Question 1 Visualize in 2D

Find orthogonal matrix $U \in R^{d\times 2}$ that maximize variance of data x_i (l_2-norm squared of AA^Tx_i) onto 2D space K

$$\hat{U} = \arg \max_{A \in R^{d\times 2} \mid A^TA = I_2} \sum_{i=1}^{n} \| AA^T x_i \|_2^2$$

we can find $\hat{U}$ by singular value decomposition of data $X \in R^{d\times n}$

$X = USV^T$ $\quad U \in R^{d\times d}$ $\quad S \in R^{d\times d}$ $\quad V^T \in R^{d\times n}$

$\Rightarrow \hat{U} = U_2$ the first 2 column of U

then calculate coordinate α

$\tilde{\alpha} = U^T x_i = (\alpha_1, \alpha_2)$

then we plot on a scatter plot

$$\begin{cases} x_1 \text{ axis}: & \alpha_1 \\ x_2 \text{ axis}: & \alpha_2 \end{cases}$$

Qustion 2 : low-rank approximation

we want to find a low-rank matrix approximation of X

$$\hat{X}_2 = \arg \min_{Y \mid rank(Y) \le 2} \| Y - X \|_F^2$$

$$\Rightarrow \hat{X}_2 = U_2 S_2 V_2^T$$