1. $m + m = 1050$

$[m, m] = 6468$

$(p, q) = 1$

$m = \frac{6468}{p}$, $m = \frac{6468}{q}$

$6468 = 6 \cdot 1050 + 168$

$1050 = 6 \cdot 168 + \boxed{42} \to$ PMP

$168 = 4 \cdot 42 + 0$

$\frac{6468}{p} + \frac{6468}{q} = 1050$

$\frac{6468q + 6468p}{p \cdot q} = 1050$

$6468(q+p) = 1050(p \cdot q) \,|\, (6468, 1050) = 42$

$154(q+p) = 25(p \cdot q)$

$\begin{cases} q + p = 25 \\ p \cdot q = 154 \end{cases} \Rightarrow \begin{cases} q = 25 - p \\ p \cdot q = 154 \Rightarrow p \cdot (25 - p) = 154 \Rightarrow -p^2 + 25p - 154 = 0 \ /\cdot(-1) \end{cases}$

$p^2 - 25p + 154 = 0$

$1 \quad -25 \quad 154$

$p_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{25 \pm \sqrt{625 - 616}}{2 \cdot 1} = \frac{25 \pm 3}{2} = \begin{matrix} \nearrow p_1 = 11 \\ \searrow p_2 = 14 \end{matrix}$

$q_1 = 25 - 11 = 14$

$q_2 = 25 - 14 = 11$

$m_1 = \frac{6468}{11} = 588$ $\qquad m_1 = \frac{6468}{14} = 462$

$m_2 = \frac{6468}{14} = 462$ $\qquad m_2 = \frac{6468}{11} = 588$

$\boxed{588 + 462 = 1050}$

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2.

9

$m - n = 102$

$[m, n] = 1547$

$(p, q) = 1$

$m = \frac{1547}{p}, \quad n = \frac{1547}{q}$

$\frac{1547}{p} - \frac{1547}{q} = 102$

$\frac{1547q - 1547p}{p \cdot q} = 102$

$1547(q - p) = 102 \cdot (p \cdot q) \quad /(1547, 102) = 17$

$91(q - p) = 6(p \cdot q)$

$1547 = 15 \cdot 102 + \boxed{17}$

$102 = 6 \cdot 17 + 0$

$\begin{cases} q - p = 6 \\ p \cdot q = 91 \end{cases} \Rightarrow \begin{cases} q = 6 + p \\ p \cdot q = 91 \Rightarrow p \cdot (6 + p) = 91 \Rightarrow p^2 + 6p - 91 = 0 \end{cases}$

$1 \quad 6 \quad -91$

$p_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 + 364}}{2} = \frac{-6 \pm 20}{2} = \begin{cases} p_1 = -13 \\ p_2 = 7 \end{cases}$

$q_1 = 6 + (-13) = -7$

$q_2 = 6 + 7 = 13$

$m_1 = \frac{1547}{-13} = -119$

$m_2 = \frac{1547}{7} = 221$

$m_1 = \frac{1547}{-7} = -221$

$m_2 = \frac{1547}{13} = 119$

$m_1 - m_2 = 102$

$-119 - (-221) = 102$

$\boxed{-119 + 221 = 102}$

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9

3. $(a,b)=27,\ [a,b]=324,\ a=\frac{3}{4}\cdot b$

$$[a,b]=\frac{a\cdot b}{(a,b)}$$

$$\boxed{a=81,\ b=108}$$

$$324=\frac{\frac{3}{4}b\cdot b}{27} \Big/ \cdot 27$$

$$8748=\frac{3}{4}\cdot b^2 \Big/ \cdot\frac{4}{3}$$

$$11664=b^2 \Big/ \sqrt{\ }$$

$$\underline{b=108}$$

$$a=\frac{3}{4}b=\frac{3\cdot 108}{4}=81$$

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4. $(a,b)=7,\quad a^2+b^2=490\quad ,(x,y)=1$

