A prescribed wake aerodynamic model for vertical axis wind turbines

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A new aerodynamic model for the prediction of vertical axis wind turbine performance is introduced. The model is fully three dimensional and is derived from consideration of both momentum and vortex theories. In the calculation process the turbine wake is modelled by a series of shed and trailing vortices. The overall shape of the wake, however, is determined from momentum theory. Comparison is made between the new model and a free vortex method. Although the accuracy levels of the two techniques are equivalent, the prescribed wake model is more than two orders of magnitude faster. The prescribed wake model is also shown to compare well with field data. Finally, the future development of the model is discussed.

NOTATION

C_n	blade normal force coefficient
C_p	turbine power coefficient
C_t	blade tangential force coefficient
U_b	velocity at turbine blade (m/s)
U_f	incoming velocity (m/s)
U_i	induced velocity (m/s)
U_w	velocity in turbine wake (m/s)
U_∞	freestream velocity (m/s)
α	blade effective incidence

Subscripts

d	downwind
f	far wake
u	upwind
2	second wake cycle

1 INTRODUCTION

When calculating the flow field around a vertical axis wind turbine, many parameters have to be considered (**1–4**). One particularly potent feature of the flow, however, is the presence of the trailing wake generated by the rotation of the blades. The manner in which this wake is modelled can, therefore, greatly influence the quality of prediction obtained from an aerodynamic model.

Aerodynamic prediction methods for vertical axis wind turbines generally fall into the categories of either momentum (**5–7**) or vortex models (**5**, **8**, **9**). Of these, momentum models present the simplest solution methodologies and are computationally quicker than their vortex counterparts. Several types of momentum model exist, but all calculate the induced axial velocity at each blade position by equating the time-averaged force on the turbine blades to the momentum flux through a prescribed number of streamtubes. Although some momentum models include the effects of streamtube expansion, the upstream influence of the turbine wake is not inherently modelled. As a result, the induced effect of the wake is neglected and estimates of instantaneous blade incidence tend to be inaccurate. This feature seriously limits the application of these methods in cases where calculated airloads provide inputs to structural analysis schemes.

Prediction schemes which are based on vortex theory represent the turbine wake by a series of vorticity elements whose strengths are dictated by the variation in blade loading with time. The influence that each vortex filament has on the entire flow field is then calculated according to the Biot–Savart law. In this way, the influence of the turbine wake is implicit in the calculation scheme. This, generally, results in more accurate estimates of blade loadings but incurs a substantial increase in computation time.

Several different vortex methods have been used to predict the flow around a vertical axis wind turbine. Of these, the free vortex method due to Strickland *et al.* (**8**) is possibly the most comprehensive. In it, a three-dimensional wake consisting of shed and trailing vortices is systematically built up as the turbine rotates. This wake is unconstrained and so is free to distort under its own influence. The wake shape that develops is very representative of the real flow (Fig. 1) and so the technique produces accurate predictions. The main drawback of this method is the substantial amount of computational time required to achieve a solution. The present investigation centres on the development of an algorithm which displays accuracy levels akin to the free vortex method but at a fraction of the computational cost.

As demonstrated by Wilson and Walker (**9**), it is possible to combine vortex and momentum theories to produce a hybrid model which displays the favourable features of both types of technique. It is, however, possible to extend this approach towards the development of a method that reproduces the accuracy levels of the free vortex model. This new, prescribed wake, model (**10**) is around two orders of magnitude faster than the original free vortex method.

2 GENERAL DESCRIPTION OF MODEL

The prescribed wake model uses a combination of vortex and momentum theory to obtain a solution for the flow around a vertical axis wind turbine. In the

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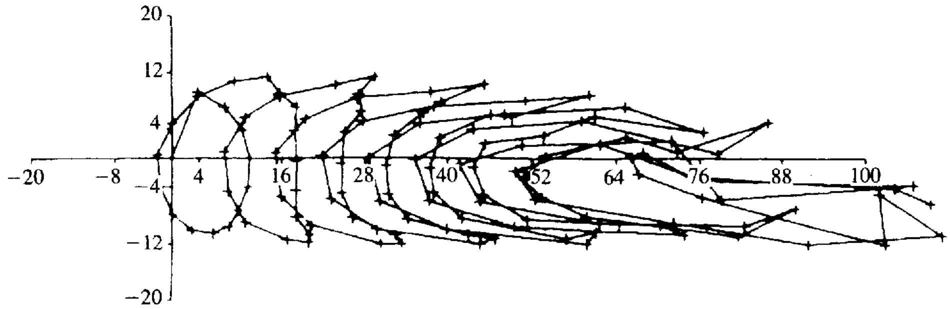

Fig. 1 Typical wake shape produced by the free vortex method

technique vortex elements, corresponding to the spanwise azimuthal blade loading variations, are shed from the turbine and follow a path prescribed by consideration of momentum theory. The induced effect that the wake has on the blade loadings is then calculated according to vortex theory.

