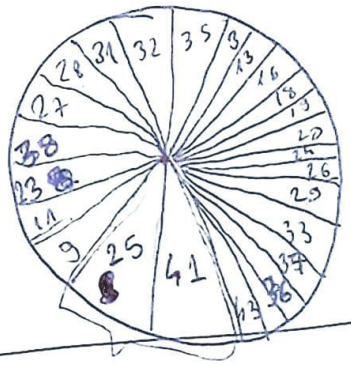
1

1.

a)

x	3	9	11	13	16	18	19	20	23	24	25	26	27	28	29	31	32	33	35	36	37	38	41	43	
n	1	2	2	1	1	1	1	1	2	1	3	1	2	2	1	2	2	1	2	1	1	2	3	1	N=37
p	2,7	5,4	5,4	2,7	2,7	2,7	2,7	2,7	5,4	2,7	8,1	2,7	5,4	5,4	2,7	5,4	5,4	2,7	5,4	2,7	2,7	5,4	8,1	2,7	100%
α	9,7	19,4	19,4	9,7	9,7	9,7	9,7	9,7	19,4	9,7	29,1	9,7	19,4	19,4	9,7	19,4	19,4	9,7	19,4	9,7	9,7	19,4	29,1	9,7	360°

b)



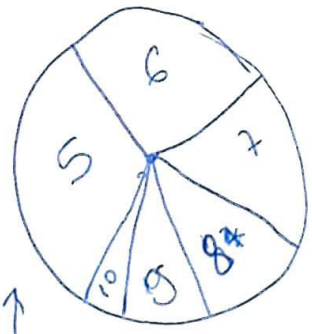
V

2.

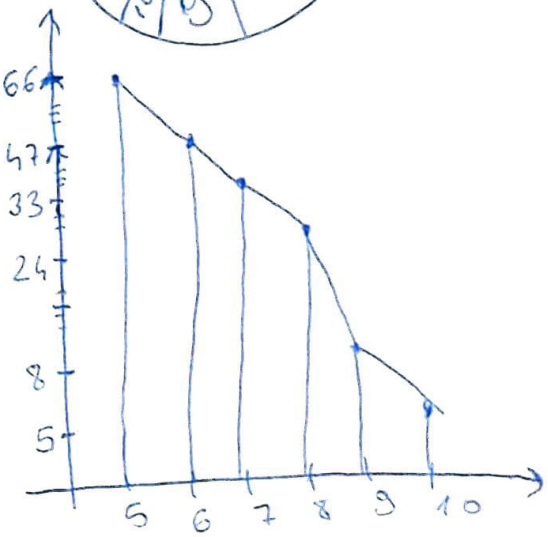
x	5	6	7	8	9	10	
n	66	47	33	24	8	5	N=183
p	36,06	25,68	18,03	13,11	4,37	2,73	100%
α	129,8	92,4	64,9	47,19	15,7	9,828	360°

V

a)



b)



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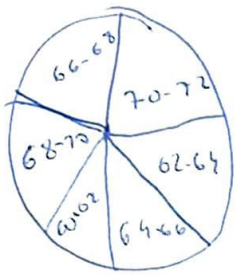
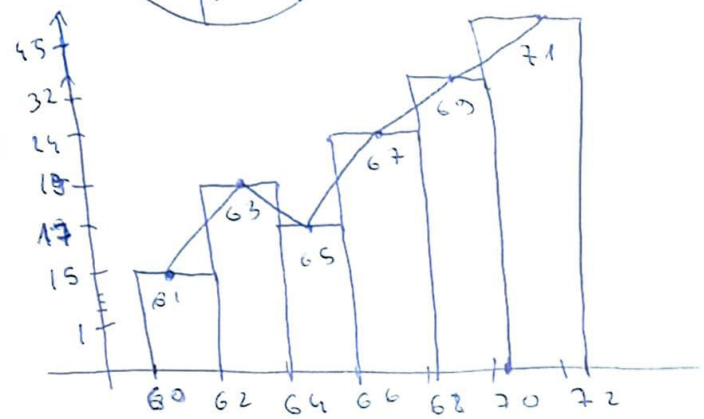
3.

x_i	60-62	62-64	64-66	66-68	68-70	70-72	
f_i	15	19	17	24	32	45	N=152
x_i	61	63	65	67	69	71	
p_i	9,86	12,5	11,18	15,78	21,05	29,60	100%
x_i	35,52	45	40,26	56,80	75,78	106,56	

a)



b)



V

4.

a) $\bar{X} = \frac{18+11+41+37+41+9+32+20+26+24+16+25+19+\dots+25}{31} =$

$= \frac{884}{31} = \underline{28,51}$

V

b) $H = \frac{31}{\frac{2}{3}+\frac{1}{11}+\frac{1}{16}+\frac{2}{18}+\frac{1}{20}+\frac{2}{23}+\frac{1}{24}+\frac{2}{25}+\frac{1}{26}+\frac{1}{27}+\frac{1}{28}+\frac{1}{29}+\frac{2}{31}+\frac{2}{32}+\dots}$

