FM320 Quantitative Finance (MT) -- Course Project

Topic selection and instructions:

- Topic 1 - Univariate analysis
- Choose one of the securities to work with. For that security, estimate univariate conditional volatility models of the kind we studied in Topic 2 of the course. Perform statistical analysis to decide which univariate model constitutes the best model for that particular series.

Stock selection and company overview:

- Stock selected -- Chevron Corporation (CVX)
- Chevron Corporation is an integrated oil and gas company based in the United States. Before the dissolution of Standard Oil Co. Inc. (the Sherman Antitrust Act), which was owned by Rockefeller and was accused of being an illegal monopoly in 1911, Chevron managed businesses in California and was named Standard Oil Co. (California). Soon after the dissolution, Chevron became one of the "Seven Sisters" and dominated the world oil industry in the early 20th century. Today, Chevron operates its upstream businesses (including oil and gas exploration and production), its downstream businesses (consist of manufacturing and selling fuels and gasoline by-products), as well as alternative energies. Its most significant operation areas include the West Coast of North America, the U.S. Gulf Coast, Southeast Asia and Australia.

Part 1: Statistics for Returns; Visualization of Returns and Distribution Tests.

Code Reference:

```
*** Part 2: Computing Returns and Statistics, Producing Display ***
*** Part 3: Histograms ***
*** Part 7: QQ Plots ***
```

1. Display statistics for log returns

	1	2
1	' '	'CVX'
2	'Average'	5.2641e−04
3	'Std. Dev.'	0.0153
4	'Max'	0.1894
5	'Min'	−0.1334
6	'Skewness'	0.0919
7	'Kurtosis'	11.4442
8	'Average (Ann.)'	0.1327
9	'Std. Dev. (Ann.)'	0.2432
10	'Date Max'	20081013
11	'Date Min'	20081015
12	'JB Stat.'	2.2406e+04
13	'JB P−Val.'	1.0000e−03

The statistics in the table are calculated based on the daily log returns of CVX stock, data extracted from 02 Jan 1990 to 29 Nov 2019. The daily stock return has an average of 0.0526%, and the average annualized stock return is 13.27%, which is almost twice of the average annualized SPX return (7.33%) (data given in the lecture notes page 25). The standard deviation of CVX return is 1.53% per day and 24.32% per year, which is also higher than that of SPX (15.43% Annualized). Therefore, I conclude that the stock of Chevron Corporation outperforms the US stock market in the long run in terms of its returns, but meanwhile exposes investor to a higher level of volatility.

The CVX stock price is significantly affected by the 2007-2008 financial crisis. As can be seen from the table, the stock exhibited maximum return (18.94%) on 13th October, 2008 and suffered from largest drop (minimum return of -13.34%) after two days. The reason behind the fact that the financial crisis had a negative impact on oil and gas industry is that the deflation and liquidation happened just after the crisis took oil and gas assets lower, and this steep decline in prices resulted in falling revenues for companies in this sector (including Chevron Corporation).

2. Non-normality Tests

I would like to conduct tests to examine whether the CVX return is normally distributed. It frequently happens in stock prices that the return series exhibits fat tails when the likelihood of extreme outcomes is higher than that of a normally distributed random variable. When this situation occurs, it is helpful to check if the assumptions for the univariate volatility model holds.

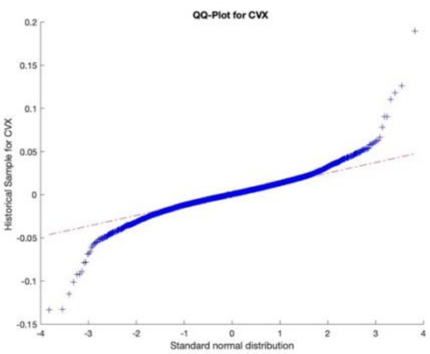
2.1 Skewness and Kurtosis

The skewness of CVX stock return is 0.0919 and the kurtosis of return is 11.4442, and for a random variable which has standard normal distribution its skewness is 0 and kurtosis is 3. Therefore, it can be proposed that the CVX stock return is not normally distributed, and this can be verified in our further normality tests, namely Jarque-Bera test and QQ-Plot against normal distribution.

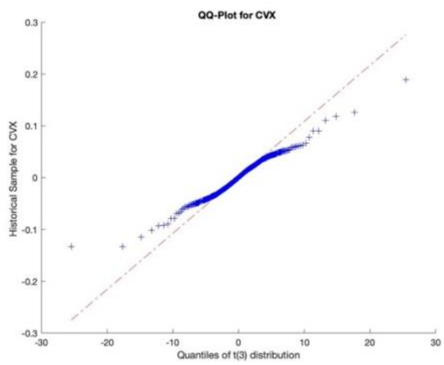
2.2 Jarque-Bera Test

The Jarque-Bera test is a statistical approach to test for normality. The null hypothesis suggests that the return series is normally distributed, and this is satisfied if the JB stat returns 0 or a small test statistic with a large p-value. Nevertheless, the t-statistic for CVX returns is given by 22406 and its p-value is 0.1%, and so I conclude that it is significant enough (with 1% significance value) to reject H_0. In the next section I will use graphical method to visualize that the CVX log return indeed has heavy tails.

