2018

Driton BILALLI

Ushtrime me detyra të zgjidhura nga

MATEMATIKA

Limite
Derivate
Integrale

Parathënia

Kjo përmbledhje detyrash të zgjidhura u dedikohet të gjithë nxënëseve, studentëve dhe të gjithë atyre të cilët ne planprogramin e tyre përfshinë, limitet, derivatet dhe integralet.

Kemi bëre përpjekjet maksimale që të përfshihen një numër i relativisht i madh i llojeve të ndryshme të limiteve, derivateve dhe integraleve, duke përdorur shembujt të ndryshe me qëllim te kuptohen me lehte detyra.

Kjo përmbledhje detyrash përmban 777 detyra të zgjidhura ne detaje dhe të ndare ne :

limiti i vargut, limiti i funksionit, derivate, rregullat e Lopitalit, integrale të pacaktuara, integrale të caktuar.

Jemi plotësisht të vetëdijshme se mund të jenë përvjedhur gabimet të rastit, prandaj lexuesit i kërkojmë ndjesë dhe jo vetëm kaq. Nga ju lexuese të nderuar presim që pa asnjë hezitim të na shkruani ne emailin *dbilalli@hotmail.com*

Vërejtjet, kritikat dhe sugjerimet ne lidhej me librin, janë të mirëpritura.

Me respekt

Driton BILALLI

Përmbajtja:

1. LIMITE

1.1 Limiti i vargut

Numri a quhet limit i vargut (x_n) nëse për çdo $\varepsilon > 0$ ekziston indeksi $n_0 = n_0(\varepsilon)$ i tillë që për çdo n, $n > n_0 \Rightarrow |x_n - a| < \varepsilon$. Shënojmë $a = \lim\limits_{n\to\infty} x_n$. Simbolikisht

$$a = \lim_{n\to\infty} x_n \Leftrightarrow (\forall \varepsilon > 0)\left(\exists n_0 = n_0(\varepsilon)\right)(\forall n)\left(n > n_0 \Rightarrow |x_n - a| < \varepsilon\right)$$

Vetitë e vargjeve konvergjente.

(1) $\left(\lim\limits_{n\to\infty} x_n = a \wedge a \neq b\right) \Rightarrow \neg\left(\lim\limits_{n\to\infty} x_n = b\right)$

(2) $x_n = y_n \,(n \in N) \Rightarrow \lim\limits_{n\to\infty} x_n = \lim\limits_{n\to\infty} y_n$

(3) $\exists a\left(a = \lim\limits_{n\to\infty} x\right)_n \Rightarrow (\exists k, K \in R)(k \leq x_n \leq K)$

(4) Le të jetë (x_{n_k}) cilido nënvarg i vargut konvergjent (x_n), atëherë

$$a = \lim_{n\to\infty} x_n \Rightarrow a = \lim_{k\to\infty} x_{n_k}$$

(5) $\left((x_n \leq y_n \leq z_n)(n \in N)\right) \wedge (\exists a \in R)\left(\lim\limits_{n\to\infty} x_n = \lim\limits_{n\to\infty} z_n = a\right) \Rightarrow \lim\limits_{n\to\infty} y_n = a$

Rregullat e kalimit me limit. Le të jenë (x_n) dhe (y_n) vargje konvergjente. Atëherë edhe vargjet $(x_n \pm y_n)$, $(x_n y_n)$, $\left(\dfrac{x_n}{y_n}\right)(y_n \neq 0)$ dhe $|x_n|$ janë konvergjente dhe vlejnë relacionet:

(1) $\lim\limits_{n\to\infty}(x_n \pm y_n) = \lim\limits_{n\to\infty} x_n \pm \lim\limits_{n\to\infty} y_n$

(2) $\lim\limits_{n\to\infty}(x_n y_n) = \lim\limits_{n\to\infty} x_n \cdot \lim\limits_{n\to\infty} y_n$

(3) $\lim\limits_{n\to\infty} \dfrac{x_n}{y_n} = \dfrac{\lim\limits_{n\to\infty} x_n}{\lim\limits_{n\to\infty} y_n}$

(4) $\lim\limits_{n\to\infty} |x_n| = \left|\lim\limits_{n\to\infty} x_n\right|$

$1^0 \lim_{n\to\infty}(a_n \pm b_n) = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n$ $\quad$ $2^0 \lim_{n\to\infty}(a_n \cdot b_n) = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$

$3^0 \lim_{n\to\infty}\frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}\left(\lim_{n\to\infty} b_n \neq 0\right)$ $\quad$ $4^0 \lim_{n\to\infty}(a_n)^k = \left(\lim_{n\to\infty} a_n\right)^k (k>0)$

$5^0 \lim_{n\to\infty}|a_n| = \left|\lim_{n\to\infty} a_n\right|$ $\quad$ $6^0 \lim_{n\to\infty}\log_b(a_n) = \log_b\left(\lim_{n\to\infty} a_n\right)(a_n>0)$

$7^0 \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

Thuhet se vargu është i Koshit: (x_n), nëse:

$(\forall \varepsilon > 0)(\exists n_0 = n_0(\varepsilon))(\forall m,n \in N)(m,n > n_0 \Rightarrow |x_m - x_n| < \varepsilon)$

Kriteri i Koshit. Vargu (x_n) konvergjon atëherë dhe vetëm atëherë kur ai është i Koshit.

Konvergjenca e vargjeve monotone. Shqyrtimi i konvergjencës së vargjeve monotone kryesisht bazohet në këtë pohim: Çdo varg monoton dhe i kufizuar është konvergjent.

Progresioni (vargu) aritmetik

Vargu aritmetik quhet vargu tek i cili diferenca e çdo dy kufizave të njëpasnjëshme është konstante.

Termi i përgjithshëm i vargut aritmetik është: $a_n = a_1 + (n-1)\cdot d$ ku d -është konstantë.

Shuma e n-termave të parë të vargut aritmetik është:

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)\cdot d\right] \text{ ose } S_n = \frac{n}{2}(a_1 + a_n)$$

Progresioni (vargu) gjeometrik

Vargu i numrave në të cilin herësi i çdo dy kufizave të njëpasnjëshme është konstant quhet progresion gjeometrik:

Termi i përgjithshëm është:

$$a_n = a_1 \cdot q^{n-1} \text{ ku } q \text{ -është herësi}$$

Shuma e n-termave të parë të vargut gjeometrik është:

$S_n = a_1 \cdot \frac{1-q^n}{1-q}$

Detyra të zgjidhura:

Detyra 1: $\lim\limits_{n\to\infty}\left(4+\frac{1}{2^n}\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(4+\frac{1}{2^n}\right)=\lim_{n\to\infty}4+\lim_{n\to\infty}\frac{1}{2^n}=4+0=4$$

Detyra 2: $\lim\limits_{n\to\infty}\frac{2n^3-3n^2+n}{6n^3}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{2n^3-3n^2+n}{6n^3}=\lim_{n\to\infty}\left(\frac{1}{3}-\frac{1}{2n}+\frac{1}{6n^2}\right)=\lim_{n\to\infty}\frac{1}{3}-\frac{1}{2}\lim_{n\to\infty}\frac{1}{n}+\frac{1}{6}\lim_{n\to\infty}\frac{1}{n^2}=\frac{1}{3}-\frac{1}{2}\cdot 0+\frac{1}{6}\cdot 0=\frac{1}{3}$$

Detyra 3: $\lim\limits_{n\to\infty}\frac{n+1}{2n+1}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{n+1}{2n+1}=\lim_{n\to\infty}\frac{\frac{n+1}{n}}{\frac{2n+1}{n}}=\frac{\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)}{\lim\limits_{n\to\infty}\left(2+\frac{1}{n}\right)}=\frac{\lim\limits_{n\to\infty}1+\lim\limits_{n\to\infty}\frac{1}{n}}{\lim\limits_{n\to\infty}2+\lim\limits_{n\to\infty}\frac{1}{n}}=\frac{1+0}{2+0}=\frac{1}{2}$$

Detyra 4: $\lim\limits_{n\to\infty}\frac{7n-1}{4n+3}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{7n-1}{4n+3}=\lim_{n\to\infty}\frac{\frac{7n-1}{n}}{\frac{4n+3}{n}}=\lim_{n\to\infty}\frac{\frac{7n}{n}-\frac{1}{n}}{\frac{4n}{n}+\frac{3}{n}}=\lim_{n\to\infty}\frac{7-\frac{1}{n}}{4+\frac{3}{n}}=\frac{7-0}{4+0}=\frac{7}{4}$$

Detyra 5: $\lim\limits_{n\to\infty}\frac{2n+1}{n+5}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{2n+1}{n+5}=\lim_{n\to\infty}\frac{\frac{2n+1}{n}}{\frac{n+5}{n}}=\lim_{n\to\infty}\frac{\frac{2n}{n}+\frac{1}{n}}{\frac{n}{n}+\frac{5}{n}}=\lim_{n\to\infty}\frac{2+\frac{1}{n}}{1+\frac{5}{n}}=\frac{2+0}{1+0}=\frac{2}{1}=2$$

Detyra 6: $\lim\limits_{n\to\infty}\frac{n^2-2n+1}{n^2+4n+3}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{n^2-2n+1}{n^2+4n+3}=\lim_{n\to\infty}\frac{\frac{n^2-2n+1}{n^2}}{\frac{n^2+4n+3}{n^2}}=\lim_{n\to\infty}\frac{1-\frac{2}{n}+\frac{1}{n^2}}{1+\frac{4}{n}+\frac{3}{n^2}}=\frac{1-0+0}{1+0+0}=\frac{1}{1}=1$$

Detyra 7: $\lim\limits_{n\to\infty}\frac{2n^2+3}{3n^2-4}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{2n^2+3}{3n^2-4}=\lim_{n\to\infty}\frac{\frac{2n^2}{n^2}+\frac{3}{n^2}}{\frac{3n^2}{n^2}-\frac{4}{n^2}}=\lim_{n\to\infty}\frac{2+\frac{3}{n^2}}{3-\frac{4}{n^2}}=\frac{2+\frac{3}{\infty^2}}{3-\frac{4}{\infty^2}}=\frac{2}{3}$$

Detyra 8: $\lim\limits_{n\to\infty}\frac{4n^2+5}{2n^2-3}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{4n^2+5}{2n^2-3}=\lim_{n\to\infty}\frac{\frac{4n^2}{n^2}+\frac{5}{n^2}}{\frac{2n^2}{n^2}-\frac{3}{n^2}}=\lim_{n\to\infty}\frac{4+\frac{5}{n^2}}{2-\frac{3}{n^2}}=\frac{4+\frac{5}{\infty^2}}{2-\frac{3}{\infty^2}}=\frac{4}{2}=2$$

Detyra 9: $\lim\limits_{n\to\infty}\frac{1}{n^2}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{1}{n^2}=\lim_{n\to\infty}\frac{1}{n\cdot n}=\lim_{n\to\infty}\left(\frac{1}{n}\cdot\frac{1}{n}\right)=\lim_{n\to\infty}\frac{1}{n}\cdot\lim_{n\to\infty}\frac{1}{n}=0\cdot 0=0$$

Detyra 10: $\lim\limits_{n\to\infty}\frac{5n-3}{n^2-2n+5}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{5n-3}{n^2-2n+5}=\lim_{n\to\infty}\frac{\frac{5n}{n^2}-\frac{3}{n^2}}{\frac{n^2}{n^2}-\frac{2n}{n^2}+\frac{5}{n^2}}=\lim_{n\to\infty}\frac{\frac{5}{n}-\frac{3}{n^2}}{1-\frac{2}{n}+\frac{5}{n^2}}=\frac{0}{1}=0$$

Detyra 11: $\lim\limits_{n\to\infty}\dfrac{n^2+n+1}{3n^2+1}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{n^2+n+1}{3n^2+1}=\lim_{n\to\infty}\frac{\frac{n^2}{n^2}+\frac{n}{n^2}+\frac{1}{n^2}}{\frac{3n^2}{n^2}+\frac{1}{n^2}}=\lim_{n\to\infty}\frac{1+\frac{1}{n}+\frac{1}{n^2}}{3+\frac{1}{n^2}}=\frac{1}{3}$$

Detyra 12: $\lim\limits_{n\to\infty}\dfrac{5n^3+9n^2+2n-5}{n^2-6n+3}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{5n^3+9n^2+2n-5}{n^2-6n+3}=\lim_{n\to\infty}\frac{\frac{5n^3}{n^3}+\frac{9n^2}{n^3}+\frac{2n}{n^3}-\frac{5}{n^3}}{\frac{n^2}{n^3}-\frac{6n}{n^3}+\frac{3}{n^3}}=\lim_{n\to\infty}\frac{5+\frac{9}{n}+\frac{2}{n^2}-\frac{5}{n^3}}{\frac{1}{n}-\frac{6}{n^2}+\frac{3}{n^3}}=\frac{5}{0}=\infty$$

Detyra 13: $\lim\limits_{n\to\infty}\dfrac{n^2+2n}{n+3}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{n^2+2n}{n+3}=\lim_{n\to\infty}\frac{\frac{n^2+2n}{n^2}}{\frac{n+3}{n^2}}=\lim_{n\to\infty}\frac{1+\frac{2}{n}}{\frac{1}{n}+\frac{3}{n^2}}=\frac{1}{0}=\infty$$

Detyra 14: $\lim\limits_{n\to\infty}\dfrac{(n-3)^2}{n^2+4n+3}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{(n-3)^2}{n^2+4n+3}=\lim_{n\to\infty}\frac{n^2-6n+9}{n^2+4n+3}=\lim_{n\to\infty}\frac{\frac{n^2-6n+9}{n^2}}{\frac{n^2+4n+3}{n^2}}=\lim_{n\to\infty}\frac{1+\frac{6}{n}+\frac{9}{n^2}}{1+\frac{4}{n}+\frac{3}{n^2}}=\frac{1-0+0}{1+0+0}=\frac{1}{1}=1$$

Detyra 15: $\lim\limits_{n\to\infty}\dfrac{n^2+2n-1}{(n-1)^2}$

Zgjidhje: $\lim\limits_{n\to\infty}\dfrac{n^2+2n-1}{(n-1)^2}=\lim\limits_{n\to\infty}\dfrac{n^2+2n-1}{n^2-2n+1}=\lim\limits_{n\to\infty}\dfrac{\frac{n^2+2n-1}{n^2}}{\frac{n^2-2n+1}{n^2}}=\lim\limits_{n\to\infty}\dfrac{1+\frac{2}{n}-\frac{1}{n^2}}{1-\frac{2}{n}+\frac{1}{n^2}}=\dfrac{1+0-0}{1-0+0}=\dfrac{1}{1}=1$

Detyra 16: $\lim\limits_{n\to\infty}\frac{n^2-1}{n^4-1}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{n^2-1}{n^4-1}=\lim_{n\to\infty}\frac{\frac{n^2-1}{n^4}}{\frac{n^4-1}{n^4}}=\lim_{n\to\infty}\frac{\frac{1}{n^2}-\frac{1}{n^4}}{1-\frac{1}{n^4}}=\frac{0}{1}=0$$

Detyra 17: $\lim\limits_{n\to\infty}\frac{5-(3n+2)}{2(3n+2)}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{5-(3n+2)}{2(3n+2)}=\lim_{n\to\infty}\frac{5-3n-2}{6n+4}=\lim_{n\to\infty}\frac{3-3n}{6n+4}=\lim_{n\to\infty}\frac{\frac{3}{n}-\frac{3n}{n}}{\frac{6n}{n}+\frac{4}{n}}=\lim_{n\to\infty}\frac{\frac{3}{n}-3}{6+\frac{4}{n}}=-\frac{3}{6}=-\frac{1}{2}$$

Detyra 18: $\lim\limits_{n\to\infty}\frac{5n^2-4n+7}{17n^2+n-6}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{5n^2-4n+7}{17n^2+n-6}=\lim_{n\to\infty}\frac{\frac{5n^2}{n^2}-\frac{4n}{n^2}+\frac{7}{n^2}}{\frac{17n^2}{n^2}+\frac{n}{n^2}-\frac{6}{n^2}}=\lim_{n\to\infty}\frac{5-\frac{4}{n}+\frac{7}{n^2}}{17+\frac{1}{n}-\frac{6}{n^2}}=\frac{5-0+0}{17+0-0}=\frac{5}{17}$$

Detyra 19: $\lim\limits_{n\to\infty}\left(\frac{1}{1-n}-\frac{3}{1-n^3}\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(\frac{1}{1-n}-\frac{3}{1-n^3}\right)=\lim_{n\to\infty}\frac{1+n+n^2-3}{1-n^3}=\lim_{n\to\infty}\frac{n^2+n-2}{(1-n)(1+n+n^2)}=\lim_{n\to\infty}\frac{(n-1)(n+2)}{(1-n)(1+n+n^2)}=$$

$$=\lim_{n\to\infty}\frac{-(n+2)}{(1+n+n^2)}=\frac{-\frac{n}{n^2}-\frac{2}{n^2}}{\frac{1}{n^2}+\frac{n}{n^2}+\frac{n^2}{n^2}}=0$$

Detyra 20: $\lim_{n\to\infty}\frac{(n+1)+(n+2)+(n+3)}{n^3+\frac{1}{2}n^2+\frac{3}{4}n+\frac{5}{6}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{(n+1)+(n+2)+(n+3)}{n^3+\frac{1}{2}n^2+\frac{3}{4}n+\frac{5}{6}}=\lim_{n\to\infty}\frac{n^3+6n^2+11n+6}{n^3+\frac{1}{2}n^2+\frac{3}{4}n+\frac{5}{6}}=\lim_{n\to\infty}\frac{\frac{n^3}{n^3}+\frac{6n^2}{n^3}+\frac{11n}{n^3}+\frac{6}{n^3}}{\frac{n^3}{n^3}+\frac{1}{2n^3}n^2+\frac{3}{4n^3}n+\frac{5}{6n^3}}=$$

$$=\frac{1+\frac{6}{n}+\frac{11}{n^2}+\frac{6}{n^3}}{1+\frac{1}{2n}+\frac{3}{4n^2}+\frac{5}{6n^3}}=\frac{1+0+0+0}{1+0+0+0}=\frac{1}{1}=1$$

Detyra 21: $\lim_{n\to\infty}\frac{n(n+1)(n+2)(n+3)}{(n+1)(n+2)(n+4)}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{n(n+1)(n+2)(n+3)}{(n+1)(n+2)(n+4)}=\lim_{n\to\infty}\frac{(n^2+n)(n+2)(n+3)}{(n^2+3n+2)(n+4)}=\lim_{n\to\infty}\frac{(n^3+3n^2+2n)(n+3)}{(n^3+7n^2+12n+8)}=$$

$$=\lim_{n\to\infty}\frac{n^4+6n^3+11n^2+6n}{n^3+7n^2+12n+8}=\lim_{n\to\infty}\frac{\frac{n^4}{n^4}+\frac{6n^3}{n^4}+\frac{11n^2}{n^4}+\frac{6n}{n^4}}{\frac{n^3}{n^4}+\frac{7n^2}{n^4}+\frac{12n}{n^4}+\frac{8}{n^4}}=\lim_{n\to\infty}\frac{1+\frac{6}{n}+\frac{11}{n^2}+\frac{6n}{n^3}}{\frac{1}{n}+\frac{7}{n^2}+\frac{12}{n^3}+\frac{8}{n^4}}=\frac{1}{0}=\infty$$

Detyra 22: $\lim_{n\to\infty}\frac{1+3+5+...+(2n-1)}{(n+1)^2}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{1+3+5+...+(2n-1)}{(n+1)^2}=\left|\begin{array}{l}S_n=1+3+5+...+(2n-1)=\frac{n}{2}\left[2a_1+(n-1)d\right]=\\=\frac{n}{2}\left[2\cdot 1+(n-1)2\right]=\frac{n}{2}\left[2+2n-2\right]=n^2\end{array}\right|$$

$$\lim_{n\to\infty}\frac{n^2}{n^2+n+1}=\lim_{n\to\infty}\frac{\frac{n^2}{n^2}}{\frac{n^2}{n^2}+\frac{n}{n^2}+\frac{1}{n^2}}=\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}+\frac{1}{n^2}}=\frac{1}{1+0+0}=\frac{1}{1}=1$$

Detyra 23: $\lim\limits_{n\to\infty}\frac{(n+1)(3n+2)(5n-7)}{n^3}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{(n+1)(3n+2)(5n-7)}{n^3}=\lim_{n\to\infty}\frac{15n^3+4n^2-25n-14}{n^3}=\lim_{n\to\infty}\frac{\frac{15n^3}{n^3}+\frac{4n^2}{n^3}-\frac{25n}{n^3}-\frac{14}{n^3}}{\frac{n^3}{n^3}}=$$

$$=\lim_{n\to\infty}\left(15+\frac{4}{n}-\frac{25}{n^2}-\frac{14}{n^3}\right)=15$$

Detyra 24: $\lim\limits_{n\to\infty}\frac{n^2-1}{1+2+3+...+n}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{n^2-1}{1+2+3+...+n}=\left|S_n=\frac{n}{2}(a_1+a_n)=\frac{n}{2}(1+n)=\frac{n^2+n}{2}\right|=$$

$$=\lim_{n\to\infty}\frac{n^2-1}{\frac{n^2+n}{2}}=\lim_{n\to\infty}\frac{2n^2-1\,|:n^2}{n^2+n\,|:n^2}=\lim_{n\to\infty}\frac{\frac{2n^2}{n^2}-\frac{1}{n^2}}{\frac{n^2}{n^2}+\frac{n}{n^2}}=\lim_{n\to\infty}\frac{2-\frac{1}{n^2}}{1+\frac{1}{n}}=\frac{2}{1}=2$$

Detyra 25: $\lim\limits_{n\to\infty}\frac{2+4+6+...+2n}{1+3+5+...+(2n-1)}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{2+4+6+...+2n}{1+3+5+...+(2n-1)}=\frac{\lim\limits_{n\to\infty}(2+4+6+...+2n)}{\lim\limits_{n\to\infty}\left[1+3+5+...+(2n-1)\right]}=$$

$$=\left|\begin{array}{l}S_n=\frac{n}{2}(a_1+a_n)=\frac{n}{2}(2+2n)=\frac{2n+2n^2}{2}=n^2+n\\ S_n=\frac{n}{2}(1+2n-1)=\frac{2n^2}{2}=n^2\end{array}\right|=\lim_{n\to\infty}\frac{n^2+n}{n^2}=\lim_{n\to\infty}\frac{n^2+n\,|:n^2}{n^2\,|:n^2}=$$

$$=\lim_{n\to\infty}\frac{\frac{n^2}{n^2}+\frac{n}{n^2}}{\frac{n^2}{n^2}}=\lim_{n\to\infty}\frac{1+\frac{1}{n}}{1}=\frac{1}{1}=1$$

Detyra 26: $\lim_{n\to\infty}\frac{1^2+2^2+3^2+...+(n-1)^2}{n^3}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{1^2+2^2+3^2+...+(n-1)^2}{n^3}=\lim_{n\to\infty}\frac{\frac{1}{6}n(n+1)(2n+1)}{n^3}=\lim_{n\to\infty}\frac{2n^3+3n^2+n}{6n^3}=$$

$$=\lim_{n\to\infty}\frac{n(2n^2+3n+1)}{6n^3}=\lim_{n\to\infty}\frac{(2n^2+3n+1)}{6n^2}=\lim_{n\to\infty}\frac{\frac{2n^2}{n^2}+\frac{3n}{n^2}+\frac{1}{n^2}}{\frac{6n^2}{n^2}}=\lim_{n\to\infty}\frac{2+\frac{3}{n}+\frac{1}{n^2}}{6}=$$

$$=\frac{2+\frac{3}{\infty}+\frac{1}{\infty^2}}{6}=\frac{2}{6}=\frac{1}{3}$$

Detyra 27: $\lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+...+\frac{n-1}{n^2}\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+...+\frac{n-1}{n^2}\right)=\lim_{n\to\infty}\frac{1+2+3+...+n-1}{n^2}=\lim_{n\to\infty}\frac{\frac{n(n-1)}{2}}{n^2}=\lim_{n\to\infty}\frac{n^2-n}{2n^2}=$$

$$=\lim_{n\to\infty}\frac{\frac{n^2}{n^2}-\frac{n}{n^2}}{\frac{2n^2}{n^2}}=\lim_{n\to\infty}\frac{1-\frac{1}{n}}{2}=\frac{1}{2}$$

Detyra 28: $\lim_{n\to\infty}\left(\frac{27}{100}+\frac{27}{100^2}+...+\frac{27}{100^n}\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(\frac{27}{100}+\frac{27}{100^2}+...+\frac{27}{100^n}\right)=27\lim_{n\to\infty}\left(\frac{1}{100}+\frac{1}{100^2}+...+\frac{1}{100^n}\right)=27\lim_{n\to\infty}\frac{1}{100}\cdot\frac{1-\left(\frac{1}{100}\right)^n}{1-\frac{1}{100}}=$$

$$=\frac{27}{100}\lim_{n\to\infty}\frac{1-\frac{1}{100^n}}{\frac{99}{100}}=\frac{27}{100}\cdot\frac{1}{\frac{99}{100}}=\frac{27}{99}=\frac{3}{11}$$

Detyra 29: $\lim\limits_{n\to\infty}\left[\frac{1+2+3+4+...+n}{n+2}-\frac{n}{2}\right]$

Zgjidhje:

$$\lim_{n\to\infty}\left[\frac{1+2+3+4+...+n}{n+2}-\frac{n}{2}\right]=\lim_{n\to\infty}\left[\frac{\frac{n}{2}(1+n)}{n+2}-\frac{n}{2}\right]=\lim_{n\to\infty}\left[\frac{n(1+n)}{2(n+2)}-\frac{n}{2}\right]=$$

$$=\lim_{n\to\infty}\left[\frac{n+n^2-n^2-2n}{2(n+2)}\right]=\lim_{n\to\infty}\frac{-n}{2n+4}=\lim_{n\to\infty}\frac{-\frac{n}{n}}{\frac{2n}{n}+\frac{4}{n}}=\lim_{n\to\infty}\frac{-1}{2+\frac{4}{n}}=\frac{-1}{2+0}=-\frac{1}{2}$$

Detyra 30: $\lim\limits_{n\to\infty}\left[\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+...+\frac{n}{n^2}\right]$

Zgjidhje:

$$\lim_{n\to\infty}\left[\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+...+\frac{n}{n^2}\right]=\lim_{n\to\infty}\left[\frac{1+2+3+...+n}{n^2}\right]=\lim_{n\to\infty}\frac{\frac{n}{2}(1+n)}{n^2}=$$

$$=\lim_{n\to\infty}\frac{n+n^2}{2n^2}=\lim_{n\to\infty}\frac{\frac{1}{n}+1}{2}=\frac{0+1}{2}=\frac{1}{2}$$

Detyra 31: $\lim\limits_{n\to\infty}\sqrt{a\cdot\sqrt{a\cdot\sqrt{a\cdots\sqrt{a}}}}$

Zgjidhje:

$$\lim_{n\to\infty}\sqrt{a\cdot\sqrt{a\cdot\sqrt{a\cdots\sqrt{a}}}}=\lim_{n\to\infty}a^{\frac{1}{2}}\cdot a^{\frac{1}{4}}\cdot a^{\frac{1}{8}}\cdots a^{\frac{1}{2^n}}=\lim_{n\to\infty}a^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...+\frac{1}{2^n}}=$$

$$=\left|\begin{array}{c}\text{Vargu është gjeometrik perdorim shprehjen: } S_n=b_1\dfrac{1-q^n}{1-q}\\ S_n=\dfrac{1}{2}\cdot\dfrac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}=\dfrac{1}{2}\cdot\dfrac{1-\frac{1}{2^n}}{\frac{1}{2}}=1-\dfrac{1}{2^n}\end{array}\right|=\lim_{n\to\infty}a^{1-\frac{1}{2^n}}=a^{1-\frac{1}{2^\infty}}=a^1=a$$