There are two significant differences between the new approach and that of the free vortex technique. Firstly, the convection of the vortex system is prescribed rather than calculated and, secondly, the full wake structure is generated at the beginning of the calculation and not by a gradual time-stepping process. This removes the major computational effort associated with the free vortex approach and results in a much faster prediction scheme.

In the free vortex method the turbine wake is generated over a given time period by introducing trailing and shed vortices at each time step. In addition, the manner in which the wake develops is influenced by every vortex filament in the wake. A computational overhead is, therefore, incurred by the necessity to calculate the influence of the wake on itself and on the turbine blade loadings at every calculation time step. Another consequence of this technique is that the vortex filament strengths on the initial wake elements, which were calculated when little or no wake was present, will be unrepresentative of the steady state solution. It is, therefore, necessary to produce a final wake size which ensures that influence of these elements on the blade loadings is negligible. As a result, larger than necessary wakes are produced.

Since the wake shape is prescribed in the new method, the influence that the wake filaments have on each other is not calculated directly. Instead, it is considered that accurate prescription of the wake shape should adequately account for this effect. This form of wake modelling also removes the requirement to build up the wake in a step-by-step manner and so it is possible to prescribe a wake consisting of many cycles at the outset of the calculation. In addition, the existence of unrepresentative vortex filament strengths at the rear of the wake has been removed by updating all filament strengths in the wake as the calculation progresses. By constantly updating wake elements in this way, it has been possible to accelerate convergence and thus significantly reduce the computation time associated with the model.

At some tip-speed ratios, both techniques are subject to problems associated with vortex–vortex and blade–vortex interactions. In the free vortex method, the high induced velocities associated with such an interaction can significantly, and unrealistically, distort the wake shape. By prescribing the wake shape in the new technique, this aspect of the interaction problem is effectively removed and convergence is facilitated.

2.1 Vortex modelling

The vortex method which forms the core of the prescribed wake model is illustrated in Fig. 2. In it, the spanwise blade loading distribution is approximated by a series of bound vortex segments of constant strength. In this way, a vorticity imbalance is created between adjoining bound vortex segments. This is resolved by the creation of trailing vortex filaments whose strengths are defined by the difference in vorticity from one bound vortex element to the next. The strengths of the vortex filaments trailing from the blade tips are simply equivalent to those of the corresponding bound vorticity segments.

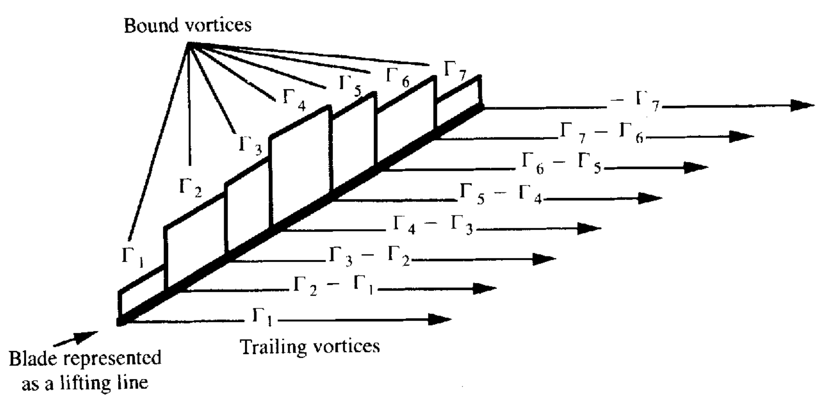

Fig. 2 Basic vortex model of turbine blade

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While this approach is adequate for a finite wing fixed at a given incidence to an oncoming flow, the change of turbine blade incidence with time necessitates inclusion of the influence of the associated variation of blade bound vorticity. This is achieved by considering the blade incidence variation in discrete time steps and producing shed vortices, equivalent to the resulting change in circulation, from each blade segment at every time step. In this way, a lattice of shed and trailing vortex elements is generated behind the turbine blade. It should be noted that constructing the model in this manner ensures that the Helmholtz continuity principle is satisfied at each lattice node point.

As indicated above, the vortex systems that trail from the turbine blade are constrained to follow a predetermined path derived from momentum theory. Thus, in the initial stages of the calculation procedure, although the wake shape is known, the strengths of the vortex elements in the wake and on the blades must be calculated. It is therefore necessary to adopt an iterative procedure in which the wake shape is fixed and the values of shed and trailing vorticity in the wake are systematically adjusted to correspond to the variations in circulation and spanwise bound vorticity on the turbine blades. The starting values for the iteration process are determined from the loadings associated with the variation of blade geometric incidence. The particular scheme adopted involves considering the induced effect of the wake at a given blade position, via the Biot–Savart relationship, and then updating the corresponding shed and trailing vorticity terms before moving to the next blade position. By only changing the specific vorticity values that correspond to the azimuthal position under consideration, the remainder of the vortex wake effectively damps the iteration process and so enhances the convergence characteristics of the system.