$\dots + \frac{1}{33}+\frac{1}{35}+\frac{1}{36}+\frac{1}{37}+\frac{2}{38}+\frac{3}{41}+\frac{1}{43}+\frac{1}{44}$

$= \frac{31}{0,22+0,09+0,06+0,11+0,05+0,08+0,04+0,08+0,03+0,03+0,03+0,03+0,06+0,06+0,03+0,02+0,02+0,05+0,07+0,02+0,02}$

$= \frac{31}{1300} = \underline{23,8462}$

4. c) Moda = 41

$Me = X_{\frac{N+1}{2}} = X_{\frac{32}{2}} = X_{16} = 29$ N-Teki

5.

X	18	30	36	42	48	54	
fi	3	4	10	12	3	1	N=33

a) Mesi Aritmetik:

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i f_i}{N} = \frac{54+120+360+504+144+54}{33} = \frac{1236}{33} = 37,45$$

- Mesi Gjeometrik: $G = 10^A$, $A = \frac{\sum_{i=1}^{n} f_i \lg X_i}{N}$

X	18	30	36	42	48	54	
fi	3	4	10	12	3	1	N=33
lgX	1,255298	1,477162	1,556350	1,623303	1,681300	1,732458	
fi lgX	3,77	5,91	15,56	19,48	5,04	1,73	Σ=51,5

$\frac{51,5}{33} = 1,56060$

$G = 10^{1,56060} = 36,358$ | $G = 36,358$

- Mesi Harmonik: $H = \frac{N}{\sum_{i=1}^{n} \frac{f_i}{X_i}}$

$H \leq G \leq \bar{X}$

$30,10 \leq 36,35 \leq 37,45$

$$H = \frac{33}{\frac{3}{18}+\frac{4}{30}+\frac{10}{36}+\frac{12}{42}+\frac{3}{48}+\frac{1}{54}} = \frac{33}{0,16+0,13+0,28+0,29+0,06+0,02}$$

$$= \frac{33}{0,94} = 35,10$$

b) Mesi Kotrer: $\bar{X}_2 = \sqrt{\frac{\sum f_i X_i^2}{N}}$

$$\bar{X}_2 = \sqrt{\frac{972+3600+12360+21168+6912+2916}{33}} = \sqrt{\frac{48528}{33}} = \sqrt{1470,}$$

$= 38,35$

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5.

b) - Mean kubik: $\bar{X}_3 = \sqrt[3]{\frac{\sum X^3 \cdot f_i}{N}}$

$$\bar{X}_3 = \sqrt[3]{\frac{17496 + 108000 + 466560 + 689056 + 331776 + 157464}{33}}$$

$$\bar{X}_3 = \sqrt[3]{\frac{1970352}{33}} = \sqrt[3]{59676.636} = 39,085$$ ✓

c) Modus = 42 ✓

Median = 36 ✓

N-Teke | $K = \frac{N+1}{2} = \frac{34}{2} = 17$

X	18	30	36	42	48	54	
f	3	4	10	12	3	1	N=33
f.k	3	7	17	29	32	33	

↑ fp => $X_p = 36$

6.

X	60-62	62-64	64-66	66-68	68-70	70-72	72-74	74-76	
f	8	7	28	30	15	15	8	9	N=120
X'	61	63	65	67	69	71	73	75	

a) ~~Mean geometric~~

$$\bar{X} = \frac{\sum X'_i f_i}{N} = \frac{488 + 441 + 1820 + 2010 + 1035 + 1065 + 584 + 675}{120}$$

$$= \frac{8118}{120} = 67,65$$ ✓

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3

6.

a) Mesi Gjeometrik:

X	60-62	62-64	64-66	66-68	68-70	70-72	72-74	74-76	
f	8	7	28	30	15	15	8	9	N=120
X'	61	63	65	67	69	71	73	75	
lgX'	1,78533	1,79934	1,81291	1,82607	1,83885	1,85126	1,86332	1,87506	
flgX'	14,28264	12,59538	50,76148	54,7821	27,57275	27,7689	14,90656	16,87554	Σ=219,55535

$$A = \frac{219,55535}{120} = 1,829628$$

$$G = 10^{1,829628} = 67,5504$$ ✓

- Mesi Harmonik:

$$H = \frac{120}{\frac{8}{61} + \frac{7}{63} + \frac{28}{65} + \frac{30}{67} + \frac{15}{69} + \frac{15}{71} + \frac{8}{73} + \frac{9}{75}} =$$

$$= \frac{120}{0,13 + 0,11 + 0,43 + 0,45 + 0,22 + 0,21 + 0,11 + 0,12} = \frac{120}{1,78} = 67,41$$ ✓

$$H \leq G \leq \bar{X}$$

b) - Mesi Katrorë

$$\bar{X}_2 = \sqrt{\frac{29768 + 27783 + 111300 + 134670 + 71415 + 75615 + 42632 + 50625}{120}}$$

$$\bar{X}_2 = \sqrt{\frac{550808}{120}} = \sqrt{4590,06} = 67,75$$ ✓

- Mesi Kubik

$$\bar{X}_3 = \sqrt[3]{\frac{1815848 + 1750329 + 7689500 + 9022890 + 4927635 + 5368665 + 3112136 + 3796875}{120}}$$

$$= \sqrt[3]{\frac{37483878}{120}} = \sqrt[3]{312365,65} = 67,85$$ ✓

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6.