2.3 QQ-Plot vs. Normal



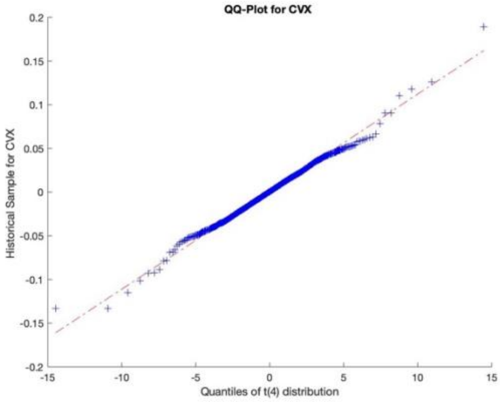
In principle, the quantiles of the historical sample for CVX should be a linear transformation of the standard normal quantiles if the return series follows normal distributions. However, the data points deviate from the linear line at extreme values: the points are lower than the line when the historical returns are lower than -0.02 and higher than the line when the returns are higher than 0.025. This shows that the distribution of CVX returns has fatter tails than a normal distribution. In this case, the probability of extremely high or low returns are higher than that of normal. Having fat tails will have large impacts on building up our model: the extreme outcomes tend to have a disproportional influence on financial matters, and having assumed normality often leads to underestimation of risk so investors could not take the right positions, but the applications of non-normal model will require more data and is usually complex.

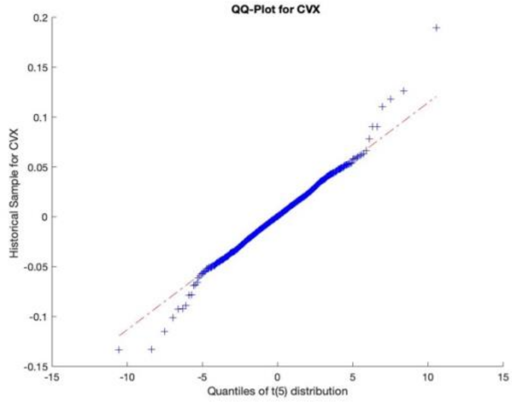


2.4 QQ-Plot vs. t(3)

It is remarkable that student t distributions have heavier tails than normal distributions, and the larger the degrees of freedom parameter "mu" is, the less heavy the tails would be and the closer the t distribution is to a normal distribution. In the bottom right plot of page 2, the quantiles of CVX returns deviate up when the returns are low and deviate down when exhibiting high return, showing the CVX stock returns do not have as heavy tails as a t(3) distribution.

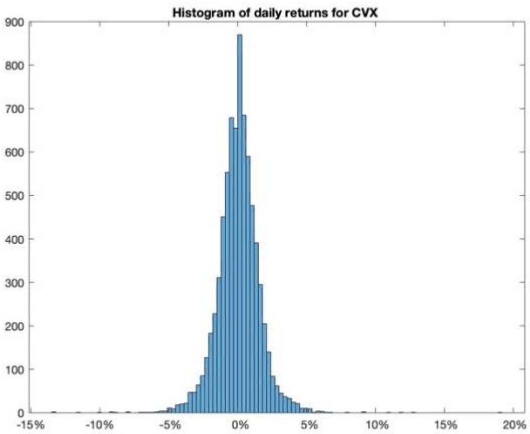
2.5 QQ-Plot vs. t(4) and QQ-Plot vs. t(5)





The QQ-Plot of CVX returns against student t(5) distribution is almost linear except for a dozen of very extreme points (as shown on the right graph), thus the empirical distribution of CVX returns is a linear function of the student t(5) distribution.

3. Histogram of return series



The histogram of daily log returns for CVX is almost symmetric around 0 and slightly positively skewed (the series has skewness 0.0919), it also has longer positive tails than negative ones, this is because Chevron Corporation experienced higher positive returns in magnitude than negative returns.

