbine rotor decay within 2 to 3 diameters due to the action of ambient turbulence (ref. 2). The bulk swirl introduced by the rotor is small, the swirl angle being less than 10 degrees.

Large scale turbulence is generated in the annular shear layer of the wake and this spreads into the core of the wake, reaching the centreline between 3 and 5 diameters downstream. Beyond 5 diameters downstream the wake profile is roughly Gaussian and the centreline deficit decays monotonically, with the rate of decay strongly dependent on the ambient turbulence intensity. Turbulence profiles across rotor wakes in the main decay region sometimes show off-axis peaks where the velocity shear is a maximum (at Nasudden for instance, ref. 3) but intriguingly this is not always the case (as at Aldborough, ref. 4). The axial turbulent velocity on the centreline decays towards the ambient level with an exponent around -2/3 in most cases. The lengthscale of the turbulence introduced by the wake is of the order of the rotor diameter.

The total momentum deficit in the wake is determined by the thrust coefficient of the wind turbine - and is thus a function of the operating conditions (tip-speed-ratio, pitch angle etc.). The rate of recovery of the wake downstream of the near wake region is governed by both the turbulence generated in the wake shear layer and the level of ambient turbulence in the atmosphere. The atmospheric turbulence is a function of the site - primarily surface roughness - the hub height of the turbine (turbulence levels are higher near the ground) and atmospheric stability. Although strong winds are normally associated with neutral stability turbines operating near the cut-in wind speed (which occurs for a relatively large proportion of the turbine operating time) could be operating in more stable conditions. These conditions will give rise to reduced turbulence intensities and hence a more persistent wake, and will also correspond to high thrust coefficient values.

Comparison of wind tunnel wake measurements with available field data indicate that wake deficits measured in the field are often smaller than those from wind tunnel experiments. This is thought to be because variability of the wind - particularly wind direction - gives rise to wake meandering, which means that a measured deficit averaged over a few minutes will in fact be averaged over a portion of the wake profile.

DEVELOPMENT OF THE EQUATIONS

The wake will be considered to be an axisymmetric, fully turbulent, wake flow with zero circumferential velocities (since swirl is small), and the flowfield will be assumed to be stationary with time. Pressure gradients in the co-flowing fluid outside the wake will be assumed to be negligible. Beyond the first few diameters downstream the gradients of mean quantities in the radial direction will be very much greater than the gradients in the axial direction. The Navier-Stokes equation can then be replaced with its equivalent thin shear layer approximation and the viscous terms dropped, giving:

$$U\,\partial U/\partial x + V\partial U/\partial r = -(1/r)\,\partial(r\overline{uv})/\partial r \qquad (1)$$

The turbulent viscosity concept is used to describe the shear stresses with an eddy viscosity defined by:

$$-\overline{uv} = \epsilon\,\partial U/\partial r, \qquad \epsilon = l_w(x)\,U_w(x) + \epsilon_a \qquad (2)$$

where l_w and U_w are suitable length and velocity scales describing the wake shear layer, and ϵ_a is the ambient turbulence contribution to the eddy viscosity. The length and velocity scales are taken to be proportional to the wake width b and the velocity difference $U_o - U_c$ across the wake shear layer; these scales are therefore characteristic of the downstream distance x and independent of r.

The contribution ϵ_a to the eddy viscosity is given by the same parameter used by boundary layer modellers to describe momentum transfer in the atmosphere - the eddy diffusivity of momentum k_M (ref. 5) which has the same dimensions as ϵ and is defined in an analogous way.

A modification is required to the equation for the eddy viscosity in the near wake - up to about 5 diameters downstream of the rotor. This is because there is a lack of equilibrium between the mean velocity field and the turbulence field in the near wake region, and the relation (2) does not apply (ref. 6).

More understanding is required of the non-equilibrium nature of the flow in this region before it can be successfully modelled. A clue, however, to the treatment of the near wake region is given by turbulence data which indicates the build-up of turbulence in the shear layer of turbine wakes. The effect of this build-up of turbulence appears consistent with a filter function F of the form:

$$F = \begin{cases} 0.65 + [(x - 4.5)/23.32]^{1/3} & x < 5.5 \\ 1 & x > 5.5 \end{cases} \qquad (3)$$

which gives the equation for the eddy viscosity as:

$$\epsilon = F[k_1\, b\,(U_o - U_c) + K_M] \qquad (4)$$

Numerical solution of the equations

The momentum equation (1) is solved, along with the equation for radial velocity, using equation (4) to calculate the eddy viscosity terms. The solution scheme adopted by the author is an implicit numerical finite difference scheme, with a simple forward difference for the axial advection term. The solution routine only requires the inversion of a tri-diagonal matrix and it can be performed quickly on a small desk-top computer.