c) $\text{Moda} = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \cdot d = 66 + \frac{2}{15} \cdot 2 = 66 + \frac{4}{15} = 66 + 0{,}267$

$= \underline{66{,}267}$ ✓

$L = 66$

$d = 2$

$\Delta_1 = 30 - 28 = 2$

$\Delta_2 = 30 - 15 = 15$

Mediana = 67 ✓

X	60-62	62-64	64-66	66-68	68-70	70-72	72-74	74-76	
f	8	7	28	30	15	15	8	9	N=120
f_k	8	15	43	73	88	103	111	120	
x'	61	63	65	67	69	71	73	75	

$73 > 60 \Rightarrow M_e = X'_p = 67$

N-sistē

$k = \frac{120}{2} = 60$

7. 2, 2, 3, 3, 4, 5, 5, 6, 6, 7, 8, 9, 9

a) $\bar{X} = \frac{4+6+4+10+12+7+8+18}{13} = \frac{69}{13} = \underline{5{,}31}$

$d = \frac{|2-5{,}31| \cdot 2 + |3-5{,}31| \cdot 2 + |4-5{,}31| + |5-5{,}31| \cdot 2 + |6-5{,}31) \cdot 2 + |7-5{,}31|}{13}$

$+ |8-5{,}31| + |9-5{,}31) \cdot 2 = \frac{6{,}62 + 4{,}68 + 1{,}31 + 0{,}31 + 1{,}38 + 1{,}69 + 2{,}69 + 7{,}38}{13}$

$= \frac{26{,}37}{13} = \underline{2{,}03}$ ✓

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7.

b)

$$s = \sqrt{\frac{(2-5{,}31)^2 \cdot 2 + (3-5{,}31)^2 \cdot 2 + (4-5{,}31)^2 + (5-5{,}31)^2 \cdot 2 + (6-5{,}31)^2 \cdot 2 + (7-5{,}31)^2 + (8-5{,}31)^2 + (9-31)^2}{13}}$$

$$s = \sqrt{\frac{21{,}91 + 10{,}67 + 1{,}72 + 0{,}19 + 0{,}95 + 2{,}86 + 7{,}24 + 27{,}23}{13}} =$$

$$s = \sqrt{\frac{72{,}77}{13}} = \sqrt{5{,}598} = 2{,}37 \quad \checkmark$$

8. 1, 2, 2, 3, 3, 4, 5, 5, 6, 7, 8, 9 $\qquad \bar{x} = \frac{55}{12} = 4{,}58$

a) $$d = \frac{|1-4{,}58| + |2-4{,}58| \cdot 2 + |3-4{,}58| \cdot 2 + |4-4{,}58| + |5-4{,}58| \cdot 2 + |6-4{,}58| + |7-4{,}58| + |8-4{,}58|}{12}$$

$$= \frac{3{,}58 + 5{,}16 + 3{,}16 + 0{,}58 + 0{,}84 + 1{,}42 + 2{,}42 + 3{,}42 + 4{,}42}{12} =$$

$$= \frac{25}{12} = 2{,}083 \quad \checkmark$$

b)

$$s = \sqrt{\frac{12{,}81 + 13{,}31 + 4{,}99 + 0{,}34 + 0{,}35 + 2{,}02 + 5{,}86 + 11{,}70 + 19{,}54}{12}}$$

$$s = \sqrt{\frac{70{,}92}{12}} = \sqrt{5{,}91} = 2{,}43 \quad \checkmark$$

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9.

x	18	30	36	42	48	54	
f	3	4	10	12	3	1	$N = 33$

a) $d = \frac{\sum_{i=1}^{n} |x_i - \bar{x}| \cdot f_i}{N}$

$\bar{x} = \frac{54 + 120 + 360 + 504 + 144 + 54}{33} = \frac{1236}{33} = 37{,}45$

$d = \frac{58{,}35 + 29{,}8 + 14{,}5 + 54{,}6 + 31{,}65 + 16{,}55}{33} = \frac{205{,}45}{33} = 6{,}23$ ✓

b) $\sigma = \sqrt{\frac{\sum (x - \bar{x})^2 \cdot f_i}{N}}$

$\sigma = \sqrt{\frac{1134{,}90 + 222{,}01 + 21{,}07 + 869{,}50 + 3339{,}1 + 273{,}90}{33}}$ ✓