$$a=7\cdot x\ ,\ b=7\cdot y$$

$$(7x)^2+(7y)^2=490$$

$$49x^2+49y^2=490 \Big/ :49$$

$$x^2+y^2=10$$

x	y	(x,y)=1	$x^2+y^2=10$	
1	3	1	10	✓
3	1	1	10	✓

$$\boxed{\begin{aligned} a&=7\cdot 1=7\\ b&=7\cdot 3=21\end{aligned}}$$

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5. $m-n=38$

$[m,n]=2717$

$(p,q)=1$

$m=\dfrac{2717}{p}$, $n=\dfrac{2717}{q}$

$2717 = 71\cdot 38 + \boxed{19}$

$38 = 2\cdot 19 + 0$

$$\frac{2717}{p}-\frac{2717}{q}=38$$

$$\frac{2717q-2717p}{p\cdot q}=38$$

$$2717(q-p)=38\cdot(p\cdot q) \quad \Big| \; (2717,38)=19$$

$$143(q-p)=2(p\cdot q)$$

$$\begin{cases} q-p=2 \\ p\cdot q=143 \end{cases} \Rightarrow \begin{cases} q=2+p \\ p\cdot q=143 \end{cases} \Rightarrow p\cdot(2+p)=143 \Rightarrow \underset{1}{p^2}+\underset{2}{2p}\;\underset{-143}{-143}=0$$

$$p_{1/2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-2\pm\sqrt{4+572}}{2}=\frac{-2\pm 24}{2}= \begin{cases} p_1=-13 \\ p_2=11 \end{cases}$$

$q_1=2+(-13)=-11$

$q_2=2+11=13$

$m_1=\dfrac{2717}{-13}=-209$, $n_1=\dfrac{2717}{-11}=-247$

$m_2=\dfrac{2717}{11}=247$, $n_2=\dfrac{2717}{13}=209$

$-209+(-247)$

9

6. $\begin{cases} x/y = (210, 77) \\ x+y = [168, 244] \end{cases} \Rightarrow \begin{cases} \frac{x}{y} = 7 \\ x+y = 1464 \end{cases} \Rightarrow \begin{cases} x = 7y \\ x+y = 1464 \end{cases}$

$\Rightarrow \begin{cases} x = 7y \\ 7y + y = 1464 \Rightarrow 8y = 1464 \Rightarrow y = 183 \end{cases}$

$\boxed{\begin{aligned} y &= 183 \\ x &= 1281 \end{aligned}}$

$210 = 2 \cdot 77 + 56$
$77 = 1 \cdot 56 + 21$
$56 = 2 \cdot 21 + 14$
$21 = 1 \cdot 14 + 7$
$14 = 2 \cdot 7 + 0$

