Can we predict house prices using known features of each house and a supervised learning approach?

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1 Background:

House prices are an important part of the economy and usually reflect trends in it. They can be influenced by the physical condition of the house and by other attributes such as location (Bin 2004). Prices are important for homeowners, prospective buyers and estate agents, prediction methods could help lead to more informed decisions to each of these stakeholders. Gao et al. (2019) suggest that prediction models may be useful for narrowing down the range of available houses for prospective buyers and allowing sellers to predict optimal times to list their houses on the market. Prediction accuracy would be important in all of these situations as inaccurate models would not be trusted by their users.

In most countries there is some form of house price index that measures changes in prices (Lu et al. 2017). This contains a summary of all transactions that take place but not the individual features of each house sold, therefore it cannot be used to make predictions of house price.

Something else to take into account is that it may be difficult for prospective buyers to visualise how square footage measurements of a house are calculated or how this measurement translates into physical size if they have not visited the house themselves. Buyers therefore rely on factors such as the number of bedrooms, bathrooms or house age to get an idea of the value of the house. This analysis will focus on which features of a house have the largest influence on the prediction of house prices. This report does not look at the effect of time on house prices. It is already well known that house prices increase every year (Alfiyatin et al. 2017).

Many house price prediction models have been created using machine learning methods. The hedonic price model is the most extensively researched and uses regression methods (Gao et al. 2019). The goal of regression methods is to build an equation which defines y, the dependent variable as a function of the x variable(s). This equation can then be used for prediction of y when given unseen values of x. Machine learning methods use data in a 'training set' to build a model, this model is then used to make predictions on an unseen 'test set' of data. The accuracy of models can be calculated by taking the predicted values from the actual values.

1.1 Objectives:

- Understand which attributes of houses given in the data set can be used to effectively construct a prediction model for house price (dependent variable).
- Minimize the differences between predicted and actual house prices by using model selection to choose the most accurate model.

2 Data:

2.1 Data Description

The dataset chosen for this analysis is from houses sold in 2014 in Washington, USA. It contains the sale price (US dollars) along with attributes of each house such as number of bedrooms, number of bathrooms, etc. There were 4600 observations with 17 variables in the original data set downloaded from [Kaggle] (https://www.kaggle.com/shree1992/housedata). Although the dataset is from 2014, it was a particularly interesting data set because it contains a large amount of information and an interesting selection of variables. A more recent data set, or one from the UK could not be found, and so this analysis will go ahead with this data set.

2.2 Data Preparation and Exploration

The first task was preparing and cleaning the dataset. This served two purposes, firstly to get to know the different variables in the data and existing patterns or correlations between them and secondly to carry out feature selection. Missing values were searched for and removed and any observations in which price was equal to zero were removed. The cleaned dataset was exported ready for use in the main analysis. This cleaned data set contained 4492 observations and 11 variables (Table 1). Some alterations were made to existing variables, these are listed in the descriptions of table 1 alongside the variable.

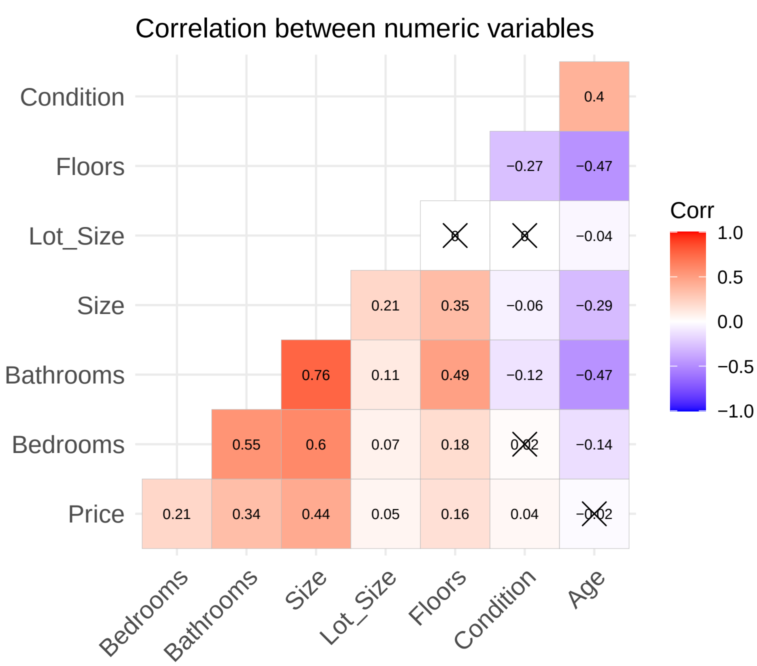
Table 1: Description of all variables present in the cleaned dataset and explanations of how they were calculated if they have been altered from the original dataset