Detyra 32: $\lim\limits_{n\to\infty}\dfrac{1^2+3^2+...+(2n-1)^2}{2^2+4^2+...+(2n)^2}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{1^2+3^2+...+(2n-1)^2}{2^2+4^2+...+(2n)^2}=\lim_{n\to\infty}\frac{1^2+2^2+3^2+4^2+...+(2n)^2-\left(2^2+4^2+...+(2n)^2\right)}{2^2+4^2+...+(2n)^2}=$$

$$=\lim_{n\to\infty}\frac{\dfrac{2n(2n+1)(4n+1)}{6}-2^2\left(1+2^2+3^2+...+n^2\right)}{2^2\left(1+2^2+3^2+...+n^2\right)}=\lim_{n\to\infty}\frac{\dfrac{2n(2n+1)(4n+1)}{6}-4\cdot\dfrac{n(n+1)(2n+1)}{6}}{4\cdot\dfrac{n(n+1)(2n+1)}{6}}=$$

$$=\lim_{n\to\infty}\frac{\dfrac{2n\left(8n^2+6n+1\right)-4n\left(2n^2+3n+1\right)}{6}}{\dfrac{4n(n+1)(2n+1)}{6}}=\lim_{n\to\infty}\frac{2n\left(8n^2+6n+1\right)-4n\left(2n^2+3n+1\right)}{4n(n+1)(2n+1)}=$$

$$=\lim_{n\to\infty}\frac{4n^2-1}{2(n+1)(2n+1)}=\lim_{n\to\infty}\frac{(2n-1)(2n+1)}{2(n+1)(2n+1)}=\lim_{n\to\infty}\frac{2n-1}{2n+2}=\frac{2}{2}=1$$

Detyra 33: $\lim\limits_{n\to\infty}\dfrac{1-2+3-4+5-6+...+2n}{\sqrt{n^2+1}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{1-2+3-4+5-6+...+2n}{\sqrt{n^2+1}}=\lim_{n\to\infty}\frac{1+3+5+...+(2n-1)-(2-4+3+...+2n)}{\sqrt{n^2+1}}=$$

$$\left|\begin{array}{l}\text{Vargu është aritmetik: } S_n=\dfrac{n}{2}\left(a_1+a_n\right)\\ S_n=\dfrac{n}{2}(1+2n-5)=\dfrac{n}{2}\cdot 2n=n^2\\ S_n=\dfrac{n}{2}(2+2n)=\dfrac{n}{2}\cdot 2(1+n)=n+n^2\end{array}\right|=\lim_{n\to\infty}\frac{n^2-\left(n+n^2\right)}{\sqrt{n^2+1}}=\lim_{n\to\infty}\frac{n^2-n-n^2}{\sqrt{n^2+1}}=\lim_{n\to\infty}\frac{-n}{\sqrt{n^2+1}}=$$

$$=\lim_{n\to\infty}\frac{\dfrac{-n}{n}}{\sqrt{\dfrac{n^2+1}{n^2}}}=-\frac{1}{\sqrt{1+\dfrac{1}{n^2}}}=-\frac{1}{1}=-1$$

Detyra 34: $\lim\limits_{n\to\infty}\left(\sqrt{2}\cdot\sqrt[4]{2}\cdot\sqrt[8]{2}\cdot.......\cdot\sqrt[2n]{2}\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(\sqrt{2}\cdot\sqrt[4]{2}\cdot\sqrt[8]{2}\cdot.......\cdot\sqrt[2n]{2}\right)=\lim_{n\to\infty}\left(2^{\frac{1}{2}}\cdot2^{\frac{1}{4}}\cdot2^{\frac{1}{8}}\cdot.......\cdot2^{\frac{1}{2n}}\right)=\lim_{n\to\infty}2^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}....\frac{1}{2n}}=\lim_{n\to\infty}2^{1-\left(\frac{1}{2}\right)^n}=2^1=2$$

Detyra 35: $\lim\limits_{n\to\infty}\left(\sqrt{a}\cdot\sqrt[4]{a}\cdot\sqrt[8]{a}\cdot...\cdot\sqrt[2^n]{a}\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(\sqrt{a}\cdot\sqrt[4]{a}\cdot\sqrt[8]{a}\cdot...\cdot\sqrt[2^n]{a}\right)=\lim_{n\to\infty}\left(a^{\frac{1}{2}}\cdot a^{\frac{1}{4}}\cdot a^{\frac{1}{8}}\cdot...\cdot a^{\frac{1}{2^n}}\right)=\lim_{n\to\infty}a^{\frac{1}{2}\cdot\frac{1\left(\frac{1}{2}\right)^n}{1\cdot\frac{1}{2}}}=\lim_{n\to\infty}a^{1\cdot\frac{1}{2^n}}=a$$

Detyra 36: $\lim\limits_{n\to\infty}\dfrac{1+3+5+...+(2n-1)}{n+1}-\dfrac{2n+1}{2}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{1+3+5+...+(2n-1)}{n+1}-\frac{2n+1}{2}=\left|\begin{array}{l}\text{Nga induksioni matematike:}\\1+3+5+...+(2n-1)=n^2\end{array}\right|=\lim_{n\to\infty}\frac{n^2}{n+1}-\frac{2n+1}{2}=$$

$$=\lim_{n\to\infty}\frac{2n^2-(2n+1)(n+1)}{2(n+1)}=\lim_{n\to\infty}\frac{2n^2-\left(2n^2+3n+1\right)}{2(n+1)}=\lim_{n\to\infty}\frac{2n^2-2n^2-3n-1}{2(n+1)}=$$

$$=\lim_{n\to\infty}\frac{-3n-1}{2(n+1)}=-\lim_{n\to\infty}\frac{3n-1}{2n+2}=-\lim_{n\to\infty}\frac{n\left(3+\frac{1}{n}\right)}{n\left(2+\frac{2}{n}\right)}=-\lim_{n\to\infty}\frac{3+\frac{1}{n}}{2+\frac{2}{n}}=-\frac{3}{2}$$

Detyra 37: $\lim\limits_{n\to\infty}\dfrac{1^2+2^2+3^2+...+n^2}{n^3}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{1^2+2^2+3^2+...+n^2}{n^3}=\lim_{n\to\infty}\frac{\frac{n(n+1)\cdot(2n+1)}{6}}{n^3}=\lim_{n\to\infty}\frac{\left(n^2+n\right)\cdot(2n+1)}{6n^3}=\lim_{n\to\infty}\frac{2n^3+n^2+2n^2+n}{6n^3}=$$

$$=\lim_{n\to\infty}\frac{\frac{2n^3}{n^3}+\frac{n^2}{n^3}+\frac{2n^2}{n^3}+\frac{n}{n^3}}{\frac{6n^3}{n^3}}=\lim_{n\to\infty}\frac{2+\frac{1}{n}+\frac{2}{n}+\frac{1}{n^2}}{6}=\frac{2}{6}=\frac{1}{3}$$

Detyra 38: $\lim\limits_{n\to\infty}\left[\dfrac{1+3+5+...+(2n-1)}{n+1}-\dfrac{2n+1}{2}\right]$

Zgjidhje:

$$\lim_{n\to\infty}\left[\frac{1+3+5+...+(2n-1)}{n+1}-\frac{2n+1}{2}\right]=\lim_{n\to\infty}\left[\frac{\left[1+(2n-1)\cdot\frac{n}{2}\right]}{n+1}-\frac{2n+1}{2}\right]=\lim_{n\to\infty}\left[\frac{n^2}{n+1}-\frac{2n+1}{2}\right]=$$

$$=\lim_{n\to\infty}\frac{-3n-1}{2(n+1)}=\lim_{n\to\infty}\frac{-\frac{3n}{n}-\frac{1}{n}}{\frac{2n}{n}+\frac{1}{n}}=\lim_{n\to\infty}\frac{-3-\frac{1}{n}}{2+\frac{1}{n}}=-\frac{3}{2}$$

Detyra 39: $\lim\limits_{n\to\infty}\left[1-\dfrac{1}{3}+\dfrac{1}{9}-\dfrac{1}{27}+...+\dfrac{(-1)^{n-1}}{3^{n-1}}\right]$

Zgjidhje:

$$\lim_{n\to\infty}\left[1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+...+\frac{(-1)^{n-1}}{3^{n-1}}\right]=\left|\begin{array}{l}\text{Vargu i dhënë është varg gjeometrik}\\ a_1=1, q=-\dfrac{1}{3}\ \text{dhe}\ S_n=a_1\cdot\dfrac{q^n-1}{q-1}=1\cdot\dfrac{\left(-\frac{1}{3}\right)^n-1}{-\frac{1}{3}-1}\end{array}\right|=$$

$$=\lim_{n\to\infty}\frac{\left(-\frac{1}{3}\right)^n-1}{-\frac{4}{3}}=-\frac{3}{4}\cdot\lim_{n\to\infty}\left[\left(-\frac{1}{3}\right)^n-1\right]=-\frac{3}{4}\cdot(0-1)=\frac{3}{4}$$

Detyra 40: $\lim\limits_{n\to\infty}\left[\dfrac{1-2+3-4+...+(2n-2)-2n}{\sqrt{n^2+1}}\right]$

Zgjidhje:

$$\lim_{n\to\infty}\left[\frac{1-2+3-4+...+(2n-2)-2n}{\sqrt{n^2+1}}\right]=\lim_{n\to\infty}\frac{1+3+(2n-1)-(2+4+...+2n)}{\sqrt{n^2+1}}=$$

$$=\left|\begin{array}{l}a_1=1, a_2=3, a_n=2n-1, d=a_2-a_1=2\\ S_n=\dfrac{n}{2}(2a_1+(n-1)2)=\dfrac{n}{2}\cdot(2+2n-1)=n\end{array}\right|=\lim_{n\to\infty}\frac{n}{\sqrt{n^2+1}}=\lim_{n\to\infty}\frac{\frac{n}{n}}{\sqrt{\frac{n^2}{n^2}+\frac{1}{n^2}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+0}}=\frac{1}{1}=1$$

Detyra 41: $\lim\limits_{n\to\infty}\dfrac{1+\frac{1}{2}+\ldots+\frac{1}{2^n}}{1+\frac{1}{3}+\ldots+\frac{1}{3^n}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{1+\frac{1}{2}+\ldots+\frac{1}{2^n}}{1+\frac{1}{3}+\ldots+\frac{1}{3^n}}=\left|\begin{array}{l}1+\frac{1}{2}+\ldots+\frac{1}{2^n}\ \text{varg gjeometrik}\ a_1=1,a_2=\frac{1}{2},q=\frac{1}{2}\\ S_n=1\cdot\dfrac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}=\dfrac{1-\left(\frac{1}{2}\right)^n}{\frac{1}{2}}=2\left(1-\frac{1}{2^n}\right)\\ 1+\frac{1}{3}+\ldots+\frac{1}{3^n}\ \text{varg gjeometrik}\ a_1=1,a_2=\frac{1}{3},q=\frac{1}{3}\\ S_n=1\cdot\dfrac{1-\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}}=\dfrac{1-\left(\frac{1}{3}\right)^n}{\frac{2}{3}}=\dfrac{3\left(1-\frac{1}{3^n}\right)}{2}\end{array}\right|=$$

$$=\lim_{n\to\infty}\frac{2\left(1-\frac{1}{2^n}\right)}{\dfrac{3\left(1-\frac{1}{3^n}\right)}{2}}=\lim_{n\to\infty}\frac{4\left(1-\frac{1}{2^n}\right)}{3\left(1-\frac{1}{3^n}\right)}=\frac{4}{3}$$

Detyra 42: $\lim\limits_{n\to\infty}\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots+\frac{1}{3^n}\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots+\frac{1}{3^n}\right)=\left|\begin{array}{l}a_1=\frac{1}{3},a_2=\frac{1}{9}=\frac{1}{3^2},a_3=\frac{1}{27}=\frac{1}{3^3}\ldots\ldots.a_n=\frac{1}{3^n},\quad q=\dfrac{a_2}{a_1}=\dfrac{\frac{1}{9}}{\frac{1}{3}}=\frac{3}{9}=\frac{1}{3}\\ S_n=\frac{1}{3}\cdot\dfrac{1-\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}}=\frac{1}{3}\cdot\dfrac{1-\left(\frac{1}{3}\right)^n}{\frac{2}{3}}=\frac{1}{3}\cdot\dfrac{3\left(1-\left(\frac{1}{3}\right)^n\right)}{2}=\dfrac{1-\left(\frac{1}{3}\right)^n}{2}\end{array}\right|=$$

$$=\lim_{n\to\infty}\frac{1-\left(\frac{1}{3}\right)^n}{2}=\frac{1-\left(\frac{1}{3}\right)^\infty}{2}=\frac{1}{2}$$

Detyra 43: $\lim\limits_{n\to\infty}\left(\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+...+\frac{1}{4^n}\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+...+\frac{1}{4^n}\right)=\left|\begin{array}{l} a_1=\frac{1}{4},a_2=\frac{1}{16}=\frac{1}{4^2},a_3=\frac{1}{64}=\frac{1}{4^3}.......a_n=\frac{1}{4^n}, \quad q=\frac{a_2}{a_1}=\frac{\frac{1}{16}}{\frac{1}{4}}=\frac{4}{16}=\frac{1}{4} \\ S_n=\frac{1}{4}\cdot\frac{1-\left(\frac{1}{4}\right)^n}{1-\frac{1}{4}}=\frac{1}{4}\cdot\frac{1-\left(\frac{1}{3}\right)^n}{\frac{3}{4}}=\frac{1}{4}\cdot\frac{4\left(1-\left(\frac{1}{4}\right)^n\right)}{3}=\frac{1-\left(\frac{1}{4}\right)^n}{3} \end{array}\right|=$$

$$=\lim_{n\to\infty}\frac{1-\left(\frac{1}{4}\right)^n}{3}=\frac{1-\left(\frac{1}{4}\right)^\infty}{3}=\frac{1}{3}$$

Detyra 44: $\lim\limits_{n\to\infty}\frac{1+a+a^2+...+a^n}{1+b+b^2+...+b^n}\quad(|a|<1,|b|<1)$

Zgjidhje:

$$\lim_{n\to\infty}\frac{1+a+a^2+...+a^n}{1+b+b^2+...+b^n}=\left|\begin{array}{l} 1+a+a^2+...+a^n-\text{vargu është gjeometrik } a_1=1,a_2=a,q=a \\ S_n=1\cdot\frac{1-a^n}{1-a}=\frac{1-a^n}{1-a} \\ 1+b+b^2+...+b^n \\ S_n=1\cdot\frac{1-b^n}{1-b}=\frac{1-b^n}{1-b} \end{array}\right|=$$

$$=\lim_{n\to\infty}\frac{\frac{1-a^n}{1-a}}{\frac{1-b^n}{1-b}}=\frac{1-b}{1-a}\cdot\lim_{n\to\infty}\frac{1-a^n}{1-b^n}=\frac{1-b}{1-a}\cdot\lim_{n\to\infty}\frac{a^n\left(\frac{1}{a^n}-1\right)}{b^n\left(\frac{1}{b^n}-1\right)}=\frac{1-b}{1-a}$$

Detyra 45: $\lim\limits_{n\to\infty}\left(\sqrt{n^2+1}\right)-\left(\sqrt{n^2-1}\right)$

Zgjidhje: $\lim\limits_{n\to\infty}\left(\sqrt{n^2+1}\right)-\left(\sqrt{n^2-1}\right)\cdot\frac{\left(\sqrt{n^2+1}\right)+\left(\sqrt{n^2-1}\right)}{\left(\sqrt{n^2+1}\right)+\left(\sqrt{n^2-1}\right)}=\lim\limits_{n\to\infty}\frac{\left(\sqrt{n^2+1}\right)^2-\left(\sqrt{n^2-1}\right)^2}{\left(\sqrt{n^2+1}\right)+\left(\sqrt{n^2-1}\right)}=$

$$=\lim_{n\to\infty}\frac{n^2+1-n^2+1}{\left(\sqrt{n^2+1}\right)+\left(\sqrt{n^2-1}\right)}=\lim_{n\to\infty}\frac{2}{\left(\sqrt{n^2+1}\right)+\left(\sqrt{n^2-1}\right)}=\frac{2}{\infty}=0$$

Detyra 46: $\lim_{n\to\infty}\sqrt{n^2+3n}-\sqrt{n^2+2n}$

Zgjidhje:

$$\lim_{n\to\infty}\sqrt{n^2+3n}-\sqrt{n^2+2n}\cdot\frac{\sqrt{n^2+3n}+\sqrt{n^2+2n}}{\sqrt{n^2+3n}+\sqrt{n^2+2n}}=\lim_{n\to\infty}\frac{\left(\sqrt{n^2+3n}\right)^2-\left(\sqrt{n^2+2n}\right)^2}{\sqrt{n^2+3n}+\sqrt{n^2+2n}}=\lim_{n\to\infty}\frac{n^2+3n-n^2-2n}{\sqrt{n^2+3n}+\sqrt{n^2+2n}}=$$

$$=\lim_{n\to\infty}\frac{n}{\sqrt{n^2+3n}+\sqrt{n^2+2n}}=\lim_{n\to\infty}\frac{\frac{n}{n}}{\frac{\sqrt{n^2+3n}+\sqrt{n^2+2n}}{\sqrt{n^2}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{3}{n}}+\sqrt{1+\frac{2}{n}}}=\frac{1}{\sqrt{1+0}+\sqrt{1+0}}=\frac{1}{1+1}=\frac{1}{2}$$

Detyra 47: $\lim_{n\to\infty}\left(\sqrt{n^2+2n}+n\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(\sqrt{n^2+2n}-n\right)\cdot\frac{\sqrt{n^2+2n}+n}{\sqrt{n^2+2n}+n}=\lim_{n\to\infty}\frac{\left(\sqrt{n^2+2n}\right)^2-n^2}{\sqrt{n^2+2n}+n}=\lim_{n\to\infty}\frac{n^2+2n-n^2}{\sqrt{n^2+2n}+n}=\lim_{n\to\infty}\frac{\frac{2n}{n}}{\sqrt{\frac{n^2}{n^2}+\frac{2n}{n^2}}+\frac{n}{n}}=$$

$$=\lim_{n\to\infty}\frac{2}{\sqrt{1+\frac{2n}{n^2}}+1}=\frac{2}{1+1}=\frac{2}{2}=1$$

Detyra 48: $\lim_{n\to\infty}\left(\sqrt{n^2+2n+2}-\sqrt{n^2-4n+3}\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(\sqrt{n^2+2n+2}-\sqrt{n^2-4n+3}\right)\cdot\frac{\sqrt{n^2+2n+2}+\sqrt{n^2-4n+3}}{\sqrt{n^2+2n+2}+\sqrt{n^2-4n+3}}=\lim_{n\to\infty}\frac{n^2+2n+2-n^2+4n-3}{\sqrt{n^2+2n+2}+\sqrt{n^2-4n+3}}=$$

$$=\lim_{n\to\infty}\frac{6n-1}{\sqrt{n^2+2n+2}+\sqrt{n^2-4n+3}}=\lim_{n\to\infty}\frac{\frac{6n}{n}-\frac{1}{n}}{\sqrt{\frac{n^2}{n^2}+\frac{2n}{n^2}+\frac{2}{n^2}}+\sqrt{\frac{n^2}{n^2}-\frac{4n}{n^2}+\frac{3}{n^2}}}=$$

$$=\lim_{n\to\infty}\frac{6-\frac{1}{n}}{\sqrt{1+\frac{2}{n}+\frac{2}{n^2}}+\sqrt{1-\frac{4}{n}+\frac{3}{n^2}}}=\frac{6-\frac{1}{\infty}}{\sqrt{1+\frac{2}{\infty}+\frac{2}{\infty^2}}+\sqrt{1-\frac{4}{\infty}+\frac{3}{\infty^2}}}=\frac{6}{\sqrt{1}+\sqrt{1}}=\frac{6}{2}=3$$

Detyra 49: $\lim_{n\to\infty}\left(\sqrt{n+3}-\sqrt{n}\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(\sqrt{n+3}-\sqrt{n}\right)\cdot\frac{\sqrt{n+3}+\sqrt{n}}{\sqrt{n+3}+\sqrt{n}}=\lim_{n\to\infty}\frac{\left(\sqrt{n+3}\right)^2-\left(\sqrt{n}\right)^2}{\sqrt{n+3}+\sqrt{n}}=\lim_{n\to\infty}\frac{n+3-n}{\sqrt{n+3}+\sqrt{n}}=\frac{3}{\infty}=0$$

Detyra 50: $\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}=\lim_{n\to\infty}\frac{\left(\sqrt{n+1}\right)^2-\left(\sqrt{n}\right)^2}{\sqrt{n+1}+\sqrt{n}}=\lim_{n\to\infty}\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}}=\lim_{n\to\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}=\frac{1}{\infty}=0$$

Detyra 51: $\lim_{n\to\infty}\frac{\sqrt{n^2+1}}{n+1}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\sqrt{n^2+1}}{n+1}=\lim_{n\to\infty}\frac{\sqrt{\frac{n^2}{n^2}+\frac{1}{n^2}}}{\frac{n}{n}+\frac{1}{n}}=\lim_{n\to\infty}\frac{\sqrt{1+\frac{1}{n^2}}}{1+\frac{1}{n}}=\frac{1}{1}=1$$

Detyra 52: $\lim_{n\to\infty}\frac{\sqrt[5]{7n^7+2n^2+1}+n}{n-\sqrt[3]{n^4+1}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\sqrt[5]{7n^7+2n^2+1}+n}{n-\sqrt[3]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt[5]{\frac{7n^7}{n^5}+\frac{2n^2}{n^5}+\frac{1}{n^5}}+\frac{n}{n}}{\frac{n}{n}-\sqrt[3]{\frac{n^4}{n^3}+\frac{1}{n^3}}}=\lim_{n\to\infty}\frac{\sqrt[5]{7n^2+\frac{2}{n^3}+\frac{1}{n^5}}+1}{1-\sqrt[3]{n+\frac{1}{n^3}}}=$$

$$=\lim_{n\to\infty}\frac{\sqrt[5]{7n^2}+1}{1-\sqrt[3]{n}}=\lim_{n\to\infty}\frac{\sqrt[5]{\frac{7n^2}{n^{15}}}+\frac{1}{\sqrt[3]{n}}}{\frac{1}{\sqrt[3]{n}}-\sqrt[3]{\frac{n}{n}}}=\frac{0+0}{0-1}=\frac{0}{-1}=0$$

Detyra 53: $\lim\limits_{n\to\infty}\dfrac{\left(\sqrt{n^2+1}+n\right)^2}{\sqrt[3]{n^6+1}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\left(\sqrt{n^2+1}+n\right)^2}{\sqrt[3]{n^6+1}}=\lim_{n\to\infty}\frac{n^2+1+2n\sqrt{n^2+1}+n^2}{\sqrt[3]{n^6+1}}=\lim_{n\to\infty}\frac{2n^2+2n\sqrt{n^2+1}+1}{\sqrt[3]{n^6+1}}=$$

$$=\lim_{n\to\infty}\frac{2+\dfrac{2\sqrt{n^2+1}}{n}+\dfrac{1}{n^2}}{\sqrt[3]{\dfrac{n^6}{n^6}+\dfrac{1}{n^6}}}=\lim_{n\to\infty}\frac{2+2\sqrt{1+\dfrac{1}{n^2}}+\dfrac{1}{n^2}}{\sqrt[3]{1+\dfrac{1}{n^6}}}=\frac{2+2\sqrt{1+0}+0}{\sqrt[3]{1+0}}=\frac{2+2}{1}=\frac{4}{1}=4$$

Detyra 54: $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^2+1}}{n+1}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^2+1}}{n+1}=\lim_{n\to\infty}\frac{\dfrac{\sqrt[3]{n^2+1}}{\sqrt[3]{n}}}{\dfrac{n+1}{n}}=\lim_{n\to\infty}\frac{\sqrt[3]{\dfrac{1}{n}+\dfrac{1}{n^3}}}{1+\dfrac{1}{n}}=\frac{0+0}{1+0}=\frac{0}{1}=0$$

Detyra 55: $\lim\limits_{n\to\infty}\dfrac{\sqrt[3]{n^3+4n+2}}{\sqrt{n^2+2n-1}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^3+4n+2}}{\sqrt{n^2+2n-1}}=\lim_{n\to\infty}\frac{\dfrac{\sqrt[3]{n^3+4n+2}}{\sqrt[3]{n^3}}}{\dfrac{\sqrt{n^2+2n-1}}{\sqrt{n^2}}}=\lim_{n\to\infty}\frac{\sqrt[3]{1+\dfrac{4}{n^2}+\dfrac{2}{n^3}}}{\sqrt{1+\dfrac{2}{n}-\dfrac{1}{n^2}}}=\frac{\sqrt[3]{1+0+0}}{\sqrt{1+0-0}}=\frac{1}{1}=1$$

Detyra 56: $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+3}-\sqrt{n+1}}{\sqrt{n+1}+\sqrt{n+2}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\sqrt{n+3}-\sqrt{n+1}}{\sqrt{n+1}+\sqrt{n+2}}=\lim_{n\to\infty}\frac{\sqrt{\dfrac{n+3}{n}}-\sqrt{\dfrac{n+1}{n}}}{\sqrt{\dfrac{n+1}{n}}+\sqrt{\dfrac{n+2}{n}}}=\lim_{n\to\infty}\frac{\sqrt{1+\dfrac{3}{n}}-\sqrt{1+\dfrac{1}{n}}}{\sqrt{1+\dfrac{1}{n}}+\sqrt{1+\dfrac{2}{n}}}=\frac{1-1}{1+1}=\frac{0}{2}=0$$

Detyra 57: $\lim\limits_{n\to\infty}\dfrac{\sqrt[4]{n^3-2n^2+1}+\sqrt[3]{n^4+1}}{\sqrt[4]{n^6+6n^5+2}-\sqrt[5]{n^7+3n^3+1}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\sqrt[4]{n^3-2n^2+1}+\sqrt[3]{n^4+1}}{\sqrt[4]{n^6+6n^5+2}-\sqrt[5]{n^7+3n^3+1}}=\lim_{n\to\infty}\frac{\dfrac{\sqrt[4]{n^3-2n^2+1}}{\sqrt{n^3}}+\dfrac{\sqrt[3]{n^4+1}}{\sqrt{n^3}}}{\dfrac{\sqrt[4]{n^6+6n^5+2}}{\sqrt{n^3}}-\dfrac{\sqrt[5]{n^7+3n^3+1}}{\sqrt{n^3}}}=\lim_{n\to\infty}\frac{\sqrt{\dfrac{n^3-2n^2+1}{n^3}}+\sqrt[3]{\dfrac{n^4+1}{\sqrt[3]{n^6}}}}{\dfrac{\sqrt[4]{n^6+6n^5+2}}{\sqrt[4]{n^6}}-\dfrac{\sqrt[5]{n^2+3n^3+1}}{\sqrt[5]{n^{\frac{15}{2}}}}}=$$

$$=\lim_{n\to\infty}\frac{\sqrt{\dfrac{n^3}{n^3}-\dfrac{2n^2}{n^3}+\dfrac{1}{n^3}}+\sqrt[3]{\dfrac{n^4}{n^6}+\dfrac{1}{n^6}}}{\sqrt{\dfrac{n^6}{n^6}+\dfrac{6n^5}{n^6}+\dfrac{2}{n^6}}-\sqrt[5]{\dfrac{n^2}{n^{\frac{15}{2}}}+\dfrac{3n^3}{n^{\frac{15}{2}}}+\dfrac{1}{n^{\frac{15}{2}}}}}=\lim_{n\to\infty}\frac{\sqrt{1-\dfrac{2}{n}+\dfrac{1}{n^3}}+\sqrt[3]{\dfrac{1}{n^2}+\dfrac{1}{n^6}}}{\sqrt{1+\dfrac{6}{n}+\dfrac{2}{n^6}}-\sqrt[5]{\dfrac{1}{n^{0.5}}+\dfrac{3}{n^{4.5}}+\dfrac{1}{n^{\frac{15}{2}}}}}=\frac{1}{1}=1$$

