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The use of the asymptotic acceleration potential method for horizontal axis windturbine rotor aerodynamics

G.J.W. van Bussel

Institute for Windenergy, Delft University of Technology, Stevinweg 1, 2628 CN Delft, The Netherlands

Summary

The acceleration potential method is introduced as a powerful approach to develop computational tools for aerodynamic calculations on horizontal axis windturbine rotors. The basic equations are given as well as a general analytical first-order asymptotic solution. Three computer codes are described in some detail as well as their application. In its simplest appearance the method is equivalent to a lifting line method with a wake relaxing in axial direction. The code in which this model is implemented, PREDICHAT, has been used extensively for calculations of performance and (stationary) blade loads. A more elaborated code, VIAX, has been developed specifically for the calculation of axial velocities in the near wake. Finally an approach is presented for the computation of loads under dynamic inflow conditions. Some results of the various codes are presented.

1. Introduction

For the calculation of aerodynamic loads on a windturbine rotor various different models are used. However, with only a few exceptions, all of them make use of a blade-element momentum approach to calculate the blade loads. More elaborate models use lifting line or lifting surface models. The latter however are often too complicated for practical use as a design tool, and are unsuitable for implementation into larger computational models for the calculation of blade movements, total loads (structural loads included) etc.

Within the Institute for Windenergy a different approach has been developed a number of years ago (van Bussel et al. [1]) which uses a stationary inviscid aerodynamic model based upon the asymptotic acceleration potential theory. It follows the outlines given by van Holten [6] for the situation of a helicopter rotor in forward flight. In the model the rotorblades are represented by spanwise and chordwise pressure distributions. So the approach is equivalent to a lifting surface model. An advantage however is that the asymptotic acceleration method gives a kind of intrinsic possibility to simplify or elaborate the models in specific areas of interest. In its simplest appearance the method is equivalent to a lifting line method with an axially delinearized wake.

A more elaborated code has been developed specifically for the calculation of the near wake. With this code it is possible to calculate e.g. the complete velocity distribution in the wake of the rotor. At the moment a dynamic version of the method is in development which gives the possibility to incorporate the effects of variations in the windspeed, and the dynamic effects caused by pitching of the blades.

2. Basic equations for the acceleration potential method

The two important equations for incompressible inviscid flow are:

$$\nabla V=0 \tag{1}$$

the continuity equation expressing the conservation of mass, and the Euler equation:

$$\rho\frac{\mathrm{D}V}{\mathrm{Dt}}=-\nabla p \tag{2}$$

for the conservation of momentum.

When the fluid is also assumed to be irrotational:

$$\nabla\times V=0 \tag{3}$$

the following equations can be derived:

$$\begin{aligned}
&\frac{\partial u}{\partial t}+\frac{1}{2}\frac{\partial}{\partial x}\,(u^2+v^2+q^2)=-\frac{1}{\rho}\frac{\partial p}{\partial x},\\
&\frac{\partial v}{\partial t}+\frac{1}{2}\frac{\partial}{\partial y}\,(u^2+v^2+q^2)=-\frac{1}{\rho}\frac{\partial p}{\partial y},\\
&\frac{\partial w}{\partial t}+\frac{1}{2}\frac{\partial}{\partial z}\,(u^2+v^2+w^2)=-\frac{1}{\rho}\frac{\partial p}{\partial z}.
\end{aligned} \tag{4}$$

The undisturbed windspeed W is assumed to be parallel to the z-axis. The rotorplane is situated in the x, y plane. If it is assumed that the perturbations in the velocities in x, y and z directions are small with respect to the undisturbed windspeed W then the linearized equation (4) can be written as

$$\frac{\partial u'}{\partial t}+W\frac{\partial u'}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x},$$
$$\frac{\partial v'}{\partial t}+W\frac{\partial v'}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}, \tag{5}$$
$$\frac{\partial w'}{\partial t}+W\frac{\partial w'}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}.$$

Here the prime indicates a velocity perturbation. The first of these equations is now differentiated with respect to x; the second with respect to y and the third with respect to z. Then they are added and finally the continuity equation, Eq. (1) is substituted. This results into:

$$\nabla^2 p=\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}=0. \tag{6}$$

This Laplace equation and Eq. (5) show that the pressure, or more accurately the pressure perturbation is acting as an acceleration potential function.