<u>Part 2: Volatility clustering and tests for autocorrelations</u>

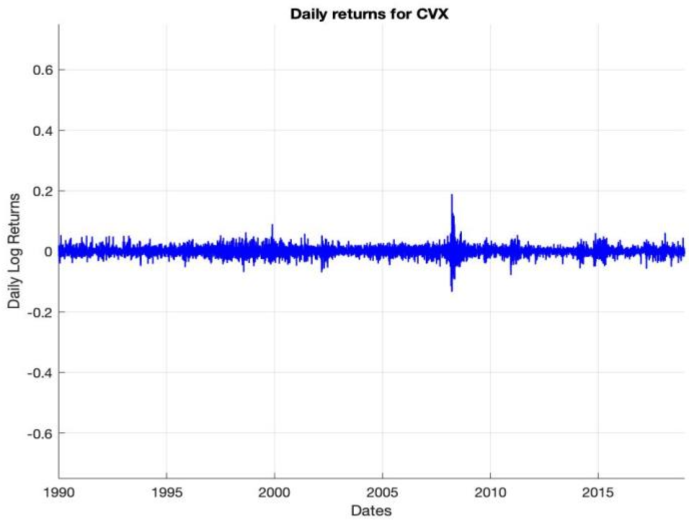
Code reference:

```
*** Part 4: Exploring Autocorrelations ***
*** Part 5: Moving Variance ***
```

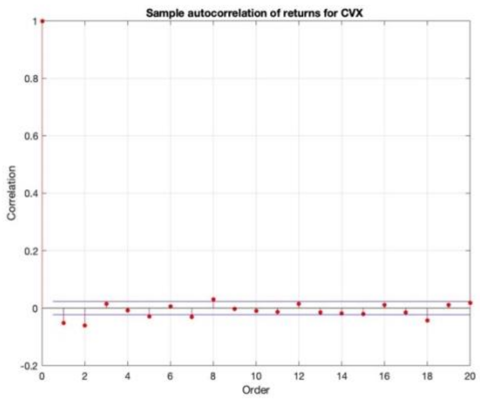
1. Time series of CVX returns chart

The chart on top of page 4 shows the daily log returns of CVX. There are implications of volatility clustering -- the spikes in year 2000 to year 2002 (which arises from Chevon's $45 billion acquisition deal of Texaco and creates the world's fourth largest publicly traded oil company with approximately $95 billion market value) eventually died down in late 2003.

After experiencing the significant oil and gas price drop during the financial crisis (oil price fell from $147 to $33 within 8 months and gas price fell from $14 to $4 in the same period), the volatility of Chevron's stock fluctuated sharply in year 2015 (in which Chevron announced to sell a 30 percent holding in its Canadian shale holdings and alleged cutting up to 7,000 jobs), and it eventually ceased in year 2013.

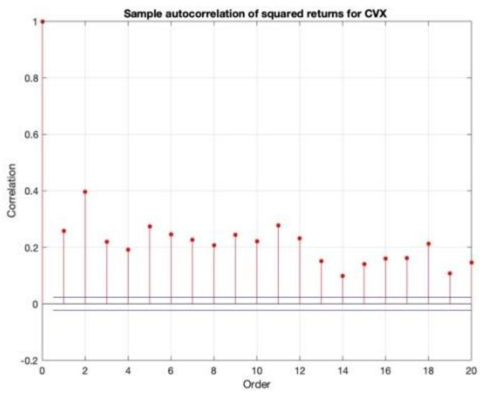


2. Autocorrelation Function of Log Returns



Autocorrelation function measures how a time series is correlated with its own lagged values, if autocorrelation is significant, it shows predictability in the time series. The graph on the left aims to see if there is any predictability in Chevron Corporation's log returns. As some of the points lie out of the blue lines (95% significance level of white noise) and most of the points stay within the blue lines, I conclude that there is little correlation between returns of CVX stock today and returns in the previous days, and thus investors could not predict the trends and make profits easily.

3. Autocorrelation Function of Squared Returns



On the graph on the left, the correlation of squared returns for CVX today and squared returns for CVX days before (of order ranging from 1 to 20) has all the points lying outside the blue lines, thus I can conclude that with 5% significance level, the squared returns are highly and positively correlated (positively because squared of returns can only be non-negative). Furthermore, I observe that as the number of lags gradually increases, the correlation index drops from a high of 0.4 to a low of 0.1, implying that observations more distant in the past is less correlated to squared returns today and thus has less predictability in today's squared returns. The serial correlation of squared returns proves the existence of volatility clustering, arising from investors' reaction to market.

4. Stylized Facts of Financial Returns

Up to now, we have obtained patterns of log returns of CVX stock. These includes:
Volatility clusters, shocks will persist for a long time and will eventually die out.
CVX log returns have heavier tails than normal distributions, t(5) distribution is appropriate.
To further analyze these trends and better predict volatility, we aim to estimate volatility models and examine their suitability for Chevron Corporation.

Part 3: Standard Model Estimations and Performance:

Using Matlab to estimate standard models (namely EWMA, ARCH(1), ARCH(2), ARCH(10) and GARCH(1,1)) to predict volatility of CVX, fitting our predictions with real return series and evaluate on these models.