Detyra 58: $\lim\limits_{n\to\infty}\dfrac{\sqrt{n+\sqrt{n+\sqrt{n}}}}{\sqrt{n+1}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\sqrt{n+\sqrt{n+\sqrt{n}}}}{\sqrt{n+1}}=\lim_{n\to\infty}\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n+1}}=\sqrt{\lim_{n\to\infty}\frac{n+\sqrt{n+\sqrt{n}}}{n+1}}=\sqrt{\lim_{n\to\infty}\frac{\dfrac{n}{n}+\dfrac{\sqrt{n+\sqrt{n}}}{n}}{\dfrac{n}{n}+\dfrac{1}{n}}}=$$

$$=\sqrt{\lim_{n\to\infty}\frac{1+\dfrac{\sqrt{n+\sqrt{n}}}{\sqrt{n}^2}}{1+\dfrac{1}{n}}}=\sqrt{\lim_{n\to\infty}\frac{1+\sqrt{\dfrac{n+\sqrt{n}}{n^2}}}{1+\dfrac{1}{n}}}=\sqrt{\lim_{n\to\infty}\frac{1+\sqrt{\dfrac{n}{n^2}+\dfrac{\sqrt{n}}{n^2}}}{1+\dfrac{1}{n}}}=\sqrt{\lim_{n\to\infty}\frac{1+\sqrt{\dfrac{1}{n}+\dfrac{\sqrt{n}}{\sqrt{n^4}}}}{1+\dfrac{1}{n}}}=$$

$$=\sqrt{\lim_{n\to\infty}\frac{1+\sqrt{\dfrac{1}{n}+\sqrt{\dfrac{n}{n^4}}}}{1+\dfrac{1}{n}}}=\sqrt{\lim_{n\to\infty}\frac{1+\sqrt{\dfrac{1}{n}+\sqrt{\dfrac{1}{n}}}}{1+\dfrac{1}{n}}}=\sqrt{\frac{1+\sqrt{0+\sqrt{0}}}{1}}=\sqrt{\frac{1+0}{1}}=\sqrt{\frac{1}{1}}=\sqrt{1}=1$$

Detyra 59: $\lim\limits_{n\to\infty}\dfrac{\sqrt{2n^2-n+3}}{n+3}$

Zgjidhje: $\lim\limits_{n\to\infty}\dfrac{\sqrt{2n^2-n+3}}{n+3}=\lim\limits_{n\to\infty}\dfrac{\sqrt{\dfrac{2n^2-n+3}{n^2}}}{\dfrac{n+3}{n}}=\lim\limits_{n\to\infty}\dfrac{\sqrt{2-\dfrac{1}{n}+\dfrac{3}{n^2}}}{1+\dfrac{3}{n}}=\dfrac{\sqrt{2-0+0}}{1+0}=\dfrac{\sqrt{2}}{1}=\sqrt{2}$

Detyra 60: $\lim\limits_{n\to\infty}\dfrac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}=\lim_{n\to\infty}\sqrt{\frac{n}{n+\sqrt{n+\sqrt{n}}}}=\lim_{n\to\infty}\sqrt{\frac{n}{n+\sqrt{n+\sqrt{n}}}\cdot\frac{n-\sqrt{n+\sqrt{n}}}{n-\sqrt{n+\sqrt{n}}}}=$$

$$=\lim_{n\to\infty}\sqrt{\frac{n\left(n-\sqrt{n+\sqrt{n}}\right)}{n^2-\left(n+\sqrt{n}\right)}}=\lim_{n\to\infty}\sqrt{\frac{n^2-n\sqrt{n+\sqrt{n}}}{n^2-\left(n+\sqrt{n}\right)}}=\lim_{n\to\infty}\sqrt{\frac{n^2-\sqrt{n^3+n^2\sqrt{n}}}{n^2-n-\sqrt{n}}}=$$

$$=\lim_{n\to\infty}\sqrt{\frac{n^2-\sqrt{n^3+\sqrt{n^4n}}}{n^2-n-\sqrt{n}}}=\lim_{n\to\infty}\sqrt{\frac{n^2-\sqrt{n^3+\sqrt{n^5}}}{n^2-n-\sqrt{n}}}\cdot\frac{\sqrt{\frac{1}{n^2}}}{\sqrt{\frac{1}{n^2}}}=\lim_{n\to\infty}\sqrt{\frac{\frac{n^2}{n^2}-\frac{\sqrt{n^3+\sqrt{n^5}}}{n^2}}{\frac{n^2}{n^2}-\frac{n}{n^2}-\frac{\sqrt{n}}{n^2}}}=\lim_{n\to\infty}\sqrt{\frac{1-\sqrt{\frac{n^3}{n^4}+\frac{\sqrt{n^5}}{n^4}}}{1-\frac{1}{n}-\sqrt{\frac{n}{n^4}}}}=$$

$$=\lim_{n\to\infty}\sqrt{\frac{1-\sqrt{\frac{1}{n}+\sqrt{\frac{n^5}{n^8}}}}{1-\frac{1}{n}-\sqrt{\frac{1}{n^3}}}}=\lim_{n\to\infty}\sqrt{\frac{1-\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}}{1-\frac{1}{n}-\sqrt{\frac{1}{n^3}}}}=\sqrt{\frac{1-\sqrt{0+\sqrt{0}}}{1-0-\sqrt{0}}}=\frac{1}{1}=1$$

Detyra 61: $\lim\limits_{n\to\infty}\dfrac{n+1}{\sqrt[3]{n^2+1}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{n+1}{\sqrt[3]{n^2+1}}=\lim_{n\to\infty}\frac{\frac{n+1}{n}}{\sqrt[3]{\frac{n^2+1}{n^3}}}=\lim_{n\to\infty}\frac{1+\frac{1}{n}}{\sqrt[3]{\frac{1}{n}+\frac{1}{n^3}}}=\frac{1+0}{0}=\frac{1}{0}=\infty$$

Detyra 62: $\lim\limits_{n\to\infty}\dfrac{\sqrt{9n^2-2n+3}}{\sqrt[3]{8n^3-2n-4}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\sqrt{9n^2-2n+3}}{\sqrt[3]{8n^3-2n-4}}=\lim_{n\to\infty}\frac{\sqrt{\frac{9n^2-2n+3}{n^2}}}{\sqrt[3]{\frac{8n^3-2n-4}{n^3}}}=\lim_{n\to\infty}\frac{\sqrt{9-\frac{2}{n}+\frac{3}{n^2}}}{\sqrt[3]{8-\frac{2}{n^2}-\frac{4}{n^3}}}=\frac{\sqrt{9-0+0}}{\sqrt[3]{8-0-0}}=\frac{\sqrt{9}}{\sqrt[3]{8}}=\frac{3}{2}$$

Detyra 63: $\lim\limits_{n\to\infty}\frac{\sqrt[4]{n^2-2n+5}}{n^2-2n}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\sqrt[4]{n^2-2n+5}}{n^2-2n}=\lim_{n\to\infty}\frac{\sqrt[4]{\frac{n^2-2n+5}{n^8}}}{\frac{n^2-2n}{n^2}}=\lim_{n\to\infty}\frac{\sqrt[4]{\frac{1}{n^6}-\frac{2}{n^7}+\frac{5}{n^8}}}{1-\frac{2}{n}}=\frac{\sqrt[4]{0}}{1}=\frac{0}{1}=0$$

Detyra 64: $\lim\limits_{n\to\infty}\frac{\sqrt[3]{n^3+2n-1}}{n+2}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^3+2n-1}}{n+2}=\lim_{n\to\infty}\sqrt[3]{\frac{n^3+2n-1}{(n+2)^3}}=\lim_{n\to\infty}\sqrt[3]{\frac{n^3+2n-1}{n^3+6n^2+12n+8}}=\lim_{n\to\infty}\sqrt[3]{\frac{\frac{n^3}{n^3}+\frac{2n}{n^3}-\frac{1}{n^3}}{\frac{n^3}{n^3}+\frac{6n^2}{n^3}+\frac{12n}{n^3}+\frac{8}{n^3}}}=$$

$$\lim_{n\to\infty}\sqrt[3]{\frac{1+\frac{2}{n^2}-\frac{1}{n^3}}{1+\frac{6}{n}+\frac{12}{n^2}+\frac{8}{n^3}}}=\sqrt[3]{\frac{1+\frac{2}{\infty^2}-\frac{1}{\infty^3}}{1+\frac{6}{\infty}+\frac{12}{\infty^2}+\frac{8}{\infty^3}}}=\sqrt[3]{\frac{1+0-0}{1+0+0+0}}=\sqrt[3]{1}=1$$

Detyra 65: $\lim\limits_{n\to\infty}\frac{2n+4}{\sqrt{9n^2+2}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{2n+4}{\sqrt{9n^2+2}}=\lim_{n\to\infty}\frac{n\left(2+\frac{4}{n}\right)}{\sqrt{n^2\left(9+\frac{2}{n^2}\right)}}=\lim_{n\to\infty}\frac{n\left(2+\frac{4}{n}\right)}{|n|\sqrt{9+\frac{2}{n^2}}}=\lim_{n\to\infty}\frac{n\left(2+\frac{4}{n}\right)}{n\sqrt{9+\frac{2}{n^2}}}=\lim_{n\to\infty}\frac{2+\frac{4}{n}}{\sqrt{9+\frac{2}{n^2}}}=\frac{2}{\sqrt{9}}=\frac{2}{3}$$

Detyra 66: $\lim\limits_{n\to\infty}\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+2}+\sqrt{n}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+2}+\sqrt{n}}=\lim_{n\to\infty}\frac{\sqrt{\frac{n}{n}}+\sqrt{\frac{1}{n}}-\sqrt{\frac{n}{n}}}{\sqrt{\frac{n}{n}}+\sqrt{\frac{2}{n}}+\sqrt{\frac{n}{n}}}=\frac{\sqrt{1}+\sqrt{0}-\sqrt{1}}{\sqrt{1}+\sqrt{0}+\sqrt{1}}=\frac{1+0-1}{1+0+1}=\frac{0}{2}=0$$

Detyra 67: $\lim\limits_{n\to\infty}\frac{\sqrt{n^2+3n}}{\sqrt[3]{n^3-2n^2}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\sqrt{n^2+3n}}{\sqrt[3]{n^3-2n^2}}=\lim_{n\to\infty}\frac{\frac{\sqrt{n^2+3n}}{n}}{\frac{\sqrt[3]{n^3-2n^2}}{n}}=\lim_{n\to\infty}\frac{\frac{\sqrt{n^2+3n}}{\sqrt{n^2}}}{\frac{\sqrt[3]{n^3-2n^2}}{\sqrt[3]{n^3}}}=\lim_{n\to\infty}\frac{\sqrt{\frac{n^2+3n}{n^2}}}{\sqrt[3]{\frac{n^3-2n^2}{n^3}}}=\lim_{n\to\infty}\frac{\sqrt{1+\frac{3}{n}}}{\sqrt[3]{1-\frac{2}{n}}}=\frac{1}{1}=1$$

Detyra 68: $\lim\limits_{n\to\infty}n^2\left(n-\sqrt{n^2+1}\right)$

Zgjidhje:

$$\lim_{n\to\infty}n^2\left(n-\sqrt{n^2+1}\right)\cdot\frac{n+\sqrt{n^2+1}}{n+\sqrt{n^2+1}}=\lim_{n\to\infty}\frac{n^2\left(n-\sqrt{n^2+1}\right)\left(n+\sqrt{n^2+1}\right)}{n+\sqrt{n^2+1}}=\lim_{n\to\infty}\frac{n^2\left(n^2-\left(n^2+1\right)\right)}{n+\sqrt{n^2+1}}=$$

$$=\lim_{n\to\infty}\frac{n^2\left(n^2-n^2-1\right)}{n+\sqrt{n^2+1}}=-\lim_{n\to\infty}\frac{n^2}{n+\sqrt{n^2+1}}=-\lim_{n\to\infty}\frac{\frac{n^2}{n^2}}{\frac{n}{n^2}+\frac{\sqrt{n^2+1}}{n^2}}=-\lim_{n\to\infty}\frac{1}{\frac{1}{n}+\sqrt{1+\frac{1}{n^4}}}=-\infty$$

Detyra 69: $\lim\limits_{n\to\infty}\left(\sqrt[3]{n+1}-\sqrt[3]{3n}\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(\sqrt[3]{n+1}-\sqrt[3]{3n}\right)\cdot\frac{\sqrt[3]{(n+1)^2}+\sqrt[3]{(n+1)\cdot 3n}+\sqrt[3]{(3n)^2}}{\sqrt[3]{(n+1)^2}+\sqrt[3]{(n+1)\cdot 3n}+\sqrt[3]{(3n)^2}}=\lim_{n\to\infty}\frac{n+1-3n}{\sqrt[3]{(n+1)^2}+\sqrt[3]{(n+1)\cdot 3n}+\sqrt[3]{(3n)^2}}=$$

$$=\lim_{n\to\infty}\frac{1-2n}{\sqrt[3]{(n+1)^2}+\sqrt[3]{(n+1)\cdot 3n}+\sqrt[3]{(3n)^2}}=0$$

Detyra 70: $\lim\limits_{n\to\infty}\sqrt[2n]{3^{2n-1}}$

Zgjidhje:

$$\lim_{n\to\infty}\sqrt[2n]{3^{2n-1}}=\lim_{n\to\infty}3^{\frac{2n-1}{2n}}=\lim_{n\to\infty}3^{1+\frac{1}{2n}}=\lim_{n\to\infty}3\cdot 3^{\frac{1}{2n}}=3\lim_{n\to\infty}3^{\frac{1}{2n}}=3\cdot 1=3$$

Detyra 71: $\lim\limits_{n\to\infty}\left(\sqrt[3]{(n+2)^2}-\sqrt[3]{(n-2)^2}\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(\sqrt[3]{(n+2)^2}-\sqrt[3]{(n-2)^2}\right)\cdot\frac{\left(\sqrt[3]{(n+2)^4}+\sqrt[3]{(n+2)^2(n-2)^2}+\sqrt[3]{(n-2)^4}\right)}{\left(\sqrt[3]{(n+2)^4}+\sqrt[3]{(n+2)^2(n-2)^2}+\sqrt[3]{(n-2)^4}\right)}=$$

$$=\lim_{n\to\infty}\frac{(n+2)^2-(n-2)^2}{\left(\sqrt[3]{(n+2)^4}+\sqrt[3]{(n+2)^2(n-2)^2}+\sqrt[3]{(n-2)^4}\right)}=\lim_{n\to\infty}\frac{n^2+4n+4-\left(n^2-4n+4\right)}{\left(\sqrt[3]{(n+2)^4}+\sqrt[3]{(n+2)^2(n-2)^2}+\sqrt[3]{(n-2)^4}\right)}=$$

$$=\lim_{n\to\infty}\frac{n^2+4n+4-n^2+4n-4}{\left(\sqrt[3]{(n+2)^4}+\sqrt[3]{(n+2)^2(n-2)^2}+\sqrt[3]{(n-2)^4}\right)}=\lim_{n\to\infty}\frac{8n}{\left(\sqrt[3]{(n+2)^4}+\sqrt[3]{(n+2)^2(n-2)^2}+\sqrt[3]{(n-2)^4}\right)}=$$

$$=\lim_{n\to\infty}\frac{8n}{\left(\sqrt[3]{\left(1+\frac{2}{n}\right)^4}+\sqrt[3]{\left(1+\frac{2}{n}\right)^2\left(1-\frac{2}{n}\right)^2}+\sqrt[3]{\left(1-\frac{2}{n}\right)^4}\right)}=\frac{0}{3}=0$$

Detyra 72: $\lim\limits_{n\to\infty}\left(\sqrt[3]{(n+1)^2}-\sqrt[3]{(n-1)^2}\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(\sqrt[3]{(n+1)^2}-\sqrt[3]{(n-1)^2}\right)\cdot\frac{\left(\sqrt[3]{(n+1)^4}+\sqrt[3]{(n+1)^2(n-1)^2}+\sqrt[3]{(n-1)^4}\right)}{\left(\sqrt[3]{(n+1)^4}+\sqrt[3]{(n+1)^2(n-1)^2}+\sqrt[3]{(n-1)^4}\right)}=$$

$$=\lim_{n\to\infty}\frac{(n+1)^2-(n-1)^2}{\left(\sqrt[3]{(n+1)^4}+\sqrt[3]{(n+1)^2(n-1)^2}+\sqrt[3]{(n-1)^4}\right)}=\lim_{n\to\infty}\frac{n^2+2n+1-\left(n^2-2n+1\right)}{\left(\sqrt[3]{(n+1)^4}+\sqrt[3]{(n+1)^2(n-1)^2}+\sqrt[3]{(n-1)^4}\right)}=$$

$$=\lim_{n\to\infty}\frac{n^2+2n+1-n^2+2n-1}{\left(\sqrt[3]{(n+1)^4}+\sqrt[3]{(n+1)^2(n-1)^2}+\sqrt[3]{(n-1)^4}\right)}=\lim_{n\to\infty}\frac{4n}{\left(\sqrt[3]{(n+1)^4}+\sqrt[3]{(n+1)^2(n-1)^2}+\sqrt[3]{(n-1)^4}\right)}=$$

$$=\lim_{n\to\infty}\frac{4n}{\left(\sqrt[3]{\left(1+\frac{1}{n}\right)^4}+\sqrt[3]{\left(1+\frac{1}{n}\right)^2\left(1-\frac{1}{n}\right)^2}+\sqrt[3]{\left(1-\frac{1}{n}\right)^4}\right)}=\frac{0}{3}=0$$

Detyra 73: $\lim_{n\to\infty}\left(\sqrt[n]{4}-16\right)$

Zgjidhje: $\lim_{n\to\infty}\left(\sqrt[n]{4}-16\right)=\lim_{n\to\infty}\left(4^{\frac{1}{n}}-16\right)=4^0-16=1-16=-15$

Detyra 74: $\lim_{n\to\infty}\left(12-\frac{3}{2n}\right)^{2008}$

Zgjidhje:

$$\lim_{n\to\infty}\left(12-\frac{3}{2n}\right)^{2008}=12-0=12^{2008}$$

Detyra 75: $\lim_{n\to\infty} n\cdot\left[1-\sqrt{\left(1-\frac{a}{n}\right)\left(1-\frac{b}{n}\right)}\right]$

Zgjidhje:

$$\lim_{n\to\infty} n\cdot\left[1-\sqrt{\left(1-\frac{a}{n}\right)\left(1-\frac{b}{n}\right)}\right]\cdot\frac{1+\sqrt{\left(1-\frac{a}{n}\right)\left(1-\frac{b}{n}\right)}}{1+\sqrt{\left(1-\frac{a}{n}\right)\left(1-\frac{b}{n}\right)}}=\lim_{n\to\infty} n\cdot\frac{1^2-\left(\sqrt{\left(1-\frac{a}{n}\right)\left(1-\frac{b}{n}\right)}\right)^2}{\underbrace{1+\sqrt{\left(1-\frac{a}{n}\right)\left(1-\frac{b}{n}\right)}}_{A}}=$$

$$=\lim_{n\to\infty} n\cdot\frac{1-\left(1-\frac{a}{n}\right)\left(1-\frac{b}{n}\right)}{A}=\lim_{n\to\infty}\frac{n\cdot\left(1-\frac{n-a}{n}\cdot\frac{n-b}{n}\right)}{A}=\lim_{n\to\infty}\frac{n-n\left(\frac{n-a}{n}\cdot\frac{n-b}{n}\right)}{A}=$$

$$=\lim_{n\to\infty}\frac{n-\frac{n-a}{n}\cdot\frac{n-b}{n}}{A}=\lim_{n\to\infty}\frac{n-\frac{(n-a)(n-b)}{n}}{A}=\lim_{n\to\infty}\frac{\frac{n^2-n^2+nb+na-ab}{n}}{A}=\lim_{n\to\infty}\frac{\frac{nb+na-ab}{n}}{A}=$$

$$=\lim_{n\to\infty}\frac{\frac{nb+na-ab}{n}}{1+\sqrt{\left(1-\frac{a}{n}\right)\left(1-\frac{b}{n}\right)}}=\lim_{n\to\infty}\frac{\frac{nb}{n}+\frac{na}{n}-\frac{ab}{n}}{1+\sqrt{\left(1-\frac{a}{n}\right)\left(1-\frac{b}{n}\right)}}=\frac{b+a-\frac{ab}{n}}{1+\sqrt{\left(1-\frac{a}{\infty}\right)\left(1-\frac{b}{\infty}\right)}}=\frac{b+a}{1+\sqrt{1}}=\frac{a+b}{2}$$

Detyra 76: $\lim_{n\to\infty}4^{(n-2)(n-3)}$

Zgjidhje:

$$\lim_{n\to\infty}4^{(n-2)(n-3)}=\lim_{n\to\infty}4^{n^2-5n+6}=\lim_{n\to\infty}4^{\frac{n^2}{n^2}-\frac{5n}{n^2}+\frac{6}{n^2}}=\lim_{n\to\infty}4^{1-\frac{5}{n}+\frac{6}{n^2}}=4^{1-0+0}=4^1=4$$

Detyra 77: $\lim\limits_{n\to\infty}\frac{(n+4)\sqrt{2}}{n+3}$

Zgjidhje: $\lim\limits_{n\to\infty}\frac{(n+4)\sqrt{2}}{n+3}=\lim\limits_{n\to\infty}\frac{\frac{(n+4)\sqrt{2}}{n}}{\frac{n+3}{n}}=\lim\limits_{n\to\infty}\frac{\sqrt{2}+\frac{4\sqrt{2}}{n}}{1+\frac{3}{n}}=\frac{\sqrt{2}+0}{1+0}=\sqrt{2}$

Detyra 78: $\lim\limits_{n\to\infty}\frac{(n+2)!+(n+1)!}{(n+3)!}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{(n+2)!+(n+1)!}{(n+3)!}=\lim_{n\to\infty}\frac{(n+2)\cdot(n+1)!+(n+1)!}{(n+3)\cdot(n+2)\cdot(n+1)!}=\lim_{n\to\infty}\frac{(n+1)![n+2+1]}{(n+3)\cdot(n+2)\cdot(n+1)!}=$$

$$=\lim_{n\to\infty}\frac{n+3}{(n+3)\cdot(n+2)}=\lim_{n\to\infty}\frac{1}{(n+2)}=\frac{\lim\limits_{n\to\infty}1}{\lim\limits_{n\to\infty}(n+2)}=0$$

Detyra 79: $\lim\limits_{n\to\infty}\frac{(n-1)!-(n+1)!}{(n+1)!}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{(n-1)!-(n+1)!}{(n+1)!}=\lim_{n\to\infty}\frac{(n-1)!-(n+1)\cdot n\cdot(n-1)!}{(n+1)\cdot n\cdot(n-1)!}=\lim_{n\to\infty}\frac{(n-1)![1-(n+1)\cdot n]}{(n+1)(n-1)!\cdot n}=$$

$$=\lim_{n\to\infty}\frac{1-n^2-n}{n^2+n}=\lim_{n\to\infty}\frac{-n^2-n+1}{n^2+n}=-1$$

Detyra 80: $\lim\limits_{n\to\infty}\frac{n!+(n+1)!}{(n+2)!-(n+1)!}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{n!+(n+1)!}{(n+2)!-(n+1)!}=\lim_{n\to\infty}\frac{n!+(n+1)n!}{(n+2)(n+1)n!-(n+1)n!}=\lim_{n\to\infty}\frac{n!(1+n+1)}{n!\left(n^2+3n+2-n-1\right)}=\lim_{n\to\infty}\frac{n+2}{n^2+2n+1}=$$

$$=\lim_{n\to\infty}\frac{\frac{n}{n^2}+\frac{2}{n^2}}{\frac{n^2}{n^2}+\frac{2n}{n^2}+\frac{1}{n^2}}=\lim_{n\to\infty}\frac{\frac{1}{n}+\frac{2}{n^2}}{1+\frac{2}{n}+\frac{1}{n^2}}=\frac{0}{1}=0$$

Detyra 81: $\lim\limits_{n\to\infty}\frac{n!}{(n+1)!-n!}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{n!}{(n+1)!-n!}=\lim_{n\to\infty}\frac{n!}{(n+1)n!-n!}=\lim_{n\to\infty}\frac{n!}{n!(n+1-1)}=\lim_{n\to\infty}\frac{1}{n}=0$$

Detyra 82: $\lim\limits_{n\to\infty}\frac{(n+1)!+(n+2)!}{n!-(n+3)!}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{(n+1)!+(n+2)!}{n!-(n+3)!}=\lim_{n\to\infty}\frac{(n+1)n!+(n+2)(n+1)n!}{n!-(n+3)(n+2)(n+1)n!}=\lim_{n\to\infty}\frac{n!\left(n+1+n^2+3n+2\right)}{n!\left[1-\left(n^3+6n^2+11n+6\right)\right]}=$$

$$=\lim_{n\to\infty}\frac{n^2+4n+3}{1-n^3-6n^2-11n-6}=\lim_{n\to\infty}\frac{n^2+4n+3}{-n^3-6n^2-11n-5}=\lim_{n\to\infty}\frac{\frac{n^2}{n^3}+\frac{4n}{n^3}+\frac{3}{n^3}}{-\frac{n^3}{n^3}-\frac{6n^2}{n^3}-\frac{11n}{n^3}-\frac{5}{n^3}}=$$

$$=\lim_{n\to\infty}\frac{\frac{1}{n}+\frac{4}{n^2}+\frac{3}{n^3}}{-1-\frac{6}{n}-\frac{11}{n^2}-\frac{5}{n^3}}=\frac{0+0+0}{-1-0-0-0}=\frac{0}{-1}=0$$

Detyra 83: $\lim\limits_{n\to\infty}\frac{3(n+1)!-6n!}{6n!-18(n-1)!}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{3(n+1)!-6n!}{6n!-18(n-1)!}=\lim_{n\to\infty}\frac{3(n+1)n(n-1)!-6n(n-1)!}{6n(n-1)!-18(n-1)!}=\lim_{n\to\infty}\frac{3(n+1)\left[n(n-1)-n(n-1)\right]}{6n(n-1)(n-3)}=$$

$$=\lim_{n\to\infty}\frac{\left(n^2+n-n^2+n\right)}{2(n-3)}=\lim_{n\to\infty}\frac{2n}{2n-6}=\lim_{n\to\infty}\frac{\frac{2n}{n}}{\frac{2n}{n}-\frac{6}{n}}=\frac{2}{2}=1$$

Detyra 84: $\lim\limits_{n\to\infty}\frac{3^n+1}{3^n}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{3^n+1}{3^n}=\lim_{n\to\infty}\frac{3^n\left(1+\frac{1}{3^n}\right)}{3^n}=\lim_{n\to\infty}\left[\left(1+\frac{1}{3}\right)^n\right]=1+\left(\frac{1}{3}\right)^{\infty}=1+0=1$$

Detyra 85: $\lim\limits_{n\to\infty}\frac{2^{n+1}+3^{n+1}}{2^n+3^n}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{2^{n+1}+3^{n+1}}{2^n+3^n}=\lim_{n\to\infty}\frac{2\cdot 2^n+3\cdot 3^n}{2^n+3^n}=\lim_{n\to\infty}\frac{\frac{2\cdot 2^n+3\cdot 3^n}{3^n}}{\frac{2^n+3^n}{3^n}}=\lim_{n\to\infty}\frac{2\cdot\left(\frac{2}{3}\right)^n+3\cdot 1}{\left(\frac{2}{3}\right)^n+1}=\frac{0+3}{0+1}=\frac{3}{1}=3$$