INPUT PARAMETERS

Fixing the value of k_1

The model as described contains one constant, k_1, which is expected to be a property of the shear layer and largely independent of ambient turbulence and the details of the wake producing object. Model predictions were compared with wind tunnel data at low ambient turbulence intensity to fix the value of k_1. The experiment was performed with an axisymmetric simulator suspended on fine wires in the centre of the wind tunnel. Comparisons of wake decay were made at thrust coefficients of 0.79, 0.62 and 0.31, and good agreement was found in all 3 cases when a value of 0.015 was used for k_1. Comparison is shown for one thrust coefficient in Fig. 1, and a comparison is also made with rotor data collected under similar conditions; the agreement illustrates that the constant is equally valid for rotors.

Initial wake parameters

It is envisaged that the solution will be started at a downstream distance of about 2 diameters, i.e. at such a distance that pressure gradients no longer dominate the flow. The wake decay rate is comparatively insensitive to the initial velocity profile and a Gaussian profile $1-U/U_o = D_M \exp(-3.56(r/b)^2)$ will normally be used. Necessary input parameters are the initial velocity deficit D_M and wake width b. Available experimental data on centreline velocity deficits at 2 diameters indicate that simulators may behave slightly different to rotors in this respect, as shown in Fig. 2; the field data from wind turbines also show some departures from the wind tunnel rotor data, perhaps due to averaging effects. An equation based on the wind tunnel studies of rotors is:

$$D_M = C_T - 0.05 - (16\ C_T - 0.5)\ A/1000 \tag{5}$$

where A is the ambient turbulence intensity (%).

The wake width b can then be calculated by conservation of momentum:

$$b = [3.56\ C_T/(8\ D_M\ (1 - 0.5\ D_M))]^{\frac{1}{2}} \tag{6}$$

Methods of estimating K_M

K_M can be described in terms of the normal boundary layer parameters. In the surface layer up to 100 m or so in height,

