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Problem 1

$$\frac{\partial f(x)}{\partial x_{(i)}} = \lim_{h\to 0} \frac{[x_{(1)}y_{(1)} + \cdots + (x_{(i)}+h)y_{(i)} + \cdots + x_{(p)}y_{(p)}] - [x_{(1)}y_{(1)} + \cdots + x_{(i)}y_{(i)} + \cdots + x_{(p)}y_{(p)}]}{h}$$

$$= \lim_{h\to 0} \frac{h\, y_{(i)}}{h}$$

$$= y_{(i)}$$

$$\Rightarrow \frac{\partial f(x)}{\partial x_{(i)}} = y_{(i)}$$

Problem 2

$$\text{trace}(AB) = \sum_{i=1}^{N}(AB)_{ii} = \sum_{i=1}^{N}\sum_{j=1}^{N} A_{ij}B_{ji} = \sum_{j=1}^{N}\sum_{i=1}^{N} B_{ji}A_{ij} = \sum_{j=1}^{N}(BA)_{jj} = \text{trace}(BA)$$

Problem 3

$\pi(j)$ is the vector in the list of vectors $\mu_1, \cdots, \mu_K \in R^P$ that has the smallest Euclidean distance between vector v_j.

Problem 4

$$\mu_k = \frac{\sum_{i|\pi(i)=k} v_i}{t}$$

where t is the number of elements in the set $S = \{v_i : \pi(i) = k\}$

$\sum_{i|\pi(i)=k} v_i$ is the sum of all the elements in the set S

proof:

define function $f(x) = \sum_{i|\pi(i)=k} \|v_i - x\|_2^2$

we want to find vector x that minimizes $f(x)$

set the derivative of $f(x)$ to 0 and solve for x to find μ_k

$$\frac{df(x)}{dx} = \frac{d \sum_{i|\pi(i)=k} \|v_i - x\|_2^2}{dx} = \frac{d \sum_{i|\pi(i)=k} \sum_{j=1}^{P} (v_{i(j)} - x_{(j)})^2}{dx} = \sum_{i|\pi(i)=k} \sum_{j=1}^{P} 2(x_{(j)} - v_{i(j)})$$

$$\text{set } \sum_{i|\pi(i)=k} \sum_{j=1}^{P} 2(x_{(j)} - v_{i(j)}) = 0$$

$$\sum_{i|\pi(i)=k} (x - v_i) = 0$$

$$t x = \sum_{i|\pi(i)=k} v_i$$

$$\Rightarrow \mu_k = x = \frac{\sum_{i|\pi(i)=k} v_i}{t}$$

where t is the number of elements in the set $S = \{v_i : \pi(i) = k\}$