Detyra 86: $\lim\limits_{n\to\infty}\frac{2^n+5^n}{2^n+5^n}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{2^n+5^n}{2^n+5^n}=\lim_{n\to\infty}\frac{\frac{2^n+5^n}{5^n}}{\frac{2^n+5^n}{5^n}}=\lim_{n\to\infty}\frac{\left(\frac{2}{5}\right)^n+1}{\left(\frac{2}{5}\right)^n+1}=\frac{0+1}{0+1}=\frac{1}{1}=1$$

Detyra 87: $\lim\limits_{n\to\infty}\frac{1-5^{n+2}}{3-5^n}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{1-5^{n+2}}{3-5^n}=\lim_{n\to\infty}\frac{1-5^n\cdot 5^2}{3-5^n}=\lim_{n\to\infty}\frac{1-25\cdot 5^n}{3-5^n}=\lim_{n\to\infty}\frac{5^n\left(\frac{1}{5^n}-25\right)}{5^n\left(\frac{3}{5^n}-1\right)}=\lim_{n\to\infty}\frac{\frac{1}{5^n}-25}{\frac{3}{5^n}-1}=\frac{-25}{-1}=25$$

Detyra 88: $\lim\limits_{n\to\infty}\frac{3\cdot 10^n+5\cdot 10^{2n}}{6\cdot 10^{n-1}+10^{2n-1}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{3\cdot 10^n+5\cdot 10^{2n}}{6\cdot 10^{n-1}+10^{2n-1}}=\lim_{n\to\infty}\frac{3\cdot 10^n+5\cdot 10^{2n}}{\frac{6}{10}\cdot 10^n+\frac{1}{10}\cdot 10^{2n}}=\lim_{n\to\infty}\frac{\frac{3\cdot 10^n+5\cdot 10^{2n}}{10^{2n}}}{\frac{\frac{6}{10}\cdot 10^n+\frac{1}{10}\cdot 10^{2n}}{10^{2n}}}=\lim_{n\to\infty}\frac{\frac{3}{10^n}+5\cdot 1}{\frac{6}{10\cdot 10^n}+\frac{1}{10}\cdot 1}=$$

$$=\frac{0+5}{0+\frac{1}{10}}=\frac{5}{\frac{1}{10}}=50$$

Detyra 89: $\lim\limits_{n\to\infty}\dfrac{3^{n+1}+5^{n+1}}{3^n+5^n}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{3^{n+1}+5^{n+1}}{3^n+5^n}=\lim_{n\to\infty}\frac{3^n\cdot 3+5^n\cdot 5}{3^n+5^n}=\lim_{n\to\infty}\frac{5^n\left(\frac{3^n}{5^n}\cdot 3+5\right)}{5^n\left(\frac{3^n}{5^n}+1\right)}=\lim_{n\to\infty}\frac{3\cdot\left(\frac{3^n}{5^n}\right)+5}{\left(\frac{3^n}{5^n}\right)+1}=\frac{5}{1}=5$$

Detyra 90: $\lim\limits_{n\to\infty}\dfrac{3\cdot 5^{n^2}+4\cdot 5^{2n^2}}{10\cdot 5^{n^2-1}-2\cdot 5^{2n^2+1}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{3\cdot 5^{n^2}+4\cdot 5^{2n^2}}{10\cdot 5^{n^2-1}-2\cdot 5^{2n^2+1}}=\lim_{n\to\infty}\frac{3\cdot 5^{n^2}+4\cdot 5^{2n^2}}{10\cdot\frac{1}{5}\cdot 5^{n^2}-2\cdot 5\cdot 5^{2n^2}}=\lim_{n\to\infty}\frac{3\cdot 5^{n^2}+4\cdot 5^{2n^2}}{5\cdot 5^{n^2}-10\cdot 5^{2n^2}}=\lim_{n\to\infty}\frac{\frac{3\cdot 5^{n^2}+4\cdot 5^{2n^2}}{5^{2n^2}}}{\frac{5\cdot 5^{n^2}-10\cdot 5^{2n^2}}{5^{2n^2}}}=$$

$$=\lim_{n\to\infty}\frac{\frac{3}{5n^2}+4\cdot 1}{\frac{5}{5^{n^2}}-10\cdot 1}=\frac{0+4}{0-10}=-\frac{4}{10}=-\frac{2}{5}$$

Detyra 91: $\lim\limits_{n\to\infty}\dfrac{5-2^n}{5\cdot 2^n+1}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{5-2^n}{5\cdot 2^n+1}=\lim_{n\to\infty}\frac{\frac{5}{2^n}-1}{5+\frac{1}{2^n}}=\frac{0-1}{5+0}=-\frac{1}{5}$$

Detyra 92: $\lim\limits_{n\to\infty}\dfrac{5-2^{-n}+4\cdot 5^{-n}}{3n+2+3^{-n}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{5-2^{-n}+4\cdot 5^{-n}}{3n+2+3^{-n}}=\lim_{n\to\infty}\frac{5-\frac{1}{2^n}+\frac{4}{5^n}}{3n+2+\frac{1}{3^n}}=\lim_{n\to\infty}\frac{5}{3n+2}=\frac{0}{3}=0$$

Detyra 93: $\lim\limits_{n\to\infty} n\cdot\left(\ln\left(n+1\right)-\ln n\right)$

Zgjidhje:

$$\lim_{n\to\infty} n\cdot\left(\ln\left(n+1\right)-\ln n\right)=\lim_{n\to\infty} n\cdot\left(\ln\frac{n+1}{n}\right)=\lim_{n\to\infty} n\cdot\left(\ln\left(1+\frac{1}{n}\right)\right)=\lim_{n\to\infty}\ln\left(1+\frac{1}{n}\right)^n=$$

$$=\ln\left(\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n\right)\ln e=1$$

Detyra 94: $\lim\limits_{n\to\infty} n\left(\ln\frac{n}{n+2}\right)$

Zgjidhje:

$$\lim_{n\to\infty} n\left(\ln\frac{n}{n+2}\right)=\lim_{n\to\infty}\ln\left(\frac{1}{\frac{n+2}{n}}\right)^n=\lim_{n\to\infty}\ln\left(\frac{1}{1+\frac{2}{n}}\right)^n=\lim_{n\to\infty}\ln\frac{1}{\left(1+\frac{2}{n}\right)^n}=\ln\frac{1}{e^2}=\ln e^{-2}=$$

$$=-2\ln e=-2\cdot 1=-2$$

Detyra 95: $\lim\limits_{n\to\infty}\left(\frac{n+2}{n+1}\right)^n$

Zgjidhje:

$$\lim_{n\to\infty}\left(\frac{n+2}{n+1}\right)^n=\lim_{n\to\infty}\left(1+\frac{n+2}{n+1}-1\right)^n=\lim_{n\to\infty}\left(1+\frac{n+2-n-1}{n+1}\right)^n=\lim_{n\to\infty}\left(1+\frac{1}{n+1}\right)^n=$$

$$=\left[\underbrace{\lim_{n\to\infty}\left(1+\frac{1}{n+1}\right)^{n+1}}_{e}\right]^{\frac{1}{n+1}\cdot n}=e^{\lim\limits_{n\to\infty}\frac{n}{n+1}}=e^{\lim\limits_{n\to\infty}\frac{\frac{n}{n}}{\frac{n}{n}+\frac{1}{n}}}=e^{\lim\limits_{n\to\infty}\frac{1}{1+\frac{1}{n}}}=e^1=e$$

Detyra 96: $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}$

Zgjidhje:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}=\left[\underbrace{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n}_{e}\right]^2=e^2$$

Detyra 97: $\lim\limits_{n\to\infty} n\left[\ln(n+3)-\ln n\right]$

Zgjidhje:

$$\lim_{n\to\infty} n\left[\ln(n+3)-\ln n\right]=\lim_{n\to\infty} n\cdot\ln\frac{n+3}{3}=\lim_{n\to\infty}\ln\left(\frac{n+3}{n}\right)^n=\ln\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^n=\ln\lim_{n\to\infty}\left(1+\frac{1}{\frac{n}{3}}\right)^n=$$

$$=\ln\left[\underbrace{\lim_{n\to\infty}\left(1+\frac{1}{\frac{n}{3}}\right)^{\frac{n}{3}}}_{e}\right]^{\frac{3}{n}\cdot n}=\ln e^3=3\ln e=3\cdot 1=3$$

Detyra 98: $\lim\limits_{n\to\infty}\left(\frac{n^2+4}{n^2-4}\right)^{n^2+1}$

Zgjidhje:

$$\lim_{n\to\infty}\left(\frac{n^2+4}{n^2-4}\right)^{n^2+1}=\lim_{n\to\infty}\left(1+\frac{n^2+4}{n^2-4}-1\right)^{n^2+1}=\lim_{n\to\infty}\left(1+\frac{n^2+4-n^2+4}{n^2-4}\right)^{n^2+1}=\lim_{n\to\infty}\left(1+\frac{8}{n^2-4}\right)^{n^2+1}=$$

$$=\lim_{n\to\infty}\left(1+\frac{1}{\frac{n^2-4}{8}}\right)^{\frac{n^2-4}{8}\cdot\frac{8}{n^2-4}\cdot\left(n^2+1\right)}=e^{\lim\limits_{n\to\infty}\frac{8n^2+8}{n^2-4}}=e^{\lim\limits_{n\to\infty}\frac{\frac{8n^2}{n^2}+\frac{8}{n^2}}{\frac{n^2}{n^2}-\frac{4}{n^2}}}=e^{\lim\limits_{n\to\infty}\frac{8+\frac{8}{n^2}}{1-\frac{4}{n^2}}}=e^{\frac{8}{1}}=e^8$$

Detyra 99: $\lim\limits_{n\to\infty}\left(\frac{n-2}{n+2}\right)^n$

Zgjidhje:

$$\lim_{n\to\infty}\left(\frac{n-2}{n+2}\right)^n=\lim_{n\to\infty}\left(1+\frac{n-2}{n+2}-1\right)^n=\lim_{n\to\infty}\left(1+\frac{n-2-n-2}{n+2}\right)^n=\lim_{n\to\infty}\left(1-\frac{4}{n+2}\right)^n=$$

$$=\lim_{n\to\infty}\left(1+\frac{1}{-\frac{n+2}{4}}\right)^{-\frac{n+2}{4}\left(-\frac{4}{n+2}\cdot n\right)}=\left[\lim_{n\to\infty}\left(1+\frac{1}{-\frac{n+2}{4}}\right)^{-\frac{n+2}{4}}\right]^{-\lim\limits_{n\to\infty}\frac{4n}{n+2}}=e^{-4}$$

Detyra 100: $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+10}$

Zgjidhje:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+10}=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+10\cdot\frac{1}{n}\cdot\frac{n}{1}}=e^{\lim\limits_{n\to\infty}(n+10)n}=e^{\lim\limits_{n\to\infty}n^2+10n}=e^{\lim\limits_{n\to\infty}\frac{n^2}{n^2}+\frac{10n}{n^2}}=e^{\lim\limits_{n\to\infty}1+\frac{10}{n}}=e^1=e$$

Detyra 101: $\lim\limits_{n\to\infty}\left(1+\frac{1}{4n}\right)^{2n-1}$

Zgjidhje:

$$\lim_{n\to\infty}\left(1+\frac{1}{4n}\right)^{2n-1}=\lim_{n\to\infty}\left(1+\frac{1}{4n}\right)^{4n\cdot\frac{2n-1}{4n}}=\left[\lim_{n\to\infty}\left(1+\frac{1}{4n}\right)^{4n}\right]^{\lim\limits_{n\to\infty}\frac{2n-1}{4n}}=e^{\frac{1}{2}}=\sqrt{e}$$

Detyra 102: $\lim\limits_{n\to\infty}\left(1-\frac{5}{n}\right)^{n}$

Zgjidhje:

$$\lim_{n\to\infty}\left(1-\frac{5}{n}\right)^{n}=\lim_{n\to\infty}\left[1+\frac{1}{\left(-\frac{n}{5}\right)}\right]^{n}=\left[\lim_{n\to\infty}\left(1+\frac{1}{\left(-\frac{n}{5}\right)}\right)^{-\frac{n}{5}}\right]^{-\frac{5}{n}\cdot n}=e^{\lim\limits_{n\to\infty}-\frac{5n}{n}}=e^{\lim\limits_{n\to\infty}-\frac{\frac{5n}{n}}{\frac{n}{n}}}=e^{-5}=\frac{1}{e^5}$$

Detyra 103: $\lim\limits_{n\to\infty}\frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}}=\lim_{n\to\infty}\frac{n\ln\left(1+\frac{1}{n}\right)}{1}=\lim_{n\to\infty}\ln\left(1+\frac{1}{n}\right)^{n}=\ln\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}=\ln e=1$$

Detyra 104: $\lim\limits_{n\to\infty}\left(1+\frac{1}{2n}\right)^{n}$

Zgjidhje:

$$\lim_{n\to\infty}\left(1+\frac{1}{2n}\right)^{n}=\lim_{n\to\infty}\left(1+\frac{1}{2n}\right)^{2n\cdot\frac{1}{2}}=\left(\lim_{n\to\infty}\left(1+\frac{1}{2n}\right)^{2n}\right)^{\frac{1}{2}}=e^{\frac{1}{2}}=\sqrt{e}$$

Detyra 105: $\lim\limits_{n\to\infty}\left(1-\frac{1}{2n}\right)^{n}$

Zgjidhje:

$$\lim_{n\to\infty}\left(1-\frac{1}{2n}\right)^{n}=\lim_{n\to\infty}\left(1-\frac{1}{2n}\right)^{2n\cdot\frac{1}{2}}=\left(\lim_{n\to\infty}\left(1-\frac{1}{2n}\right)^{2n}\right)^{\frac{1}{2}}=e^{-\frac{1}{2}}$$

Detyra 106: $\lim\limits_{n\to\infty}\left(1-\frac{1}{n^2}\right)^{n}$

Zgjidhje:

$$\lim_{n\to\infty}\left(1-\frac{1}{n^2}\right)^{n}=\left[\lim_{n\to\infty}\left(1-\frac{1}{n^2}\right)^{n^2}\right]^{\frac{1}{n^2}\cdot n}=e^{-\lim\limits_{n\to\infty}\frac{1}{n}}=e^{0}=1$$

Detyra 107: $\lim\limits_{n\to\infty}\left(1+\frac{2}{n}\right)^{n-1}$

Zgjidhje:

$$\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^{n-1}=\lim_{n\to\infty}\left(1+\frac{1}{\frac{n}{2}}\right)^{n-1}=\left[\lim_{n\to\infty}\left(1+\frac{1}{\frac{n}{2}}\right)^{\frac{n}{2}}\right]^{\frac{2}{n}\cdot(n-1)}=e^{\lim\limits_{n\to\infty}\left(2-\frac{2}{n}\right)}=e^{2}$$

Detyra 108: $\lim\limits_{n\to\infty}\ln\left(1+\frac{1}{n}\right)^{n}$

Zgjidhje:

$$\lim_{n\to\infty}\ln\left(1+\frac{1}{n}\right)^{n}=\ln\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}=\ln e=1$$

Detyra 109: $\lim\limits_{n\to\infty}\left(1+\frac{3}{n+1}\right)^{2n}$

Zgjidhje:

$$\lim_{n\to\infty}\left(1+\frac{3}{n+1}\right)^{2n}=\lim_{n\to\infty}\left(1+\frac{1}{\frac{n+1}{3}}\right)^{2n}=\left[\lim_{n\to\infty}\left(1+\frac{1}{\frac{n+1}{3}}\right)^{\frac{n+1}{3}}\right]^{\frac{3}{n+1}\cdot 2n}=e^{\lim\limits_{n\to\infty}\frac{6n}{n+1}}=e^{6}$$

Detyra 110: $\lim\limits_{n\to\infty}\left(1+\frac{1}{4n+1}\right)^{2n-2}$

Zgjidhje:

$$\lim_{n\to\infty}\left(1+\frac{1}{4n+1}\right)^{2n-2}=\left[\lim_{n\to\infty}\left(1+\frac{1}{4n+1}\right)^{4n+1}\right]^{\frac{2n-2}{4n+1}}=e^{\lim\limits_{n\to\infty}\frac{2n-2}{4n+1}}=e^{\lim\limits_{n\to\infty}\frac{\frac{2n}{n}-\frac{2}{n}}{\frac{4n}{n}+\frac{1}{n}}}=e^{\frac{2}{4}}=e^{\frac{1}{2}}=\sqrt{e}$$

Detyra 111: $\lim\limits_{n\to\infty}\ln\left(1+\frac{4}{n+1}\right)^{n}$

Zgjidhje:

$$\lim_{n\to\infty}\ln\left(1+\frac{4}{n+1}\right)^{n}=\ln\lim_{n\to\infty}\left(1+\frac{4}{n+1}\right)^{n}=\ln\left[\lim_{n\to\infty}\left(1+\frac{1}{\frac{n+1}{4}}\right)^{\frac{n+1}{4}}\right]^{\frac{4n}{n+1}}=\ln e^{\lim\limits_{n\to\infty}\frac{4n}{n+1}}=\ln e^{\lim\limits_{n\to\infty}\frac{\frac{4n}{n}}{\frac{n+1}{n}}}=\ln e^{4}$$

Detyra 112: $\lim\limits_{n\to\infty}\frac{\log_8 8+\log_8 64+\ldots+\log_8 8^n}{n^2}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\log_8 8+\log_8 64+\ldots+\log_8 8^n}{n^2}=\lim_{n\to\infty}\frac{\log_8 8+\log_8 8^2+\ldots+\log_8 8^n}{n^2}=$$

$$=\lim_{n\to\infty}\frac{1\cdot\log_8 8+2\cdot\log_8 8+\ldots+n\cdot\log_8 8}{n^2}=\lim_{n\to\infty}\frac{1+2+\ldots+n}{n^2}=\lim_{n\to\infty}\frac{\frac{n(n+1)}{2}}{n^2}=\frac{1}{2}$$

Detyra 113: $\lim\limits_{n\to\infty}\dfrac{\log_3 3+\log_3 9+\log_3 27+...+\log_3 3^n}{n^2}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{\log_3 3+\log_3 9+\log_3 27+...+\log_3 3^n}{n^2}=\lim_{n\to\infty}\frac{\log_3 3+\log_3 3^2+\log_3 3^3+...+\log_3 3^n}{n^2}=$$

$$=\lim_{n\to\infty}\frac{\log_3 3+2\log_3 3+3\log_3 3+...+n\log_3 3}{n^2}=\lim_{n\to\infty}\frac{1+2+3+...+n}{n^2}=\lim_{n\to\infty}\frac{\frac{n^2+n}{2}}{n^2}=$$

$$=\lim_{n\to\infty}\frac{n^2+n}{2n^2}=\lim_{n\to\infty}\frac{\frac{n^2}{n^2}+\frac{n}{n^2}}{\frac{2n^2}{n^2}}=\lim_{n\to\infty}\frac{1+\frac{1}{n}}{2}=\frac{1}{2}$$

Detyra 114: $\lim\limits_{n\to\infty}\ln\left(\dfrac{n^2-2n+7}{n^2-n+5}\right)^{n+1}$

Zgjidhje:

$$\lim_{n\to\infty}\ln\left(\frac{n^2-2n+7}{n^2-n+5}\right)^{n+1}=\ln\lim_{n\to\infty}\left(1+\frac{n^2-2n+7}{n^2-n+5}-1\right)^{n+1}=\ln\lim_{n\to\infty}\left(1+\frac{n^2-2n+7-n^2+n-5}{n^2-n+5}\right)^{n+1}=$$

$$=\ln\lim_{n\to\infty}\left(1+\frac{2-n}{n^2-n+5}\right)^{n+1}=\ln\left[\lim_{n\to\infty}\left(1+\frac{1}{\frac{n^2-n+5}{2-n}}\right)^{\frac{n^2-n+5}{2-n}}\right]^{\frac{(n+1)(2-n)}{n^2-n+5}}=\ln e^{\lim\limits_{n\to\infty}\frac{(n+1)(2-n)}{n^2-n+5}}=\ln e^{-1}=-1$$

Detyra 115: $\lim\limits_{n\to\infty}\ln\left(\dfrac{n+2}{n-3}\right)^{n}$

Zgjidhje:

$$\lim_{n\to\infty}\ln\left(\frac{n+2}{n-3}\right)^{n}=\ln\lim_{n\to\infty}\left(\frac{n+2}{n-3}\right)^{n}=\ln\lim_{n\to\infty}\left(1+\frac{n+2}{n-3}-1\right)^{n}=\ln\lim_{n\to\infty}\left(1+\frac{n+2-n+3}{n-3}\right)^{n}=$$

$$=\ln\lim_{n\to\infty}\left(1+\frac{5}{n-3}\right)^{n}=\ln\left[\lim_{n\to\infty}\left(1+\frac{1}{\frac{n-3}{5}}\right)^{\frac{n-3}{5}}\right]^{\frac{5n}{n-3}}=\ln e^{\lim\limits_{n\to\infty}\frac{5n}{n-3}}=\ln e^{5}=5$$

Detyra 116: $\lim\limits_{n\to\infty}\frac{1^4+2^4+3^4+...+n^4}{n^5}-\lim\limits_{n\to\infty}\frac{1+2^3+3^3+...+n^3}{n^5}$

Zgjidhje:

$$\lim_{n\to\infty}\frac{1^4+2^4+3^4+...+n^4}{n^5}-\lim_{n\to\infty}\frac{1+2^3+3^3+...+n^3}{n^5}=\lim_{n\to\infty}\frac{n(n+1)(2n+1)\left(3n^2+3n-1\right)}{30n^5}-\lim_{n\to\infty}\frac{n^2(n+1)^2}{4n^5}=$$

$$=\lim_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\left(3+\frac{3}{n}-\frac{1}{n^2}\right)}{30}-\lim_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)^2}{4n}=\frac{1\cdot2\cdot3}{30}-0=\frac{6}{30}=\frac{1}{5}$$

Detyra 117: $\lim\limits_{n\to\infty}\left(\frac{1}{1\cdot2}-\frac{1}{1\cdot3}+\frac{1}{2\cdot3}-\frac{1}{3\cdot5}+...+\frac{1}{n(n+1)}-\frac{1}{(2n-1)(2n+1)}\right)$

Zgjidhje:

$$\lim_{n\to\infty}\left(\frac{1}{1\cdot2}-\frac{1}{1\cdot3}+\frac{1}{2\cdot3}-\frac{1}{3\cdot5}+...+\frac{1}{n(n+1)}-\frac{1}{(2n-1)(2n+1)}\right)=$$

$$=\lim_{n\to\infty}\left(\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+...+\frac{1}{n(n+1)}\right)-\lim_{n\to\infty}\left(\frac{1}{1\cdot3}-\frac{1}{3\cdot5}+...+\frac{1}{(2n-1)(2n+1)}\right)=$$

$$=\lim_{n\to\infty}\left[\left(1-\frac{1}{n+1}\right)-\frac{1}{2}\left(1-\frac{1}{2n+1}\right)\right]=\lim_{n\to\infty}\left[\frac{n}{n+1}-\frac{n}{2n+1}\right]=\lim_{n\to\infty}\frac{n^2}{2n^2+3n+1}=$$

$$=\frac{\frac{n^2}{n^2}}{\frac{2n^2}{n^2}+\frac{3n}{n^2}+\frac{1}{n^2}}=\frac{1}{2+\frac{3}{n}+\frac{1}{n^2}}=\frac{1}{2+0+0}=\frac{1}{2}$$

Detyra 118: Tregoni se: $\lim\limits_{n\to\infty}\frac{a^n}{n!}=0$

Zgjidhje: Le të jetë $m\in\mathrm{N}$ i tillë që $m+1>|a|$, atëherë

$$0<\left|\frac{a^n}{n!}-0\right|=\frac{|a|^n}{n!}=\frac{|a|}{1}\cdot\frac{|a|}{2}\cdots\frac{|a|}{m}\cdot\frac{|a|}{m+1}\cdots\frac{|a|}{n}=\frac{|a|^m}{m!}\cdot\frac{|a|}{m+1}\cdots\frac{|a|}{n}$$

$$\le\frac{|a|^m}{m!}\cdot\frac{|a|}{m+1}\cdots\frac{|a|}{m+1}=\frac{|a|^m}{m!}\cdot\left(\frac{|a|}{m+1}\right)^{n-m}\to(n\to\infty)$$

$$\Rightarrow\left|\frac{a^n}{n!}-0\right|\to0(n\to\infty)\Rightarrow\lim_{n\to\infty}\frac{a^n}{n!}=0$$

Detyra 119: Tregoni se: $\lim_{n\to\infty}\frac{1}{\sqrt[n]{n!}}=0$

Zgjidhje:

Më parë, me induksion matematik tregojmë se $n!>\left(\frac{n}{3}\right)^n \left(n\in N\right)$. Për $n=1$

jobarazimi është i vërtetë, sepse $1>\frac{1}{3}$. Supozojmë se jobarazimi është i vërtetë për n atëherë

$$\left(n+1\right)!=\left(n+1\right)n!>\left(\frac{n}{3}\right)^n\left(n+1\right)=\left(\frac{n+1}{3}\right)^{n+1}\frac{3}{\left(1+\frac{1}{n}\right)^n}>\left(\frac{n}{3}\right)^n\left(n+1\right)$$

$=\left(\frac{n+1}{3}\right)^{n+1}\frac{3}{\left(1+\frac{1}{n}\right)^n}>\left(\frac{n+1}{3}\right)^{n+1}$. Jobarazimi i fundit është i vërtetë, sepse

$$\left(1+\frac{1}{n}\right)^n=1+\frac{n}{n}+\frac{n(n-1)}{2!}\frac{1}{n^2}+...+\frac{1}{n^n}=1+1+\frac{1}{2!}\left(1-\frac{1}{n}\right)+...+\frac{1}{n!}\left(1-\frac{1}{n}\right)...\left(1-\frac{n-1}{n}\right)$$

$$<1+1+\frac{n(n-1)}{2!}\frac{1}{n^2}+...+\frac{1}{n^n}=1+1+\frac{1}{2!}\left(1-\frac{1}{n}\right)+...+\frac{1}{n!}\left(1-\frac{1}{n}\right)...\left(1-\frac{n-1}{n}\right)$$

$$<1+1+\frac{1}{2!}+...+\frac{1}{n!}<1+1+\frac{1}{2}+...+\frac{1}{2^{n-1}}<1+1+\frac{1}{2}+...+\frac{1}{2^{n-1}}+...=3.$$

Në fund, nga $0<\frac{1}{\sqrt[n]{n!}}<\frac{1}{\sqrt[n]{\left(\frac{n}{3}\right)^n}}=\frac{3}{n}$, sipas teoremës mbi limitin e tri vargjeve rrjedh se $\lim_{n\to\infty}\frac{1}{\sqrt[n]{n!}}=0$

Detyra 120: Tregoni se: $\lim_{n\to\infty}\frac{n^2}{a^n}=0\left(a>1\right)$

Zgjidhje:

Meqenëse $a>1$ atëherë $a=1+h\left(h>0\right)$, prej nga

$$a^n=\left(1+h\right)^n=1+nh+\frac{n(n-1)}{2}h^2+\frac{n(n-1)(n-2)}{6}h^3+...+h^n>\frac{n(n-1)(n-2)}{6}h^3$$

$$\Rightarrow 0<\frac{n^2}{a^n}<\frac{n^2}{\frac{n(n-1)(n-2)}{6}h^3}=\frac{6}{h^3n\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)}<\frac{6}{h^3n\frac{1}{2^2}}=\frac{24}{h^3n}$$

$\Rightarrow\left|\frac{n^2}{a^n}-0\right|=\left|\frac{n^2}{a^n}\right|<\frac{24}{h^3n}=\varepsilon$, sepse për $n>5, 1-\frac{1}{n}<\frac{1}{2}\wedge 1-\frac{2}{n}<\frac{1}{2}$.