3. Asymptotic considerations

In finding the solution of Laplace equations asymptotic expansion techniques are often used. The lifting line approach is such an example. Often the span of a lifting surface is large with respect to the chord (e.g. an aircraft wing). At some distance of the wing (in the *far field*) the experienced accelerations will be equivalent to those felt by a line on which the load is concentrated. So in this area the wing can be modelled with a pressure dipole line. Close to the surface the experienced accelerations will be dominated by the chordwise load distribution. So in the *near field* the experienced accelerations will be almost *two-dimensional*, and can be modelled with two-dimensional pressure distributions.

In the method used here both asymptotic approximations are combined with the use of a *common field* term. This third term is needed to cancel the far field term in the vicinity of the lifting surface, as well as the near field term far away from the aerodynamic active surface. In such a way both terms are matched to give an expression which is valid in the whole field.

4. The first-order boundary value problem and its solution

In the application of asymptotics, a choice is always made for the order of magnitude that will be neglected. In order to keep the expressions relatively simple, only the first order problem and its solution are presented here.

The first (obvious) boundary condition is the fact that the pressure perturbations should vanish far away from the aerodynamical active surface:

$$p \to 0 \quad \text{when} \quad x^2+y^2+z^2 \to \infty. \tag{7}$$

The second boundary condition can be described as a linearized kinematic boundary condition: for a flat plate aerofoil located in the y_b plane this yields:

$$\frac{\partial p}{\partial y_b}=0 \tag{8}$$

on the rotorblade. Here a local x_b, y_b, z_b coordinate system is used. The projection of the rotor blade is assumed to be situated in the x_b, z_b plane (the rotor plane), the blade surface is described as a surface function with function values along the y_b axis.

Finally the Kutta–Joukowski condition has to be satisfied,

$$p \to -\infty, \tag{9}$$

at the leading edge of the rotorblade in such a way that

$$\frac{w}{\Omega r+u}=\theta_p(z_b) \tag{10}$$

on the rotor blade.

In Eqs. (8) and (10) the index b indicates a coordinate system related to the blade. The expression $\theta_p(z_b)$ represents the setting angle of the blade, i.e. the local angle with respect to the plane of rotation. For a fast running windturbine rotor and under the assumption that the velocity in the rotorplane is small with respect to the rotational velocity Ωr the expressions (8) and (10) can be written as

$$\frac{\partial p}{\partial z}=0 \tag{11}$$

on the rotorblade, and

$$\frac{w}{\Omega r}=\theta_p(z_b) \tag{12}$$

on the rotor blade.

The general first-order solution for the problem (6), (7) and (11) can be written as

$$\frac{p}{\frac{1}{2}\rho w^2} = -\frac{1}{\pi}\frac{l(z_b)}{\frac{1}{2}\rho W^2 c(z_b)}\frac{\sin\varphi}{\cosh\eta+\cos\varphi}$$

$$+\frac{1}{2\pi}\frac{l(z_b)}{\frac{1}{2}\rho W^2 c(z_b)}\frac{c(z_b)}{r_b}\sin\chi \tag{13}$$

$$+\frac{1}{\pi}\sum_{n=1}^{\infty} A_n P_n^1(\cos\theta)\, Q_n^1(\cosh\nu)\sin\chi.$$

In this equation $l(z_b)$ is the spanwise lift distribution over the blade. The following expression is found:

$$l(z_b) = \tfrac{1}{2}\rho W^2 b[1-(z_b/\tfrac{1}{2}b)^2]^{1/2}\sum_{n=1}^{\infty} A_n P_n^1(z_b/\tfrac{1}{2}b). \tag{14}$$

In (13) the expression $c(z_b)$ yields the chord distribution and b indicates the span of the blade.

The first expression on the right hand side of Eq. (13) is the near field term written in local elliptical coordinates φ and η. The third expression is the far field term, written in prolate spheroidal coordinates θ, ν and χ; and the middle expression in the right hand side is the common field expression written in circular cylinder coordinates z_b, r_b and χ. The P_n^1 and Q_n^1 functions represent associate Legendre functions of the first and second kind. Legendre functions are the *natural* solutions for problems written in prolate spheroidal coordinates. With Eq. (14) it can be seen that close to the blade the common field expression exhibits a singular behaviour (caused by Q_n^1). This behaviour is also found in the far field term, but with the opposite sign. Thus the total expression does not have this essential singularity. The coefficients A_n are not determined at this stage. This is done by application of the final boundary condition (12).