168, 244	2
84, 122	2
42, 61	2
21, 61	3
7, 61	7
1, 1	61
	1464

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9-2

1) $3^{86} \equiv 1 \pmod 4$ ✓

$3 \equiv 3 \pmod 4$

$3^2 \equiv 1 \pmod 4 \,/^{43}$

$3^{86} \equiv 1 \pmod 4$

---

2) $5^{125} \equiv 5 \pmod{13}$

$5 \equiv 5 \pmod{13}$

$5^2 \equiv 12 \pmod{13}$

$5^3 \equiv 8 \pmod{13}$

$5^4 \equiv 1 \pmod{13}$

$5^5 \equiv 5 \pmod{13} \,/^{25}$

$5^{125} \equiv 5^{25} \pmod{13}$

$5 \equiv 5 \pmod{13}$

$5^2 \equiv 12 \pmod{13}$

$5^3 \equiv 8 \pmod{13}$

$5^4 \equiv 1 \pmod{13}$

$5^5 \equiv 5 \pmod{13} \,/^{5}$

$5^{25} \equiv 5 \pmod{13}$

---

3) $9^{275} \equiv \underline{18} \pmod{21}$

$9 \equiv 9 \pmod{21}$

$9^2 \equiv 18 \pmod{21}$

$9^3 \equiv 15 \pmod{21}$

$9^4 \equiv 9 \pmod{21}$

$9^5 \equiv 18 \pmod{21} \,/^{55}$

$9^{275} \equiv 18^{55} \pmod{21}$

$18^{55} \equiv \underline{18} \pmod{21}$

$18 \equiv 18 \pmod{21}$

$18^2 \equiv 9 \pmod{21}$

$18^3 \equiv 15 \pmod{21}$

$18^4 \equiv 18 \pmod{21}$

$18^5 \equiv 9 \pmod{21} \,/^{11}$

$18^{55} \equiv 9^{11} \pmod{21}$

$9^{11} \equiv \underline{18} \pmod{21}$

~~$9^{3} \equiv 2 \pmod{4}$~~

3 1 3 8 1 0 5 9 6 0 9

4) $3^{2021} \equiv x \pmod{23}$

17 7 14 7

$3 \equiv 3 \pmod{23}$

$3^2 \equiv 9 \pmod{23}$

$3^3 \equiv 4 \pmod{23}$

$3^4 \equiv 12 \pmod{23}$

$3^5 \equiv 13 \pmod{23}$

$3^6 \equiv 16 \pmod{23}$

$3^7 \equiv 2 \pmod{23}$

$3^8 \equiv 6 \pmod{23}$

$3^9 \equiv 18 \pmod{23}$

$3^{10} \equiv 8 \pmod{23}$

$3^{11} \equiv 1 \pmod{23}$ / 183

$3^{2013} \equiv 1 \pmod{23}$

⋮

$3^{2014} \equiv 3 \pmod{23}$

⋮

$3^{2015} \equiv 9 \pmod{23}$

$3^{2016} \equiv 4 \pmod{23}$

$3^{2017} \equiv 12 \pmod{23}$

$3^{2018} \equiv 13 \pmod{23}$

$3^{2019} \equiv 16 \pmod{23}$

$3^{2020} \equiv 2 \pmod{23}$

$3^{2021} \equiv 6 \pmod{23}$

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9-2

5.

5. $2^{46} \equiv x \pmod 9 \Rightarrow 2^{46} \equiv 7 \pmod 9$

$2 \equiv 2 \pmod 9$

$2^2 \equiv 4 \pmod 9$ /23

$2^3 \equiv 8 \pmod 9$

$2^4 \equiv 7 \pmod 9$

$2^5 \equiv 5 \pmod 9$

$2^{46} \equiv 4^{23} \equiv \pmod 9$

$4^{23} \equiv x \pmod 9$

$4 \equiv 4 \pmod 9$

$4^2 \equiv 7 \pmod 9$

$4^3 \equiv 1 \pmod 9$ /7

$4^{21} \equiv 1 \pmod 9$

$4^{22} \equiv 4 \pmod 9$

$4^{23} \equiv 7 \pmod 9$

---

6. $3^{65} + 5^{87} \equiv x \pmod{11}$

$3 \equiv 3 \pmod{11}$	$5 \equiv 5 \pmod{11}$
$3^2 \equiv 9 \pmod{11}$	$5^2 \equiv 5 \pmod{11}$
$3^3 \equiv 5 \pmod{11}$	$5^3 \equiv 4 \pmod{11}$ /29
$3^4 \equiv 4 \pmod{11}$	$5^{87} \equiv 4^{29} \pmod{11}$
$3^5 \equiv 1 \pmod{11}$ /13	$4 \equiv 4 \pmod{11}$
$3^{65} \equiv 1 \pmod{11}$	$4^2 \equiv 5 \pmod{11}$
	$4^3 \equiv 9 \pmod{11}$
	$4^4 \equiv 3 \pmod{11}$
	$4^5 \equiv 1 \pmod{11}$ /5
	$4^{25} \equiv 1 \pmod{11}$
	⋮
	$4^{29} \equiv 3 \pmod{11}$

6. $1 + 3 \equiv 4 \pmod{11}$

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7.