Prej nga rrjedh se për çdo $\varepsilon<0$ jobarazimi $\left|\frac{n^2}{a^n}-0\right|<\varepsilon$

plotësohet për çdo $n>\max\left\{5,\left[\frac{24}{h^3\varepsilon}\right]\right\}=n_0$

1.2. Limiti i funksionit

Përkufizim: Le të jetë (a,b) interval i fundmë ose i pafundmë dhe $x_0 \in (a,b)$ dhe le të jetë $f:(a,b)\to \mathrm{R}$ ose $f:(a,b)\setminus\{x_0\}\to \mathrm{R}$.

(1) Numri A quhet limit i funksionit f në pikën x_0 nëse për çdo $\varepsilon > 0$, ekziston $\delta = \delta(\varepsilon) > 0$ i tillë që për çdo $x \in (a,b)$, $|x - x_0| < \delta \Rightarrow |f(x) - A| < \varepsilon$.

Do të përdorim shënimin $\lim\limits_{x\to x_0} f(x) = A$.

(2) Funksioni f ka limit ∞ në pikën x_0 nëse për çdo $M > 0$ ekziston $\delta(\varepsilon) > 0$ i tillë që për çdo x, $|x - x_0| < \delta \Rightarrow |f(x)| > M$.

Shkruajmë $\lim\limits_{x\to x_0} f(x) = \infty$.

(3) Numri A quhet limit i funksionit f kur $x\to\infty$, nëse për çdo $\varepsilon > 0$, ekziston $t > 0$ i tillë që për çdo x, $x > t \Rightarrow |f(x) - A| < \varepsilon$.

Shkruajmë $\lim\limits_{x\to\infty} f(x) = A$.

Rregullat e kalimit me limit

Supozojmë se c është një konstant dhe $\lim\limits_{x\to a} f(x)$ dhe $\lim\limits_{x\to a} g(x)$ ekzistojnë, atëherë barazimet që vijojnë janë të vërteta:

1^0 $\lim\limits_{x\to a} c = c$

2^0 $\lim\limits_{x\to a} x = a$

3^0 $\lim\limits_{x\to a} x^n = a^n$ ku n është një numër natyror

4^0 $\lim\limits_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$

5^0 $\lim\limits_{x\to a}\left[f(x) \pm g(x)\right] = \lim\limits_{x\to a} f(x) \pm \lim\limits_{x\to a} g(x)$

6^0 $\lim\limits_{x\to a}\left[c\cdot f(x)\right] = c\cdot \lim\limits_{x\to a} f(x)$

$7^0 \lim\limits_{x\to a}\left[f(x)\cdot g(x)\right]=\lim\limits_{x\to a} f(x)\cdot\lim\limits_{x\to a} g(x)$

$8^0 \lim\limits_{x\to a}\dfrac{f(x)}{g(x)}=\dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$ në qoftë se $\lim\limits_{x\to a} g(x)\neq 0$

$9^0 \lim\limits_{x\to a}\left[f(x)\right]^n=\left[\lim\limits_{x\to a} f(x)\right]^n$ ku n është një numër natyror.

$10^0 \lim\limits_{x\to a}\sqrt[n]{f(x)}=\sqrt[n]{\lim\limits_{x\to a} f(x)}$ ku n është numër natyror

Disa limite të rëndësishme

$\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1;$

$\lim\limits_{x\to 0}\dfrac{\tan x}{x}=1;$

$\lim\limits_{x\to 0}\dfrac{e^x-1}{x}=1;$

$\lim\limits_{x\to 0}\dfrac{a^x-1}{x}=\ln a;$

$\lim\limits_{x\to 0}\dfrac{\ln(1+x)}{x}=1;$

$\lim\limits_{x\to 0}\dfrac{\log_a(1+x)}{x}=\dfrac{1}{\ln a};$

$\lim\limits_{x\to \infty}\dfrac{(1+x)^m-1}{x}=m;$

$\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}=\mathrm{e};$

$\lim\limits_{x\to 0}\left(1+\dfrac{1}{x}\right)^x=e.$

Veprimet matematike me limite të funksionit

Nëse funksionet $f(x)$ dhe $g(x)$ kanë për limit numrat A dhe B, kur x tenton kah një numër i fundmë a[1] d.m.th $\lim_{x\to a} f(x) = A$ dhe $\lim_{x\to a} g(x) = B$, atëherë kanë vend rregullat e veprimeve me limite:

1) Limiti i shumës dhe diferencës se funksioneve:

$$\lim_{x\to a}\left[f(x) \pm g(x)\right] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = A + B$$

2) Limiti i herësit të funksionit:

$$\lim_{x\to a}\left[f(x) \cdot g(x)\right] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) = A \cdot B$$

3) Limiti i fuqisë:

$$\lim_{x\to a} f^k(x) = \left[\lim_{x\to a} f(x)\right]^k = A^k$$

4) Konstantja mund të nxjerrët jashtë limiti:

$$\lim_{x\to a}\left[k \cdot f(x)\right] = k \cdot \lim_{x\to a} f(x) = k \cdot A \quad (k \neq 0)$$

5) Limiti i rrënjës:

$$\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)} = \sqrt[n]{A}, \quad (A \geq 0)$$

6) Limiti eksponencial dhe logaritmik:

$$\lim_{x\to a} k^{f(x)} = k^{\lim_{x\to a} f(x)} = k^A, \quad \lim_{x\to a} \log_c\left[f(x)\right] = \log_c\left[\lim_{x\to a} f(x)\right], c \in \mathrm{R}^+ \setminus \{1\}$$

[1] Rregullat vlejnë edhe kur a tenton $\pm\infty$

Detyra të zgjidhura:

Detyra 1: $\lim_{x\to 1}(2x+3)$

Zgjidhje:

$$\lim_{x\to 1}(2x+3)=2\cdot 1+3=2+3=5$$

Detyra 2: $\lim_{x\to 2}\frac{2x^2-2}{x-1}$

Zgjidhje:

$$\lim_{x\to 2}\frac{2x^2-2}{x-1}=\lim_{x\to 2}\frac{2\left(x^2-1\right)}{x-1}=\lim_{x\to 2}\frac{2(x-1)\cdot(x+1)}{x-1}=\lim_{x\to 2}2(x+1)=2(2+1)=2\cdot 3=6$$

Detyra 3: $\lim_{x\to 1}\frac{x+1}{2}$

Zgjidhje:

$$\lim_{x\to 1}\frac{x+1}{2}=\frac{1+1}{2}=\frac{2}{2}=1$$

Detyra 4: $\lim_{x\to 2}\frac{x^2-4}{x^2-3x+2}$

Zgjidhje:

$$\lim_{x\to 2}\frac{x^2-4}{x^2-3x+2}=\lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)(x-1)}=\lim_{x\to 2}\frac{x+2}{x-1}=\frac{2+2}{2-1}=\frac{4}{1}=4$$

Detyra 5: $\lim_{x\to 4}\frac{x^2+7x-44}{x^2-6x+8}$

Zgjidhje:

$$\lim_{x\to 4}\frac{x^2+7x-44}{x^2-6x+8}=\lim_{x\to 4}\frac{(x-4)(x+11)}{(x-4)(x-2)}=\lim_{x\to 4}\frac{x+11}{x-2}=\frac{4+11}{4-2}=\frac{15}{2}$$

Detyra 6: $\lim\limits_{x\to 5}\dfrac{x^2-7x+10}{25-x^2}$

Zgjidhje:

$$\lim_{x\to 5}\frac{x^2-7x+10}{25-x^2}=\lim_{x\to 5}\left(\frac{(x-5)(x-2)}{-(x-5)(x+5)}\right)=\lim_{x\to 5}\left(-\frac{x-2}{x+5}\right)=-\frac{5-2}{5+5}=-\frac{3}{10}$$

Detyra 7: $\lim\limits_{x\to -2}\dfrac{3x+6}{x^3+8}$

Zgjidhje:

$$\lim_{x\to -2}\frac{3x+6}{x^3+8}=\lim_{x\to -2}\frac{3(x+2)}{(x+2)(x^2-2x+4)}=\lim_{x\to -2}\frac{3}{x^2-2x+4}=\frac{3}{(-2)^2-2\cdot(-2)+4}=\frac{3}{12}=\frac{1}{4}$$

Detyra 8: $\lim\limits_{x\to 4}\dfrac{32-2x^2}{x^2-11x+28}$

Zgjidhje:

$$\lim_{x\to 4}\frac{32-2x^2}{x^2-11x+28}=\lim_{x\to 4}\frac{-2x^2+32}{x^2-11x+28}=\lim_{x\to 4}\frac{-2(x+4)(x-4)}{(x-4)(x-7)}=\lim_{x\to 4}\frac{-2(x+4)}{(x-7)}=\frac{-2(4+4)}{4-7}=\frac{-16}{-3}=\frac{16}{3}$$

Detyra 9: $\lim\limits_{x\to \frac{1}{2}}\dfrac{8x^3-1}{6x^2-5x+1}$

Zgjidhje:

$$\lim_{x\to \frac{1}{2}}\frac{8x^3-1}{6x^2-5x+1}=\lim_{x\to \frac{1}{2}}\frac{(2x)^3-1^3}{6x^2-5x+1}=\lim_{x\to \frac{1}{2}}\frac{(2x-1)(4x^2+2x+1)}{(3x-1)(2x-1)}=\lim_{x\to \frac{1}{2}}\frac{4x^2+2x+1}{3x-1}=$$

$$=\frac{4\cdot\frac{1}{2}+2\cdot\frac{1}{2}+1}{3\cdot\frac{1}{2}-1}=\frac{1+1+1}{\frac{1}{2}}=\frac{3}{\frac{1}{2}}=6$$

Detyra 10: $\lim\limits_{x\to \frac{1}{2}}\dfrac{2x-1}{8x^2+2x-3}$

Zgjidhje:

$$\lim_{x\to \frac{1}{2}}\frac{2x-1}{8x^2+2x-3}=\lim_{x\to \frac{1}{2}}\frac{2x-1}{(4x+3)(2x-1)}=\lim_{x\to \frac{1}{2}}\frac{1}{4x+3}=\frac{1}{4\cdot\frac{1}{2}+3}=\frac{1}{2+3}=\frac{1}{5}$$

Detyra 11: $\lim\limits_{x\to\frac{1}{2}}\frac{4x^2-4x+1}{4x^2-1}$

Zgjidhje:

$$\lim_{x\to\frac{1}{2}}\frac{4x^2-4x+1}{4x^2-1}=\lim_{x\to\frac{1}{2}}\frac{(2x-1)^2}{(2x+1)(2x-1)}=\lim_{x\to\frac{1}{2}}\frac{2x-1}{2x+1}=\frac{2\cdot\frac{1}{2}-1}{2\cdot\frac{1}{2}+1}=\frac{1-1}{1+1}=\frac{0}{2}=0$$

Detyra 12: $\lim\limits_{x\to\frac{1}{3}}\frac{27x^3-1}{3x-1}$

Zgjidhje:

$$\lim_{x\to\frac{1}{3}}\frac{27x^3-1}{3x-1}=\lim_{x\to\frac{1}{3}}\frac{(3x-1)(9x^2+3x+1)}{3x-1}=\lim_{x\to\frac{1}{3}}(9x^2+3x+1)=9\cdot\left(\frac{1}{3}\right)^2+3\cdot\frac{1}{3}+1=3$$

Detyra 13: $\lim\limits_{x\to 2}\frac{x^3-2x-4}{x^3-8}$

Zgjidhje:

$$\lim_{x\to 2}\frac{x^3-2x-4}{x^3-8}=\lim_{x\to 2}\frac{(x-2)(x^2+2x+2)}{(x-2)(x^2+2x+4)}=\lim_{x\to 2}\frac{x^2+2x+2}{x^2+2x+4}=\frac{2^2+2\cdot 2+2}{2^2+2\cdot 2+4}=\frac{4+4+2}{4+4+4}=\frac{10}{12}=\frac{5}{6}$$

Detyra 14: $\lim\limits_{x\to 2}\frac{x^2+x-6}{x^2-4}$

Zgjidhje:

$$\lim_{x\to 2}\frac{x^2+x-6}{x^2-4}=\lim_{x\to 2}\frac{(x+3)(x-2)}{(x-2)(x+2)}=\lim_{x\to 2}\frac{x+3}{x+2}=\frac{2+3}{2+2}=\frac{5}{4}$$

Detyra 15: $\lim\limits_{x\to a}\frac{x^3-a^3}{x-a}$

Zgjidhje:

$$\lim_{x\to a}\frac{x^3-a^3}{x-a}=\lim_{x\to a}\frac{(x-a)(x^2+ax+a^2)}{x-a}=\lim_{x\to a}(x^2+ax+a^2)=a^2+a\cdot a+a^2=3a^2$$

Detyra 16: $\lim\limits_{x\to 1}\frac{\sqrt{x}-1}{x-1}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}=\lim_{x\to 1}\frac{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}{(x-1)\left(\sqrt{x}+1\right)}=\lim_{x\to 1}\frac{x-1}{(x-1)\left(\sqrt{x}+1\right)}=\lim_{x\to 1}\frac{1}{\left(\sqrt{x}+1\right)}=\frac{1}{\sqrt{1}+1}=\frac{1}{2}$$

Detyra 17: $\lim\limits_{y\to 2}\frac{7y^3-56}{3(y-2)}$

Zgjidhje:

$$\lim_{y\to 2}\frac{7y^3-56}{3(y-2)}=\lim_{y\to 2}\frac{7(y-2)\left(y^2+2y+4\right)}{3(y-2)}=\lim_{y\to 2}\frac{7\left(y^2+2y+4\right)}{3}=\frac{7\left(2^2+2\cdot 2+4\right)}{3}=$$

$$=\frac{7(4+4+4)}{3}=\frac{7\cdot 12}{3}=\frac{84}{3}=28$$

Detyra 18: $\lim\limits_{x\to 0}\frac{5x}{3-\sqrt{9+x}}$

Zgjidhje:

$$\lim_{x\to 0}\frac{5x}{3-\sqrt{9+x}}\cdot\frac{3+\sqrt{9+x}}{3+\sqrt{9+x}}=\lim_{x\to 0}\frac{5x\left(3+\sqrt{9+x}\right)}{9-(9+x)}=\lim_{x\to 0}\frac{5x\left(3+\sqrt{9+x}\right)}{9-9-x}=\lim_{x\to 0}\frac{5x\left(3+\sqrt{9+x}\right)}{-x}=$$

$$=\lim_{x\to 0}-5\left(3+\sqrt{9+x}\right)=-5\left(3+\sqrt{9+0}\right)=-5\left(3+\sqrt{9}\right)=-5(3+3)=-5\cdot 6=-30$$

Detyra 19: $\lim\limits_{x\to 0}\frac{9-\sqrt{81-5x}}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{9-\sqrt{81-5x}}{x}\cdot\frac{9+\sqrt{81-5x}}{9+\sqrt{81-5x}}=\lim_{x\to 0}\frac{81-(81-5x)}{x\left(9+\sqrt{81-5x}\right)}=\lim_{x\to 0}\frac{81-81+5x}{x\left(9+\sqrt{81-5x}\right)}=$$

$$=\lim_{x\to 0}\frac{5x}{x\left(9+\sqrt{81-5x}\right)}=\lim_{x\to 0}\frac{5}{\left(9+\sqrt{81-5x}\right)}=\frac{5}{\left(9+\sqrt{81-5\cdot 0}\right)}=\frac{5}{\left(9+\sqrt{81}\right)}=\frac{5}{(9+9)}=\frac{5}{18}$$

Detyra 20: $\lim\limits_{x\to 3}\dfrac{2-\sqrt{x+1}}{x-3}$

Zgjidhje:

$$\lim_{x\to 3}\frac{2-\sqrt{x+1}}{x-3}\cdot\frac{2+\sqrt{x+1}}{2+\sqrt{x+1}}=\lim_{x\to 3}\frac{4-(x+1)}{(x-3)\left(2+\sqrt{x+1}\right)}=\lim_{x\to 3}\frac{4-x-1}{(x-3)\left(2+\sqrt{x+1}\right)}=\lim_{x\to 3}\frac{-(x-3)}{(x-3)\left(2+\sqrt{x+1}\right)}=$$

$$=\lim_{x\to 3}\frac{-1}{\left(2+\sqrt{x+1}\right)}=-\frac{1}{\left(2+\sqrt{3+1}\right)}=-\frac{1}{\left(2+\sqrt{4}\right)}=-\frac{1}{(2+2)}=-\frac{1}{4}$$

Detyra 21: $\lim\limits_{x\to 3}\dfrac{\sqrt{x+2}-\sqrt{5}}{x-3}$

Zgjidhje:

$$\lim_{x\to 3}\frac{\sqrt{x+2}-\sqrt{5}}{x-3}\cdot\frac{\sqrt{x+2}+\sqrt{5}}{\sqrt{x+2}+\sqrt{5}}=\lim_{x\to 3}\frac{x+2-5}{(x-3)\left(\sqrt{x+2}+\sqrt{5}\right)}=\lim_{x\to 3}\frac{x-3}{(x-3)\left(\sqrt{x+2}+\sqrt{5}\right)}=$$

$$=\lim_{x\to 3}\frac{1}{\left(\sqrt{x+2}+\sqrt{5}\right)}=\frac{1}{\left(\sqrt{3+2}+\sqrt{5}\right)}=\frac{1}{\left(\sqrt{5}+\sqrt{5}\right)}=\frac{1}{2\sqrt{5}}$$

Detyra 22: $\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{4x-4}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\sqrt{x+3}-2}{4x-4}\cdot\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}=\lim_{x\to 1}\frac{\left(\sqrt{x+3}^{2}-2^{2}\right)}{(4x-4)\left(\sqrt{x+3}+2\right)}=\lim_{x\to 1}\frac{x+3-4}{(4x-4)\left(\sqrt{x+3}+2\right)}=$$

$$=\lim_{x\to 1}\frac{x-1}{4(x-1)\left(\sqrt{x+3}+2\right)}=\lim_{x\to 1}\frac{1}{4\left(\sqrt{x+3}+2\right)}=\frac{1}{4\left(\sqrt{1+3}+2\right)}=\frac{1}{4\left(\sqrt{4}+2\right)}=$$

$$=\frac{1}{4(2+2)}=\frac{1}{4(4)}=\frac{1}{16}$$

Detyra 23: $\lim\limits_{x\to 3}\dfrac{9-x^{2}}{\sqrt{3x}-3}$

Zgjidhje: $\lim\limits_{x\to 3}\dfrac{9-x^{2}}{\sqrt{3x}-3}=\lim\limits_{x\to 3}\dfrac{(3-x)(3+x)}{\sqrt{3x}-3}\cdot\dfrac{\sqrt{3x}+3}{\sqrt{3x}+3}=\lim\limits_{x\to 3}\dfrac{(3-x)(3+x)\left(\sqrt{3x}+3\right)}{3x-9}=$

$$=\lim_{x\to 3}\frac{(3-x)(3+x)\left(\sqrt{3x}+3\right)}{-3(3-x)}=\lim_{x\to 3}\frac{(3+x)\left(\sqrt{3x}+3\right)}{-3}=\frac{(3+3)\left(\sqrt{3\cdot 3}+3\right)}{-3}=\frac{6\cdot 6}{-3}=\frac{36}{-3}=-12$$

Detyra 24: $\lim\limits_{x\to 1}\dfrac{\sqrt{2x^2-4}-\sqrt{6x^2-20}}{x-2}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\sqrt{2x^2-4}-\sqrt{6x^2-20}}{x-2}\cdot\frac{\sqrt{2x^2-4}+\sqrt{6x^2-20}}{\sqrt{2x^2-4}+\sqrt{6x^2-20}}=\lim_{x\to 1}\frac{2x^2-4-6x^2+20}{(x-2)\left(\sqrt{2x^2-4}+\sqrt{6x^2-20}\right)}=$$

$$=\lim_{x\to 1}\frac{-4x^2+16}{(x-2)\left(\sqrt{2x^2-4}+\sqrt{6x^2-20}\right)}=\lim_{x\to 1}\frac{-4\left(x^2-4\right)}{(x-2)\left(\sqrt{2x^2-4}+\sqrt{6x^2-20}\right)}=$$

$$=\lim_{x\to 1}\frac{-4(x-2)(x+2)}{(x-2)\left(\sqrt{2x^2-4}+\sqrt{6x^2-20}\right)}=\lim_{x\to 1}\frac{-4(x+2)}{\left(\sqrt{2x^2-4}+\sqrt{6x^2-20}\right)}=$$

$$=\frac{-4(1+2)}{\sqrt{2\cdot 1^2-4}+\sqrt{6\cdot 1^2-20}}=\frac{-4(3)}{\sqrt{4-4}+\sqrt{36-20}}=\frac{-12}{\sqrt{16}}=-\frac{12}{4}=-3$$

Detyra 25: $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2+5}-3}{x-2}$

Zgjidhje:

$$\lim_{x\to 2}\frac{\sqrt{x^2+5}-3}{x-2}\cdot\frac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3}=\lim_{x\to 2}\frac{x^2+5-9}{(x-2)\left(\sqrt{x^2+5}+3\right)}=\lim_{x\to 2}\frac{x^2-4}{(x-2)\left(\sqrt{x^2+5}+3\right)}=$$

$$=\lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)\left(\sqrt{x^2+5}+3\right)}=\lim_{x\to 2}\frac{(x+2)}{\left(\sqrt{x^2+5}+3\right)}=\frac{2+2}{\sqrt{2^2+5}+3}=\frac{4}{\sqrt{4+5}+3}=\frac{4}{\sqrt{9}+3}=\frac{4}{3+3}=\frac{4^{:2}}{6^{:2}}=\frac{2}{3}$$

Detyra 26: $\lim\limits_{x\to 0}\dfrac{\sqrt{2+x}-\sqrt{2}}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}\cdot\frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}=\lim_{x\to 0}\frac{2+x-2}{x\left(\sqrt{2+x}+\sqrt{2}\right)}=\lim_{x\to 0}\frac{x}{x\left(\sqrt{2+x}+\sqrt{2}\right)}=\lim_{x\to 0}\frac{1}{\sqrt{2+x}+\sqrt{2}}=$$

$$=\frac{1}{\sqrt{2+0}+\sqrt{2}}=\frac{1}{2\sqrt{2}}$$

Detyra 27: $\lim\limits_{x\to 1}\frac{\sqrt{x+3}-2}{x^2-1}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\sqrt{x+3}-2}{x^2-1}\cdot\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}=\lim_{x\to 1}\frac{\left(\sqrt{x+3}-2\right)\left(\sqrt{x+3}+2\right)}{\left(x^2-1\right)\left(\sqrt{x+3}+2\right)}=\lim_{x\to 1}\frac{x+3-4}{\left(x^2-1\right)\left(\sqrt{x+3}+2\right)}=$$

$$=\lim_{x\to 1}\frac{x-1}{(x-1)(x+1)\left(\sqrt{x+3}+2\right)}=\lim_{x\to 1}\frac{1}{(x+1)\left(\sqrt{x+3}+2\right)}=\frac{1}{(1+1)\left(\sqrt{1+3}+2\right)}=\frac{1}{2\left(\sqrt{4}+2\right)}=\frac{1}{2(2+2)}=\frac{1}{8}$$

Detyra 28: $\lim\limits_{x\to 1}\frac{\sqrt{x+3}-2}{x^2-1}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\sqrt{x+3}-2}{x^2-1}\cdot\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}=\lim_{x\to 1}\frac{x+3-4}{\left(x^2-1\right)\left(\sqrt{x+3}+2\right)}=\lim_{x\to 1}\frac{x-1}{(x-1)(x+1)\left(\sqrt{x+3}+2\right)}=$$

$$=\lim_{x\to 1}\frac{1}{(x+1)\left(\sqrt{x+3}+2\right)}=\frac{1}{(1+1)\left(\sqrt{1+3}+2\right)}=\frac{1}{2(2+2)}=\frac{1}{2\cdot 4}=\frac{1}{8}$$

Detyra 29: $\lim\limits_{x\to 0}\frac{\sqrt{9+5x+4x^2}-3}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sqrt{9+5x+4x^2}-3}{x}\cdot\frac{\sqrt{9+5x+4x^2}+3}{\sqrt{9+5x+4x^2}+3}=\lim_{x\to 0}\frac{9+5x+4x^2-9}{x\left(\sqrt{9+5x+4x^2}+3\right)}=\lim_{x\to 0}\frac{x(5+4x)}{x\left(\sqrt{9+5x+4x^2}+3\right)}$$

$$=\lim_{x\to 0}\frac{(5+4x)}{\left(\sqrt{9+5x+4x^2}+3\right)}=\frac{5+4\cdot 0}{\sqrt{9+5\cdot 0+4\cdot 0^2}+3}=\frac{5}{\sqrt{9}+3}=\frac{5}{3+3}=\frac{5}{6}$$

Detyra 30: $\lim\limits_{x\to -1}\frac{x+1}{\sqrt{6x^2+3}+3x}$

Zgjidhje:

$$\lim_{x\to -1}\frac{x+1}{\sqrt{6x^2+3}+3x}\cdot\frac{\sqrt{6x^2+3}-3x}{\sqrt{6x^2+3}-3x}=\lim_{x\to -1}\frac{(x+1)\left(\sqrt{6x^2+3}-3x\right)}{6x^2+3-9x^2}=\lim_{x\to -1}\frac{(x+1)\left(\sqrt{6x^2+3}-3x\right)}{-3(x+1)(x-1)}=$$

$$=\lim_{x\to -1}\frac{\left(\sqrt{6x^2+3}-3x\right)}{-3(x-1)}=\frac{\sqrt{6(-1)^2+3}-3(-1)}{-3(-1-1)}=\frac{\sqrt{9}+3}{6}=\frac{3+3}{6}=\frac{6}{6}=1$$

Detyra 31: $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt{x}-1}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}\cdot\frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}=\lim_{x\to 1}\frac{\left(\sqrt[3]{x}^3-1^3\right)\left(\sqrt{x}+1\right)}{\left(\sqrt{x}^2-1^2\right)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}=$$

$$=\lim_{x\to 1}\frac{(x-1)\left(\sqrt{x}+1\right)}{(x-1)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}=\lim_{x\to 1}\frac{\sqrt{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}=\frac{\sqrt{1}+1}{\sqrt[3]{1^2}+\sqrt[3]{1}+1}=\frac{1+1}{1+1+1}=\frac{2}{3}$$

Detyra 32: $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x}-\sqrt[3]{3x+1}}{6x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[3]{3x+1}}{6x}\cdot\frac{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}\cdot\sqrt[3]{3x+1}+\left(\sqrt[3]{3x+1}\right)^2}{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}\cdot\sqrt[3]{3x+1}+\left(\sqrt[3]{3x+1}\right)^2}=$$

$$=\lim_{x\to 0}\frac{\left(\sqrt[3]{1+x}\right)^3-\left(\sqrt[3]{3x+1}\right)^3}{6x\underbrace{\left[\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}\cdot\sqrt[3]{3x+1}+\left(\sqrt[3]{3x+1}\right)^2\right]}_{\mathrm{A}}}=\lim_{x\to 0}\frac{1+x-3x-1}{6x(\mathrm{A})}=\lim_{x\to 0}\frac{-2x}{6x(\mathrm{A})}=$$

$$=-\frac{2}{6}\lim_{x\to 0}\frac{1}{\left(\sqrt[3]{1+x}\right)^2+\sqrt[3]{1+x}\cdot\sqrt[3]{3x+1}+\left(\sqrt[3]{3x+1}\right)^2}=-\frac{1}{3}\cdot\frac{1}{\sqrt[3]{1}^2+\sqrt{1}+\sqrt[3]{1}^2}=-\frac{1}{3}\cdot\frac{1}{1+1+1}=-\frac{1}{9}$$