5. Determination of the coefficients

The kinematic boundary condition states that the velocity found at the rotorblade should be such that the particles of air move tangential to the blade surface. Since the pressure distribution is known now the accelerations in the field are also determined, apart from the coefficients A_n. When the path of a particle of air is known, the accelerations could be integrated along it, and the result would be a certain velocity in the rotorplane. However nothing is known a priory with respect to the path, since it is determined by the pressure field. There is however a possibility to linearize the process to fulfill the final boundary condition. If it is assumed that the accelerations found along a straight, unperturbed path, travelled with a constant speed are about equal to the accelerations experienced by the real particle along its real path, then the whole process becomes linear in its coefficients A_n. This means that these coefficients can be determined from a simple set of linear equations representing the (final) kinematic boundary condition.

The situation of a multibladed rotor can now be tackled with a simple approach. The solution (13) for the one bladed rotor can simply be expanded with similar equations for the other blades, all in their own *local* blade coefficients. The determination of the coefficients A_n now implies of course more labour, but is in principle unchanged.

6. Computer codes using the asymptotic acceleration potential method

6.1. *Partial delinearization*

In the practical application of the above described method it turned out to be necessary to imply an axial delinearization in order to obtain physically consistent results. This has directly to do with the normal operational mode of a windturbine. Near the rotorplane, where the pressure fluctuations caused by the blades are most evident, the components u and v can still be considered small with respect to the undisturbed windspeed W. The axial velocity will however be significantly different from the undisturbed windspeed W. For the calculation of stationary situations it turns out to be sufficient to replace the undisturbed travelling speed W along the straight paths with a actual travelling speed in the rotorplane calculated in the first iteration step. When the calculations start using $0.6667W$ in stead of W only two iterations are necessary to obtain convergence in the result.

6.2. *PREDICHAT*

The code to calculate stationary blade loads as well as rotor torque, thrust and performance is named PREDICHAT. In this code the potential calculation according to the above described scheme is followed by a viscous routine. The routine takes (measured) two dimensional airfoil characteristics as a basis, but is able to extrapolate for large (positive and negative) angles of attack, when these are not given in the table. The calculated inflow angle forms the basis on which lift and drag are inter- or extra-polated from the tabulated results.

6.3. *VIAX*

A second code has been developed using the acceleration potential approach, for the calculation of the axial velocities in the near wake of the rotor. In this case the whole procedure is based upon the potential calculation scheme alone. Until now the code has been used primarily for comparison with experimental results from hot wire measurements, with fairly good results (Vermeer et al. [2]). An extension of the code, which also takes into account radial velocity disturbances (the code VIAXRAD) has been developed to a large extent.

6.4. Dynamic inflow code

In order to be able to calculate the effects of variations in the inflow conditions of the rotor, or the effect of blade pitching movements on the aerodynamical loads a procedure has been developed which makes use of the code PREDICHAT. Application in situations with a stepwise variation in either windspeed or pitch angle has taken place already. The procedure first calculated the experienced accelerations of particles of air arriving at the rotor blade for the initial and the ultimate situation, using the potential part of the code PREDICHAT. Then an interpolation scheme is applied, which combines *initial accelerations* with *ultimate accelerations* depending upon the time difference between the change in situation and the moment on which the loads are requested. In the end the PREDICHAT viscous routine is applied to modify the potential dynamical result.

6.5. Present developments

At the moment the effects are studied of a better representation of the accelerations found along the particles path. Although the trajectory is still kept straight, the accelerations found in the first iteration are integrated leading to a fluctuating velocity in the second iteration. With respect to the results found from PREDICHAT the effect from the inclusion of this modification are small. It is however thought that implementation into VIAX will improve the accuracy of the calculated (axial) velocities in the wake. Furthermore it will improve the calculations carried out with respect to dynamic inflow situations, because interpolation between initial end ultimate state will no longer be necessary. In fact no ultimate state or ultimate accelerations are needed anymore because the code will be able to develop the dynamic loads from the initial stationary situation and its dynamic development.