2.2 Momentum modelling

The purpose of the momentum model in the prescribed wake method is to provide initial estimates of convection velocities at each azimuthal position on the turbine. These velocities are then used to construct a basic wake shape which, in turn, is used to provide more accurate estimates of blade loadings. A further application of momentum theory is then employed to provide a final wake shape. It is therefore essential that the momentum model used is sufficiently accurate to permit a realistic estimate of the actual wake shape to be generated.

Momentum models of vertical axis wind turbines fall into the categories of single streamtube, multiple streamtube, double streamtube and double-multiple streamtube. These techniques offer various levels of sophistication, but all operate on the principle that the flow past the wind turbine can be modelled by equating the change in streamwise momentum through the turbine to the streamwise forces on the turbine blades. All of these representations of momentum theory were considered for the prescribed wake method, but it was found that wake shapes generated on the basis of the simpler momentum models were not sufficiently realistic at all tip-speed ratios. For this reason, a double-multiple streamtube model was ultimately incorporated into the prescribed wake scheme. The main advantage that this type of momentum model has over the simpler methods is its ability to differentiate between the induced velocities on the upwind and downwind passes of the turbine blade. This feature was found to be crucial when vortex convection in the near-wake region is being considered.

In the double-multiple streamtube model, the turbine is represented by two actuator discs in tandem (Fig. 3). The flow through these two discs is subdivided into a series of streamtubes, in both the horizontal and vertical directions, wherein the changes in streamwise momentum are equated to the forces on the turbine blades. Induced velocities on the upwind and downwind passes of the blades are calculated by considering each actuator disc in a consecutive manner. In order to obtain a closed solution, it is necessary to make some assumption concerning the nature of the input flow condition at the second actuator disc. The basic model assumes that the wake from the first disc is fully developed prior to consideration of the second disc.

2.3 Initial wake geometry

To develop a first estimate of the wake shape associated with a vertical axis wind turbine, it is necessary to model the rate of convection of vorticity elements downstream. In the prescribed wake method, a first approximation to the induced velocities at each blade position is provided by the double-multiple streamtube model. In the method, a simple wake geometry is derived by convecting vortex elements downstream at the resultant of the incoming and the calculated induced streamwise velocities. Although this approach is not strictly in keeping with momentum theory, since the calculated wake velocity is not used, the estimated wake shape compares favourably with that of the free vortex model.

The predicted variation of turbine power coefficient versus tip-speed ratio from this basic wake model compares well with that predicted by the free wake method. The corresponding variations in instantaneous blade

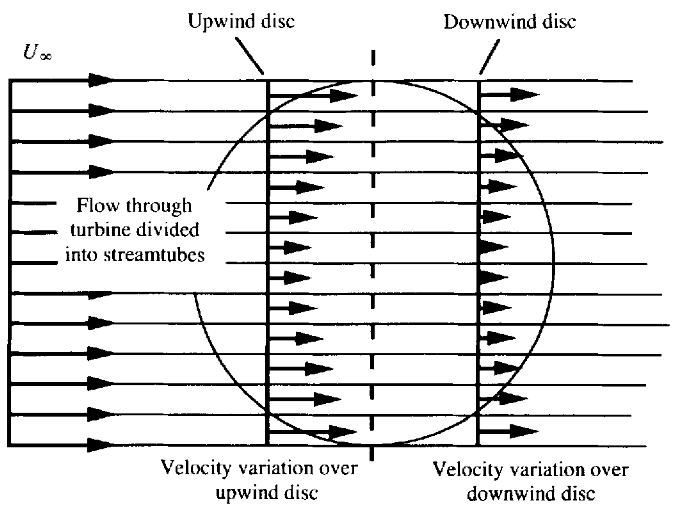

Fig. 3 Double-multiple streamtube model of vertical axis wind turbine

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loadings, however, are not modelled quite as well. A detailed comparison with the free-wake method indicates that the poor loading predictions of this wake model stem from its inability to account for near and far wake convection velocities. In particular, the near wake from the upstream blade pass, which often lies within the turbine swept perimeter and has a strong influence on the induced velocity field at the turbine, is badly modelled. This problem is alleviated by the next stage of the wake prescription process which consists of modifications to the above wake geometry based on momentum considerations and observations made by extracting pertinent information from the free vortex method.