Detyra 33: $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x^2}-1}{x^2}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sqrt[3]{1+x^2}-1}{x^2}\cdot\frac{\sqrt[3]{\left(1+x^2\right)^2}+\sqrt[3]{1+x^2}+1}{\sqrt[3]{\left(1+x^2\right)^2}+\sqrt[3]{1+x^2}+1}=\lim_{x\to 0}\frac{\sqrt[3]{1+x^2}^3-1^3}{x^2\left(\sqrt[3]{\left(1+x^2\right)^2}+\sqrt[3]{1+x^2}+1\right)}=$$

$$=\lim_{x\to 0}\frac{1+x^2-1}{x^2\left(\sqrt[3]{\left(1+x^2\right)^2}+\sqrt[3]{1+x^2}+1\right)}=\lim_{x\to 0}\frac{x^2}{x^2\left(\sqrt[3]{\left(1+x^2\right)^2}+\sqrt[3]{1+x^2}+1\right)}=$$

$$=\lim_{x\to 0}\frac{1}{\left(\sqrt[3]{\left(1+x^2\right)^2}+\sqrt[3]{1+x^2}+1\right)}=\frac{1}{\left(\sqrt[3]{\left(1+0^2\right)^2}+\sqrt[3]{1+0^2}+1\right)}=\frac{1}{1+1+1}=\frac{1}{3}$$

Detyra 34: $\lim\limits_{x\to 3}\log\frac{x-3}{\sqrt{x+6}-3}$

Zgjidhje:

$$\lim_{x\to 3}\log\frac{x-3}{\sqrt{x+6}-3}=\lim_{x\to 3}\log\frac{x-3}{\sqrt{x+6}-3}\cdot\frac{\sqrt{x+6}+3}{\sqrt{x+6}+3}=\lim_{x\to 3}\log\frac{x-3}{x-3}\left(\sqrt{x+6}+3\right)=$$
$$=\lim_{x\to 3}\log\left(\sqrt{x+6}+3\right)=\log 6$$

Detyra 35: $\lim\limits_{x\to 2}\frac{\sqrt[3]{10-x}-2}{x-2}$

Zgjidhje:

$$\lim_{x\to 2}\frac{\sqrt[3]{10-x}-2}{x-2}\cdot\frac{\sqrt[3]{(10-x)^2}+2\sqrt[3]{10-x}+2^2}{\sqrt[3]{(10-x)^2}+2\sqrt[3]{10-x}+2^2}=\lim_{x\to 2}\frac{\left(\sqrt[3]{10-x}-2\right)\left(\sqrt[3]{(10-x)^2}+2\sqrt[3]{10-x}+2^2\right)}{(x-2)\left(\underbrace{\sqrt[3]{(10-x)^2}+2\sqrt[3]{10-x}+4}_{A}\right)}=$$

$$=\lim_{x\to 2}\frac{\sqrt[3]{(10-x)^3}-2^3}{(x-2)(\mathrm{A})}=\lim_{x\to 2}\frac{10-x-8}{(x-2)(\mathrm{A})}=\lim_{x\to 2}\frac{-(x-2)}{(x-2)(\mathrm{A})}=\lim_{x\to 2}\frac{-1}{\sqrt[3]{(10-x)^2}+2\sqrt[3]{10-x}+4}=$$
$$=\frac{-1}{\sqrt[3]{(10-2)^2}+2\sqrt[3]{10-2}+4}=-\frac{1}{\sqrt[3]{64}+2\sqrt[3]{8}+4}=-\frac{1}{4+4+4}=-\frac{1}{12}$$

Detyra 36: $\lim\limits_{x\to 2}\frac{\sqrt{x+7}-3\sqrt{2x-3}}{\sqrt[3]{x+6}-2\sqrt[3]{3x-5}}$

Zgjidhje:

$$\lim_{x\to 2}\frac{\sqrt{x+7}-3\sqrt{2x-3}}{\sqrt[3]{x+6}-2\sqrt[3]{3x-5}}\cdot\frac{\sqrt{x+7}+3\sqrt{2x-3}}{\sqrt{x+7}+3\sqrt{2x-3}}\cdot\frac{\sqrt[3]{(x+6)^2}+\sqrt[3]{x+6}\,2\sqrt[3]{3x-5}+2^2\sqrt[3]{(3x-5)^2}}{\sqrt[3]{(x+6)^2}+\sqrt[3]{x+6}\,2\sqrt[3]{3x-5}+2^2\sqrt[3]{(3x-5)^2}}=$$
$$=\lim_{x\to 2}\frac{x+7-9(2x-3)}{\sqrt{x+7}+3\sqrt{2x-3}}\cdot\frac{\sqrt[3]{(x+6)^2}+\sqrt[3]{x+6}\,2\sqrt[3]{3x-5}+2^2\sqrt[3]{(3x-5)^2}}{x+6-8(3x-5)}=$$
$$=\lim_{x\to 2}\frac{-17x+34}{-23x+46}\cdot\lim_{x\to 2}\frac{\sqrt[3]{(x+6)^2}+\sqrt[3]{x+6}\,2\sqrt[3]{3x-5}+2^2\sqrt[3]{(3x-5)^2}}{\sqrt{x+7}+3\sqrt{2x+3}}=\frac{17}{23}\cdot 2=\frac{34}{23}$$

Detyra 37: $\lim\limits_{x\to 1}\frac{\sqrt{x+8}-\sqrt{8x+1}}{\sqrt{5-x}-\sqrt{7x-3}}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\sqrt{x+8}-\sqrt{8x+1}}{\sqrt{5-x}-\sqrt{7x-3}}\cdot\frac{\sqrt{x+8}-\sqrt{8x+1}}{\sqrt{5-x}-\sqrt{7x-3}}\cdot\frac{\sqrt{x+8}+\sqrt{8x+1}}{\sqrt{x+8}+\sqrt{8x+1}}\cdot\frac{\sqrt{5-x}+\sqrt{7x-3}}{\sqrt{5-x}+\sqrt{7x-3}}=$$

$$=\lim_{x\to 1}\frac{x+8-8x-1}{5-x-7x+3}\cdot\lim_{x\to 1}\frac{\sqrt{5-x}+\sqrt{7x-3}}{\sqrt{x+8}+\sqrt{8x+1}}=\frac{7}{8}\cdot\frac{4}{6}=\frac{7}{12}$$

Detyra 38: $\lim\limits_{x\to 1}\frac{\sqrt{x+8}-\sqrt[3]{3x+24}}{x-1}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\sqrt{x+8}-\sqrt[3]{3x+24}}{x-1}=\lim_{x\to 1}\frac{\sqrt{x+8}-3+3-\sqrt[3]{3x+24}}{x-1}=\lim_{x\to 1}\frac{\sqrt{x+8}-3}{x-1}+\lim_{x\to 1}\frac{3-\sqrt[3]{3x+24}}{x-1}=$$

$$=\lim_{x\to 1}\frac{\sqrt{x+8}-3}{x-1}\cdot\frac{\sqrt{x+8}+3}{\sqrt{x+8}+3}+\lim_{x\to 1}\frac{3-\sqrt[3]{3x+24}}{x-1}\cdot\frac{9+3\sqrt[3]{3x+24}+\sqrt[3]{(3x+24)^2}}{9+3\sqrt[3]{3x+24}+\sqrt[3]{(3x+24)^2}}=$$

$$=\lim_{x\to 1}\frac{x+8-9}{(x-1)\left(\sqrt{x+8}+3\right)}+\lim_{x\to 1}\frac{27-(3x+24)}{(x-1)\left(9+3\sqrt[3]{3x+24}+\sqrt[3]{(3x+24)^2}\right)}=$$

$$\lim_{x\to 1}\frac{x-1}{(x-1)\left(\sqrt{x+8}+3\right)}+\lim_{x\to 1}\frac{27-3x-24}{(x-1)\left(9+3\sqrt[3]{3x+24}+\sqrt[3]{(3x+24)^2}\right)}=$$

$$=\lim_{x\to 1}\frac{1}{\left(\sqrt{x+8}+3\right)}-\lim_{x\to 1}\frac{3}{\left(9+3\sqrt[3]{3x+24}+\sqrt[3]{(3x+24)^2}\right)}=\frac{1}{6}-\frac{3}{27}=\frac{27-18}{162}=\frac{9}{162}=\frac{1}{18}$$

Detyra 39: $\lim\limits_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x}\cdot\frac{\sqrt{2-x}+\sqrt{2+x}}{\sqrt{2-x}+\sqrt{2+x}}=\lim_{x\to 0}\frac{2-x-(2+x)}{x\left(\sqrt{2-x}+\sqrt{2+x}\right)}=\lim_{x\to 0}\frac{2-x-2-x}{x\left(\sqrt{2-x}+\sqrt{2+x}\right)}=$$

$$=\lim_{x\to 0}\frac{-2x}{x\left(\sqrt{2-x}+\sqrt{2+x}\right)}=\lim_{x\to 0}\frac{-2}{\left(\sqrt{2-x}+\sqrt{2+x}\right)}=-\frac{2}{\sqrt{2}+\sqrt{2}}=-\frac{2}{2\sqrt{2}}=-\frac{1}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$$

Detyra 40: $\lim\limits_{x\to 5}\dfrac{x^2-25}{2-\sqrt{x-1}}$

Zgjidhje:

$$\lim_{x\to 5}\frac{x^2-25}{2-\sqrt{x-1}}\cdot\frac{2+\sqrt{x-1}}{2+\sqrt{x-1}}=\lim_{x\to 5}\frac{x^2-25\left(2+\sqrt{x-1}\right)}{4-x+1}=\lim_{x\to 5}\frac{x^2-25\left(2+\sqrt{x-1}\right)}{5-x}=$$

$$=-\lim_{x\to 5}\frac{(x-5)(x+5)\left(2+\sqrt{x-1}\right)}{x-5}=-\lim_{x\to 5}(x+5)\left(2+\sqrt{x-1}\right)=-10\cdot 4=-40$$

Detyra 41: $\lim\limits_{x\to 65}\dfrac{\sqrt{x-1}-8}{\sqrt[3]{x-1}-4}$

Zgjidhje:

$$\lim_{x\to 65}\frac{\sqrt{x-1}-8}{\sqrt[3]{x-1}-4}\cdot\frac{\sqrt{x-1}+8}{\sqrt{x-1}+8}\cdot\frac{\sqrt[3]{(x-1)^2}+4\sqrt[3]{x-1}+16}{\sqrt[3]{(x-1)^2}+4\sqrt[3]{x-1}+16}=\lim_{x\to 65}\frac{(x-65)\left(\sqrt[3]{(x-1)^2}+4\sqrt[3]{x-1}+16\right)}{(x-65)\left(\sqrt{x-1}+8\right)}=$$

$$=\lim_{x\to 65}\frac{\sqrt[3]{(x-1)^2}+4\sqrt[3]{x-1}+16}{\sqrt{x-1}+8}=\frac{16+16+16}{8+8}=\frac{48}{16}=3$$

Detyra 42: $\lim\limits_{x\to 4}\dfrac{x^2-16}{3-\sqrt{x^2-7}}$

Zgjidhje:

$$\lim_{x\to 4}\frac{x^2-16}{3-\sqrt{x^2-7}}\cdot\frac{3+\sqrt{x^2-7}}{3+\sqrt{x^2-7}}=\lim_{x\to 4}\frac{\left(x^2-16\right)\left(3+\sqrt{x^2-7}\right)}{\left(3-\sqrt{x^2-7}\right)\left(3+\sqrt{x^2-7}\right)}=\lim_{x\to 4}\frac{\left(x^2-16\right)\left(3+\sqrt{x^2-7}\right)}{9-\left(x^2-7\right)}=$$

$$=\lim_{x\to 4}\frac{\left(x^2-16\right)\left(3+\sqrt{x^2-7}\right)}{16-x^2}=-\lim_{x\to 4}\left(3+\sqrt{x^2-7}\right)=-\left(3+\sqrt{16-7}\right)=-\left(3+\sqrt{9}\right)=-(3+3)=-6$$

Detyra 43: $\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}-2}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}\cdot\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}=\lim_{x\to 0}\frac{\left(\sqrt{4+x}-2\right)\left(\sqrt{4+x}+2\right)}{x\left(\sqrt{4+x}+2\right)}=\lim_{x\to 0}\frac{\left(\sqrt{4+x}\right)^2-2^2}{x\left(\sqrt{4+x}+2\right)}=$$

$$=\lim_{x\to 0}\frac{4+x-4}{x\left(\sqrt{4+x}+2\right)}=\lim_{x\to 0}\frac{x}{x\left(\sqrt{4+x}+2\right)}=\lim_{x\to 0}\frac{1}{\left(\sqrt{4+x}+2\right)}=\frac{1}{\sqrt{4+0}+2}=\frac{1}{2+2}=\frac{1}{4}$$

Detyra 44: $\lim\limits_{x\to 7}\frac{2-\sqrt{x-3}}{x^2-49}$

Zgjidhje:

$$\lim_{x\to 7}\frac{2-\sqrt{x-3}}{x^2-49}\cdot\frac{2+\sqrt{x-3}}{2+\sqrt{x-3}}=\lim_{x\to 7}\frac{7-x}{(x-7)(x+7)}\cdot\frac{1}{\lim\limits_{x\to 7}\left(2+\sqrt{x-3}\right)}=\lim_{x\to 7}\frac{-(x-7)}{(x-7)(x+7)}\cdot\frac{1}{4}=$$

$$=\frac{1}{4}\lim_{x\to 7}\frac{-1}{(x+7)}=\frac{1}{4}\cdot\frac{(-1)}{7+7}=\frac{1}{4}\cdot\frac{(-1)}{14}=-\frac{1}{56}$$

Detyra 45: $\lim\limits_{x\to 0}\frac{\sqrt{t^2+9}-3}{t^2}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sqrt{t^2+9}-3}{t^2}\cdot\frac{\sqrt{t^2+9}+3}{\sqrt{t^2+9}+3}=\lim_{x\to 0}\frac{\left(t^2+9\right)-9}{t^2\left(\sqrt{t^2+9}+3\right)}=\lim_{x\to 0}\frac{t^2+9-9}{t^2\left(\sqrt{t^2+9}+3\right)}=\lim_{x\to 0}\frac{t^2}{t^2\left(\sqrt{t^2+9}+3\right)}=$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{t^2+9}+3}=\frac{1}{\sqrt{0^2+9}+3}=\frac{1}{\sqrt{9}+3}=\frac{1}{3+3}=\frac{1}{6}$$

Detyra 46: $\lim\limits_{x\to 1}\left[\frac{1}{x^2-1}-\frac{2}{x^4-1}\right]$

Zgjidhje:

$$\lim_{x\to 1}\left[\frac{1}{x^2-1}-\frac{2}{x^4-1}\right]=\lim_{x\to 1}\left[\frac{1}{x^2-1}-\frac{2}{\left(x^2-1\right)\left(x^2+1\right)}\right]=\lim_{x\to 1}\frac{x^2+1-2}{\left(x^2-1\right)\left(x^2+1\right)}=$$

$$=\lim_{x\to 1}\frac{x^2-1}{\left(x^2-1\right)\left(x^2+1\right)}=\lim_{x\to 1}\frac{1}{\left(x^2+1\right)}=\frac{1}{1+1}=\frac{1}{2}$$

Detyra 47: $\lim\limits_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$

Zgjidhje:

$$\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)=\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{(1-x)\left(1+x+x^2\right)}\right)=\lim_{x\to 1}\frac{1+x+x^2-3}{(1-x)\left(1+x+x^2\right)}=$$

$$=\lim_{x\to 1}\frac{x^2+x-2}{(1-x)\left(1+x+x^2\right)}=\lim_{x\to 1}\frac{(x+2)(-1)(1-x)}{(1-x)\left(1+x+x^2\right)}=-\lim_{x\to 1}\frac{x+2}{1+x+x^2}=-\frac{1+2}{1+1+1}=-1$$

Detyra 48: $\lim\limits_{x\to 2}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right)$

Zgjidhje:

$$\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right)=\lim_{x\to 2}\left[\frac{x^2-4-4(x-2)}{(x-2)(x^2-4)}\right]=\lim_{x\to 2}\left[\frac{x^2-4-4x+8}{(x-2)(x^2-4)}\right]=\lim_{x\to 2}\left[\frac{x^2-4-4x+8}{(x-2)(x^2-4)}\right]=$$

$$=\lim_{x\to 2}\left[\frac{x^2-4x+4}{(x-2)(x^2-4)}\right]=\lim_{x\to 2}\left[\frac{(x-2)^2}{(x-2)(x^2-4)}\right]=\lim_{x\to 2}\left[\frac{x-2}{(x-2)(x+2)}\right]=\lim_{x\to 2}\left(\frac{1}{x+2}\right)=\frac{1}{2+2}=\frac{1}{4}$$

Detyra 49: $\lim\limits_{x\to 0}\left(\frac{x^3-3x+1}{x-4}+1\right)$

Zgjidhje:

$$\lim_{x\to 0}\left(\frac{x^3-3x+1}{x-4}+1\right)=\lim_{x\to 0}\frac{x^3-3x+1+x-4}{x-4}=\lim_{x\to 0}\frac{x^3-2x-3}{x-4}=\frac{0^3-2\cdot 0-3}{x-4}=\frac{-3}{-4}=\frac{3}{4}$$

Detyra 50: $\lim\limits_{x\to 0}\left[\frac{(1+x)(1+2x)(1+3x)-1}{x}\right]$

Zgjidhje:

$$\lim_{x\to 0}\left[\frac{(1+x)(1+2x)(1+3x)-1}{x}\right]=\lim_{x\to 0}\frac{6x^3+11x^2+6x+1-1}{x}=\lim_{x\to 0}\frac{x(6x^2+11x+6)}{x}=$$

$$=\lim_{x\to 0}(6x^2+11x+6)=(6\cdot 0^2+11\cdot 0+6)=6$$

Detyra 51: $\lim\limits_{x\to 1}\left(\frac{x+2}{x^2-5x+4}+\frac{x-4}{3x^2-9x+6}\right)$

Zgjidhje:

$$\lim_{x\to 1}\left(\frac{x+2}{x^2-5x+4}+\frac{x-4}{3x^2-9x+6}\right)=\lim_{x\to 1}\left[\frac{x+2}{(x-4)(x-1)}+\frac{x-4}{3(x-2)(x-1)}\right]=$$

$$=\lim_{x\to 1}\frac{3(x+2)\cdot(x-2)+(x-4)\cdot(x-4)}{3(x-4)\cdot(x-2)\cdot(x-1)}=\lim_{x\to 1}\frac{3(x^2-4)+x^2-8x+16}{3(x-4)\cdot(x-2)\cdot(x-1)}=$$

$$=\lim_{x\to 1}\frac{3x^2-12+x^2-8x+16}{3(x-4)\cdot(x-2)\cdot(x-1)}=\lim_{x\to 1}\frac{4x^2-8x+4}{3(x-4)\cdot(x-2)\cdot(x-1)}=\lim_{x\to 1}\frac{4(x-1)^2}{3(x-4)\cdot(x-2)\cdot(x-1)}=$$

$$=\lim_{x\to 1}\frac{4(x-1)}{3(x-4)\cdot(x-2)}=\frac{4(1-1)}{3(1-4)\cdot(1-2)}=\frac{4\cdot(0)}{3(-3)\cdot(-1)}=0$$

Detyra 52: $\lim\limits_{x\to\infty}\dfrac{x^3-3}{3x^3+2x+1}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{x^3-3}{3x^3+2x+1}=\lim_{x\to\infty}\frac{\frac{x^3}{x^3}-\frac{3}{x^3}}{\frac{3x^3}{x^3}+\frac{2x}{x^3}+\frac{1}{x^3}}=\lim_{x\to\infty}\frac{1-\frac{3}{x^3}}{3+\frac{2}{x^2}+\frac{1}{x^3}}=\frac{1}{3}$$

Detyra 53: $\lim\limits_{x\to\infty}\dfrac{2x^3+x^2+5}{3x^3+x-1}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{2x^3+x^2+5\big|:x^3}{3x^3+x-1\big|:x^3}=\lim_{x\to\infty}=\frac{\frac{2x^3}{x^3}+\frac{x^2}{x^3}+\frac{5}{x^3}}{\frac{3x^3}{x^3}+\frac{x}{x^3}-\frac{1}{x^3}}=\lim_{x\to\infty}=\frac{2+\frac{1}{x}+\frac{5}{x^3}}{3+\frac{1}{x^2}-\frac{1}{x^3}}=\frac{2}{3}$$

Detyra 54: $\lim\limits_{x\to\infty}\dfrac{x^2+x-2}{2x^2-x+4}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{x^2+x-2\big|:x^2}{2x^2-x+4\big|:x^2}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}+\frac{x}{x^2}-\frac{2}{x^2}}{\frac{2x^2}{x^2}-\frac{x}{x^2}+\frac{4}{x^2}}=\lim_{x\to\infty}\frac{1+\frac{1}{x}-\frac{2}{x^2}}{2-\frac{1}{x}+\frac{4}{x^2}}=\frac{1+0-0}{2-0+0}=\frac{1}{2}$$

Detyra 55: $\lim\limits_{x\to\infty}\dfrac{7x^4+2x^3-14}{5x^4+x^3+x^2+x-1}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{7x^4+2x^3-14\big|:x^4}{5x^4+x^3+x^2+x-1\big|:x^4}=\lim_{x\to\infty}\frac{\frac{7x^4}{x^4}+\frac{2x^3}{x^4}-\frac{14}{x^4}}{\frac{5x^4}{x^4}+\frac{x^3}{x^4}+\frac{x^2}{x^4}+\frac{x}{x^4}-\frac{1}{x^4}}=\lim_{x\to\infty}\frac{7+\frac{2}{x}-\frac{14}{x^4}}{5+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}-\frac{1}{x^4}}=\frac{7}{5}$$

Detyra 56: $\lim\limits_{x\to\infty}\dfrac{5x^4-7x^2+5x-4}{x^4+x^2+x+1}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{5x^4-7x^2+5x-4}{x^4+x^2+x+1}\Bigg|:x^4=\lim_{x\to\infty}\frac{\frac{5x^4}{x^4}-\frac{7x^2}{x^4}+\frac{5x}{x^4}-\frac{4}{x^4}}{\frac{x^4}{x^4}+\frac{x^2}{x^4}+\frac{x}{x^4}+\frac{1}{x^4}}=\lim_{x\to\infty}\frac{5-\frac{7}{x^2}+\frac{5}{x^3}-\frac{4}{x^4}}{1+\frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^4}}=\frac{5}{1}=5$$

Detyra 57: $\lim\limits_{x\to\infty}\dfrac{(x+3)(x+4)(x+5)}{x^4+x-11}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{(x+3)(x+4)(x+5)}{x^4+x-11}=\lim_{x\to\infty}\frac{x^3+7x+12}{x^4+x-11}\Bigg|:x^4=\lim_{x\to\infty}\frac{\frac{x^3}{x^4}+\frac{7x}{x^4}+\frac{12}{x^4}}{\frac{x^4}{x^4}+\frac{x}{x^4}-\frac{11}{x^4}}=\lim_{x\to\infty}\frac{\frac{1}{x}+\frac{7}{x^3}+\frac{12}{x^4}}{1+\frac{1}{x^3}-\frac{11}{x^4}}=\frac{0}{1}=0$$

Detyra 58: $\lim\limits_{x\to\infty}\dfrac{3x^3-3x^2+x}{2x^3+x-1}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{3x^3-3x^2+x}{2x^3+x-1}\Bigg|:x^3=\lim_{x\to\infty}\frac{\frac{3x^3}{x^3}-\frac{3x^2}{x^3}+\frac{x}{x^3}}{\frac{2x^3}{x^3}+\frac{x}{x^3}-\frac{1}{x^3}}=\lim_{x\to\infty}\frac{3-\frac{3}{x}+\frac{1}{x^2}}{2+\frac{1}{x^2}-\frac{1}{x^3}}=\frac{3}{2}$$

Detyra 59: $\lim\limits_{x\to\infty}\left(\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2}\right)$

Zgjidhje:

$$\lim_{x\to\infty}\left(\frac{x^3}{3x^2-4}-\frac{x^2}{3x+2}\right)=\lim_{x\to\infty}\frac{2x^3+4x^2}{9x^3+6x^2-12x-8}=\lim_{x\to\infty}\frac{2+\frac{4}{x}}{9+\frac{6}{x}-\frac{12}{x^2}-\frac{8}{x^3}}=\frac{2}{9}$$

Detyra 60: $\lim\limits_{x\to\infty}\left(\sqrt{9x^2+1}-3x\right)$

Zgjidhje:

$$\lim_{x\to\infty}\left(\sqrt{9x^2+1}-3x\right)\cdot\frac{\sqrt{9x^2+1}+3x}{\sqrt{9x^2+1}+3x}=\lim_{x\to\infty}\frac{\left(\sqrt{9x^2+1}-3x\right)\left(\sqrt{9x^2+1}+3x\right)}{\sqrt{9x^2+1}+3x}=$$

$$=\lim_{x\to\infty}\frac{9x^2+1-9x^2}{\sqrt{9x^2+1}+3x}=\lim_{x\to\infty}\frac{1}{\sqrt{9x^2+1}+3x}=0$$

Detyra 61: $\lim\limits_{x\to\infty}\dfrac{2\sqrt{x}+3\sqrt[3]{x}+5\sqrt[5]{x}}{\sqrt{3x-2}+\sqrt[3]{2x-3}}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{2\sqrt{x}+3\sqrt[3]{x}+5\sqrt[5]{x}}{\sqrt{3x-2}+\sqrt[3]{2x-3}}=\lim_{x\to\infty}\frac{2+\frac{3}{\sqrt[6]{x}}+\frac{5}{\sqrt[10]{x}}}{\sqrt{3-\frac{2}{x}}+\sqrt[6]{\frac{4}{x}-\frac{12}{x^2}+\frac{9}{x^3}}}=\frac{2}{\sqrt{3}}$$

Detyra 62: $\lim\limits_{x\to\infty} x\left(\sqrt{x^2+1}-x\right)$

Zgjidhje:

$$\lim_{x\to\infty} x\left(\sqrt{x^2+1}-x\right)\cdot\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}=\lim_{x\to\infty}\frac{x\left(x^2+1-x^2\right)}{\sqrt{x^2+1}+x}=\lim_{x\to\infty}\frac{x}{\sqrt{x^2+1}+x}=1$$

Detyra 63: $\lim\limits_{x\to\infty}\sqrt{x^2+x}-x$

Zgjidhje:

$$\lim_{x\to\infty}\sqrt{x^2+x}-x\cdot\frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x}=\lim_{x\to\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x}=\lim_{x\to\infty}\frac{x}{\sqrt{x^2+x}+x}=\lim_{x\to\infty}\frac{x}{\sqrt{x^2\left(1+\frac{1}{x}\right)}+x}=$$

$$=\lim_{x\to\infty}\frac{x}{x\sqrt{1+0}+x}=\lim_{x\to\infty}\frac{x}{2x}=\frac{1}{2}$$

Detyra 64: $\lim\limits_{x\to\infty}\left(3x-\sqrt{9x^2-10x+1}\right)$

Zgjidhje:

$$\lim_{x\to\infty}\left(3x-\sqrt{9x^2-10x+1}\right)\cdot\frac{3x+\sqrt{9x^2-10x+1}}{3x+\sqrt{9x^2-10x+1}}=\lim_{x\to\infty}\frac{\left(3x-\sqrt{9x^2-10x+1}\right)\left(3x+\sqrt{9x^2-10x+1}\right)}{3x+\sqrt{9x^2-10x+1}}=$$

$$=\lim_{x\to\infty}\frac{9x^2-9x^2+10x+1}{3x+\sqrt{9x^2-10x+1}}=\lim_{x\to\infty}\frac{10x+1}{3x+\sqrt{9x^2-10x+1}}=\lim_{x\to\infty}\frac{\frac{10x}{x}+\frac{1}{x}}{\frac{3x}{x}+\sqrt{\frac{9x^2}{x^2}-\frac{10x}{x^2}+\frac{1}{x^2}}}=$$

$$=\lim_{x\to\infty}\frac{10+\frac{1}{x}}{3+\sqrt{9-\frac{10}{x}+\frac{1}{x^2}}}=\frac{10+0}{3+\sqrt{9-0+0}}=\frac{10}{3+3}=\frac{10}{6}=\frac{5}{3}$$

