Yale University
Department of Statistics and Data Science

Statistics 365/565

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Notes: This part of the homework recaps derivatives, adding matrices, and functions.

Notation: $[k] = \{1, 2, \ldots, k\}$. For a matrix $A \in \mathbb{R}^{m \times n}$ we will let $A_{(i,:)}$ denote the i^{th} row and $A_{(:,j)}$ denote the j^{th} column. **Both will be treated as column vectors.**

Problem 1: The inner-product between two vectors $x, y \in \mathbb{R}^p$ is defined as $\langle x, y \rangle = x^T y = \sum_{i=1}^{p} x_{(i)} y_{(i)}$. Let $f(x) = \langle x, y \rangle$. Compute $\frac{\partial f(x)}{\partial x_{(i)}}$. That is the partial derivative of $f(x)$ with respect to the i^{th} coordinate.

Problem 2: Let A and B two matrices of commensurate dimensions. Show that $\mathrm{trace}(AB) = \mathrm{trace}(BA)$.

The following problems are relevant when we study clustering. In clustering the goal is to find "representative" data points for different parts of your data. When you have those representative data points then you need to understand which of your original data points belongs with which representative.

Problem 3: Suppose that you have a list of vectors (data points) $v_1, v_2, \ldots, v_n \in \mathbb{R}^p$ and another list of vectors $\mu_1, \ldots, \mu_k \in \mathbb{R}^p$. Define a function $\pi(j) = \arg\min_{i \in k} \|\mu_i - v_j\|_2^2$. In words, what is $\pi(j)$? One sentence answer.

Problem 4: Suppose that you have a list of vectors $v_1, v_2, \ldots, v_n \in \mathbb{R}^p$. Suppose you have a function $\pi(j) : [n] \mapsto [k]$. That is $\pi(j)$ maps some integer between 1 and n and maps it to another integer between 1 and k. Compute

$$\mu_k = \arg\min_x \sum_{i|\pi(i)=k} \|v_i - x\|_2^2$$