Variable	Description of Variable
price	House sale price in thousands of US dollars, the original data was divided by 1000 to give this value
bedrooms	Numeric variable, number of bedrooms
bathrooms	Numeric variable, number of bathrooms
sqft_living	Numeric variable, area of house in square feet
sqft_lot	Numeric variable, area of whole housing lot in square feet
floors	Numeric variable, number of floors in the house
condition	Numeric variable, condition of house from 1 to 5
if_basement	Binary variable, 1 = if house has a basement, 0 = if house has no basement, this was originally showing the size of the basement, but not all houses had basements so it was changed into a binary variable
house_age	House Age in years, calculated by 2014 minus the year the house was built
if_renovated	Binary variables showing 1 = if house has been renovated, 0 = if no renovation, again this was originally showing the year of renovation however not all houses had been renovated and so it was changed to binary
city	Factor variable (32 levels) giving location of house to the nearest city in Washington, USA, any city that had less than 10 houses in it was removed

2.3 Data Visualisation

The cleaned data set was explored to look for any correlations between variables (Figure 1). The dependent variable (price) had a positive correlation with sqft_living, number of bathrooms and number of bedrooms. The strongest relationship seen was between sqft_living and number of bathrooms. It is interesting to note that the only non significant correlation involving price is between price and house age.

Figure 1: A correlation map showing the correlations between variables (excluding city, if_basement and if_renovated), the colours represent strength of correlation, the crosses show where the correlation is not significant.



3 Methods

A supervised learning approach was chosen throughout this analysis. In a supervised learning approach there is a continuous response variable, in this case house price, and a number of predictor variables. In all the approaches tried house price was a quantitative variable.

The dataset was split randomly into two parts, a training set containing 80% of the observations and a test set containing the remaining 20%. The training set was used to train all of the models and the test set to assess accuracy of the models.

3.1 Linear Regression

Firstly, a simple linear regression model was created using price as the dependent variable (y) and square foot living area (sqft_living) as the independent variable (x). Sqft_living was chosen because it was shown to be the most correlated variable to house price in the exploratory data analysis. A simple linear regression model uses the *lm()* function. This linear regression takes a model with equation

price = B0 + B1(sqft_living) + E

and estimates coefficients which produce a line of best fit, minimising the difference between predicted and actual values. This type of simple linear regression is known as ordinary least squares. In the equation above, B0 is the intercept, B1 is the coefficient produced by the model and E is the error term.

This simple model was then expanded to allow all the other variables in the dataset to act as predictor variables, this is multiple linear regression. The model output showed that 17 of these variables had a significant impact ($P<0.5$) on the price. The coefficients of each of these variables are shown below.

Table 2: Names and Coefficient estimates of variables that were significant in the multiple linear regression involving all predictor variables.

	Coef Estimates	P-Value
(Intercept)	-300.0848039	0.0004652
bedrooms	-58.3531453	0.0000065
bathrooms	68.2541014	0.0012979
sqft_living	0.2609683	0.0000000
condition	36.6402055	0.0231854
if_basement1	-58.8254709	0.0091500
house_age	1.0007524	0.0342776
cityBellevue	402.9644709	0.0000000
cityIssaquah	192.8171552	0.0023546
cityKent	192.0829810	0.0023943
cityKirkland	286.7290160	0.0000059
cityMedina	1191.9241550	0.0000000
cityMercerisland	529.6076728	0.0000000
cityRedmond	243.3809709	0.0000647
citySammamish	217.8271899	0.0008843
citySeattle	313.5117092	0.0000000
cityShoreline	158.5865198	0.0215383
cityWoodinville	143.9101433	0.0460584

It should be noted that *if_basement*, *if_renovated* and *city* were all factor variables, and so when they were fit into a model, they were converted into dummy variables, with one for each factor level. This is why some of the variables selected above are not the same as those variables shown in the cleaned data set. This also raises the idea that location has a large influence on house price as 11 of the variables shown above are all dummy variables originating from *city*.