Code reference:

```
*** Part 8: Compute an EWMA volatility for CVX return series ***
*** Part 9: Estimate ARCH models for CVX return series ***
*** Part 10: Estimate a GARCH(1,1) model for CVX return series ***
*** Part 11: Produce a chart with log returns and +/- 2 Std. Dev. for
    models ***
```

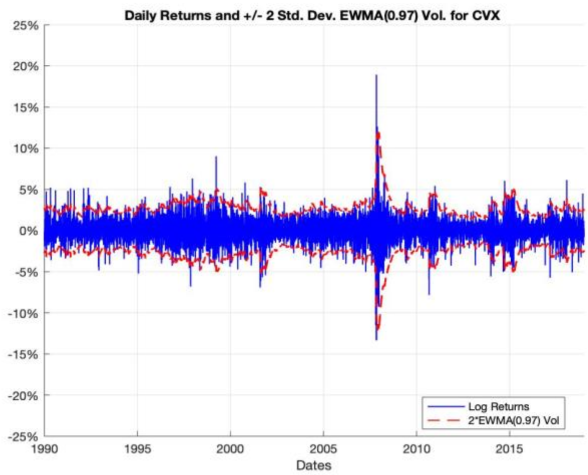
1. Moving Average Models

In this section we will estimate the Exponentially Weighted Moving Average model for CVX volatility only, because the simple Moving Average model -- $\sigma_t^2 = \sum_{i=1}^{W_E} r_{t-i}^2$ -- has significant disadvantages: when the moving window is chosen to be small, there is only a small number of observations incorporated in the estimation, and as a result the model is more responsive to individual shocks, resulting in very unstable volatility estimates; when the moving window is a large number, whenever a shock happens, it takes much time for the shock to die down, so the volatility is over-estimated during this period, forcing investors to take small positions over a period of good opportunities.

The exponentially weighted moving average model (EWMA) is given by the specification:

$$\sigma_t^2 = (1-\lambda) r_{t-1}^2 + \lambda \sigma_{t-1}^2$$

where λ is the decay parameter and is chosen to be 0.97 -- this large value of λ is chosen to incorporate greater persistence of past shocks. The performance chart of EWMA volatility estimates is given below:



Pros (of EWMA model):
It has removed some of the drawbacks of simple moving average model: EWMA volatility estimates have incorporated all the past information, namely stock returns and conditional volatility in the model, and the current conditional variance estimate is a weighted average of previous day stock return squared and previous conditional variance. Also, it allows current information to take a higher weight

whilst past information to have exponentially decaying weights. Thus, the EWMA volatility is more stable than Moving Average model and it adapts to the real observations well.

Cons:

Unlike ARCH or GARCH models, knowledge of the model parameter λ is insufficient to pin down the unconditional variance of the model, which is an important characteristic, and its unconditional variance is in fact infinite. What's more, the conditional variance given by the model is not mean-reverting.

2. ARCH Models

2.1 ARCH (1) model

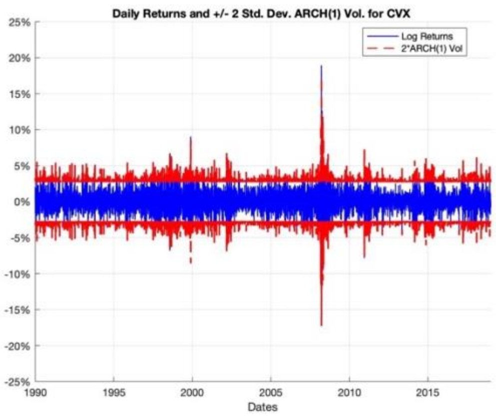
The display statistics for the model parameters and the performance of ARCH (1) model is given below:

	1	2
1	''	'CVX'
2	'Constant'	1.8239e-04
3	'ARCH(1)'	0.2029
4	'Variance'	2.2881e-04
5	'Unconditional Vol. (Ann.)'	0.2401

Model specification:

$$\sigma_t^2 = 0.000182 + 0.203 r_{t-1}^2$$

The annualized unconditional volatility is 24.01%.



Pros:

The ARCH models have the common advantage that they incorporate past shocks on volatility estimates, and ARCH(1) has settled down the drawbacks of an EWMA model: knowledge of the model parameter (ARCH term is smaller than 1) is sufficient to pin down the unconditional variance of the model, and this unconditional variance is finite. The conditional variance is mean-reverting, and it has unconditionally heavy tails even though our underlying assumption is that the standardized residual is conditionally normal, which is a feature in consistent with the stylized patterns we have analyzed in section 2 of part 1.

Cons:

The estimates have seen a lower bound, and most of the observed time series lie between the bounds.

The model estimation is bad in terms of its stability, which means a small number of parameters have brought us very unstable volatility estimates, and to settle down this problem we then try ARCH (2) and ARCH (10) models.