$$K_M = ku_*\ z/\phi_m(z/L) \tag{7}$$

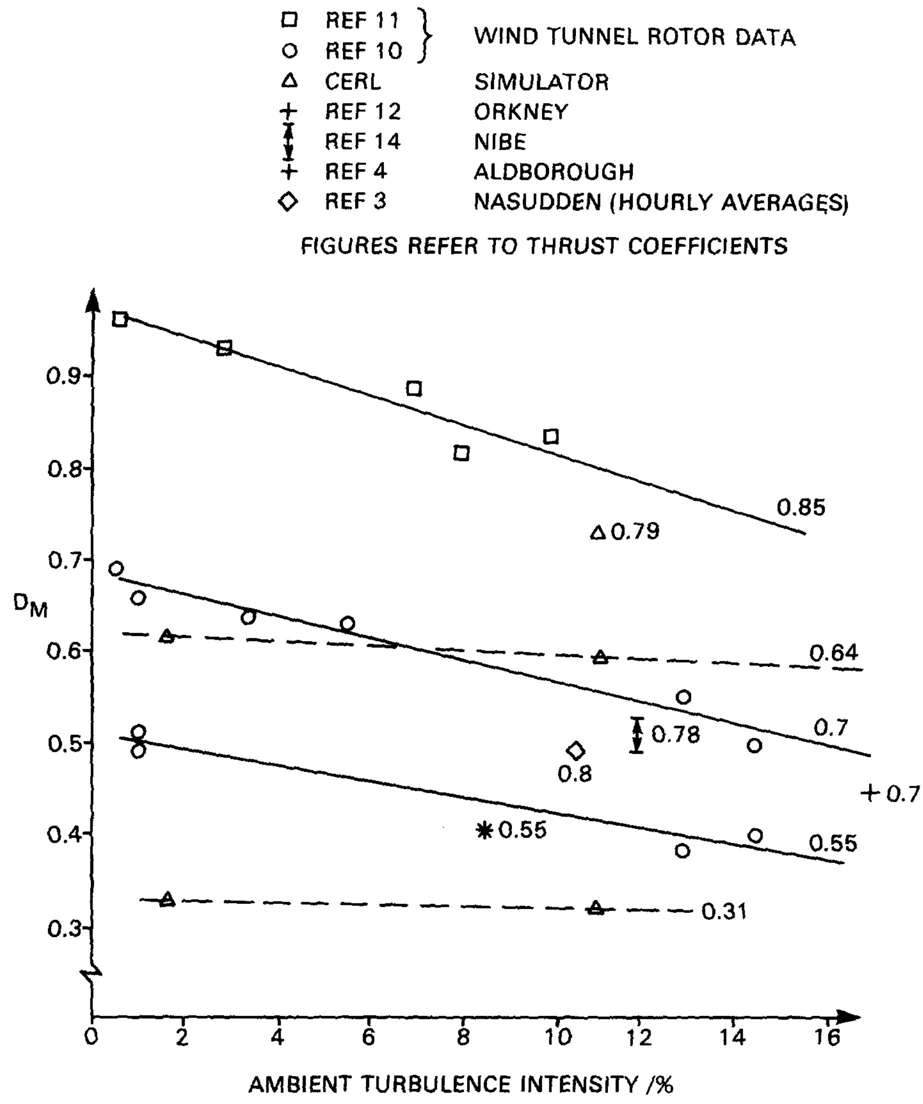

Fig. 1. Initial velocity deficit as a function of ambient turbulence and rotor thrust.

where u_* is the friction velocity and ϕ_m (=1 for neutral conditions) reflects the influence of stability on the mixing processes. If a logarithmic profile is taken as representative of neutral conditions:

$$U = (u_* /k) \ln (z/z_o) \tag{8}$$

this equation links the value of ϵ_a in a simple way to the turbine hub height and the surface roughness. In neutral conditions the complete equation for the eddy viscosity is:

$$\epsilon /(U_H D) = Fk_1 (b/D) (1 - U_c/U_H) + Fk^2/\ln (z_H/z_o) \tag{9}$$

Empirical relationships must be used to determine K_M in the case of stable conditions (see, for example, ref. 5).

THE TREATMENT OF WAKE MEANDERING

The wake model as described above predicts the wake velocity field assuming the externally imposed windfield is stationary with time. In fact this will never be perfectly true for a wind turbine operating in the atmospheric boundary layer, although the approximation becomes reasonable in stable conditions. In neutral and unstable atmospheric conditions the wake will meander relative to an observer fixed on the ground due to fluctuations in wind direction. This will result in the centreline velocity deficit measured by the fixed observer, $\hat{d}$, being less than the stationary value because the measurement point sweeps across a region of the wake profile during the averaging period of the measurement.

An expression has been developed by Ainslie (ref. 7) to correct for wake meandering; this correction gives the deficit measured by an observer as:

$$\hat{d} = d_o [1 + 7.12 (\sigma_\theta x /b)^2]^{-\frac{1}{2}} \tag{10}$$

where d_o is the uncorrected centreline deficit and σ_θ is the standard deviation of wind direction fluctuations, measured over the same averaging time as the wake measurements (but not including very short timescale fluctuations).

A particular field experiment may include sufficient background meteorological data to enable σ_θ to be estimated. However, for predictive purposes a simple estimate is given by the approximate relation:

$$\sigma_\theta = \sigma_v/U \tag{11}$$

Since estimates of σ_v are available for neutral conditions as a function of surface roughness and height (ref. 8, for instance) a table of approximate values of σ_θ can be drawn up (Table 1). These are the values used for the model predictions in the next section.

MODEL PREDICTIONS

The predictions of the wake model are shown in figs. 3 and 4. The calculations are for neutral stability conditions, a wind turbine hub height of around 50 m and surface roughness of 0.05 m (i.e. gentle grassy farmland). Figure 3 shows the wake decay for various thrust coefficient values without any meandering correction; the results thus correspond to a near-instantaneous profile, and can be compared with data from wind tunnel experiments undertaken using a simulated atmospheric boundary layer (which does not include the long-timescale meandering effects). The agreement is very encouraging.