7. Results

Figures 1, 2 and 3 show some results from calculations with PREDICHAT, VIAX and the dynamic inflow scheme. In general it can be said that the torque, axial force and power calculated with PREDICHAT show reasonable good agreement with other predictions (from both blade element momentum theory models as from more elaborated models), see van Bussel [3]. Based upon these comparisons no real preference can be given to any physical model. Taking a more detailed view however shows that PREDICHAT, as well as other elaborate models, are able to determine realistic distributions of blade loads, angle of incidence distributions etc. Figure 1 shows an example taken from van Bussel [4] in which the results from several experimental approaches for open air measurements are compared. Also the predictions with PROPCODE (Tangler [5]) and PREDICHAT (van Bussel [3]) are depicted.

The comparison of the VIAX predictions show reasonable good agreement

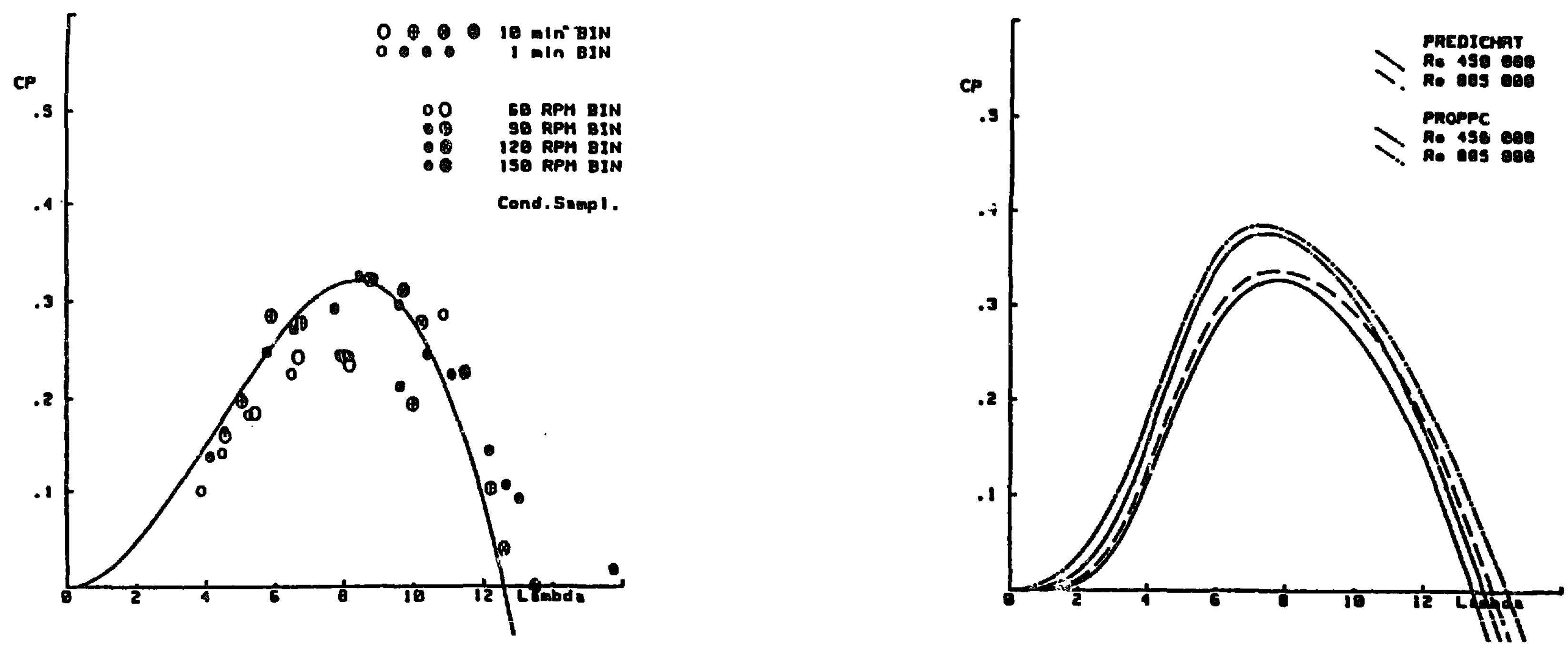

Fig. 1. C_p–λ results from three different measurement methods for open air experiments (a), and theoretical predictions (b) from PROPCODE and PREDICHAT.