$9^{43}+8^{121}\equiv X \pmod 6$

$9\equiv 3 \pmod 6$

$9^2\equiv 3 \pmod 6) /21$

$9^{42}\equiv 3^{21} \pmod 6$

$3\equiv 3 \pmod 6$

$3^2\equiv 3 \pmod 6$

$3^3\equiv 3 \pmod 6) /7$

$3^{21}\equiv 3^7 \pmod 6$

$3^{21}\equiv \underline{3} \pmod 6$

3

$8^{121}\equiv \pmod 6$

$8\equiv 2 \pmod 6$

$8^2\equiv 4 \pmod 6$

$8^3\equiv 2 \pmod 6) /40$

$8^{120}\equiv 2^{40} \pmod 6$

$8^{121}\equiv 2^{41} \pmod 6$

$2^{41}\equiv \pmod 6$

$2\equiv 2 \pmod 6$

$2^2\equiv 4 \pmod 6$

$2^3\equiv 2 \pmod 6) /13$

$2^{39}\equiv 2^{13} \pmod 6$

⋮

$2^{41}\equiv 2^{15} \pmod 6$

$2^3\equiv 2 \pmod 6) /5$

$2^5\equiv \underline{2} \pmod 6$

$3+2\equiv 5 \pmod 6$

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8.

$13^{16}+4^{25}\cdot 12^{15}\equiv X \pmod 7$

$13\equiv 6 \pmod 7$

$13^2\equiv 1 \pmod 7) /8$

$13^{16}\equiv \underline{1} \pmod 7$

1

$4\equiv 4 \pmod 7$

$4^2\equiv 2 \pmod 7$

$4^3\equiv \underline{1} \pmod 7$

$4^4\equiv 4 \pmod 7$

$4^5\equiv 2 \pmod 7) /5$

$4^{25}\equiv \underline{4} \pmod 7$

4

$12\equiv 5 \pmod 7$

$12^2\equiv 4 \pmod 7$

$12^3\equiv 6 \pmod 7) /5$

$12^{15}\equiv \underline{6} \pmod 7$

6

$1+4\cdot 6\equiv 4 \pmod 7$

19204 8234

9. $19^{91} - 91^{19} \cdot 3^{21} \equiv x \pmod 3$

$19^{91} \equiv x \pmod 3$	$91^{19} \equiv x \pmod 3$	$3^{21} \equiv 0 \pmod 3$	$1 - 0 \equiv 1 \pmod 3$
$19 \equiv 1 \pmod 3 \,/\,7$	$91 \equiv 1 \pmod 3 \,/\, 19$	0	
$19^7 \equiv 1 \pmod 3 \,/\, 13$	$91^{19} \equiv 1 \pmod 3$		
$19^{91} \equiv 1 \pmod 3$	1		
1			

19204 8234

10. $7^{33} + 5^{171} \equiv x \pmod 6$

$7 \equiv 1 \pmod 6 \,/\, 33$	$5^{171} \equiv x \pmod 6$	$1 + 5 \equiv 0 \pmod 6$
$7^{33} \equiv 1 \pmod 6$	$5 \equiv 5 \pmod 6$	
1	$5^2 \equiv 1 \pmod 6$	
	$5^3 \equiv 5 \pmod 6 \,/\, 57$	
	$5^{171} \equiv 5^{57} \pmod 6$	
	$5^3 \equiv 5 \pmod 6 \,/\, 19$	
	$5^{57} \equiv 5^{19} \pmod 6$	
	$5^2 \equiv 1 \pmod 6 \,/\, 9$	
	$5^{18} \equiv 1 \pmod 6$	
	$5^{19} \equiv 5 \pmod 6$	
	5	

19204 8234

9-3

1. $\bar{7} + \bar{x} + \bar{4} = \bar{5}$ në Z_{11}

$\bar{7} + \bar{x} = \bar{1} \,/\cdot \bar{8}$

$\bar{x} = \bar{8}$

$x \equiv 8 \pmod{11}$

---

2. $\bar{3} + \bar{x} + \bar{9} = \bar{6} /\cdot \bar{2}$ në Z_5

$\bar{3} + \bar{x} = \bar{1} - \bar{4}$

$\bar{3} + x = \bar{2} \,/\cdot \bar{2}$

$\underline{x = \bar{4}}$

$x \equiv 4 \pmod{5}$

$\bar{1} - \bar{4} = \bar{y}$

$\bar{1} = \bar{y} + \bar{4}$

$+$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$
$\bar{1}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{0}$
$\bar{4}$	$\bar{4}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$