Figure 4 shows the wake decay for various thrust coefficient values, but including the effects of wake meandering on the averaged wake deficits. These calculations therefore correspond to the wake deficits measured in field experiments, and available experimental data is included in Fig. 4. Again, the overall agreement is very good, and it supports the use of a wake meandering correction. Detailed comparisons on a case-by-case basis requires good quality information on the meteorological conditions which is not always available. The indication of the comparison in Fig. 4, however, is that the wake model as formulated, and making use of the input parameters suggested in this paper, provides a very good estimate of the wake velocity deficits which will be encountered in practice.

FUTURE DEVELOPMENTS

The main uncertainties requiring further work relate to the near wake region. The development of the shear stresses (and hence the eddy viscosity) in the near wake is still not well understood, and the empirical equation (3) has had to be used in this model. The initial wake velocity deficits used at 2 diameters come from a parameterisation of available experimental data; this again leads to uncertainty in the predicted deficits in the first 3 or 4 diameters downstream, although the uncertainty will reduce as more field data becomes available.

The model as presented here is essentially a 2-dimensional model, taking advantage of axisymmetric symmetry. It would be interesting to pursue the model in a 3-dimensional formulation, which could then include vertical profiles of windspeed and turbulence.

The wake model describes the flowfield in the wake of a single turbine. For small windfarms the interactions will be between pairs of turbines, and the relevant effects can be calculated directly. However, for groups of more than about 5 turbines the interactions will involve more than 2 turbines, and effects such as the growth of one wake within the flowfield of an upstream wake become important. The method of dealing with this is usually to take single wake

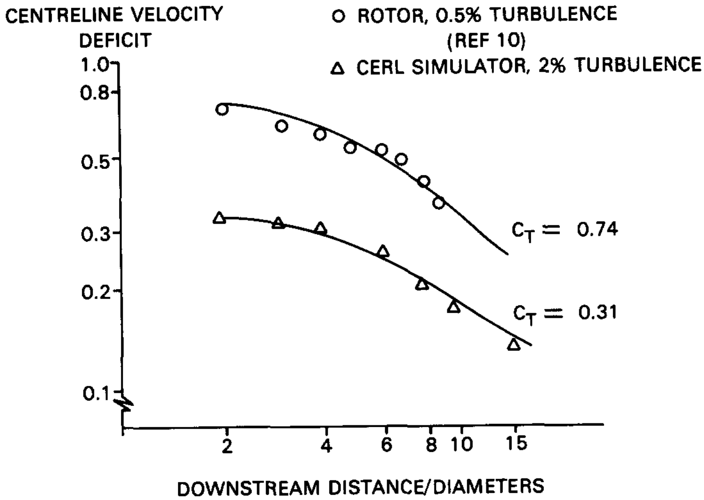

Fig. 2. Comparison of model calculations with wind tunnel data at low ambient turbulence.

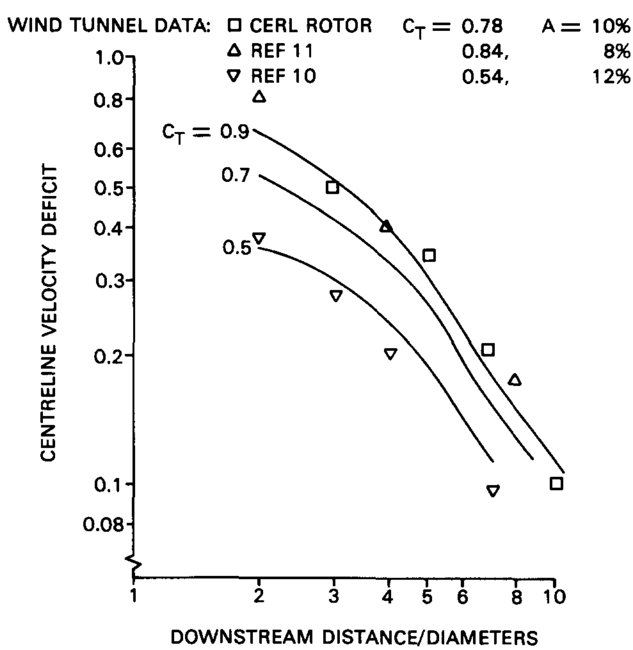

Fig. 3. Wake decay as a function of rotor thrust for a typical neutral boundary layer (no meandering correction). Calculations for z_H = 50 m, z_o = 0.05 m, $K_M / (U_H D)$ = 0.023, A = 14%.