2.4 Streamwise wake geometry modifications

Momentum theory disctates that the wake velocity of a vertical axis wind turbine is related to the velocity at a corresponding blade position by

$$U_w = 2U_b - U_f \tag{1}$$

where the incoming velocity U_f may depend on whether the upstream or downstream blade pass is being considered. This equation represents the classical result that one-half of the velocity defect occurs upstream of the actuator disc and one-half downstream. Application of equation (1), via double-disc momentum theory, gives the wake velocity behind the upwind blade in a given streamtube as

$$U_{w_u} = 2U_{b_u} - U_\infty \tag{2}$$

Introducing the velocity induced at the turbine blade as

$$U_{i_u} = U_{b_u} - U_\infty \tag{3}$$

equation (2) becomes

$$U_{w_u} = U_\infty + 2U_{i_u} \tag{4}$$

Following equation (1) and assuming that the incoming velocity to the downstream blade pass is the wake convection velocity from the upstream pass, the wake convection speed behind a downstream blade is given by

$$U_{w_d} = 2U_{b_d} - U_{w_u} \tag{5}$$

If the velocity induced on the downstream blade is referenced to the freestream velocity, as for the upstream blade in equation (3), combining equations (3), (4) and (5) yields

$$U_{w_d} = U_\infty + 2U_{i_d} - 2U_{i_u} \tag{6}$$

Equations (4) and (6), therefore, provide relationships between the induced velocities at the blades and the wake convection velocities.

Actual convection velocities, corresponding to a series of wake cycles, have been extracted from the predictions of the free vortex model. The convection velo-

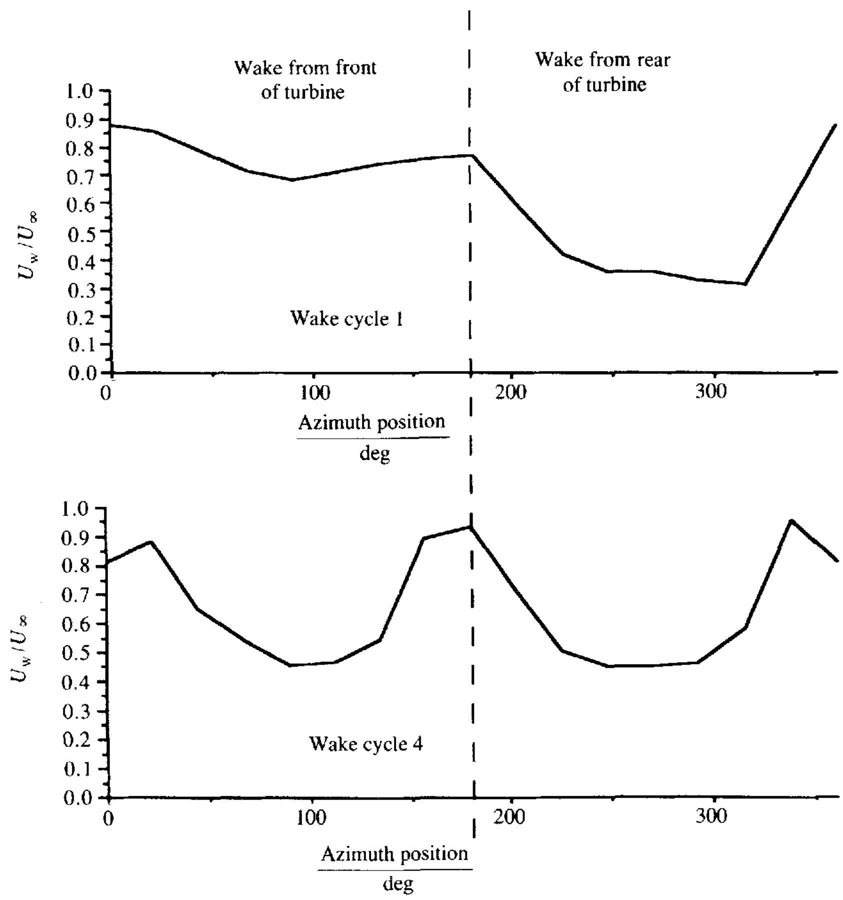

Fig. 4 Mid-span wake convection velocities predicted by the free vortex method at tip-speed ratio 4