2.2 ARCH (2) Model

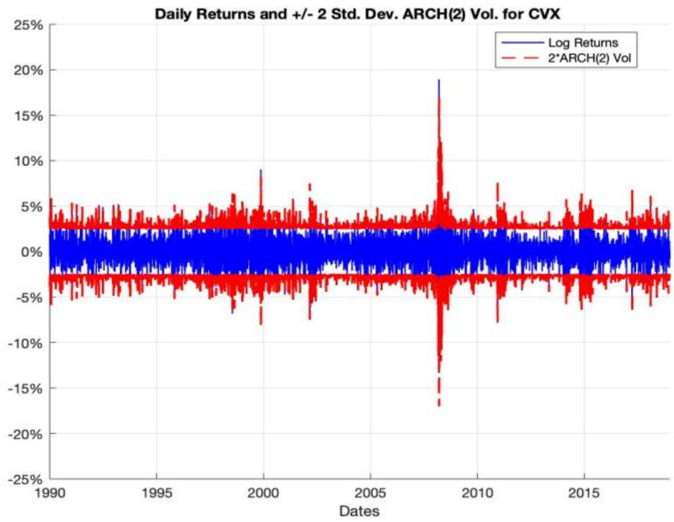
Model specification:

$$\sigma_t^2 = 0.00015 + 0.1436 r_{t-1}^2 + 0.1881 r_{t-2}^2$$

The p-value for each of the parameters to be estimated (constant term, ARCH (1) term and ARCH (2) term) is very small in value, implying that all the parameters in here are non-negligible in terms of their statistical significance.

The annualized unconditional volatility estimate is 23.78%, very similar to that of ARCH (1).

	1	2
1	' '	'CVX'
2	'Constant'	1.5001e-04
3	'ARCH(1)'	0.1436
4	'ARCH(2)'	0.1881
5	'Sum of Alph...	0.3317
6	'Variance'	2.2448e-04
7	'Condition...	0.2378
8	'T-Statistics'	27.9186
9	[]	7.9275
10	[]	6.5494
11	'P-Values'	0
12	[]	2.2204e-15
13	[]	5.7755e-11



Pros:

ARCH (2) model shares the same advantages as of ARCH (1) model except that it does not have unconditionally heavy tails. Also, comparing to ARCH (1) model the ARCH (2) estimates tend to reduce instability, but only by a little. We will focus on the disadvantages of implementing this model later.

Cons:

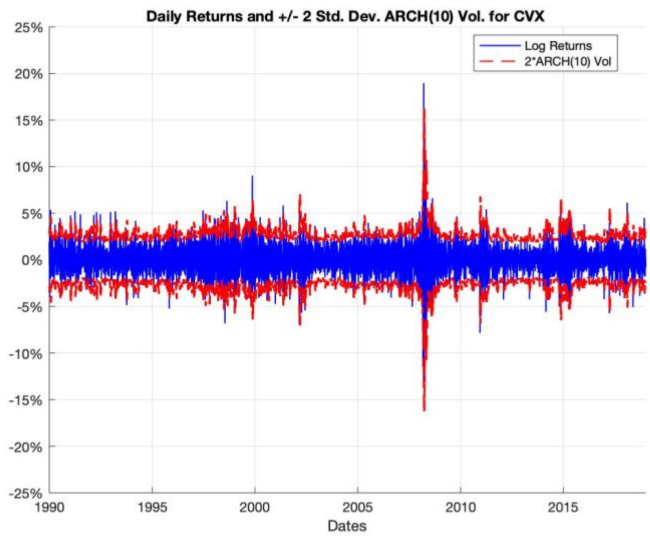
The chart has shown very poor adaptability of ARCH (2) model: there is still lower bounds for the volatility estimates, which are driven by the constant term in our model specification.

Volatility estimates are very unstable which could lead to practical problems. So we once again increase our number of parameters to see if there could be any improvement.

2.3 ARCH (10) Model

The display statistics and the performance chart are given below:

	1	2
1	' '	'CVX'
2	'Constant'	7.6122e-05
3	'ARCH(1)'	0.0646
4	'ARCH(2)'	0.0684
5	'ARCH(3)'	0.0583
6	'ARCH(4)'	0.0586
7	'ARCH(5)'	0.0980
8	'ARCH(6)'	0.0765
9	'ARCH(7)'	0.0576
10	'ARCH(8)'	0.0554
11	'ARCH(9)'	0.0509
12	'ARCH(10)'	0.0628
13	'Sum of Coefficients'	0.6511
14	'Variance'	2.1821e-04
15	'Unconditional Vol. (...	0.2345



Model specification:

$$\sigma_t^2 = 0.0000761 + 0.0646r_{t-1}^2 + 0.0684r_{t-2}^2 + 0.0583r_{t-3}^2 + 0.0586r_{t-4}^2 + 0.0980r_{t-5}^2 + 0.0765r_{t-6}^2 + 0.0576r_{t-7}^2 + 0.0554r_{t-8}^2 + 0.0509r_{t-9}^2 + 0.0628r_{t-10}^2$$

The unconditional annualized volatility is 23.45%, which is slightly less than the ARCH (2) estimate. We will analyse the performance of the ARCH (10) model next.