$\Rightarrow \bar{1} = \bar{4} + \bar{2}$

$y = \bar{2}$

---

3) $\bar{8}\,\bar{x} + \bar{1} = \bar{3}$ në Z_9

$8\bar{x} = \bar{2} / \bar{8}$

$\cancel{8}\,\bar{x} = \bar{7}$

$x \equiv 7 \pmod{9}$

---

4. $\bar{6}\,\bar{x} + \bar{2} = \bar{10}$ në Z_7

$\bar{6}\,\bar{x} = \bar{8} \,/\, \bar{7}$

$\bar{6}\,\bar{x} = \bar{1}$

$x = \bar{6}$

$x \equiv 6 \pmod{7}$

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1. $\begin{cases} x \equiv 11 \pmod 5 \\ x \equiv 3 \pmod 7 \end{cases} \Rightarrow \begin{cases} x \equiv 1 \pmod 5 \\ x-3 = 7k, k \in \mathbb{Z} \end{cases} \Rightarrow \begin{cases} x \equiv 1 \pmod 5 \\ x = 7k+3 \end{cases} \Rightarrow \begin{cases} 1 \equiv 1 \pmod 5 \\ 7k+3 \equiv 1 \pmod 5 \end{cases} \Rightarrow$

$\Rightarrow 7k+4 \equiv 2 \pmod 5$

$\overline{7k} + \overline{4} = \overline{2}$

$\overline{7}\,\overline{k} = \overline{2} - \overline{4}$

$\overline{7}\,\overline{k} = \overline{3} \,\big|\cdot \overline{3}$ $\qquad (\overline{2}+\overline{5}) - \overline{4} = \overline{7} - \overline{4} = \overline{3}$

$\overline{k} = \overline{4}$ $\qquad x = 7k+3 = 3+7(5t+4) = 3+35t+28 = 31+35t$

$k \equiv 4 \pmod 5$

$k-4 = 5t$ $\qquad x \equiv 31 \pmod{35}$

$k = 5t+4$

---

2. $\begin{cases} x \equiv 2 \pmod 3 \\ x \equiv 4 \pmod 2 \end{cases} \Rightarrow \begin{cases} x \equiv 2 \pmod 3 \\ x \equiv 0 \pmod 2 \end{cases} \Rightarrow \begin{cases} x \equiv 2 \pmod 3 \\ x = 2k, k \in \mathbb{Z} \end{cases} \Rightarrow \begin{cases} 2 \equiv 2 \pmod 3 \\ 2k \equiv 2 \pmod 3 \end{cases}$

$2k+2 \equiv 4 \pmod 3$ $\qquad x = 2k = 2(3t+1) = 6t+2$

$2k+2 \equiv 1 \pmod 3$

$\overline{2}\,\overline{k} + \overline{2} = \overline{1}$ $\qquad x \equiv 2 \pmod 6$

$\overline{2}\,\overline{k} = \overline{1} - \overline{2}$ $\qquad \overline{1} - \overline{2} = (\overline{1}+\overline{3}) - \overline{2} = \overline{2}$

$\overline{2}\,\overline{k} = \overline{2} \,\big|\cdot \overline{2}$

$\overline{k} = \overline{1}$

$k \equiv 1 \pmod 3$

$k = 3t+1$

19204834

3. $\begin{cases} x \equiv 6 \pmod 5 \\ x \equiv 8 \pmod{11} \end{cases} \Rightarrow \begin{cases} x \equiv 1 \pmod 5 \\ x \equiv 8 \pmod{10} \end{cases} \Rightarrow \begin{cases} x - 1 = 5k,\ k \in \mathbb{Z} \\ x \equiv 8 \pmod{11} \end{cases} \Rightarrow \begin{cases} x = 5k+1 \\ x \equiv 8 \pmod{11} \end{cases}$

$\Rightarrow \begin{cases} 5k+1 \equiv 8 \pmod{11} \\ 8 \equiv 8 \pmod{11} \end{cases} \Rightarrow \begin{matrix} 5k+9 \equiv 16 \pmod{11} \\ 5k+9 \equiv 5 \pmod{11} \end{matrix}$