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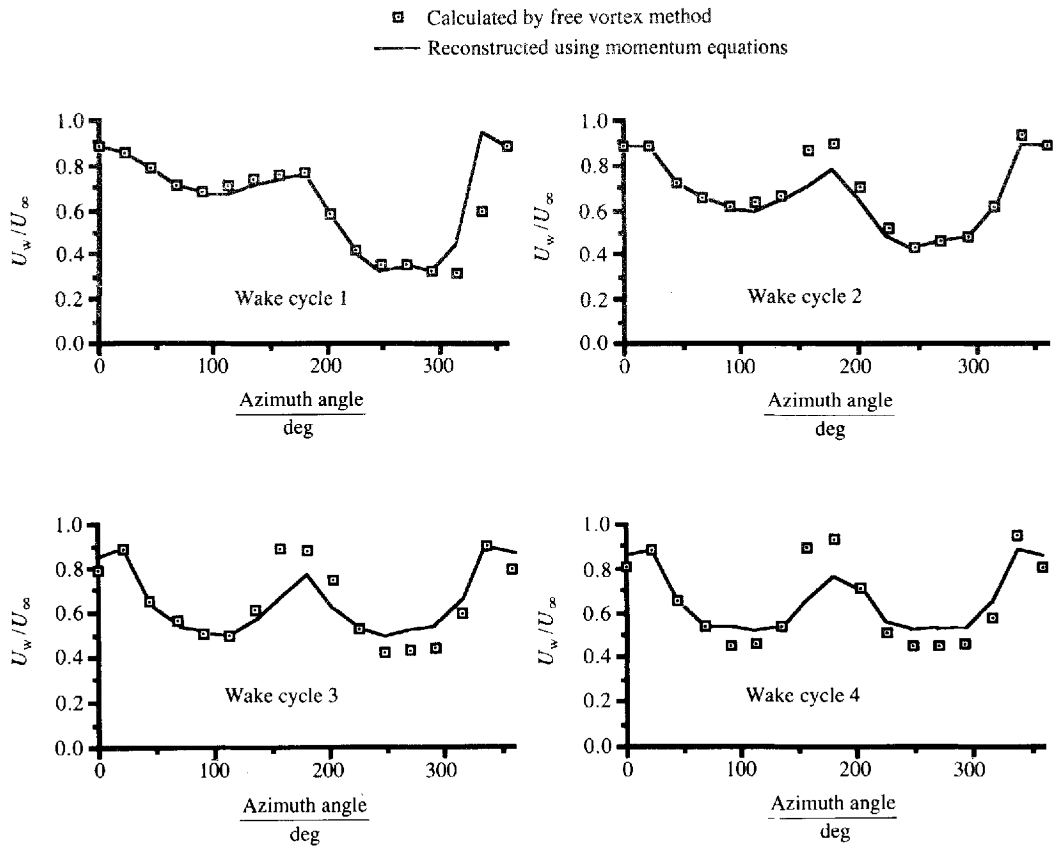

Fig. 5 Wake vortex velocities calculated by the free vortex method and momentum equations at the blade mid-span

cities in the first and fourth wake cycles, at tip-speed ratio 4, are presented in Fig. 4 for the mid-span position. From this figure it is apparent that, as predicted above, different convection velocities corresponding to the upstream and downstream portions of the wake cycle do exist in the near wake region. This condition is, however, not sustained into the far wake where the two halves of the wake cycle are equivalent.

As a consequence of the spanwise loading variations on the turbine blades, there is a slight variation in the magnitudes of the wake convection velocities towards the blade tips. This variation is accompanied by changes in blade loading but the general convection velocity behaviour exhibited in Fig. 4 is observed in all cases.

A detailed study of the relationship between induced velocities predicted at the blades and the convection velocities in the near wake was undertaken to establish the level of agreement between momentum theory and the free vortex model. In this study, the induced velocities predicted at the blades by the free vortex method were applied to equations (4) and (6) to produce estimates of the wake convection velocities. These convection velocities were then compared to those actually predicted by the free-vortex method. It was found that substantial agreement existed between the velocities predicted for the first cycle of the vortex model and those obtained from the momentum theory. In particular, the upstream wake convection velocities were almost identical for the range of tip-speed ratios considered. At tip-speed ratios greater than four, however, equation (6) produced slightly lower convection speeds than those measured from the vortex method.

As indicated above, the basic momentum analysis used to derive equation (6) assumes full expansion of the wake from the upstream pass prior to calculation of the downstream losses. This assumption, therefore, does not include any upstream influence of the downstream blade and its associated vortex system. It is apparent, from the free vortex method, that the wake structure is very concentrated near the turbine at high tip-speed ratios and thus does exert a strong influence on upstream conditions. The momentum theory above was therefore modified to include this effect. A satisfactory expression for the incoming velocity to the downstream blade was found to be

$$U_{f_d} = U_{b_d} - U_{i_u} \tag{7}$$

In this way, the incoming velocity to the downstream pass is influenced by the velocity at the downstream blade and the velocity induced on the upstream pass. The downstream wake convection velocity then becomes

$$U_{w_d} = U_\infty + U_{i_d} + U_{i_u} \tag{8}$$

Substitution of the induced blade velocities, predicted by the free vortex model, into equations (4) and (8) results in an accurate reproduction of the first vortex wake cycle.