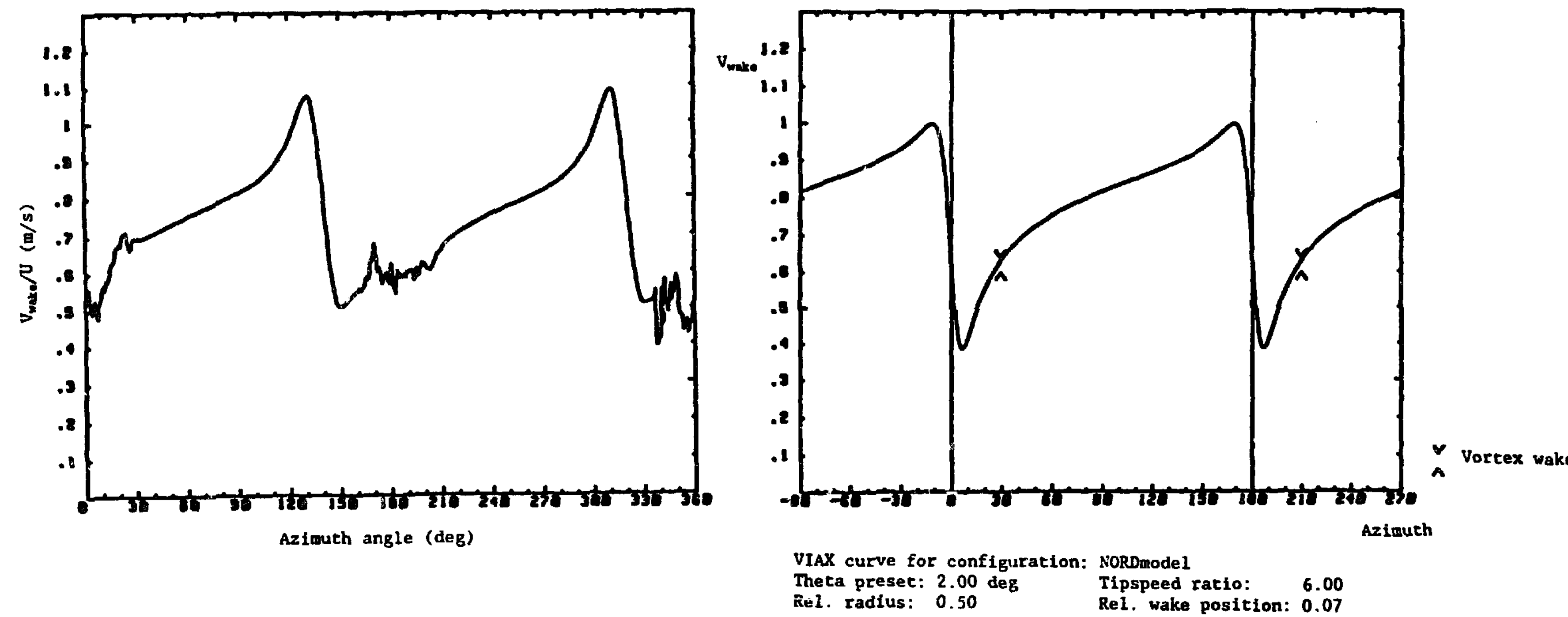

Fig. 2. Measured (a) and calculated axial velocities (b) in the very near wake of a model rotor. Ignore the shift in azimuthal position.