$\hat{5}\hat{k} + \hat{9} = \hat{5}$

$\hat{5}\hat{k} = \hat{5} - \hat{9}$

$\hat{5}\hat{k} = \hat{7} \mid \cdot \hat{9}$

$\hat{k} = \hat{9}$

$k \equiv 8 \pmod{11}$

$k - 8 = 11t$

$\underline{k = 11t + 8}$

$x = 5k+1 = 5(11t+8)+1 = 55t+40+1 = 41+55t$

$x \equiv 41 \pmod{55}$

---

4. $\begin{cases} x \equiv 7 \pmod 9 \\ x \equiv 3 \pmod 2 \end{cases} \Rightarrow \begin{cases} x \equiv 7 \pmod 9 \\ x \equiv 1 \pmod 2 \end{cases} \Rightarrow \begin{cases} x \equiv 7 \pmod 9 \\ x - 1 = 2k,\ k \in \mathbb{Z} \end{cases} \Rightarrow \begin{cases} x \equiv 7 \pmod 5 \\ x = 2k+1 \end{cases}$

$\Rightarrow \begin{cases} 7 \equiv 7 \pmod 9 \\ 2k+1 \equiv 7 \pmod 9 \end{cases} \overset{(+)}{\Rightarrow} 2k+8 \equiv 14 \pmod 5 \Rightarrow 2k+8 \equiv 5 \pmod 9$

$\hat{2}\hat{k} + \hat{8} = \hat{5}$

$\hat{2}\hat{k} = \hat{5} - \hat{8}$

$\hat{2}\hat{k} = \hat{6} \mid \cdot \hat{5}$

$\hat{k} = \hat{3}$

$\underline{k = 9t + 3}$

$k \equiv 3 \pmod 9$

$x = 2k+1 = 1 + 2(9t+3)$

$= 1 + 18t + 6 = 18t + 7$

$x \equiv 7 \pmod{18}$

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10-2

1. $S = \left\{ \frac{1+2m}{1+2n} \mid m, n \in \mathbb{Z} \right\}$

1.1. $\forall x, y \in S,\ x \cdot y \in \mathbb{Z}$

$$\frac{1+2m}{1+2n} \cdot \frac{1+2a}{1+2b} = \frac{1+2a+2m+4ma}{1+2n+2b+4nb} = \frac{1+2(\overbrace{a+m+2ma}^{m})}{1+2(\underbrace{n+b+2nb}_{n})} = \frac{1+2m}{1+2n} \in \mathbb{Z}$$

1.2. $\forall x, y, z \in S,\ (x \cdot y) \cdot z = x \cdot (y \cdot z)$

$$\left(\frac{1+2m}{1+2n} \cdot \frac{1+2a}{1+2b}\right) \cdot \frac{1+2x}{1+2y} = \frac{1+2m}{1+2n}\left(\frac{1+2a}{1+2b} \cdot \frac{1+2x}{1+2y}\right)$$

$$\frac{1+2(\overbrace{a+m+2am}^{m})}{1+2(\underbrace{n+b+2bn}_{n})} \cdot \frac{1+2x}{1+2y} = \frac{1+2m}{1+2n} \cdot \frac{1+2(\overbrace{a+x+2ax}^{a})}{1+2(\underbrace{y+n+2yn}_{b})}$$

$$\frac{1+2m}{1+2n} \cdot \frac{1+2x}{1+2y} = \frac{1+2m}{1+2n} \cdot \frac{1+2a}{1+2b}$$

$$\frac{1+2(\overbrace{m+x+2mx}^{m})}{1+2(\underbrace{n+y+2ny}_{n})} = \frac{1+2(\overbrace{m+a+2ma}^{m})}{1+2(\underbrace{n+b+2nb}_{n})}$$

$$\frac{1+2m}{1+2n} = \frac{1+2m}{1+2n}$$

1.3 $\exists e \in \mathbb{Z},\ \forall x \in S,\ xe = ex = x$

$xe = x$

$$\frac{1+2m}{1+2n} \cdot e = \frac{1+2m}{1+2n}$$

$$e = \frac{\frac{1+2m}{1+2n}}{\frac{1+2m}{1+2n}}$$

$e = 1 \mid 1 \in \mathbb{Z}$

1.4 $\forall x \in S,\ \exists x' \in S$

$xx' = x'x = e$

$xx' = e$

$x' = \frac{e}{x}$

$$x' = \frac{1}{\frac{1+2m}{1+2n}}$$

$$x' = \frac{1+2n}{1+2m} \in S, \quad m, n \in \mathbb{Z}$$

$(S, *)$, * Grup me * - Shumëzim i zakonshëm

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2.