The process whereby the wake convection velocities from the upstream and downstream blade passes equalize in the far wake was also examined. In the free vortex method, a uniform convection velocity distribution, equivalent to the average of the upstream and downstream components of the first wake cycle, is established

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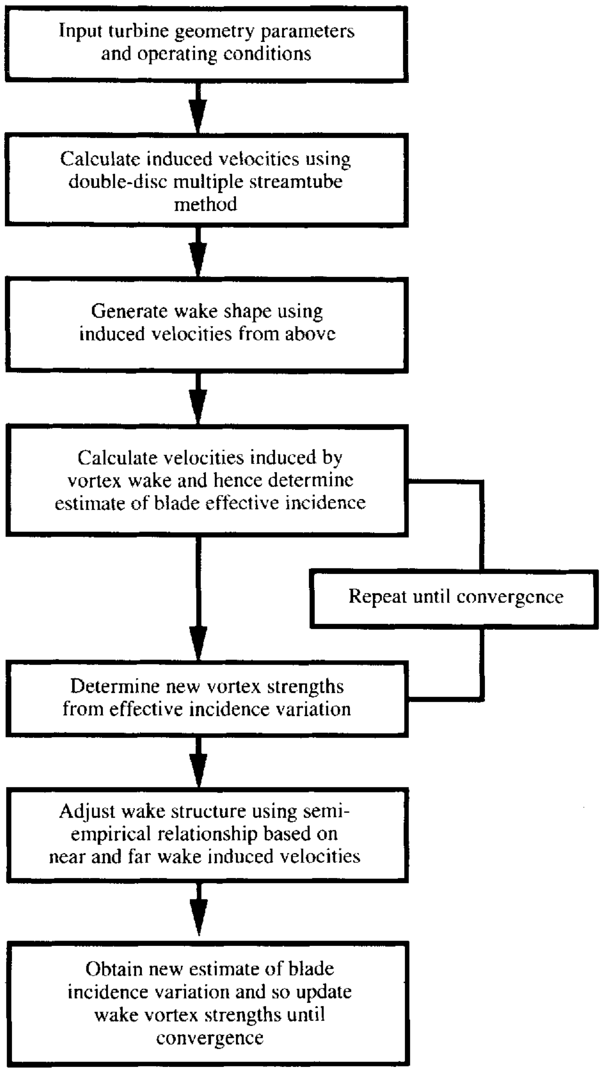

Fig. 6 Prescribed wake calculation scheme

by the third wake cycle. This equalization process appears virtually independent of the tip-speed ratio. The convection speed of the wake from the third cycle onwards can thus be related to the velocities in the near wake by

$$U_{\mathrm{wf}} = \frac{U_{\mathrm{wu}} + U_{\mathrm{wd}}}{2} \tag{9}$$

The second wake cycle is then approximated by assuming a linear variation in velocity between the first and third cycles. The velocity of the section corresponding to the upstream blade pass is given by

$$U_{\mathrm{wu2}} = \frac{U_{\mathrm{wu}} + U_{\mathrm{wf}}}{2} \tag{10}$$

and that of the downstream section by

$$U_{\mathrm{wd2}} = \frac{U_{\mathrm{wd}} + U_{\mathrm{wf}}}{2} \tag{11}$$

The mid-span wake convection velocities calculated via equations (4), (8), (9), (10) and (11), based on the blade induced velocities from the free vortex method, are compared at tip-speed ratio 4 with those obtained directly from the free vortex method in Fig. 5. As is apparent from the figure, the above equations permit accurate reconstruction of the wake shape once the induced velocities on the turbine blades are known. Also, on application of the above equations, the variation of blade loading along the span produces spanwise variations in wake convection velocity in line with the free vortex method. In the prescribed wake method, the estimate of blade induced velocities resulting from the basic wake shape described in Section 2.3 above is used as the basis for the modification process.

2.5 Wake expansion

The reduction in streamwise velocity of the near wake immediately behind the turbine blades results in a local expansion of the wake in the cross-stream direction. The vortex structure generated by the free wake model initially exhibits this behaviour and then contracts as the freestream energizes the slower moving air in the wake. Using the free vortex model it was possible to establish that, while the initial wake expansion was very significant to the predicted performance of the turbine, the subsequent wake contraction had little influence on the result obtained. This was particularly true at high tip-speed ratios where a substantial portion of the near wake structure, generated by the upstream blade pass, resided within the swept volume of the turbine. The proximity of the vortex filaments to the blades, in these cases, meant that any change in the near wake structure would have a significant influence on the result obtained.

The rate of expansion of the first wake cycle was found to be equivalent to the average cross-stream velocity induced at the turbine. It was, therefore, possible to include the effects of wake expansion into the first cycle of the prescribed wake model by consideration of the induced velocity field at the turbine blades. It was found that this provided a reasonable approximation to the behaviour of the free vortex model at low to moderate tip-speed ratios where wake expansion was only apparent over the first cycle. At high tip-speed ratios, however, the free vortex model exhibited wake expansion into the second cycle. As yet, this has not been accurately modelled and, as a result, may influence the quality of prediction at high tip-speed ratios. Thus, in the current method, the first wake cycle incorporates expansion and the other cycles are fixed and do not expand or contract.