Detyra 65: $\lim\limits_{x\to\infty}\left(x+\sqrt[3]{x^2-x^3}\right)$

Zgjidhje: $$\lim_{x\to\infty}\left(x+\sqrt[3]{x^2-x^3}\right)\cdot\frac{x^2-x\sqrt[3]{x^2-x^3}+\sqrt[3]{\left(x^2-x^3\right)^2}}{x^2-x\sqrt[3]{x^2-x^3}+\sqrt[3]{\left(x^2-x^3\right)^2}}=\lim_{x\to\infty}\frac{x^3+x^2-x^3}{x^2-x\sqrt[3]{x^2-x^3}+\sqrt[3]{\left(x^2-x^3\right)^2}}=$$

$$=\lim_{x\to\infty}\frac{x^2}{x^2-x\sqrt[3]{x^2-x^3}+\sqrt[3]{\left(x^2-x^3\right)^2}}\,|:x^2=\lim_{x\to\infty}\frac{1}{1-\sqrt[3]{\frac{1}{x}-1}+\sqrt[3]{\frac{1}{x^2}-\frac{2}{x}+1}}=\frac{1}{1+1+1}=\frac{1}{3}$$

Detyra 66: $\lim\limits_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$

Zgjidhje:

$$\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x\sqrt{x}}}-\sqrt{x}\right)\cdot\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}=\lim_{x\to\infty}\frac{\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)\left(\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}\right)}{\left(\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}\right)}=$$

$$=\lim_{x\to\infty}\frac{x+\sqrt{x+\sqrt{x}}-x}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}=\lim_{x\to\infty}\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}=\lim_{x\to\infty}\frac{\sqrt{\frac{x}{x}+\sqrt{\frac{x}{x}}}}{\sqrt{\frac{x}{x}+\sqrt{\frac{x}{x}+\sqrt{\frac{x}{x}}}}+\sqrt{\frac{x}{x}}}=\frac{1}{1+1}=\frac{1}{2}$$

Detyra 67: $\lim\limits_{x\to\infty}\frac{(x+5)^{10}}{(x-1)^{9}}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{(x+5)^{10}}{(x-1)^{9}}=\lim_{x\to\infty}\frac{\left(\frac{(x+5)^{10}}{x^{10}}\right)}{\left(\frac{(x-1)^{9}}{x^{9}}\right)}=\lim_{x\to\infty}\left(\frac{\left(\frac{x-5}{x}\right)^{10}}{\left(\frac{x-1}{x}\right)^{9}}\right)\cdot\lim_{x\to\infty}x=\lim_{x\to\infty}\frac{\left(1+\frac{5}{x}\right)^{10}}{\left(1-\frac{1}{x}\right)^{9}}\cdot\lim_{x\to\infty}x=\infty$$

Detyra 68: $\lim\limits_{x\to\infty}\frac{10x^{10}+10^{10}}{\left(10x^{2}+5\right)^{5}}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{10x^{10}+10^{10}}{\left(10x^{2}+5\right)^{5}}=\lim_{x\to\infty}\frac{\frac{10x^{10}+10^{10}}{x^{10}}}{\frac{\left(10x^{2}+5\right)^{5}}{x^{10}}}=\lim_{x\to\infty}\frac{10+\frac{10^{10}}{x^{10}}}{\left(\frac{10x^{2}+5}{x^{2}}\right)^{5}}=\lim_{x\to\infty}\frac{10+\frac{10^{10}}{x^{10}}}{\left(10+\frac{5}{x^{2}}\right)^{5}}=\frac{10}{10^{5}}=\frac{1}{10^{4}}$$

Detyra 69: $\lim\limits_{x\to\infty}\frac{(x-1)^{10}(2x+3)^{15}}{(3x+1)^{25}}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{(x-1)^{10}(2x+3)^{15}}{(3x+1)^{25}}=\frac{\lim\limits_{x\to\infty}(x-1)^{10}\cdot\lim\limits_{x\to\infty}(2x+3)^{15}}{\lim\limits_{x\to\infty}(3x+1)^{25}}=\frac{\lim\limits_{x\to\infty}x^{10}\cdot\lim\limits_{x\to\infty}(2x)^{15}}{\lim\limits_{x\to\infty}(3x)^{25}}=\lim_{x\to\infty}\frac{2^{15}\cdot x^{25}}{3^{25}\cdot x^{25}}=\frac{2^{15}}{3^{25}}$$

Detyra 70: $\lim\limits_{x\to\infty}\frac{(2x-3)^{20}\cdot(3x+2)^{30}}{(2x+1)^{50}}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{(2x-3)^{20}\cdot(3x+2)^{30}}{(2x+1)^{50}}=\lim_{x\to\infty}\frac{\left(\frac{2x}{x}-\frac{3}{x}\right)^{20}\cdot\left(\frac{3x}{x}+\frac{2}{x}\right)^{30}}{\left(\frac{2x}{x}+\frac{1}{x}\right)^{50}}=\lim_{x\to\infty}\frac{\left(2-\frac{3}{x}\right)^{20}\cdot\left(3+\frac{2}{x}\right)^{30}}{\left(2+\frac{1}{x}\right)^{50}}=$$

$$=\frac{\left(2-\frac{3}{\infty}\right)^{20}\cdot\left(3+\frac{2}{\infty}\right)^{30}}{\left(2+\frac{1}{\infty}\right)^{50}}=\frac{2^{20}\cdot 3^{30}}{2^{50}}=\frac{3^{30}}{2^{50-20}}=\frac{3^{30}}{2^{30}}=\left(\frac{3}{2}\right)^{30}$$

Detyra 71: $\lim\limits_{x\to\infty}\frac{(4x-1)^{10}(3x+1)^{20}}{(6x+5)^{30}}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{(4x-1)^{10}(3x+1)^{20}}{(6x+5)^{30}}=\lim_{x\to\infty}\frac{\left(\frac{4x}{x}-\frac{1}{x}\right)^{10}\left(\frac{3x}{x}+\frac{1}{x}\right)^{20}}{\left(\frac{6x}{x}+\frac{5}{x}\right)^{30}}=\lim_{x\to\infty}\frac{\left(4-\frac{1}{x}\right)^{10}\left(3+\frac{1}{x}\right)^{20}}{\left(6+\frac{5}{x}\right)^{30}}=$$

$$=\frac{(4-0)^{10}(3+0)^{20}}{(6+0)^{30}}=\frac{4^{10}\cdot 3^{20}}{6^{30}}=\frac{2^{10}\cdot 2^{10}\cdot 3^{10}\cdot 3^{10}}{6^{10}\cdot 6^{10}\cdot 6^{10}}=\frac{6^{10}\cdot 6^{10}}{6^{10}\cdot 6^{10}\cdot 6^{10}}=\frac{1}{6^{10}}=6^{-10}$$

Detyra 72: $\lim\limits_{x\to\infty}\left[\frac{3x^2}{2x+1}-\frac{(2x-1)(3x^2+x+2)}{4x^2}\right]$

Zgjidhje:

$$\lim_{x\to\infty}\left[\frac{3x^2}{2x+1}-\frac{(2x-1)(3x^2+x+2)}{4x^2}\right]=\lim_{x\to\infty}\frac{3x^2\cdot 4x^2-(2x+1)(2x-1)(3x^2+x+2)}{(2x+1)\cdot 4x^2}=$$

$$=\lim_{x\to\infty}\frac{12x^4-12x^4-4x^3-5x^2+x+2}{8x^3+4x^2}=\lim_{x\to\infty}\frac{-4x^3-5x^2+x+2\,\big|:x^3}{8x^3+4x^2\,\big|:x^3}=\lim_{x\to\infty}\frac{\frac{-4x^3}{x^3}-\frac{5x^2}{x^3}+\frac{x}{x^3}+\frac{2}{x^3}}{\frac{8x^3}{x^3}+\frac{4x^2}{x^3}}=$$

$$=\lim_{x\to\infty}\frac{-4-\frac{5}{x}+\frac{1}{x^2}+\frac{2}{x^3}}{8+\frac{4}{x}}=-\frac{4}{8}=-\frac{1}{2}$$

Detyra 73: $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^3-2x^2+1}+\sqrt[3]{x^4+1}}{\sqrt[4]{x^6+6x^5+2}-\sqrt[5]{x^7+3x^3+1}}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{\sqrt{x^3-2x^2+1}+\sqrt[3]{x^4+1}}{\sqrt[4]{x^6+6x^5+2}-\sqrt[5]{x^7+3x^3+1}}=\left|\begin{array}{l}\left\{\dfrac{3}{2},\dfrac{4}{3},\dfrac{6}{4},\dfrac{7}{5}\right\}, \text{ku vlera më e madhe është:}\dfrac{6}{4}=\dfrac{3}{2}=\sqrt{x^3}\\ x^{\frac{3}{2}}=\sqrt{x^3};\ \sqrt[3]{\left(x^{\frac{3}{2}}\right)^3}=\sqrt[3]{x^{\frac{9}{2}}}\\ \sqrt[4]{\left(x^{\frac{3}{2}}\right)^4}=\sqrt[4]{x^{\frac{12}{2}}}=\sqrt[4]{x^6}\\ \sqrt[5]{\left(x^{\frac{3}{2}}\right)^5}=\sqrt[5]{x^{\frac{15}{2}}}\end{array}\right|=$$

$$=\lim_{x\to\infty}\frac{\sqrt{\dfrac{x^3}{x^3}-\dfrac{2x^2}{x^3}+\dfrac{1}{x^3}}+\sqrt[3]{\dfrac{x^4}{x^{\frac{9}{2}}}+\dfrac{1}{x^{\frac{9}{2}}}}}{\sqrt[4]{\dfrac{x^6}{x^6}+\dfrac{6x^5}{x^6}+\dfrac{2}{x^6}}-\sqrt[5]{\dfrac{x^7}{x^{\frac{15}{2}}}+\dfrac{3x^3}{x^{\frac{15}{2}}}+\dfrac{1}{x^{\frac{15}{2}}}}}=\lim_{x\to\infty}\frac{\sqrt{1-\dfrac{2}{x}+\dfrac{1}{x^3}}+\sqrt[3]{\dfrac{1}{\sqrt{x}}+\dfrac{1}{\sqrt{x^9}}}}{\sqrt[4]{1+\dfrac{6}{x}+\dfrac{2}{x^6}}-\sqrt[5]{\dfrac{1}{\sqrt{x}}+\dfrac{3}{\sqrt{x}}+\dfrac{1}{\sqrt{x^{15}}}}}=$$

$$=\frac{\sqrt{1-\dfrac{2}{\infty}+\dfrac{1}{\infty^3}}+\sqrt[3]{\dfrac{1}{\sqrt{\infty}}+\dfrac{1}{\sqrt{\infty^9}}}}{\sqrt[4]{1+\dfrac{6}{\infty}+\dfrac{2}{\infty^6}}-\sqrt[5]{\dfrac{1}{\sqrt{\infty}}+\dfrac{3}{\sqrt{\infty}}+\dfrac{1}{\sqrt{\infty^{15}}}}}=\frac{1+0}{1-0}=\frac{1}{1}=1$$

Detyra 74: $\lim\limits_{x\to\infty}\left(\sqrt{tx+b}-\sqrt{tx+a}\right)$

Zgjidhje:

$$\lim_{x\to\infty}\left(\sqrt{tx+b}-\sqrt{tx+a}\right)\cdot\frac{\sqrt{tx+b}+\sqrt{tx+a}}{\sqrt{tx+b}+\sqrt{tx+a}}=\lim_{x\to\infty}\frac{\left(\sqrt{tx+b}-\sqrt{tx+a}\right)\left(\sqrt{tx+b}+\sqrt{tx+a}\right)}{\sqrt{tx+b}+\sqrt{tx+a}}=$$

$$=\lim_{x\to\infty}\frac{tx+b-tx-a}{\sqrt{tx+b}+\sqrt{tx+a}}=\lim_{x\to\infty}\frac{b-a}{\sqrt{tx+b}+\sqrt{tx+a}}=0$$

Detyra 75: $\lim\limits_{x\to\infty}\dfrac{ax^2+bx+c}{px^2+qx+r}$

Zgjidhje: $\lim\limits_{x\to\infty}\dfrac{ax^2+bx+c}{px^2+qx+r}=\lim\limits_{x\to\infty}\dfrac{x^2\left(a+\dfrac{b}{x}+\dfrac{c}{x^2}\right)}{x^2\left(p+\dfrac{q}{x}+\dfrac{r}{x^2}\right)}=\lim\limits_{x\to\infty}\dfrac{a+\dfrac{b}{x}+\dfrac{c}{x^2}}{p+\dfrac{q}{x}+\dfrac{r}{x^2}}=\dfrac{a+\dfrac{b}{\infty}+\dfrac{c}{\infty^2}}{p+\dfrac{q}{\infty}+\dfrac{r}{\infty^2}}=\dfrac{a+0+0}{p+0+0}=\dfrac{a}{p}$

Detyra 76: $\lim\limits_{x\to\infty} x\left(\sqrt{x^2+2x}-2\sqrt{x^2+x}+x\right)$

Zgjidhje:

$$\lim_{x\to\infty} x\left(\sqrt{x^2+2x}-2\sqrt{x^2+x}+x\right)\cdot\frac{\sqrt{x^2+2x}+2\sqrt{x^2+x}+x}{\sqrt{x^2+2x}+2\sqrt{x^2+x}+x}=$$

$$=\lim_{x\to\infty}\frac{2x\left(\sqrt{x^2+2x}-x-1\right)}{\sqrt{x^2+2x}+2\sqrt{x^2+x}+x}\cdot\frac{\sqrt{x^2+2x}+x+1}{\sqrt{x^2+2x}+x+1}=\lim_{x\to\infty}\frac{-2x^2}{\left(\sqrt{x^2+2x}+x+2\sqrt{x^2+x}\right)\left(\sqrt{x^2+2x}+x+1\right)}=$$

$$=\lim_{x\to\infty}\frac{-2}{\left(\sqrt{1+\frac{2}{x}}+1+2\sqrt{1+\frac{1}{x}}\right)\left(\sqrt{1+\frac{2}{x}}+1+\frac{1}{x}\right)}=\frac{-2}{\left(\sqrt{1+\frac{2}{\infty}}+1+2\sqrt{1+\frac{1}{\infty}}\right)\left(\sqrt{1+\frac{2}{\infty}}+1+\frac{1}{\infty}\right)}=-\frac{2}{8}=-\frac{1}{4}$$

Detyra 77: $\lim\limits_{x\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2-x+1}\right)$

Zgjidhje:

$$\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2-x+1}\right)\cdot\frac{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}}{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}}=$$

$$=\lim_{x\to\infty}\frac{x^2+x+1-x^2+x-1}{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}}=\lim_{x\to\infty}\frac{2x}{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}}=\lim_{x\to\infty}\frac{\frac{2x}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}+\frac{1}{x^2}}+\sqrt{\frac{x^2}{x^2}-\frac{x}{x^2}+\frac{1}{x^2}}}=$$

$$=\lim_{x\to\infty}\frac{2}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}=\frac{2}{1+1}=\frac{2}{2}=1$$

Detyra 78: $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-5x+1}-x\right)$

Zgjidhje:

$$\lim_{x\to-\infty}\left(\sqrt{x^2-5x+1}-x\right)\cdot\frac{\sqrt{x^2-5x+1}+x}{\sqrt{x^2-5x+1}+x}=\lim_{x\to-\infty}\frac{\left(\sqrt{x^2-5x+1}-x\right)\left(\sqrt{x^2-5x+1}+x\right)}{\sqrt{x^2-5x+1}+x}=$$

$$=\lim_{x\to-\infty}\frac{x^2-5x+1-x^2}{\sqrt{x^2-5x+1}+x}=\lim_{x\to-\infty}\frac{-5x+1}{\sqrt{x^2-5x+1}+x}=\lim_{x\to-\infty}\frac{x\left(-5+\frac{1}{x}\right)}{|x|\left(\sqrt{1-\frac{5}{x}+\frac{1}{x^2}}+\frac{x}{|x|}\right)}=$$

$$=\lim_{x\to-\infty}\frac{x\left(-5+\frac{1}{x}\right)}{(-x)\left(\sqrt{1-\frac{5}{x}+\frac{1}{x^2}}+1\right)}=\frac{5}{2}$$

Detyra 79: $\lim\limits_{x\to\infty}\dfrac{(x+2)(x+4)\cdot x}{3x^3+x^2-2x}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{(x+2)(x+4)\cdot x}{3x^3+x^2-2x}=\lim_{x\to\infty}\frac{\left(x^2+4x+2x+8\right)\cdot x}{3x^3+x^2-2x}\lim_{x\to\infty}\frac{\left(x^2+6x+8\right)\cdot x}{3x^3+x^2-2x}=\lim_{x\to\infty}\frac{x^3+6x^2+8x}{3x^3+x^2-2x}=$$

$$=\lim_{x\to\infty}\frac{\frac{x^3}{x^3}+\frac{6x^2}{x^3}+\frac{8x}{x^3}}{\frac{3x^3}{x^3}+\frac{x^2}{x^3}-\frac{2x}{x^3}}=\lim_{x\to\infty}\frac{1+\frac{6}{x}+\frac{8}{x^2}}{3+\frac{1}{x}-\frac{2}{x^2}}=\frac{1}{3}$$

Detyra 80: $\lim\limits_{x\to\infty}\left(\sqrt{x^2-5x+6}-x\right)$

Zgjidhje:

$$\lim_{x\to\infty}\left(\sqrt{x^2-5x+6}-x\right)\cdot\frac{\sqrt{x^2-5x+6}+x}{\sqrt{x^2-5x+6}+x}=\lim_{x\to\infty}\frac{\left(\sqrt{x^2-5x+6}-x\right)\left(\sqrt{x^2-5x+6}+x\right)}{\sqrt{x^2-5x+6}+x}=$$

$$=\lim_{x\to\infty}\frac{x^2-5x+6-x^2}{\sqrt{x^2-5x+6}+x}=\lim_{x\to\infty}\frac{-5x+6}{\sqrt{x^2-5x+6}+x}=\lim_{x\to\infty}\frac{x\left(-5+\frac{6}{x}\right)}{x\left(\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}+1\right)}=-\frac{5}{2}$$

Detyra 81: $\lim\limits_{x\to a}\dfrac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}}$

Zgjidhje:

$$\lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}}=\lim_{x\to a}\left(\frac{\sqrt{x}-\sqrt{a}}{\sqrt{x^2-a^2}}+\frac{\sqrt{x-a}}{\sqrt{x^2-a^2}}\right)=\lim_{x\to a}\left(\frac{x-a}{\sqrt{x^2-a^2}\left(\sqrt{x}+\sqrt{a}\right)}+\frac{1}{\sqrt{x+a}}\right)=$$

$$=\lim_{x\to a}\left(\frac{1}{\sqrt{x}+\sqrt{a}}\sqrt{\frac{x-a}{x+a}}+\frac{1}{\sqrt{x+a}}\right)=\frac{1}{\sqrt{2a}}$$

Detyra 82: $\lim\limits_{x\to a}\dfrac{x^2-a^2}{5x^2-4ax-a^2}$

Zgjidhje:

$$\lim_{x\to a}\frac{x^2-a^2}{5x^2-4ax-a^2}=\lim_{x\to a}\frac{(x-a)(x+a)}{5x^2-5ax+ax-a^2}=\lim_{x\to a}\frac{(x-a)(x+a)}{5x(x-a)+a(x+a)}=\lim_{x\to a}\frac{(x-a)(x+a)}{(x-a)(5x+a)}=$$

$$=\lim_{x\to a}\frac{x+a}{5x+a}=\frac{a+a}{5a+a}=\frac{2a}{6a}=\frac{2}{6}=\frac{1}{3}$$

Detyra 83: $\lim\limits_{x\to 0}\dfrac{3^{2x+3}-27}{3^{x+1}-3}$

Zgjidhje:

$$\lim_{x\to 0}\frac{3^{2x+3}-27}{3^{x+1}-3}=\lim_{x\to 0}\frac{3^3\left(3^{2x}-1\right)}{3\left(3^x-1\right)}=\lim_{x\to 0}\frac{3^2\left(3^x-1\right)\left(3^x+1\right)}{\left(3^x-1\right)}=\lim_{x\to 0}3^2\left(3^x+1\right)=9\left(3^0+1\right)=$$

$$=9\left(1+1\right)=9\cdot 2=18$$

Detyra 84: $\lim\limits_{x\to a}\dfrac{3\sqrt{x}-3\sqrt{a}}{x-a}$

Zgjidhje:

$$\lim_{x\to a}\frac{3\sqrt{x}-3\sqrt{a}}{x-a}=\lim_{x\to a}\frac{3\left(\sqrt{x}-\sqrt{a}\right)}{x-a}\cdot\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}=\lim_{x\to a}\frac{3\left(\sqrt{x}-\sqrt{a}\right)\left(\sqrt{x}+\sqrt{a}\right)}{\left(x-a\right)\left(\sqrt{x}+\sqrt{a}\right)}=$$

$$=\lim_{x\to a}\frac{3\left(x-a\right)}{\left(x-a\right)\left(\sqrt{x}+\sqrt{a}\right)}=\frac{3}{\left(\sqrt{a}+\sqrt{a}\right)}=\frac{3}{2\sqrt{a}}$$

Detyra 85: $\lim\limits_{x\to y}\dfrac{3x^3y+2y^4-2xy^3-3x^2y^2}{x^3y-2y^2x^2-y^3x+2y^4}$

Zgjidhje:

$$\lim_{x\to y}\frac{3x^3y+2y^4-2xy^3-3x^2y^2}{x^3y-2y^2x^2-y^3x+2y^4}=\lim_{x\to y}\frac{3x^2y\left(x-y\right)-2y^3\left(x-y\right)}{\left(x-y\right)\left(x^2-y^2\right)-2y^2\left(x^2-y^2\right)}=\lim_{x\to y}\frac{\left(x-y\right)\left(3x^2y-2y^3\right)}{\left(x-y\right)\left(x+y\right)\left(xy-2y^2\right)}=$$

$$\lim_{x\to y}\frac{3x^2y-2y^3}{\left(x+y\right)\left(xy-2y^2\right)}=\frac{3y^2y-2y^3}{\left(y+y\right)\left(y\cdot y-2y^2\right)}=\frac{3y^3-2y^3}{2y\left(y^2-2y^2\right)}=\frac{y^3}{2y\left(-y^2\right)}=-\frac{y^3}{2y^3}=-\frac{1}{2}$$

Detyra 86: $\lim\limits_{x\to 0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=\lim_{x\to 0}\frac{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\lim_{x\to 0}\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=$$

$$=\lim_{x\to 0}\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\lim_{x\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{\sqrt{x+0}+\sqrt{x}}=\frac{1}{\sqrt{x}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

Detyra 87: $\lim\limits_{x\to -1}\frac{x+1}{\sqrt[4]{x+17}-2}$

Zgjidhje:

$$\lim_{x\to -1}\frac{x+1}{\sqrt[4]{x+17}-2}=\left|\begin{array}{l}Zëvendësojmë:\\ x+17=t^4\\ x\to -1,\qquad t\to 2\end{array}\right|=\lim_{t\to 2}\frac{t^4-17+1}{t-2}=\lim_{t\to 2}\frac{t^4-16}{t-2}=\lim_{t\to 2}\frac{\left(t^2-4\right)\left(t^2+4\right)}{t-2}=$$

$$=\lim_{t\to 2}\frac{(t-2)(t+2)\left(t^2+4\right)}{t-2}=\lim_{t\to 2}\frac{(t+2)\left(t^2+4\right)}{1}=(2+2)\left(2^2+4\right)=4\cdot 8=32$$

Detyra 88: $\lim\limits_{x\to 1}\frac{\sqrt[3]{x}-1}{x-1}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{x-1}=\left|\begin{array}{l}Zëvëndësojmë:\\ \sqrt[3]{x}=t\\ x\to 1;\ \ t\to 1\end{array}\right|=\lim_{t\to 1}\frac{t-1}{t^3-1}=\lim_{t\to 1}\frac{t-1}{(t-1)\left(t^2+t+1\right)}=\lim_{t\to 1}\frac{1}{\left(t^2+t+1\right)}=\frac{1}{\left(1^2+1+1\right)}=\frac{1}{(1+1+1)}=\frac{1}{3}$$

Detyra 89: $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}=\left|\begin{array}{l}Zëvendësojmë:\\ 1+x=t^6\Rightarrow x=t^6-1\\ x\to 0,\qquad t\to 1\end{array}\right|=\lim_{t\to 1}\frac{\sqrt{t^6}-1}{\sqrt[3]{t^6}-1}=\lim_{t\to 1}\frac{t^3-1}{t^2-1}=\lim_{t\to 1}\frac{(t-1)\left(t^2+t+1\right)}{(t-1)(t+1)}=$$

$$=\lim_{t\to 1}\frac{t^2+t+1}{t+1}=\frac{1^2+1+1}{1+1}=\frac{1+1+1}{1+1}=\frac{3}{2}$$

Detyra 90: $\lim\limits_{x\to -1}\frac{1+\sqrt[3]{x}}{1+\sqrt[5]{x}}$

Zgjidhje:

$$\lim_{x\to -1}\frac{1+\sqrt[3]{x}}{1+\sqrt[5]{x}}=\left|\begin{array}{l}Zëvendësojmë:\\ x=t^{15}\\ x\to -1, t\to -1\end{array}\right|=\lim_{t\to -1}\frac{1+t^5}{1+t^3}=\lim_{t\to -1}\frac{(1+t)\left(1-t+t^2-t^3+t^4\right)}{(1+t)\left(1-t+t^2\right)}=$$

$$=\lim_{t\to -1}\frac{\left(1-t+t^2-t^3+t^4\right)}{\left(1-t+t^2\right)}=\frac{1-(-1)+(-1)^2-(-1)^3+(-1)^4}{1-(-1)+(-1)^2}=\frac{1+1+1+1+1}{1+1+1}=\frac{5}{3}$$

Detyra 91: $\lim\limits_{x\to -1}\frac{1+\sqrt[7]{x}}{1+\sqrt[5]{x}}$

Zgjidhje:

$$\lim_{x\to -1}\frac{1+\sqrt[7]{x}}{1+\sqrt[5]{x}}=\left|\begin{array}{l}\textit{Zëvendësojmë}: \\ x=t^{35} \\ x\to -1,\ t\to -1\end{array}\right|=\lim_{t\to -1}\frac{1+\sqrt[7]{t^{35}}}{1+\sqrt[5]{t^{35}}}=\lim_{t\to -1}\frac{1+t^5}{1+t^7}=\lim_{t\to -1}\frac{(1+t)\left(1-t+t^2-t^3+t^4\right)}{(1+t)\left(1-t+t^2-t^3+t^4-t^5+t^6\right)}=$$

$$=\lim_{t\to -1}\frac{1-t+t^2-t^3+t^4}{1-t+t^2-t^3+t^4-t^5+t^6}=\frac{1-(-1)+(-1)^2-(-1)^3+(-1)^4}{1-(-1)+(-1)^2-(-1)^3+(-1)^4-(-1)^5+(-1)^6}=\frac{1}{1}=1$$

Detyra 92: $\lim\limits_{x\to 1}\frac{\sqrt[4]{x}-\sqrt[6]{x}}{\sqrt[8]{x}-\sqrt[12]{x}}$