a) $a * b = a^b$

a.1) $\forall a, b \in R^+$, $a * b \in R^+$

$a * b = a^b \in R^+, \forall a, b$

a.2) $\forall a, b, c \in R^+$, $(a*b)*c = a*(b*c)$

$a*(b*c) = (a*b)*c$

$a * b^c = a^b * c$

$a^{b^c} \neq a^{bc}$

$(S, *)$ - Strukturë, Nëse $* \Rightarrow a^b$, $(S, *)$ - Grupoid.

b) $a * b = a^2 b^2$

b.1) $\forall a, b \in R^+$, $a * b \in R^+$

$a * b = a^2 b^2$, $a^2 b^2 \in R^+$

b.2) $\forall a, b, c \in R^+$, $(a*b)*c = a*(b*c)$

$(a*b)*c = a*(b*c)$

$a^2 b^2 * c = a * b^2 c^2$

$(a^2 b^2)^2 c^2 = a^2 (b^2 c^2)^2$

$a^4 b^4 c^2 \neq a^2 b^4 c^4$

$(S, *)$, Grupoid, Kur $* \Rightarrow a * b = a^2 b^2$

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---

3. $G = \left\{ \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{a} \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}}_{b} \underbrace{\begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}}_{c} \right\}$

3.1) $\forall a, b \in G$, $a * b \in G$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} \Rightarrow \in G$

$\begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in G$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \in G$

$\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in G$

$\begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in G$

$\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in G$

3.

3.2) $\forall a,b,c \in G,\ (a \cdot b) \cdot c = a \cdot (b \cdot c)$

$$\left(\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}0&-1\\1&-1\end{bmatrix}\right)\cdot\begin{bmatrix}-1&1\\-1&0\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}\left(\begin{bmatrix}0&-1\\1&-1\end{bmatrix}\begin{bmatrix}-1&1\\-1&0\end{bmatrix}\right)$$

$$\begin{bmatrix}0&-1\\1&-1\end{bmatrix}\begin{bmatrix}-1&1\\-1&0\end{bmatrix}=\begin{bmatrix}0&-1\\1&-1\end{bmatrix}\begin{bmatrix}-1&1\\-1&0\end{bmatrix}$$

3.3) $\exists e \in G,\ \forall x \in G,\ ex = x \cdot e = x$

$x e = x \ / x^{-1}$

$x^{-1} x e = x^{-1} x$

$1 e = 1$

$e = 1 \Rightarrow e = \begin{bmatrix}1&0\\0&1\end{bmatrix} \in G$

3.4) $\forall x \in G,\ \exists x' \in G,\ x x' = x' x = e$

$x' x = e$

$x' x x^{-1} = e x^{-1}$

$x' = e x^{-1}$

a) $\begin{bmatrix}1&0\\0&1\end{bmatrix}\cdot\begin{bmatrix}1&0\\0&1\end{bmatrix}^{-1}=\begin{bmatrix}1&0\\0&1\end{bmatrix}^{-1}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$

b) $e \cdot \begin{bmatrix}0&-1\\1&-1\end{bmatrix}^{-1}=\begin{bmatrix}0&-1\\1&-1\end{bmatrix}^{-1}=\begin{bmatrix}-1&1\\-1&0\end{bmatrix}$

c) $e \cdot \begin{bmatrix}-1&1\\-1&0\end{bmatrix}^{-1}=\begin{bmatrix}-1&1\\-1&0\end{bmatrix}^{-1}=\begin{bmatrix}0&-1\\1&-1\end{bmatrix}$

$(G, *)$, grup, Nëse $*$ → Shumëzim me Matrica.