2.6 The prescribed wake model

In the prescribed wake method, the technique outlined in Section 2.3 above is used to generate a simple wake geometry from which estimates of the velocities at the turbine blades are obtained. These estimates represent a significant improvement on the double-multiple streamtube momentum model, but still fall short of the accuracy obtained via the free vortex method. A second level modification is, therefore, applied to the wake structure. In this process, the calculated estimates of induced velocities at the blades are used, via the expressions derived

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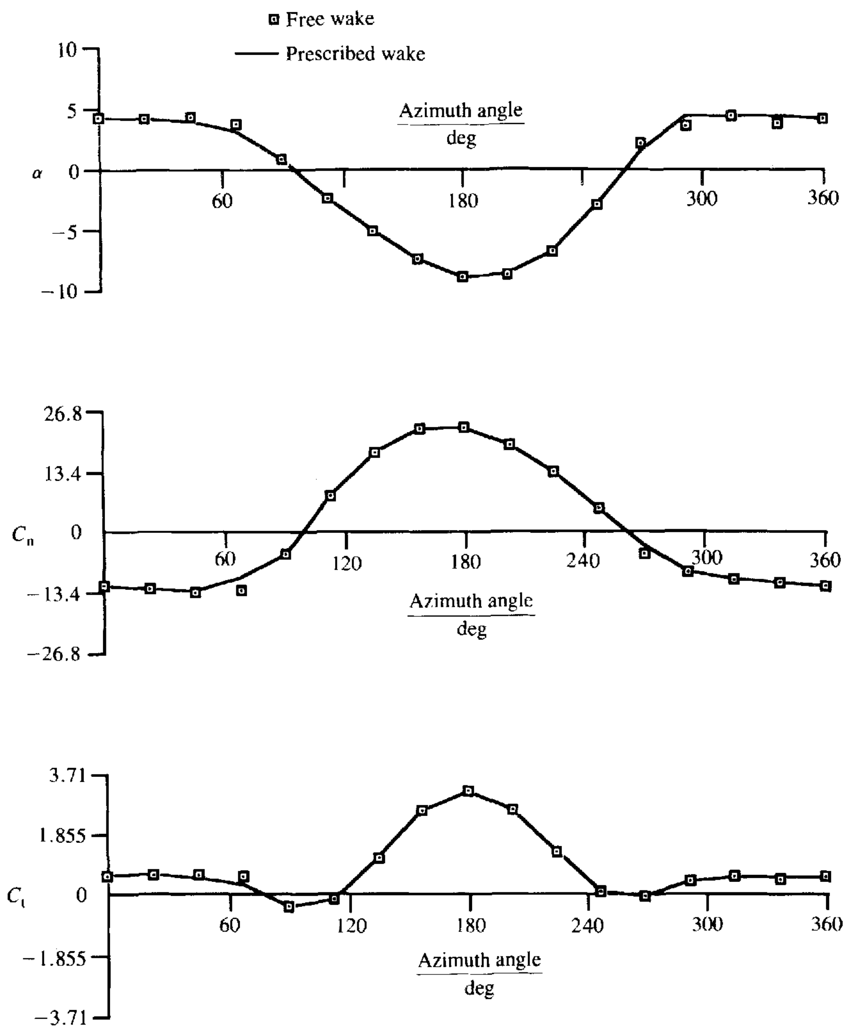

Fig. 7 Comparison of free and prescribed wake vortex methods at tip-speed ratio 5

above, to modify the wake in both the streamwise and cross-stream directions. New induced velocities are then calculated by iteration. The overall procedure is outlined in Fig. 6 and Fig. 7 illustrates the quality of agreement obtained between the prescribed wake model and the free wake method at tip-speed ratio 5. The technique achieves the same accuracy levels as the free vortex model but with a reduction in computation time of approximately two orders of magnitude.

3 COMPARISON OF THE PRESCRIBED WAKE MODEL WITH FIELD DATA

The main purpose behind the development of the prescribed wake model was to provide an accurate and computationally efficient three-dimensional modelling tool for use in the design of both Darrieus and straight-bladed vertical axis wind turbines. It was envisaged that the model would be used primarily to provide estimates of instantaneous blade forces and moments for aerodynamic–structural interaction analyses. As such, power predictions obtained from the prescribed wake method are subject to errors associated with the omission of features such as crossarms, towers or support wires from the model. The configuration of the Darrieus turbine, however, is approximated reasonably well by the 'clean' model and so it is possible to compare the predicted power coefficients with those obtained in field tests.