Multicollinearity was also explored from the multiple linear model. The variance inflation factors (VIF) for each variable was calculated which shows how much of the variance of the regression coefficient is inflated due to any multicollinearity in the model. None of the VIF were above 5, with the highest being *bathrooms* at 3.3, and so there was no problematic collinearity indicated. This was carried out using the *vif()* function in the *car* package.

Polynomial models of regression were trialed but they did not improve RMSE results, so this approach was not looked into further. The relationship between sqft_living and price looked to be linear in the exploratory data analysis so this was not a surprising development.

3.2 Variable Selection Methods

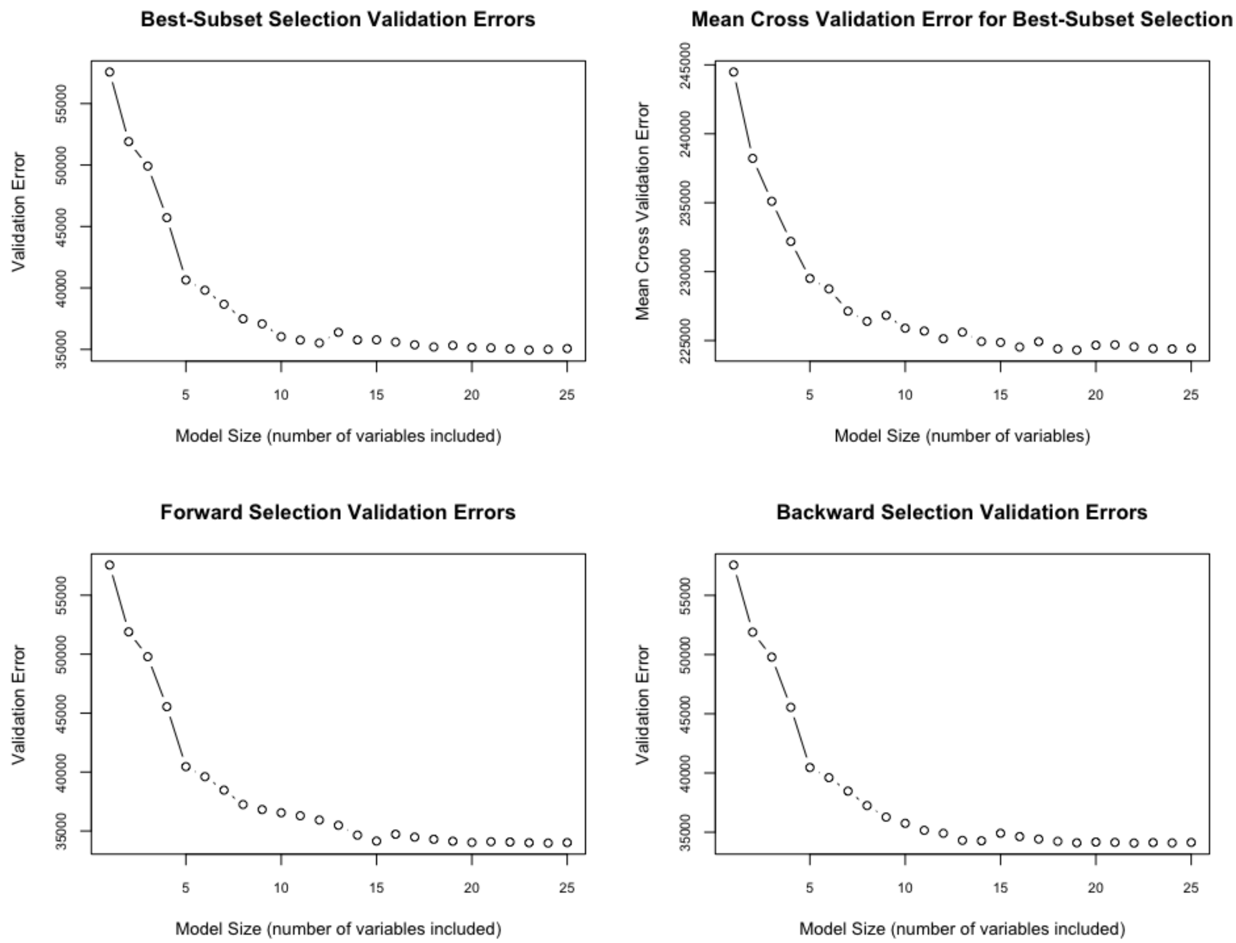
To look further into which variables were most influential on price, variable selection methods were explored. Best subset selection is a method that finds the best combinations of predictors that produce the best fit in terms of squared error. Forward stepwise selection is slightly different as it starts with no variables and one by one adds the variable which gives the smallest increase in squared error. Backward stepwise selection follows the same idea as forward selection but it starts with a full model, and iteratively removes variables until it leaves a one variable model with the lowest mean squared error.

