HW 1

1) $$\frac{\partial \langle x, y \rangle}{\partial x_{(i)}} = y_{(i)}$$

2) $$tr(AB) = \sum_{i=1}^{n} (AB)_{(ii)}$$

$A \in \mathbb{R}^{n \times d}$
$B \in \mathbb{R}^{d \times n}$

$$= \sum_{i=1}^{n} \sum_{j=1}^{d} A_{ij} B_{ji}$$

$$= \sum_{j=1}^{d} \sum_{i=1}^{n} A_{ij} B_{ji}$$

$$= \sum_{j=1}^{d} (BA)_{(jj)}$$

$$= tr(BA)$$

3) $\pi(j)$ is the point μ_i that is closest to v_j of all possible μ_i's.

4) $$\mu_K = \underset{x}{\operatorname{argmin}} \sum_{i \mid \pi(i) = K} \|v_i - x\|_2^2$$

$$\Rightarrow \nabla_x \|v_i - x\|_2^2 = (x - v_i)$$

$$\Rightarrow \nabla_x \sum_{i \mid \pi(i) = K} = \sum_{i \mid \pi(i) = K} x - v_i \quad \text{set to } 0$$

$$\sum_{i \mid \pi(i) = K} x = \sum_{i \mid \pi(i) = K} v_i \Rightarrow x = \frac{\sum_{i \mid \pi(i) = K} v_i}{\sum_{i \mid \pi(i) = K} 1}$$

$\Rightarrow$ μ_K is average over all v_i such that $\pi(i)$ is equal to K.