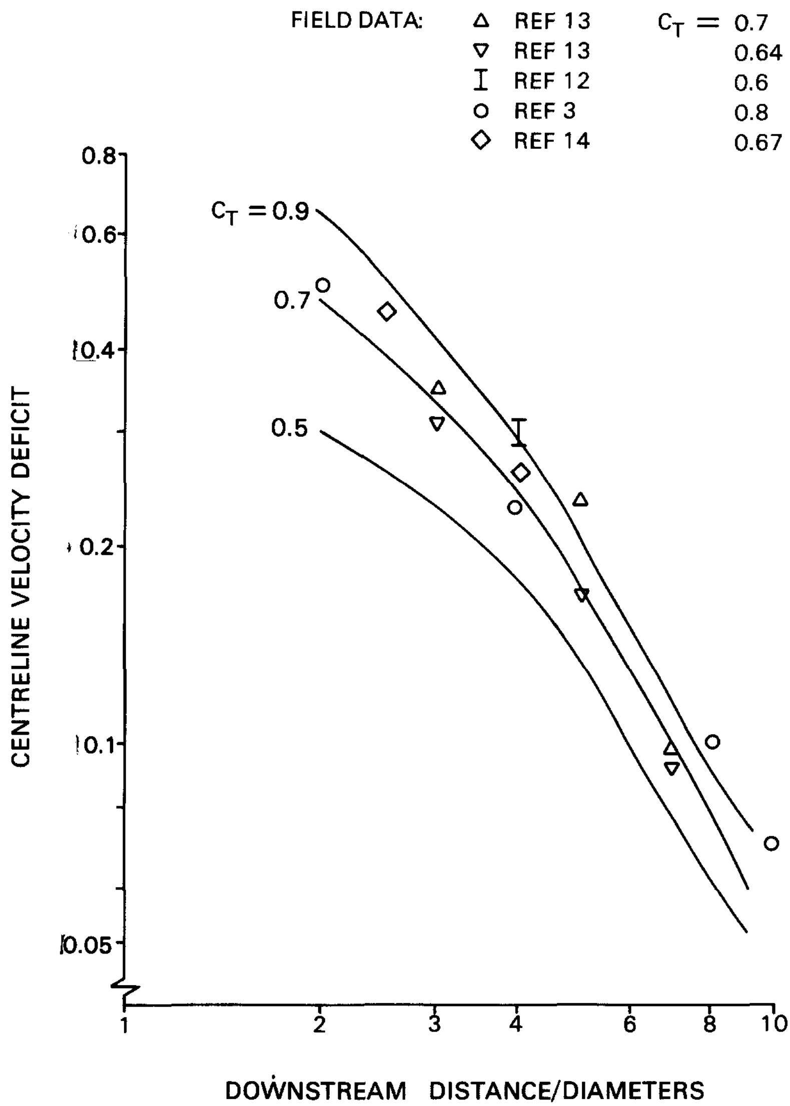

Fig. 4. Wake decay as a function of rotor thrust for a typical neutral boundary layer (including meandering correction). Parameters as for Fig 3, with σ_θ = 0.1.

characteristics as shown in figs. 3 and 4 and to superpose the wake velocity fields. This will inevitably give rise to inaccuracies (ref. 9) -for instance the higher turbulence within the windfarm due to upstream wakes is not often taken into account. What limited data is available on the behaviour of overlapping wakes indicates that the superposition assumption could be seriously in error. Clearly this is an area where much further work is needed.

CONCLUSIONS

A single wake model has been presented which aims to calculate the wake velocity field behind a wind turbine taking account of all relevant meteorological influences. Recommendations are made as to the appropriate input parameters. Comparison with available experimental data shows good agreement with both wind tunnel studies and field data, provided the effects of wake meandering are modelled in the latter case.

The model is comparatively simple and the equations can be solved speedily on a desk-top computer. It is, therefore, suggested that the model can be used to provide reliable estimates of wake deficits for use in the planning and designing of windfarms.

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TABLE 1

Wind direction fluctuation values, neutral stability

Height	σ_θ		
	Zo = 0.1 Rough farmland	Zo = 0.05 Smooth farmland	Zo = 0.01 Level grass plane
10 m	0.16	0.14	0.11
20 m	0.14	0.12	0.10
40 m	0.12	0.11	0.09
60 m	0.11	0.10	0.08
100 m	0.10	0.09	0.07