In best subset selection, cross-validation was used to choose among variables of differing sizes. This is a direct method of estimating test error. A model with 15 variables had the lowest cross-validation error. These variables were extracted and a linear model created containing only them. This linear model had a lower MSE than the simple linear model with just one variable, but was higher, therefore less accurate than the multiple linear model created with just

If we had used R2 statistics to assess the models, we would always end up with a model containing all of the variables as the 'best' one. In this analysis direct methods of test error estimation were used, however as an alternative, adjusted R2, CP or BIC criterion may have been used to indirectly estimate the test error. Validation set approach was also take to estimate test error, this gave a model with many more variables and so it was decided that cross-validation was the most appropriate method to use to help narrow down the variables to the most influential.

The validation error at different model sizes for the three kinds of variable selection are shown in Figure 2, this also includes the cross-validation error for best-subset selection. Forward and backward selection yielded almost identical results to each other with only a one variable difference to best-subset selection. The specific variables selected from best-subset selection were also very similar to the ones that showed a significant effect in the multiple linear model, with only one variable different. Forward and backward subset selection methods gave lower MSE results than best-subset selection. Figure 2 shows that all of these methods produce extremely similar results.

Figure 2: The Validation and Cross Validation Error plots for best-subset, forward and backward selection.



3.3 Ridge Regression and LASSO

Ridge Regression and the LASSO are both shrinkage methods. In the above variable selection methods, only a subset of predictors are used. In shrinkage methods all of the predictors are included in the model but the coefficient estimates are constrained towards zero, this can help to reduce variance.

Ridge Regression utilises L2 regularisation, this adds a penalty to the coefficients that is equal to the square of the coefficients. Lambda is a tuning parameter, as its magnitude increases, the shrinkage penalty has more of an impact and the coefficient estimates will be closer to zero. Selecting the right value of lambda is very important, in this analysis it was selected using cross-validation. If lambda = 0, this method would be identical to ordinary least squares. Ridge regression does however always include all of the variables in the data set, this can lead to problems with interpretation.

LASSO is another coefficient shrinkage technique in which the L0 norm is replaced with the L1 norm. LASSO stands for Least Absolute Shrinkage and Selection Operator. The L1 norm applies a penalty to the coefficients equal to the absolute value of the coefficients. In this method, lambda allows some coefficients to be set equal to 0, in which case they are dropped out of the regression model. In this way it acts as a selection method for choosing the variables with the most influence. This decreases the variance of the model but increases the bias. We can change lambda to any value, but in this analysis cross-validation was used to select the best value of lambda. This method suggested a model containing 34 variables led to the lowest RMSE. A model was also created using an alternative value of lambda. This alternative model only contained 15 variables and is therefore more easily interpretable, the RMSE was only increased slightly but was still lower than any of the other models in this analysis.

3.4 Tree Based Methods

A few different tree based methods were explored in this analysis, starting with one simple decision tree, moving through bagging, randomForests and boosting. Tree based methods have the benefit of being easy to interpret but can sometimes over-simplify things. Trees are grown using branches which split due to conditions, eventually reaching an end node that gives the outcome, in this case the outcome is House Price. Tree methods have a limit on the number of variables, so the reduced data set from forward selection was used throughout the tree methods.

Using just one decision tree gave a MSE higher than using just ordinary least squares, the pruned tree with the lowest cross validation error had the same number of branches as the original decision tree. This simple decision tree only used one variable - sqft_living, suggesting this has largest effect on house price.

3.4.1 Random Forests and Bagging

RandomForest is a method that combines together multiple decision trees, this can help to improve prediction accuracy. Bagging is a type of randomForest, also known as Bootsrap Aggregation, in which the number of predictors (m) that is considered at each split of the tree is equal to the total number of predictors (p) in the data set. Random Forests only considers a subset of the predictors, usually sqrt(p) which in this case was around 3, at each split. Lowering the number of predictors considered at each split of the tree reduces variance and in this analysis led to a much improved model compared to randomForest where m = p.