$S = \sqrt{\frac{2855{,}25}{33}} = \sqrt{86{,}52} = 9{,}30$

10.

x	5	8	9	12	14	16	
f	4	14	10	12	3	7	$N = 50$

$\bar{x} = \frac{20 + 112 + 90 + 144 + 42 + 112}{50} = \frac{520}{50} = 10{,}4$ ✓

a) $\sigma = \sqrt{\frac{116{,}64 + 80{,}64 + 19{,}4 + 30{,}72 + 38{,}88 + 219{,}52}{50}} = \sqrt{\frac{505{,}8}{50}} = \sqrt{10{,}116} = 3{,}18$ ✓

b) $d = \frac{216 + 33{,}6 + 14 + 19{,}2 + 10{,}8 + 39{,}2}{50} = \frac{138{,}4}{50} = 2{,}768$ ✓

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11.

X	60-62	62-64	64-66	66-68	68-70	70-72	72-74	74-76	
f	18	3	23	31	15	9	12	27	N=138
x_i	61	63	65	67	69	71	73	75	

a) $d = \frac{132,48+16,08+77,28+42,16+9,6+23,76+55,68+179,26}{138} = \frac{536,3}{138} = 3,89$

$\overline{X} = \frac{1098+189+1495+2077+1035+639+876+2025}{138} = \frac{9434}{138}$

$\overline{X} = 68,136$ ✓

b) $s = \sqrt{\frac{975,05+86,19+259,66+57,34+6,14+62,73+258,36+1190,15}{138}}$

$s = \sqrt{\frac{2895,62}{138}} = \sqrt{20,98} = 4,58$ ✓

c) $\frac{N}{4} = \frac{138}{4} = 34,5$

X	f	f·K	kvartili	
60-62	18	18	18	
62-64	3	21	3	
			13,5	Q1
64-66	23	44	9,5	
			25	Q2
66-68	31	75	6	
68-70	15	90		
70-72	9	99		
			4,5	Q3
72-74	12	111	7,5	
74-76	27	138		Q4

$Q_1 = 64 + 2 \cdot \frac{34,5-21}{23} = 64 + 1,17 = 65,17$

$Q_2 = 66 + 2 \cdot \frac{69-44}{31} = 66 + 1,613 = 67,61$

$Q_3 = 72 + 2 \cdot \frac{103,5-99}{12} = 72 + 0,75 = 72,75$

$Q = \frac{72,75-65,17}{2} = 3,79$ ✓

11.

d) 69

X	f	f.r	fkr
60-62	18	13,04	13,04
62-64	3	2,17	15,21
64-66	23	16,67	31,88
66-68	31	22,46	54,34
68-70]	15	10,87	65,21
70-72	9	6,52	71,73
72-74	12	8,70	80,43
74-76	27	19,57	100
	N=138		

$C = 54,33 + \left| \frac{69-60}{5} \right| 1,01$

$C = 54,33 + 2,174 = \underline{56,504}$

✓

10 20 4 8 2 34

6

12.

$\frac{148}{4} = 37$

a)

x	f	f.k	kvartiler
10-20	17	17	17
20-30	21	38	Q_1
30-40	32	70	32
40-50	13	83	Q_2
50-60	28	111	Q_3
60-70	22	133	
70-72	15	148	Q_4
	N=148		

?

$Q_1 = 20 + 10 \cdot \frac{37 - 17}{21} = 20 + 9{,}52 = 29{,}52$

$Q_2 = 40 + 10 \cdot \frac{74 - 70}{13} = 40 + 3{,}08 = 43{,}08$

$Q_3 = 50 + 10 \cdot \frac{111 - 83}{28} = 50 + 10 = 60$

$Q = \frac{60 - 29{,}52}{2} = 15{,}24$ ✓

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12.

b)

x	f	fr	fk.r
10-20	17	11,49	11,49
20-30	21	14,19	25,68
30-40	32	21,62	47,3
40-50	13	8,78	56,08
50-60	28	18,92	75
60-70	22	14,86	89,6
70-72	15	10,14	100
	148		

$x = 47$

$$C = 47{,}3 + \left| \frac{47-40}{4} \cdot 8{,}78 \right|$$

$$C = 47{,}3 + 15{,}365 = 62{,}67$$ ✓

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13. a) 4, 1, 4, 2, 4, 3, 4, 4, 4, 5, . . .

a_n: 4, 0, 4, 0, 4, 0, 4, 0 . . . $f(x) = \frac{4}{1-x^2}$

b_n: 0, 1, 0, 2, 0, 3, 0, 4, ... $\frac{X}{(1-x^2)^2}$

Duke shikuar natim e vargjeve të marrur në numrave natyrore $f(x) = \frac{1}{1-X}$ => në rastin tonë

$f(x) = \frac{1}{1-x^2}$, nga kjo rrjedh në $\frac{1}{1-x^2} \cdot \frac{X}{1-x^2} = \frac{X}{(1-x^2)^2}$