In Fig. 8 the power coefficient predictions from the prescribed wake model are compared with field data obtained from the Sandia 17 metre Darrieus turbine

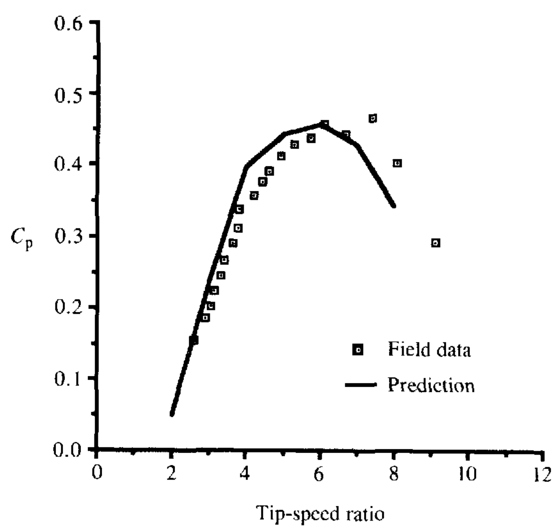

Fig. 8 Comparison of prescribed wake prediction with field data from 17 m Sandia turbine (38.7 r/min)

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(11) at a rotational speed of 38.7 r/min. As can be observed from the figure, the prediction compares reasonably well with the field data over the entire range of tip-speed ratios. It is clear, however, that the predictions obtained at high tip-speed ratios show the greatest discrepancies with the field data. This is a likely consequence of the sensitivity of the high tip-speed ratio cases to the accuracy with which the shape of the closely packed near wake structure, which may partially be contained within the turbine swept volume, is prescribed. Comparison of the prescribed wake shapes with those of the free vortex method indicate that a slight underprediction of wake convection velocities occurs at high tip-speed ratios. Although this difference is small, it produces a lower power coefficient. Additionally, the solution is particularly sensitive to differences between the input two-dimensional aerofoil tangential force coefficients and those actually experienced by the turbine blade profile. This is especially true at high tip-speed ratios where the effective incidence range of the turbine blades is low.

At low tip-speed ratios, the range of effective incidence encountered by the blades of both the Darrieus and straight-bladed turbines necessitates consideration of unsteady aerodynamics. A major component of the unsteady effect is the influence of the trailing wake. This, however, is already accounted for in the prescribed wake model and so full implementation of dynamic effects can be achieved by the inclusion of quasi-steady and apparent mass effects (1). A fully unsteady version of the prescribed wake model is currently under development.

4 CONCLUSION

A prescribed wake vortex method for the prediction of the aerodynamic loads on a vertical axis wind turbine has been developed. The method displays accuracy levels akin to a free vortex model, but is considerably more efficient. The predicted variation of power coefficient with tip-speed ratio for a Darrieus turbine agrees well with field data.

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REFERENCES

1 **Galbraith, R. A. McD., Niven, A. J.** and **Coton, F. N.** Unsteady aerodynamics and its relevance to wind turbines. *Wind Engng*, 1991, **14**(5), 286–298.
2 **Harris, A.** The role of unsteady aerodynamics in vertical axis wind turbines. In *Workshop on recent developments in the aerodynamics of wind turbines*, Nottingham, 1990 (BWEA).
3 **Galbraith, R. A. McD., Coton, F. N., Saliveros, E.** and **Kokkodis, G.** Aerofoil scale effects and their relevance to wind turbines. Ninth Annual Conference on *Wind energy*, Edinburgh, Scotland, 1987 (BWEA).
4 **Jesch, L. F.** and **Walton, D.** Reynolds number effects on the aerodynamic performance of a vertical axis wind turbine. Third International Symposium on *Wind energy systems*, Denmark, 1980, pp. 323–332 (BHRA).
5 **Strickland, J. H.** A review of aerodynamic analysis methods for vertical axis wind turbine. Fifth ASME Symposium on *Wind energy*, 1986, Vol. 2, pp. 7–17 (ASME).
6 **Paraschivoiu, I.** and **Delclaux, F.** Double multiple streamtube model with recent improvements. *J. Energy*, 1982, **7**(3), 250–255.
7 **Strickland, J. H.** A performance prediction model for the Darrieus turbine. International Symposium on *Wind energy systems*, Cambridge, 1976, paper C3 (BHRA).
8 **Strickland, J. H., Webster, B. T.** and **Nauyen, T.** A vortex model of the Darrieus turbine: an analytic and experimental study. *J. Fluids Engng*, 1979, **101**, 500–505.
9 **Wilson, R. E.** and **Walker, S. N.** Fixed wake analysis of the Darrieus rotor. SAND-81-7026, Sandia National Laboratories, 1981.
10 **Basuno, B., Coton, F. N.** and **Galbraith, R. A. McD.** On the aerodynamic modelling of vertical axis wind turbines. In *Aerodynamic aspects of wind energy convertors*, 1990 (Mechanical Engineering Publications, London).
11 **Worstell, M. H.** Aerodynamic performance of the DOE/Sandia 17-m-diameter vertical axis wind turbine. *J. Energy*, 1981, **5**(1).

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