<u>Pros:</u>

Compared to the ARCH (1) and ARCH (2) models, this model has much higher adaptability to our real observations and can capture most of the spikes in volatility well.

<u>Cons:</u>

This model seems to be constantly overestimating the conditional volatility during normal periods, as can be shown from the white space between the estimations in red and the real data in blue; and it tends to underestimate the conditional volatility during extreme volatility spikes, as can be seen that it could not fully capture the large volatility spikes in years 1992, 2000 and 2010.

2.4 Conclusion for ARCH estimates

The ARCH models have in general improved the drawbacks in EWMA models, and predicted the unconditional volatility in a well-behaved manner, however, with low levels of parameters the volatility estimates are very unstable and exhibit poor adaptability, leading to practical issues, and when we increase the number of lags (in this context, I have used ARCH(10)) to relieve this problem, the estimation uncertainty is raised a lot and the estimation is done much slowly. Moreover, the conditions on parameters for the fourth moment to exist is too restrictive. Thus, I conclude that the ARCH estimations are not suitable for my CVX stock volatility.

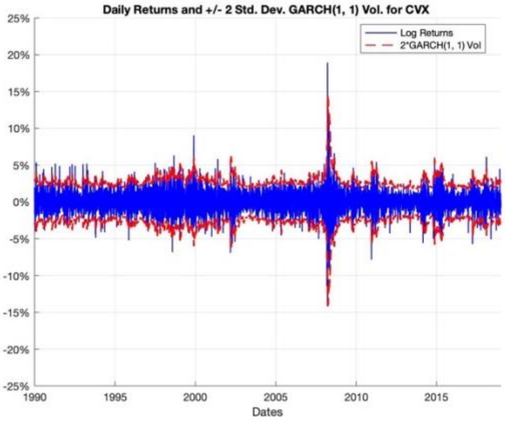
3. GARCH (1,1) Model

	1	2
1	' '	'CVX'
2	'Constant'	3.9904e−06
3	'ARCH(1)'	0.0647
4	'GARCH(1)'	0.9168
5	'Sum of Coeffs.'	0.9814
6	'Variance'	2.1510e−04
7	'Unconditional V...	0.2328

Model specification:

$$\sigma_t^2 = 3.99e^{-06} + 0.0647r_{t-1}^2 + 0.9168\sigma_{t-1}^2$$

Annualized unconditional volatility for CVX is 23.28%. The fact that the GARCH term coefficient is much larger than the ARCH coefficient implies that the previous estimates play a significant part in today's volatility estimate, and the effect of news on conditional volatility is less significant.



<u>Pros:</u>

In addition to all the benefits ARCH models have brought (including finite unconditional variance, conditional volatility to be mean-reverting, and unconditionally heavy tails), the GARCH (1,1) model effectively incorporates the impacts of past shocks even prior to time (t-2), so the memory lasts long and the variance is not going to change prominently, thus the volatility estimate is very stable. Last but not least, with the shorter lag length in GARCH (1,1), the number of parameters to be estimated is smaller, reducing estimation uncertainty and accelerating estimation process. Thus, the GARCH (1,1) is a rather satisfying model and is harmonious with real data.

Cons:

The GARCH (1,1) is too simple to capture certain characteristics such as leverage effects, and the GARCH extensions are introduced in the next part to improve on GARCH (1,1).

Part 4: GARCH Extensions for CVX Volatility Estimates

Code reference:

```
*** Part 12 - Estimating various versions of GARCH for CVX ***
```

	1	2	3	4	5
1	' '	'GARCH(1, 1)'	'GJR–GARCH'	'GARCH with t Res.'	'Power GARCH'
2	'Constant'	3.8080e–06	4.4060e–06	2.8926e–06	8.1174e–06
3	'ARCH(1)'	0.0622	0.0280	0.0615	0.0660
4	'GARCH(1)'	0.9200	0.9177	0.9259	0.9201
5	'Leverage'	NaN	0.0708	NaN	NaN
6	'Power'	NaN	NaN	NaN	1.8258
7	'Nu'	NaN	NaN	7.9946	NaN
8	't (Const)'	4.5364	4.8693	2.8332	1.0379
9	't (ARCH)'	8.2570	4.1618	7.3911	8.3832
10	't (GARCH)'	94.0543	93.8220	77.3927	96.1925
11	't (Lever.)'	NaN	5.8889	NaN	NaN
12	't (Power)'	NaN	NaN	NaN	–0.7681