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4. $G=\{1,-1,i,-i\}$

4.1) $\forall a,b\in G,\ a*b\in G$

1) $1\cdot -1=-1\in G$
2) $1\cdot i=i\in G$
3) $1\cdot(-i)=-i\in G$
4) $-1\cdot 1=-1\in G$
5) $-1\cdot i=-i\in G$
6) $-1\cdot(-i)=i\in G$
7) $i\cdot 1=i\in G$
8) $i\cdot(-1)=-i\in G$
9) $i\cdot(-i)=-i\in G$
10) $-i\cdot -(-1)=1\in G$
11) $-i\cdot(-1)=i\in G$
12) $-i\cdot i=-(-1)=1\in G$

4.2) $\forall a,b,c\in G,\ (a\cdot b)\cdot c=a\cdot(b\cdot c)$

$(1\cdot(-1))\cdot i=1\cdot(-1\cdot i)\Rightarrow -i=-i$

1) $(1\cdot(-1))\cdot i=1\cdot(-1(-1))\Rightarrow i=i$
2) $(1,(-1))(-i)=1(-1(-1))\Rightarrow$
3) $(1\cdot i)(-i)=1(i(1))\Rightarrow -i\cdot i=i\cdot 1$
4) $(1(i))(-1)=1(-i(-1))=-i(-1)=-i(-1)$
5) $(-1\cdot 1)i=-1(1\cdot i)\Rightarrow -i=i$
6) $(-1\cdot 1)\cdot(-i)=-1(1(-i))\Rightarrow i=i$
7) $(-1\cdot i)1=-1(i\cdot 1)\Rightarrow -i=i$
8) $((-1)(-i))1=-1(-i\cdot 1)\Rightarrow i=i$
9) $(1\cdot i)-i=1(i-i)\Rightarrow (1(-i)=i(-1)$
10) $(1(-i))i=1(-i\,i)\Rightarrow -ii=-ii$
11) $(1(-i))i=1(+i\,i)\Rightarrow(-i)^2=(-i)^2$
12) $(-1,i)-i=-1(i(-i))\Rightarrow i^2=i^2$
12) $(-1(-i))i=-1(-i\,i)\Rightarrow -i=-i$
13) $(i\cdot 1)-1=i(1(-1))\Rightarrow -i=-i$
14) $(i(-1))1=i(-1\cdot 1)\Rightarrow -ii=-ii$
15) $(i\cdot 1)(-i)=i(1\cdot(-i))\Rightarrow$
16) $(i(-1))1=i(-i\cdot 1)=-i\,i=-ii$
16) $(i(-1))-i(-1(-i))\Rightarrow(-i)^2=(-i)^2$
17) $(i(-1))-i(-1(-i))\Rightarrow(-i)^2=(-i)^2$
18) $(i(-i))-1=i(-i(-1))\Rightarrow(-i)^2=(-i)^2$
19) $(-i\cdot 1)-1=-i(1(-1))\Rightarrow i=i$
19) $(-i(-1))1=-i((-1)\cdot 1)\Rightarrow i=i$
20) $(-i(-1))i=-i(1\cdot i)\Rightarrow -ii=-ii$
21) $(-i\cdot 1)i=-i(i\cdot 1)\Rightarrow -ii=-ii$
22) $(-i\cdot i)1=-i(i\cdot 1)\Rightarrow i^2=i^2$
23) $(i(-1))i=-i(-1\cdot i)\Rightarrow i^2=i^2$
23) $(-i(-1))i=-i(i(-1))\Rightarrow i^2=i^2$
24) $(-i,i)-1=-i(i(-1))$

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4.3) $\exists e \in G, \forall x \in G, ex = xe = x$

$e \cdot x = x$

$e = \frac{x}{x} = 1, \; 1 \in G$

4.4) $\forall x \in G, \exists x' \in G, xx' \cancel{=} x'x = e$

$x'x = e$

$x' = \frac{e}{x} \Rightarrow x' = x^{-1}$

a) $1^{-1} = 1 \in G$

b) $-1^{-1} = -1 \in G$

c) $i^{-1} = \frac{1}{i} \in G$

d) $-i^{-1} = \frac{1}{-i} \in G$

G - Formom Grup

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