$a_n + b_n = \frac{4}{1-x^2} + \frac{X}{}$

12.

b)

X	f	fr	fk.n
10-20	17	11,49	11,49
20-30	21	14,19	25,68
30-40	32	21,62	47,3
40-50	13	8,78	56,08
50-60	28	18,92	75
60-70	22	14,86	89,6
70-72	15	10,14	100
	148		

a. $X = 47$

$$C = 47,3 + \left| \frac{47 - 40}{4} \cdot 8,78 \right|$$

$$C = 47,3 + 15,365 = 62,67 \quad \checkmark$$

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13.

a) 4, 1, 4, 2, 4, 3, 4, 4, 4, 5, ...

a_n: 4, 0, 4, 0, 4, 0, 4, 0, 4, 0 - - - - $f_1(x) = \frac{4}{1-x^2}$

b_n: 0, 1, 0, 2, 0, 3, 0, 4, 0, 5, - - - - $f_2(x) = x \cdot \frac{1}{(1-x^2)^2} = \boxed{\frac{x}{(1-x^2)^2}}$

Duke u bazuar në rreshtin e numrave natyrorë i cili fitohet nga prodhimi i të dy funksioneve 1, 1, 1, 1, ... ($\frac{1}{1-x}$) ; => ($\frac{1}{(1-x)^2}$)

Në rastin tonë rreshti 1, 0, 2, 0, 3, 0, 4, 0, 5, ... do fitohet si prodhim i funksionit $\frac{1}{1-x^2}$ 1, 0, 1, 0, 1, 0,

pra $\frac{1}{1-x^2} \cdot \frac{1}{1-x^2} = \boxed{\frac{1}{(1-x^2)^2}}$

$a_n + b_n = \frac{4}{1-x^2} + \frac{x}{(1-x^2)^2} = \boxed{\frac{4-4x^2+x}{(1-x^2)^2}}$ ✓

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7

13.

b) $0,0,0,0,0,1,4,1,4,1,4,\ldots$ $\Rightarrow \frac{X^5}{1-X} = f(x_1)$

$0,0,0,0,0,1,1,1,1,1,1,\ldots$ $\Rightarrow \frac{3\cdot X^6}{1-X^2} = f(x_2)$

$0,0,0,0,0,0,3,0,3,0,3,\ldots$

$0,0,0,0,0,1,4,1,4,1,4,\ldots$

$f(x) = f(x_1) + f(x_2) = \frac{X^5}{1-X} + \frac{3\cdot X^6}{1-X^2} = \frac{4X^6 + X^5}{1-X^2}$ ✓

c) $2,3,4,2,2,2,2,2,\ldots$

$f(X) = 2+3X+4X^2+2X^3+2X^4+2X^5+2X^6+\ldots =$

$= 2+3X+4X^2+2X^3(1+X+X^2+X^3+\ldots) =$

$= 2+3X+4X^2+\frac{2X^3}{1-X} = \frac{2+X+X^2-2X^3}{1-X}$ ✓

d) $3,5,1,1,1,1,1,1,\ldots$

$f(X) = 3+5X+X^2+X^3+X^4+X^5+X^6+\ldots =$

$= 3+5X+X^2(1+X+X^2+X^3+X^4+\ldots) =$

$= 3+5X+\frac{X^2}{1-X} = \frac{3+2X-4X^2}{1-X}$ ✓

e) $0,0,0,0,0,0,0,0,1,2,3,4,5,\ldots$

$f(X) = X^8 \cdot f_1(X) \cdot \frac{1}{1-X} = X^8 \cdot \frac{1}{(1-X)^2} = \frac{X^8}{(1-X)^2}$ ✓

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14.

a) $(1-x^3)^3 = \sum_{k=0}^{n} C_n^k \cdot x^k = \sum_{k=0}^{3} \underbrace{C_3^k}_{a_k} \cdot x^k \cdot (-1)^k$

$a_k = C_3^k \cdot (-1)^k$

$a_0 = C_3^0 = 1 \cdot (-1)^0 = 1$

$a_1 = C_3^1 = 3 \cdot (-1)^1 = -3$

$a_2 = C_3^2 = \frac{3!}{2! \cdot 1!} = \frac{3!}{2!} = 3 \cdot (-1)^2 = 3$

$a_3 = C_3^3 = \frac{3!}{3! \cdot 1} = 1 \cdot (-1)^3 = -1$ ✓

a_n: 1, -3, 3, -1, 0, 0, 0, ...