Model specifications and explanations:

- GARCH (1,1): $\sigma_t^2 = 3.81e^{-06} + 0.0622r_{t-1}^2 + 0.92\sigma_{t-1}^2$
- GJR-GARCH: $\sigma_t^2 = 4.41e^{-06} + [0.028 + 0.0708I_{[r_{t-1}<0]}]r_{t-1}^2 + 0.9259\sigma_{t-1}^2$
  This model has incorporated the leverage effect: when big negative shocks occur to CVX stock prices, conditional volatility will be more significantly influenced than big positive shocks because people react stronger to bad news.
- GARCH with t-distributed residuals:
  Assumption: the standardized residual z_t follows student t distribution with degrees of freedom 7.9946 (approximately 8).
  $\sigma_t^2 = 2.89e^{-06} + 0.0615r_{t-1}^2 + 0.926\sigma_{t-1}^2$
  The normalizing factor is 0.866.
  The assumption will later on be confirmed in the residual test.
- Power GARCH: $\sigma_t^2 = 8.12e^{-06} + 0.066r_{t-1}^{1.83} + 0.92\sigma_{t-1}^{1.83}$
  This model has allowed the possibility that the actual power of past returns and past volatilities of CVX that matters for recent conditional volatility is non-quadratic.

Part 5: Comparing Models and Selection for CVX (Parameter Tests, Likelihood Ratios and Residual Analysis)

Code reference:

```
*** Part 13 - Coefficient tests and likelihood ratio tests ***
*** Part 14 - Computing standardized residuals, testing them ***
```

1. Parameter Tests

-- Reference: page 9 table

-- To test whether specific parameters for GARCH extensions are statistically equal to 0, so that the more complex model can be reduced to the baseline (GARCH (1,1)).

- GARCH (1,1):

The test statistics for the constant term, the ARCH term and GARCH term are all greater than 2 (4.53, 8.25 and 94.05 respectively), thus all the coefficients for the basic GARCH (1,1) model are statistically significant with 5% significance level and should not be omitted. It is noteworthy that the test statistic for the GARCH term is far larger than our critical value 1.96, implying the GARCH term to be very significant in the CVX volatility estimation, and this figure has reconfirmed my conclusion to have rejected ARCH models.

- GJR-GARCH:

The test statistics for the constant term, the ARCH term and the GARCH term are all greater than 2 and thus statistically significant. A leverage term is introduced in this model and is of numerical value 0.0708, and its test statistic is 5.8889 which is also greater than 2, and thus we conclude that all the coefficients in GJR-GARCH model are statistically significant.

- GARCH with t(8)-distributed residuals:

Similarly, the constant term, the ARCH term and the GARCH term all have greater-than-2 test statistics (2.83, 7.39 and 77.39 respectively). Conclusion of t-distributed model could not be made based on parameter test and further tests should be processed.

- Power GARCH:

The ARCH-like term and the GARCH-like term are statistically significant. Nonetheless, the t-stat for the constant term is 1.0379 only, implying the constant coefficient is not statistically significant. Also, for the t-test of the power coefficient to test whether the power is significantly different from 2, the test statistic is of magnitude 0.7681 which is smaller than 2, and thus the null hypothesis should not be rejected and the power is statistically close to 2. If my likelihood ratio test also shows the Power GARCH does not statistically improve on the results but increases complexity, I will discard this model under the circumstance of CVX estimates.

2. Likelihood Ratio Tests

-- A more generalized version to test whether the restricted models are significant or not.

- Test GJR-GARCH against GARCH:
  The p-value is 0, suggesting that GJR-GARCH is a more appropriate model than GARCH (1,1) model at 1% significance level.
- Test Power GARCH against GARCH:
  The p-value is 0.3700, implying that the Power GARCH is not significantly different from the GARCH model (even at 10% significance level) and this fact has corroborated my conjecture that the Power GARCH model does not improve on my estimates for CVX volatility estimates. I will therefore reject this model.
- Test t(8)-distributed residuals against normally-distributed residuals:
  The p-value is 0, so the t-GARCH is more appropriate than GARCH (1,1).

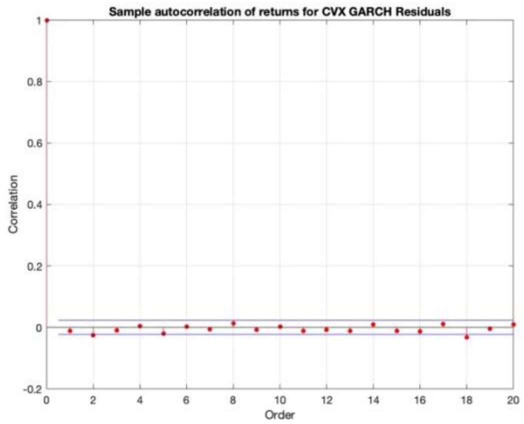
Now we have GJR-GARCH model and GARCH with t(8)-distributed residuals which are both better improvements on standard GARCH featuring on certain characteristics, to further filter down to choose one univariate model that fits best for stock CVX, I compute residual analysis to test for assumptions.