3.4.2 Boosting

Boosting is another tree based method in which trees are grown sequentially, with each tree 'learning' from the last. It learns more slowly than other approaches and can reduce overfitting. Two different variations of a boosted model were created, with different values of lambda, the tuning parameter, and although reducing lambda improved MSE, it was not competitive with the multiple linear regression.

The tree based methods, although more easily interpretable, produced some high RMSE results. The best of these models were the randomForests with *mtry=3* and a smaller number of trees, the RMSE of these was similar to the simple linear model containing only one variable.

4 Results:

Link to Github repository containing fully reproducible methods script.

The objectives of this analysis were to understand which attributes of the houses can be used to most effectively construct a prediction model for house price, and to then minimise the differences between predicted and actual house price using model selection.

For regression problems the most common way to measure accuracy of a model is by minimising test error. The model that gave the lowest RMSE and therefore was the final approach chosen in this analysis is LASSO (Table 3), however ridge regression and multiple linear regression also gave competitive RMSE. Ridge regression is a far more complicated model than the LASSO as it does not drop out any predictor variables. For the sake of interpretability, the ridge regression model will not be further analysed here. The LASSO generally performs well when there is a few variables with a large influence on response, and a number of variables with a lesser influence, which is the case in this dataset.

Table 3: Results of each model attempted, with error shown as RMSE = Root Mean Squared Error.

Model	RMSE
Simple Linear Model	239.9351
Multiple Linear Model with all availanle predictors	187.2485
Mutiple Linear Model with only 15 predictors	187.8978
Linear Model, variables selected by best-subset selection	218.6456
Linear Model, variables selected by backward stepwise selection	215.3550
Linear Model, variables selected by forward stepwise selection	214.3310
LASSO Regression Model	186.8578
Ridge Regression Model	187.5832
Basic Decision Tree	255.7033
Bagging Model of randomForest, m = p	306.8430
Bagging Model, reduced to 100 trees	296.0593
randomForest, m = 3	234.2084
randomForest, m = 30, reduced to 30 trees	233.3519
Boosting with lambda = 0.1	647.3418
Boosting with lambda = 0.001	245.5804

It is clear that location has a large influence on house price as out of the 34 variables that are left in the model, 26 of them were location dummy variables. The size of the house (sqft_living) was a non-location variable that had the largest influence on house prices. To consider which of the variables had the largest influence on price, a linear model was constructed this time without the location variables, from this the *sqft_living* was the most significant, followed by *bedrooms*, *bathrooms* and *house_age*. This agrees with the plots of influence seen from the randomForest plots that showed these variables to be most influential.

The LASSO model with the lowest RMSE contained 34 variables, however by changing the lambda value given, the variables can be dropped down to 15, with only a slight increase in RMSE (still lower than any of the other models not brought forward to the results section). These variables are very similar to those that showed significance in the multiple linear regression model carried out with all of the variables.

The difference between the RMSE of the LASSO and the RMSE of the multiple linear regression containing only 15 variables (those that were significant in the multiple linear regression containing all predictors) is very small. The multiple linear regression is again more interpretable than the LASSO. The diagnostic plots of this multiple linear regression are shown below. From the first of the diagnostic plots it is clear that the assumption of linear regression holds true due to the straight horizontal line at 0 we can see. The points follow the QQ plot line, but there are some tails at either ends, this may suggest that the residuals are not normally distributed. The third plot checks for homogenity of variance of the residuals, it is clear that the red line is not horizontal. A Breusch Pagen Test was carried on this linear regression model (P = 0.1304, df = 17) leading to the the null hypothesis being accepted, concluding there is homoscedasticity. This is good as it meant no further transformations were needed on this model. The final graph checks for any points with high leverage using Cook's distance. None of the points in this data set lie outside of this distance, although a few are near the border.

4.1 Conclusions:

In the case of this dataset, a LASSO model containing 34 gave the lowest RMSE. However a similarly acceptable low score was given by a multiple linear regression containing only 15 variables which is much easier to interpret due to it containing less variables. The predictor variables that had the largest influence on House Price in this dataset were house size (sqft_living), number of bedrooms and bathrooms and house age. By looking at the coefficients produced by the various models it is clear that some of the location dummy variables had large impacts, in particular houses in Seattle, Mercer Island, Medina and Bellevue had more expensive homes, and those in Federal Way had less expensive homes.

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