---

b) $(1-2x)^4 = \sum_{k=0}^{4} C_4^k \cdot (-2x)^k = \sum_{k=0}^{4} C_4^k \cdot (2x)^k \cdot (-1)^k = \sum_{k=0}^{4} C_4^k \cdot 2^k \cdot x^k \cdot (-1)^k$

$a_k = C_4^k \cdot 2^k \cdot (-1)^k$

$a_0 = C_4^0 \cdot 2^0 \cdot (-1)^0 = 1$

$a_1 = C_4^1 \cdot 2^1 \cdot (-1)^1 = -8$

4 · 2 · -1

$a_2 = C_4^2 \cdot 2^2 \cdot (-1)^2 = 24$

6 · 4 · 1

$a_3 = C_4^3 \cdot 2^3 \cdot (-1)^3 = -32$

4 · 8 · -1

$a_4 = C_4^4 \cdot 2^4 \cdot (-1)^4 = 16$

a_n: 1, -8, 24, -32, 16, 0, 0, 0, ... ✓

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

8

14.

c) $\frac{1}{(1-3X)^4} \Rightarrow (1-3X)^{-4}$

$(1-3X)^{-4} = \sum_{k=0}^{\infty} (-1)^k \cdot C_{4+k-1}^{k} \cdot (-3X)^k = \sum_{k=0}^{\infty} (-1)^k \cdot C_{3+k}^{k} \cdot 3^k \cdot X^k \cdot (-1)^k$

$(-1)^k \cdot (-1)^k = 1$

$= \sum_{k=0}^{\infty} \underbrace{C_{3+k}^{k} \cdot 3^k}_{a_k} \cdot X^k$

$a_k = C_{3+k}^{k} \cdot 3^k$

$a_0 = C_3^0 \cdot 3^0 = 1$

$a_1 = C_4^1 \cdot 3^1 = 4 \cdot 3 = 12$

$a_2 = C_5^2 \cdot 3^2 = 10 \cdot 9 = 90$

$a_3 = C_6^3 \cdot 3^3 = 20 \cdot 27 = 540$

$a_4 = C_7^4 \cdot 3^4 = 35 \cdot 81 = 2835$ ✓

$a_n: 1, 12, 90, 540, 2835, \ldots$

---

d) $\frac{1}{(1+X)^3} \Rightarrow (1+X)^{-3}$

$(1+X)^{-3} = \sum_{k=0}^{\infty} (-1)^k C_{3+k-1}^{k} X^k = \sum_{k=0}^{\infty} \underbrace{(-1)^k C_{2+k}^{k}}_{a_k} \cdot X^k$

$a_k = (-1)^k \cdot C_{2+k}^{k}$

$a_0 = (-1)^0 \cdot C_2^0 = 1$

$a_1 = (-1)^1 \cdot C_3^1 = -3$

$a_2 = (-1)^2 \cdot C_4^2 = 6$

$a_3 = (-1)^3 \cdot C_5^3 = -1 \cdot \frac{5!}{3! \cdot 2!} = -\frac{5 \cdot 4 \cdot 3 \cdot 2!}{6 \cdot 2!} = -\frac{60}{6} = -10$

$a_3 = -10$

$a_4 = (-1)^4 \cdot C_6^4 = \frac{6 \cdot 5 \cdot 4!}{4! \cdot 2!} = 15$ ✓

$a_n: 1, -3, 6, -10, 15, \ldots$

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14.

e) $\frac{1}{(1-2x)^2} \Rightarrow (1-2x)^{-2}$

$(1-2x)^{-2} = (-1)^k \cdot C_{1+k}^{k} \cdot (-1)^k \cdot (2)^k \cdot (x)^k =$

$= \underbrace{C_{1+k}^{k} \, 2^k}_{a_k} \cdot x^k$

$\boxed{a_k = C_{1+k}^{k} \cdot 2^k}$

$a_0 = C_1^0 \cdot 2^0 = 1$

$a_1 = C_2^1 \cdot 2^1 = 2 \cdot 2 = 4$

$a_2 = C_3^2 \cdot 2^2 = 3 \cdot 4 = 12$

$a_3 = C_4^3 \cdot 2^3 = 4 \cdot 8 = 32$

$a_4 = C_5^4 \cdot 2^4 = 5 \cdot 16 = 80$ ✓

$a_n: 1, 4, 12, 32, 80, \ldots$

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15.

a) $\begin{cases} a_n = 2a_{n-1} + 3a_{n-2} \\ a_0 = 1,\ a_1 = 3 \end{cases}$, $n \geq 2$

$a_n - 2a_{n-1} - 3a_{n-2} = 0$

$t^2 - 2t - 3 = 0$

$1 \quad -2 \quad -3$

$\Delta = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} = \begin{matrix} t_1 = 3 \\ t_2 = -1 \end{matrix}$

$a_n = C_1 t_1^n + C_2 \cdot t_2^n$

$a_n = C_1 3^n + C_2 (-1)^n$

$n = 0,\ a_0 = C_1 \cdot 3^0 + C_2 \cdot (-1)^0$

$\boxed{1 = C_1 + C_2}$

$n = 1,\ a_1 = C_1 \cdot 3^1 + C_2 (-1)^1$

$\boxed{3 = 3C_1 - C_2}$

$\begin{cases} C_1 + C_2 = 1 \\ 3C_1 - C_2 = 3 \end{cases}$ $(+) \Rightarrow 4C_1 = 4 \Rightarrow \boxed{C_1 = 1}$