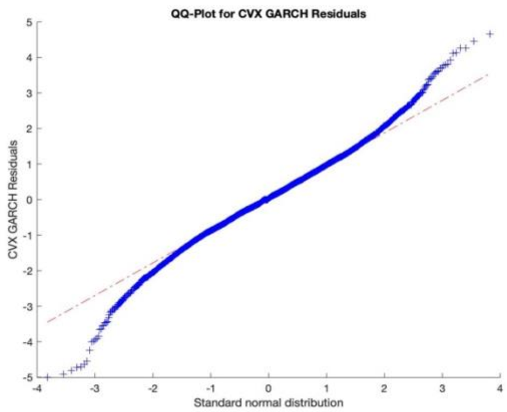
3. Residual Analysis

-- In GARCH (1,1) model with normal standardized residuals, the assumption is such that the residuals are <u>independent and identically distributed, following standard normal distribution</u>. Whilst in GARCH with t(8)-distributed residuals, the assumption is adjusted on the observation of heavy tails, it is that the residuals are <u>independent and identically distributed, following t(8) distributions.</u>

-- Want to examine the two assumptions separately to find out which assumption is true.

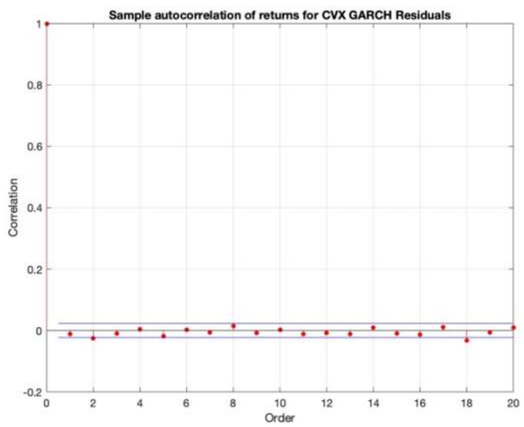
GARCH (1,1) residual:

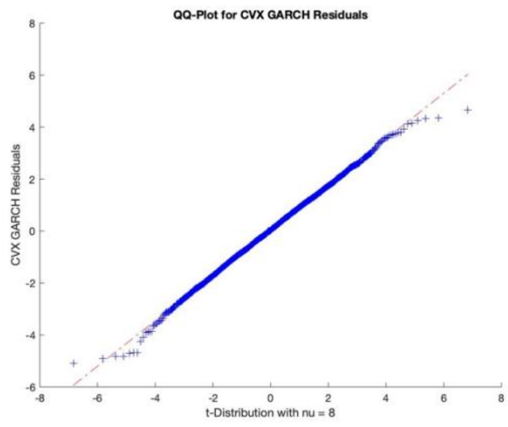




The graph displayed on the left shows that there is no serial correlation between the residuals, so the residuals are independent from each other. The chart displayed on the right illustrates that the residuals have heavier and fatter tails than a standard normal distribution, hence we conclude that the assumption for GARCH (1,1) is violated in the real dataset.

t(8)-distributed GARCH residual:





The graph on the left shows that as all red spots lie between the two blue lines, there is no predictability in CVX's residual and thus the residuals are independent to each other.

The plot on the right efficiently shows that the quantiles of CVX GARCH residuals are a linear transformation of the quantiles of t(8) distribution (except for a few of the most extreme points), and hence the residuals are t(8)-distributed, indicating that the assumptions for the residuals are both satisfied.

4. Conclusion

As under GJR-GARCH, the underlying assumption lies that the standardized residuals are normally distributed, which is violated by exhibiting heavier tails, I would conclude that the t(8)-distributed GARCH model is the best univariate volatility model for CVX stock returns among all the models we have analysed.

<u>Part 6: Summary:</u>

Our final choice of the best univariate model to estimate Chevron Corporation's stock volatility is the GARCH model with adjusted assumption:

It is assumed that the standardized residuals are independent and each of them follows a student t(8) distribution, with normalizing factor to be 0.866.

The assumption is based on the stylized facts we have observed in the first two parts that the log returns of Chevron's stock have fatter tails than a normal distribution.

The model specification is: $\boldsymbol{\sigma_t^2 = 2.89e^{-06} + 0.0615r_{t-1}^2 + 0.926\sigma_{t-1}^2}$, and the model is evaluated based on the parameter test, likelihood ratio test and residual analysis, and it is proved to be efficient.