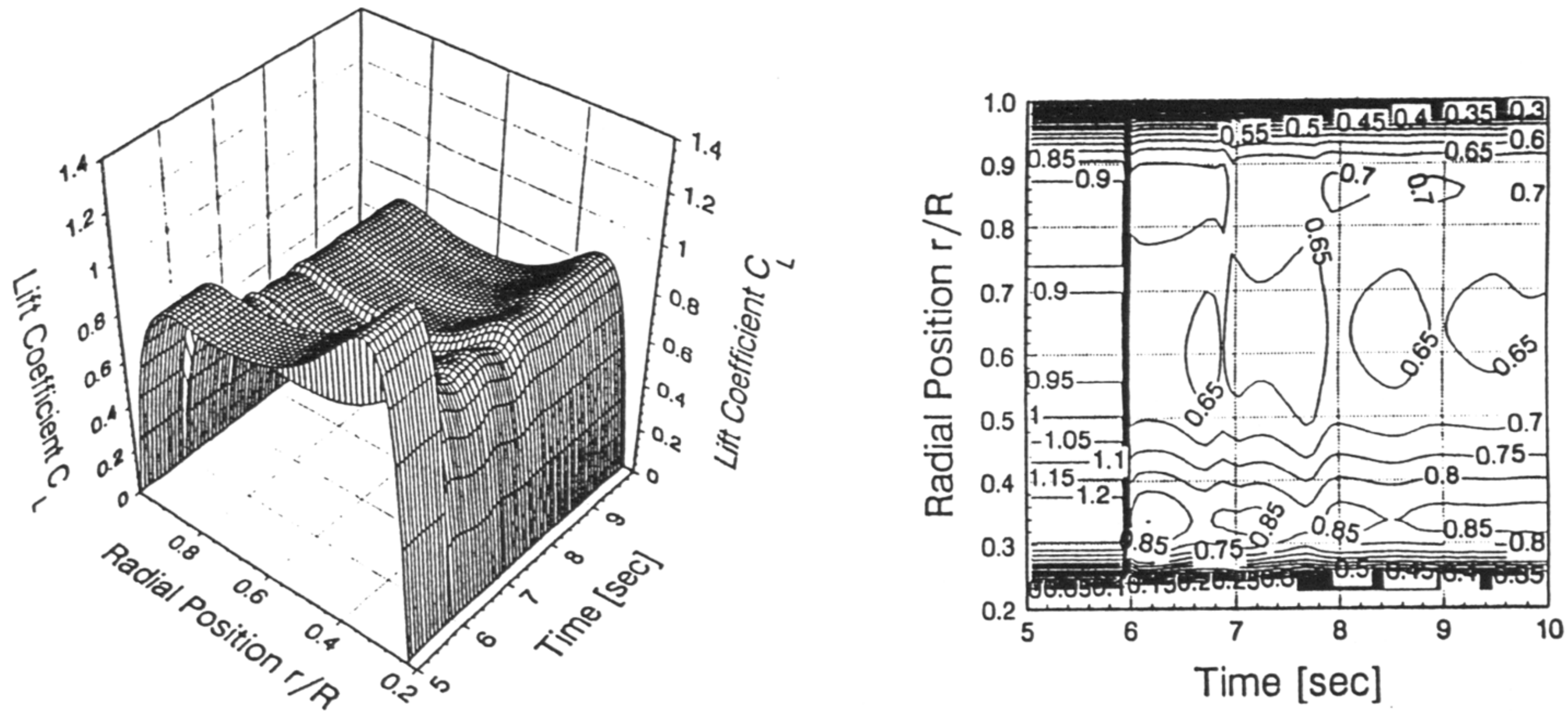

Fig. 3. Calculated variations in blade loads when a stepwise variation windspeed is assumed at $T=6$. The contour plot of the lift coefficient (b) shows clearly the effect of the shed vorticity in the wake. Blade passage frequency is 1.1 Hz.

with the experimental results. Since VIAX uses potential (inviscid) theory the *region of turbulence* in the helical sheet of vorticity behind the rotor is not predicted. Its position however is determined quite accurately. Figure 2 is taken from Vermeer et al. [2].

The calculations for dynamic inflow have not yet been compared with experimental results. The agreement with the predictions of other theoretical models was good. From the calculated results it can be seen that much detail can be obtained. In the figures the effect of shed vorticity in the wake upon the instantaneous blade load distribution can be distinguished clearly. Figure 3 is produced within the framework of a CEC-Joule project.

Al the calculations with the various codes have been carried out on PC-type computers (286, 386 and 486 processors). The amount of calculation time depends upon the application. PREDICHAT calculations run very fast (order of magnitude of some seconds) on these type of machines. The other applications take more time but in general the results are available within minutes.

8. Conclusions

The acceleration potential method give good possibilities for the development of relatively fast computer codes for several applications.

PREDICHAT overall results are comparable to blade element momentum theory method results. Where PREDICHAT is probably more time consuming, it gives also more detail which is sometimes relevant.

VIAX calculations could also be carried out with a lifting line method. The computational effort as well as computing time needed is probably less. Comparison with detailed measurements in the near wake of a model rotor, also carried out at Delft University of Technology, is promising.

Dynamic inflow calculations show the capabilities in instationary flow. The corresponding lifting line approach, and certainly lifting surface methods require much more labour and computer force.

The fact that a common (acceleration potential) basis for the various codes is used has proven to be very helpful.

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