$1 + C_2 = 1 \Rightarrow \boxed{C_2 = 0}$

$a_n = 1 \cdot 3^n + 0 \cdot (-1)^n \Rightarrow \boxed{a_n = 3^n}$ ✓

Prova:

$a_0 = 3^0 + 0 \cdot (-1)^0 = 1$ ✓

$a_1 = 3^1 = 3$ ✓

15.

b) $\begin{cases} a_n = 2a_{n-1} + 3a_{n-2} + 3^n = 0 \\ a_0 = 0,\ a_1 = 1 \end{cases}$ $n \geq 2$

I $a_n - 2a_{n-1} - 3a_{n-2} = 0$

$t^2 - 2t - 3 = 0$

$\Delta = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} = \begin{matrix} t_1 = 3 \\ t_2 = -1 \end{matrix}$

II $a_n = C_1 t_1^n + C_2 \cdot t_2^n + C_3 \cdot n \cdot 3^n$

$a_n = C_1 \cdot 3^n + C_2 \cdot (-1)^n + C_3 \cdot n \cdot 3^n$

$n=0,\ a_0 = C_1 \cdot 3^0 + C_2 \cdot (-1)^0 + C_3 \cdot 0 \cdot 3^0$

$\boxed{0 = C_1 + C_2}$

$n=1,\ a_1 = C_1 \cdot 3 + C_2 \cdot (-1) + C_3 \cdot 3^1$

$\boxed{1 = 3C_1 - C_2 + 3C_3}$

$n=2,\ a_2 = ?\ \Rightarrow a_2 = 2a_{2-1} + 3a_{2-2} + 3^2$

$a_2 = 2 \cdot 1 + 3 \cdot 3^0 + 3^2$

$\boxed{a_2 = 11}$

$a_2 = C_1 \cdot 3^2 + C_2 \cdot (-1)^2 + C_3 \cdot 2 \cdot 3^2$

$\boxed{11 = 3C_1 + C_2 + 18C_3}$

$$\Delta = \begin{vmatrix} 1 & 1 & 0 & 1 & 1 \\ 3 & -1 & 3 & 3 & -1 \\ 9 & 1 & 18 & 9 & 1 \end{vmatrix} =$$

$= (-18 + 27 + 0) - (54 + 3 + 0)$

$= 9 - 57 = -48$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & -1 & 3 & 1 & -1 \\ 11 & 1 & 18 & 11 & 1 \end{vmatrix} =$$

$= (33 + 0) - (18 + 0) = 33 - 18 = 15$

$\boxed{C_1 = \frac{15}{-48} = \frac{5}{-16}}$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 0 & 1 & 0 \\ 3 & 1 & 3 & 3 & 1 \\ 9 & 11 & 18 & 9 & 11 \end{vmatrix} =$$

$= (18 + 0 + 0) - (0 + 33 + 0)$

$= 18 - 33 = -15$

$C_2 = \frac{-15}{-48} = \frac{15}{48} = \frac{5}{16}$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 0 & 1 & 1 \\ 3 & -1 & 1 & 3 & -1 \\ 9 & 1 & 11 & 9 & 1 \end{vmatrix} =$$

$= (-11 + 9 + 0) - (33 + 1 + 0) =$

$= -2 - 34 = -36$

$C_3 = \frac{-36}{-48} = \frac{36}{48} = \frac{3}{12} = \frac{3}{4}$

Proba: $\boxed{a_n = \left(-\frac{5}{16}\right) \cdot 3^n + \frac{5}{16} \cdot (-1)^n + \frac{3}{4} \cdot n \cdot 3^n}$ ✓

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$a_0 = \left(-\frac{5}{16}\right) \cdot 3^0 + \frac{5}{16} \cdot (-1)^0 + \frac{3}{4} \cdot 0 \cdot 3^0$

$a_0 = -\frac{5}{16} + \frac{5}{16} = 0$ ✓

$a_1 = \left(-\frac{5}{16}\right) \cdot 3 + \frac{5}{16} \cdot (-1) + \frac{3}{4} \cdot 3 = -\frac{15}{16} - \frac{5}{16} + \frac{9}{4} = \frac{-20}{16} + \frac{9}{4} = \frac{-20+36}{16} = \frac{16}{16} = 1$ ✓

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15.

c) $\begin{cases} a_{n+2} = 4a_{n+1} - 4a_n + 2^n \\ a_0 = 0,\ a_1 = 0 \end{cases} \quad | n \geq 2$

$a_{n+2} - 4a_{n+1} + 4a_n = 0$

$t^2 - 4t + 4 = 0$

$\Delta = \frac{4 \pm \sqrt{16-16}}{2} = \frac{4 \pm 0}{2} = 2 = t_1$ - nemoje e nhuma fiindca (2)

$a_n = C_1 \cdot t_1^n + C_2 \cdot n \cdot t_1^n + C_3 \cdot n^2 \cdot 2^n$

$a_n = C_1 \cdot 2^n + C_2 \cdot n \cdot 2^n + C_3 \cdot n^2 \cdot 2^n$

$n = 0,\ a_0 = C_1 + C_2 \cdot 0 \cdot 2^0 + C_3 \cdot 0^2 \cdot 1$

$0 = C_1$

$n = 1,\ a_1 = C_1 \cdot 2 + C_2 \cdot 2 + C_3 \cdot 2$

$0 = 2C_1 + 2C_2 + 2C_3$

$n = 2,\ a_2 = ?$

$a_{2+0} = 4a_{0+1} - 4a_0 + 2^0$

$a_2 = 4 \cdot 0 - 4 \cdot 0 + 1$

$a_2 = 1$

$1 = 4C_1 + 8C_2 + 16C_3$

Proba: $a_n = \left(-\frac{1}{8}\right) \cdot n \cdot 2^n + \frac{1}{8} n^2 \cdot 2^n$

$a_0 = 0$ ✓

$a_1 = -\frac{2}{8} + \frac{2}{8} = 0$ ✓

$\begin{cases} C_1 = 0 \\ 2C_2 + 2C_3 = 0 \\ 8C_2 + 16C_3 = 1 \mid \cdot \frac{1}{4} \end{cases} \Rightarrow \begin{cases} 2C_2 + 2C_3 = 0 \\ 2C_2 + 4C_3 = \frac{1}{4} \end{cases} (-) \Rightarrow -2C_3 = -\frac{1}{4} \Rightarrow C_3 = \frac{1}{8}$

$2C_2 + 2 \cdot \frac{1}{8} = 0$

$2C_2 = -\frac{1}{4} \Rightarrow C_2 = -\frac{1}{8}$

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15.

d) $\begin{cases} a_{n+3} = 6a_{n+2} - 11a_{n+1} + 6a_n \\ a_0 = 2,\ a_1 = 0,\ a_2 = -2 \end{cases}$

$a_{n+3} - 6a_{n+2} + 11a_{n+1} - 6a_n = 0$

$t^3 - 6t^2 + 11t + 6 = 0$

$t^3 - 6t^2 + 6t + 5t - 5 - 1 = 0$

$(t^3 - 1) - 6t(t-1) + 5(t-1) = 0$

$(t-1)(t^2 + t + 1) - 6t(t-1) + 5(t-1) = 0$

$(t-1) \cdot ((t^2 + t + 1) - 6t + 5) = 0$

$(t-1) \cdot (t^2 - 5t + 6) = 0$

$(t-1) = 0 \Rightarrow \underline{t_1 = 1}$

$t^2 - 5t + 6 = 0 \Rightarrow \boxed{t_2 = 3 \mid t_3 = 2}$

$\boxed{a_n = C_1 \cdot t_1^n + C_2 \cdot t_2^n + C_3 \cdot t_3^n}$

$n = 0,\ 2 = C_1 \cdot 1 + C_2 \cdot 1 + C_3 \cdot 1 \Rightarrow \boxed{C_1 + C_2 + C_3 = 2}$

$n = 1,\ \boxed{0 = C_1 + 3C_2 + 2C_3}$

$n = 2,\ \boxed{-2 = C_1 + 3C_2 + 4C_3}$

$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 9 & 4 \end{vmatrix} \begin{matrix} 1 & 1 \\ 1 & 3 \\ 1 & 9 \end{matrix} = (12 + 2 + 9) - (4 + 18 + 3) = 23 - 25 = \underline{-2}$

$\Delta_1 = \begin{vmatrix} 2 & 1 & 1 \\ 0 & 3 & 2 \\ -2 & 9 & 4 \end{vmatrix} \begin{matrix} 2 & 1 \\ 0 & 3 \\ -2 & 9 \end{matrix} = (24 + 4 + 0) - (0 + 36 - 6) = 28 - 30 = \underline{-10} \quad \Big| \quad C_1 = \frac{-10}{-2} = 5$

$\Delta_2 = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & 2 \\ 1 & -2 & 4 \end{vmatrix} \begin{matrix} 1 & 2 \\ 1 & 0 \\ 1 & -2 \end{matrix} = (4 - 2) - (8 - 4) = 2 - 4 = -2 \quad \Big| \quad C_2 = \frac{-2}{-2} = 1$

$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 0 \\ 1 & 9 & -2 \end{vmatrix} \begin{matrix} 1 & 1 \\ 1 & 3 \\ 1 & 9 \end{matrix} = (-6 + 12) - (-2 + 6) = 12 - 4 = \underline{8} \quad \Big| \quad C_3 = \frac{8}{-2} = -4$

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$\boxed{a_n = 5 \cdot 1 + 1 \cdot 3^n + (-4) \cdot 2^n}$