Zgjidhje: $$\lim_{x\to 1}\frac{\sqrt[4]{x}-\sqrt[6]{x}}{\sqrt[8]{x}-\sqrt[12]{x}}=\left|\begin{array}{l}\textit{Zëvendësojmë}: \\ x=t^{24} \\ x\to 1,\ t\to 1\end{array}\right|=\lim_{t\to 1}\frac{\sqrt[4]{t^{24}}-\sqrt[6]{t^{24}}}{\sqrt[8]{t^{24}}-\sqrt[12]{t^{24}}}=\lim_{t\to 1}\frac{t^6-t^4}{t^3-t^2}=\lim_{t\to 1}\frac{t^4\left(t^2-1\right)}{t^2(t-1)}=$$

$$=\lim_{t\to 1}\frac{t^2(t-1)(t+1)}{t-1}=\lim_{t\to 1}t^2(t+1)=1^2(1+1)=2$$

Detyra 93: $\lim\limits_{x\to 1}\frac{\sqrt{x}+\sqrt[3]{x}-2}{x-1}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\sqrt{x}+\sqrt[3]{x}-2}{x-1}=\left|\begin{array}{l}\textit{Zëvendësojmë}: \\ x=t^{6} \\ x\to 1,\ t\to 1\end{array}\right|=\lim_{t\to 1}\frac{\sqrt{t^6}+\sqrt[3]{t^6}-2}{t^6-1}=\lim_{t\to 1}\frac{t^3+t^2-1}{t^6-1}=$$

$$=\lim_{t\to 1}\frac{(t-1)\left(t^2+2t+2\right)}{(t-1)\left(t^5+t^4+t^3+t^2+t+1\right)}=\lim_{t\to 1}\frac{\left(t^2+2t+2\right)}{\left(t^5+t^4+t^3+t^2+t+1\right)}=$$

$$=\frac{1^2+2\cdot 1+2}{1^5+1^4+1^3+1^2+1+1}=\frac{1+2+2}{1+1+1+1+1+1}=\frac{5}{6}$$

Detyra 94: $\lim\limits_{x\to 1}\dfrac{1-\sqrt[4]{x}}{1-\sqrt[6]{x}}$

Zgjidhje:

$$\lim_{x\to 1}\frac{1-\sqrt[4]{x}}{1-\sqrt[6]{x}}=\left|\begin{array}{l}Z\ddot{e}vend\ddot{e}sojm\ddot{e}:\\ x=t^{12}\\ x\to 1,\ t\to 1\end{array}\right|=\lim_{t\to 1}\frac{1-\sqrt[4]{t^{12}}}{1-\sqrt[6]{t^{12}}}=\lim_{t\to 1}\frac{1-t^3}{1-t^2}=\lim_{t\to 1}\frac{(1-t)\left(1+t+t^2\right)}{(1-t)(1+t)}=\lim_{t\to 1}\frac{1+t+t^2}{1+t}=$$

$$=\frac{1+1+1^2}{1+1}=\frac{3}{2}$$

Detyra 95: $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}=\left|\begin{array}{l}Z\ddot{e}vend\ddot{e}sojm\ddot{e}:\\ x=t^{12}\\ x\to 1,\ t\to 1\end{array}\right|=\lim_{t\to 1}\frac{\sqrt[3]{t^{12}}-1}{\sqrt[4]{t^{12}}-1}=\lim_{t\to 1}\frac{t^4-1}{t^3-1}=\lim_{t\to 1}\frac{(t-1)\left(t^3+t^2+t+1\right)}{(t-1)\left(t^2+t+1\right)}=$$

$$=\lim_{t\to 1}\frac{t^3+t^2+t+1}{t^2+t+1}=\frac{1^3+1^2+1+1}{1^2+1+1}=\frac{4}{3}$$

Detyra 96: $\lim\limits_{x\to 1}\left(\dfrac{3}{1-\sqrt{x}}-\dfrac{2}{1-\sqrt[3]{x}}\right)$

Zgjidhje:

$$\lim_{x\to 1}\left(\frac{3}{1-\sqrt{x}}-\frac{2}{1-\sqrt[3]{x}}\right)=\left|\begin{array}{l}Z\ddot{e}vend\ddot{e}sojm\ddot{e}:\\ x=t^{6}\\ x\to 1,\ t\to 1\end{array}\right|=\lim_{t\to 1}\left(\frac{3}{1-t^3}-\frac{2}{1-t^2}\right)=\lim_{t\to 1}\left[\frac{3}{(1-t)\left(1+t+t^2\right)}-\frac{2}{(1-t)(1+t)}\right]=$$

$$=\lim_{t\to 1}\frac{3(1+t)-2\left(1+t+t^2\right)}{(1-t)(1+t)\left(1+t+t^2\right)}=\lim_{t\to 1}\frac{3+3t-2-2t-2t^2}{(1-t)(1+t)\left(1+t+t^2\right)}=\lim_{t\to 1}\frac{-2t^2+t+1}{(1-t)(1+t)\left(1+t+t^2\right)}=$$

$$=\lim_{t\to 1}\frac{(1-t)(2t+1)}{(1-t)(1+t)\left(1+t+t^2\right)}=\lim_{t\to 1}\frac{2t+1}{(1+t)\left(1+t+t^2\right)}=\frac{2\cdot 1+1}{(1+1)\left(1+1+1^2\right)}=\frac{3}{6}=\frac{1}{2}$$

Detyra 97: $\lim\limits_{x\to -1}\dfrac{1+\sqrt[3]{x}}{1+\sqrt[5]{x}}$

Zgjidhje:

$$\lim_{x\to -1}\frac{1+\sqrt[3]{x}}{1+\sqrt[5]{x}}=\left|\begin{array}{l}\textit{Zëvendësojmë}:\\ x=t^{15}\\ x\to -1, t\to -1\end{array}\right|=\lim_{t\to -1}\frac{1+t^5}{1+t^3}=\lim_{t\to -1}\frac{(1+t)\left(1-t+t^2-t^3+t^4\right)}{(1+t)\left(1-t+t^2\right)}=$$

$$=\lim_{t\to -1}\frac{\left(1-t+t^2-t^3+t^4\right)}{\left(1-t+t^2\right)}=\frac{1-(-1)+(-1)^2-(-1)^3+(-1)^4}{1-(-1)+(-1)^2}=\frac{1+1+1+1+1}{1+1+1}=\frac{5}{3}$$

Detyra 98: $\lim\limits_{x\to 1}\dfrac{\sqrt[5]{x}-1}{\sqrt[4]{x}-1}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\sqrt[5]{x}-1}{\sqrt[4]{x}-1}=\left|\begin{array}{l}\textit{Zëvendësojmë}:\\ x=t^{20}\\ x\to 1,\ t\to 1\end{array}\right|=\lim_{t\to 1}\frac{\sqrt[5]{t^{20}}-1}{\sqrt[4]{t^{20}}-1}=\lim_{t\to 1}\frac{t^4-1}{t^5-1}=\lim_{t\to 1}\frac{(t-1)\left(t^3+t^2+t+1\right)}{(t-1)\left(t^4+t^3+t^2+t+1\right)}=$$

$$=\lim_{t\to 1}\frac{t^3+t^2+t+1}{t^4+t^3+t^2+t+1}=\frac{1^3+1^2+1+1}{1^4+1^3+1^2+1+1}=\frac{4}{5}$$

Detyra 99: $\lim\limits_{x\to 1}\left[\dfrac{1}{2\left(1-\sqrt{x}\right)}-\dfrac{1}{3\left(1-\sqrt[3]{x}\right)}\right]$

Zgjidhje:

$$\lim_{x\to 1}\left[\frac{1}{2\left(1-\sqrt{x}\right)}-\frac{1}{3\left(1-\sqrt[3]{x}\right)}\right]=\left|\begin{array}{ll}\textit{Zëvendësojmë}: x=t^6 & \\ x\to 1, & t\to 1\end{array}\right|=\lim_{t\to 1}\frac{1}{2\left(1-t^3\right)}-\frac{1}{3\left(1-t^2\right)}=$$

$$=\lim_{t\to 1}\frac{3\left(1-t^2\right)-2\left(1-t^3\right)}{6\left(1-t^3\right)\left(1-t^2\right)}=\frac{1}{6}\lim_{t\to 1}\frac{3\left(1-t^2\right)-2\left(1+t+t^2\right)}{\left(1+t+t^2\right)\left(1-t^2\right)}=\frac{1}{6}\lim_{t\to 1}\frac{-2t^2+t+1}{\left(1+t+t^2\right)\left(1-t^2\right)}=$$

$$=\frac{1}{6}\lim_{t\to 1}\frac{1-t^2+t-t^2}{\left(1+t+t^2\right)\left(1-t^2\right)}=\frac{1}{6}\lim_{t\to 1}\frac{(1-t)(1+t)+t(1-t)}{\left(1+t+t^2\right)(1-t)(1+t)}=\frac{1}{6}\lim_{t\to 1}\frac{1+t+t}{\left(1+t+t^2\right)(1+t)}=$$

$$=\frac{1}{6}\cdot\frac{1+1+1}{(1+1+1)(1+1)}=\frac{1}{6}\cdot\frac{3}{6}=\frac{1}{12}$$

Detyra 100: $\lim\limits_{x\to 0}\dfrac{e^{5x}-3e^{3x}+2}{e^{7x}-7e^{2x}+6}$

Zgjidhje:

$$\lim_{x\to 0}\frac{e^{5x}-3e^{3x}+2}{e^{7x}-7e^{2x}+6}=\left|\begin{array}{l}Z\ddot{e}v\ddot{e}nd\ddot{e}sojm\ddot{e}:\\ e^{x}=t\\ x\to 0;\quad t\to 1\end{array}\right|=\lim_{t\to 1}\frac{t^5-3t^3+2}{t^7-7t^2+6}=\lim_{t\to 1}\frac{(t-1)\left(t^4+t^3-2t^2-2t-2\right)}{(t-1)\left(t^6+t^5+t^4+t^3+t^2-6t-6\right)}=$$

$$\lim_{t\to 1}\frac{t^4+t^3-2t^2-2t-2}{t^6+t^5+t^4+t^3+t^2-6t-6}=\frac{1^4+1^3-2\cdot 1^2-2\cdot 1-2}{1^6+1^5+1^4+1^3+1^2-6\cdot 1-6}=\frac{-4}{-7}=\frac{4}{7}$$

Detyra 101: $\lim\limits_{x\to\infty}\left(\dfrac{x^k}{1+x+x^2+...+x^k}\right)^{1+2x}$ $(k\in\mathbf{N})$

Zgjidhje:

$$\lim_{x\to\infty}\left(\frac{x^k}{1+x+x^2+...+x^k}\right)^{1+2x}=\lim_{x\to\infty}\left(\frac{x^k}{\frac{x^{k+1}-1}{x-1}}\right)^{1+2x}=\lim_{x\to\infty}\left(\frac{x^k(x-1)}{x^{k+1}-1}\right)^{1+2x}=$$

$$=e^{\lim\limits_{x\to\infty}(2x+1)\ln\frac{x^k(x-1)}{x^{k+1}-1}}=e^{\lim\limits_{x\to\infty}(2x+1)\ln\frac{x^{k+1}-1(x^k-1)}{x^{k+1}-1}}=e^{\lim\limits_{x\to\infty}(2x+1)\ln\left(1-\frac{x^k-1}{x^{k+1}-1}\right)^{-\frac{x^{k+1}-1}{x^k-1}\left(-\frac{x^k-1}{x^{k+1}-1}\right)}}=$$

$$=e^{\lim\limits_{x\to\infty}(2x+1)\left(-\frac{x^k-1}{x^{k+1}-1}\right)}=e^{-2}$$

Detyra 102: $\lim\limits_{x\to -8}\dfrac{\sqrt{1-x}-3}{2+\sqrt[3]{x}}$

Zgjidhje:

$$\lim_{x\to -8}\frac{\sqrt{1-x}-3}{2+\sqrt[3]{x}}=\left|\begin{array}{l}Z\ddot{e}v\ddot{e}nd\ddot{e}sojm\ddot{e}:\\ \sqrt[3]{x}=t\ /^3\\ x=t^3\\ x\to -8;\quad t\to -2\end{array}\right|=\lim_{t\to -2}\frac{\sqrt{1-t^3}-3}{2+t}\cdot\frac{\sqrt{1-t^3}+3}{\sqrt{1-t^3}+3}=\lim_{t\to -2}\frac{1-t^3-9}{(2+t)\left(\sqrt{1-t^3}+3\right)}=$$

$$=\lim_{t\to -2}\frac{-t^3-8}{(2+t)\left(\sqrt{1-t^3}+3\right)}=-\lim_{t\to -2}\frac{t^3+8}{(2+t)\left(\sqrt{1-t^3}+3\right)}=-\lim_{t\to -2}\frac{(t+2)\left(t^2-2t+4\right)}{(2+t)\left(\sqrt{1-t^3}+3\right)}=-\lim_{t\to -2}\frac{t^2-2t+4}{\sqrt{1-t^3}+3}=$$

$$=\frac{(-2)^2-2(-2)+4}{\sqrt{1-(-2)^3}+3}=-\frac{4+4+4}{\sqrt{1+8}+3}=-\frac{12}{\sqrt{9}+3}=-\frac{12}{3+3}=-\frac{12}{6}=-2$$

Detyra 103: $\lim\limits_{y\to 2}\dfrac{\dfrac{\sqrt{5y-6}-\sqrt{3y-2}}{\sqrt{7y-5}-\sqrt{4y+1}}}{\dfrac{\sqrt{5y-1}-\sqrt{2y+5}}{\sqrt{6y-8}-\sqrt{4y-4}}}$

Zgjidhje:

$$\lim_{y\to 2}\frac{\dfrac{\sqrt{5y-6}-\sqrt{3y-2}}{\sqrt{7y-5}-\sqrt{4y+1}}}{\dfrac{\sqrt{5y-1}-\sqrt{2y+5}}{\sqrt{6y-8}-\sqrt{4y-4}}}=\lim_{y\to 2}\frac{\left(\sqrt{5y-6}-\sqrt{3y-2}\right)\cdot\left(\sqrt{6y-8}-\sqrt{4y-4}\right)}{\left(\sqrt{7y-5}-\sqrt{4y+1}\right)\cdot\left(\sqrt{5y-1}-\sqrt{2y+5}\right)}=$$

$$=\underbrace{\lim_{y\to 2}\frac{\sqrt{5y-6}-\sqrt{3y-2}}{\sqrt{7y-5}-\sqrt{4y+1}}}_{A}\cdot\underbrace{\lim_{y\to 2}\frac{\sqrt{6y-8}-\sqrt{4y-4}}{\sqrt{5y-1}-\sqrt{2y+5}}}_{B}=A\cdot B$$

$$A=\lim_{y\to 2}\frac{\sqrt{5y-6}-\sqrt{3y-2}}{\sqrt{7y-5}-\sqrt{4y+1}}\cdot\frac{\sqrt{5y-6}+\sqrt{3y-2}}{\sqrt{5y-6}+\sqrt{3y-2}}\cdot\frac{\sqrt{7y-5}+\sqrt{4y+1}}{\sqrt{7y-5}+\sqrt{4y+1}}=$$

$$=\lim_{y\to 2}\frac{(5y-6-3y+2)\cdot\left(\sqrt{7y-5}+\sqrt{4y+1}\right)}{(7y-5-4y-1)\cdot\left(\sqrt{5y-6}+\sqrt{3y-2}\right)}=\lim_{y\to 2}\frac{(2y-4)\cdot\left(\sqrt{7y-5}+\sqrt{4y+1}\right)}{(3y-6)\cdot\left(\sqrt{5y-6}+\sqrt{3y-2}\right)}=$$

$$=\lim_{y\to 2}\frac{2(y-2)\cdot\left(\sqrt{7y-5}+\sqrt{4y+1}\right)}{3(y-2)\cdot\left(\sqrt{5y-6}+\sqrt{3y-2}\right)}=\lim_{y\to 2}\frac{2\cdot\left(\sqrt{7y-5}+\sqrt{4y+1}\right)}{3\cdot\left(\sqrt{5y-6}+\sqrt{3y-2}\right)}=\frac{2\cdot\left(\sqrt{7\cdot 2-5}+\sqrt{4\cdot 2+1}\right)}{3\cdot\left(\sqrt{5\cdot 2-6}+\sqrt{3\cdot 2-2}\right)}=$$

$$=\frac{2\cdot\left(\sqrt{14-5}+\sqrt{8+1}\right)}{3\cdot\left(\sqrt{10-6}+\sqrt{6-2}\right)}=\frac{2\cdot\left(\sqrt{9}+\sqrt{9}\right)}{3\cdot\left(\sqrt{4}+\sqrt{4}\right)}=\frac{2\cdot(3+3)}{3\cdot(2+2)}=\frac{2\cdot 6}{3\cdot 4}=\frac{12}{12}=1$$

$$B=\lim_{y\to 2}\frac{\sqrt{6y-8}-\sqrt{4y-4}}{\sqrt{5y-1}-\sqrt{2y+5}}\cdot\frac{\sqrt{6y-8}+\sqrt{4y-4}}{\sqrt{6y-8}+\sqrt{4y-4}}\cdot\frac{\sqrt{5y-1}+\sqrt{2y+5}}{\sqrt{5y-1}+\sqrt{2y+5}}=$$

$$=\lim_{y\to 2}\frac{(6y-8-4y+4)\cdot\left(\sqrt{5y-1}+\sqrt{2y+5}\right)}{(5y-1-2y-5)\cdot\left(\sqrt{6y-8}+\sqrt{4y-4}\right)}=\lim_{y\to 2}\frac{(2y-4)\cdot\left(\sqrt{5y-1}+\sqrt{2y+5}\right)}{(3y-6)\left(\sqrt{6y-8}+\sqrt{4y-4}\right)}=$$

$$=\lim_{y\to 2}\frac{2(y-2)\cdot\left(\sqrt{5y-1}+\sqrt{2y+5}\right)}{3(y-2)\cdot\left(\sqrt{6y-8}+\sqrt{4y-4}\right)}=\lim_{y\to 2}\frac{2\cdot\left(\sqrt{5y-1}+\sqrt{2y+5}\right)}{3\cdot\left(\sqrt{6y-8}+\sqrt{4y-4}\right)}=\frac{2\cdot\left(\sqrt{9}+\sqrt{9}\right)}{3\cdot\left(\sqrt{4}+\sqrt{4}\right)}=\frac{2\cdot 6}{3\cdot 4}=\frac{12}{12}=1$$

$$A\cdot B=1\cdot 1=1$$

Detyra 104: $\lim\limits_{x\to\infty}\left((x+2)\ln(x+2)-2(x+1)\ln(x+1)+x\ln x\right)$

Zgjidhje:

$$\lim_{x\to\infty}\left((x+2)\ln(x+2)-2(x+1)\ln(x+1)+x\ln x\right)=\lim_{x\to\infty}\ln\frac{(x+2)^{x+2}x^x}{(x+1)^{2(x+1)}}=$$

$$=\lim_{x\to\infty}\ln\frac{x+2}{x+1}\left(\frac{x+2}{x+1}\right)^{x+1}\cdot\frac{x^x}{(x+1)^x}=\lim_{x\to\infty}\ln\frac{x+2}{x+1}+\lim_{x\to\infty}\ln\left(\frac{x+2}{x+1}\right)^{x+1}+\lim_{x\to\infty}\ln\frac{x^x}{(x+1)^x}=$$

$$=\lim_{x\to\infty}\ln\left(1+\frac{1}{x+1}\right)^{x+1}+\lim_{x\to\infty}\ln\frac{1}{\left(1+\frac{1}{x}\right)^x}=1-1=0$$

Detyra 105: $\lim\limits_{x\to\infty}\left(\frac{3x+1}{3x-2}\right)^{2x}$

Zgjidhje:

$$\lim_{x\to\infty}\left(\frac{3x+1}{3x-2}\right)^{2x}=\lim_{x\to\infty}\frac{\left(1+\frac{1}{3x}\right)^{2x}}{\left(1-\frac{2}{3x}\right)^{2x}}=\frac{\lim\limits_{x\to\infty}\left(\left(1+\frac{1}{3x}\right)^{3x}\right)^{\frac{2}{3}}}{\lim\limits_{x\to\infty}\left(\left(1-\frac{2}{3x}\right)^{-\frac{3x}{2}\cdot}\right)^{-\frac{4}{3}}}=\frac{e^{\frac{2}{3}}}{e^{-\frac{4}{3}}}=e^2$$

Detyra 106: $\lim\limits_{x\to 5}(x-4)^{\frac{1}{x-5}}$

Zgjidhje:

$$\lim_{x\to 5}[x-4]^{\frac{1}{x-5}}=\lim_{x\to 5}\left[1+(x-5)\right]^{\frac{1}{x-5}}=e$$

Detyra 107: $\lim\limits_{x\to\infty}\left(\frac{2x+3}{2x+1}\right)^{x+1}$

Zgjidhje:

$$\lim_{x\to\infty}\left(\frac{2x+3}{2x+1}\right)^{x+1}=\lim_{x\to\infty}\left(1+\frac{2}{2x+1}\right)^{x+1}=\lim_{x\to\infty}\left[\left(1+\frac{2}{2x+1}\right)^{\frac{2x+1}{2}}\right]^{\frac{2(x+1)}{2x+1}}=$$

$$=e^{\lim\limits_{x\to\infty}\frac{2(x+1)}{2x+1}}=e^1=e$$

Detyra 108: $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

Zgjidhje:

Marrim çfarëdo numri real x që $x>1$ që të ndodhet ndërmjet dy numrave të njëpasnjëshëm natyral $n,\ n+1$

$$\frac{1}{n+1}\le\frac{1}{x}<\frac{1}{n}\Rightarrow 1+\frac{1}{n+1}\le 1+\frac{1}{x}<1+\frac{1}{n}$$

$$\left(1+\frac{1}{n+1}\right)^n\le\left(1+\frac{1}{x}\right)^x<\left(1+\frac{1}{n}\right)^{n+1}$$

$$\left(1+\frac{1}{n+1}\right)^n<\left(1+\frac{1}{x}\right)^x<\left(1+\frac{1}{n}\right)^{n+1}$$

$$\lim_{x\to\infty}\left(1+\frac{1}{n+1}\right)^n<\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x<\lim_{x\to\infty}\left(1+\frac{1}{n}\right)^{n+1}$$

Meqë

$$\lim_{x\to\infty}\left(1+\frac{1}{n+1}\right)^n=\lim_{x\to\infty}\left(1+\frac{1}{n+1}\right)^n\cdot\frac{1}{\lim\limits_{x\to\infty}\left(1+\frac{1}{n+1}\right)^n}=e$$

dhe

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{n+1}=\lim_{x\to\infty}\left(1+\frac{1}{n}\right)^n\cdot\lim_{x\to\infty}\left(1+\frac{1}{n}\right)=e$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e \text{ ose } \lim_{x\to\infty}\left(1+x\right)^{\frac{1}{x}}=e$$

Detyra 109: $\lim\limits_{x\to\infty}\left(1-\frac{3}{x}\right)^x$

Zgjidhje:

$$\lim_{x\to\infty}\left(1-\frac{3}{x}\right)^x=\lim_{x\to\infty}\left(1-\frac{1}{\frac{x}{3}}\right)^{\frac{x}{3}\cdot 3}=\left(\lim_{x\to\infty}\left(1-\frac{1}{\frac{x}{3}}\right)^{\frac{x}{3}}\right)^3=\left(e^{-1}\right)^3=e^{-3}$$

Detyra 110: $\lim\limits_{x\to\infty}\left(1+\frac{1}{3x}\right)^x$

Zgjidhje:

$$\lim_{x\to\infty}\left(1+\frac{1}{3x}\right)^x=\left|\begin{matrix}3x=t, & x=\frac{t}{3}\\ x\to\infty, & t\to\infty\end{matrix}\right|=\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{\frac{t}{3}}=\lim_{t\to\infty}\left[\left(1+\frac{1}{t}\right)^t\right]^{\frac{1}{3}}=\left[\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^t\right]^{\frac{1}{3}}=e^{\frac{1}{3}}=\sqrt[3]{e}$$

Detyra 111: $\lim\limits_{x\to 0}(1+3x)^{\frac{1}{x}}$

Zgjidhje:

$$\lim_{x\to 0}(1+3x)^{\frac{1}{x}}=\left|\begin{matrix}\dfrac{1}{x}=t, & x=\dfrac{1}{t}\\ x\to 0, & t\to\infty\end{matrix}\right|=\lim_{t\to\infty}\left[1+\frac{3}{t}\right]^{t}=e^{3}$$

Detyra 112: $\lim\limits_{x\to 1}\dfrac{\ln(2-x)}{1-x}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\ln(2-x)}{1-x}=\lim_{x\to 1}\ln\left[1+(1-x)\right]^{\frac{1}{1-x}}=\ln e=1$$

Detyra 113: $\lim\limits_{x\to\infty}\left(1+\dfrac{a}{x}\right)^{x}$

Zgjidhje:

$$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{x}=\lim_{x\to\infty}\left(1+\frac{1}{\frac{x}{a}}\right)^{\frac{x}{a}\cdot a}=\left(\lim_{x\to\infty}\left(1+\frac{1}{\frac{x}{a}}\right)^{\frac{x}{a}}\right)^{a}=(e)^{a}=e^{a}$$

Detyra 114: $\lim\limits_{x\to 0}\dfrac{a^{x}-b^{x}}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{a^{x}-b^{x}}{x}=\lim_{x\to 0}\frac{a^{x}-1-b^{x}+1}{x}=\lim_{x\to 0}\frac{\left(a^{x}-1\right)-\left(b^{x}-1\right)}{x}=\lim_{x\to 0}\frac{a^{x}-1}{x}-\lim_{x\to 0}\frac{b^{x}-1}{x}=\ln a-\ln b=\ln\frac{a}{b}$$

Detyra 115: $\lim\limits_{x\to 0}\dfrac{e^{ax}-e^{bx}}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{x}=\lim_{x\to 0}\frac{e^{ax}-1+1-e^{bx}}{x}=\lim_{x\to 0}\frac{e^{ax}-1-\left(e^{bx}-1\right)}{x}=\lim_{x\to 0}\frac{e^{ax}-1}{x}-\lim_{x\to 0}\frac{e^{bx}-1}{x}=$$

$$=\lim_{x\to 0}\frac{a\left(e^{ax}-1\right)}{ax}-\lim_{x\to 0}\frac{b\left(e^{ax}-1\right)}{bx}=a\cdot\lim_{x\to 0}\frac{\left(e^{ax}-1\right)}{ax}-b\cdot\lim_{x\to 0}\frac{\left(e^{ax}-1\right)}{bx}=a\cdot 1-b\cdot 1=a-b$$

Detyra 116: $\lim\limits_{x\to a}\dfrac{a^x-x^a}{x-a}$

Zgjidhje:

$$\lim_{x\to a}\frac{a^x-x^a}{x-a}-\frac{a^x-x^a}{x-a}=\lim_{x\to a}\frac{a^{x-a}-1}{x-a}-\lim_{x\to a}a^{a-1}\,\frac{\left(1+\dfrac{x-a}{a}\right)^a}{\dfrac{x-a}{a}}=a^n\ln a-a^{a-1}\cdot a=a^a\ln\frac{a}{e}$$

Detyra 117: $\lim\limits_{x\to 0}\dfrac{a^x-1}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{a^x-1}{x}=\left|\begin{array}{l}\text{Zëvendësojmë}:\\ a^x-1=t\\ a^x=t+1\,\big|\ln\\ \ln a^x=\ln(t+1)\\ x\ln a=\ln(t+1)\\ x=\dfrac{\ln(1+t)}{\ln a}\\ x\to 0;\Rightarrow a^0-1=0\Rightarrow t\to 0\end{array}\right|=\lim_{t\to 0}\frac{t}{\dfrac{\ln(1+t)}{\ln a}}=\lim_{t\to 0}\frac{\ln a}{\dfrac{\ln(t+1)}{t}}=$$

$$=\lim_{t\to 0}\frac{\ln a}{\dfrac{1}{t}\ln(t+1)}=\lim_{t\to 0}\frac{\ln a}{\ln(t+1)^{\frac{1}{t}}}=\frac{\ln a}{\ln\lim\limits_{t\to 0}(t+1)^{\frac{1}{t}}}=\frac{\ln a}{\ln e}=\frac{\ln a}{1}=\ln a$$

Detyra 118: $\lim\limits_{x\to 0}\dfrac{a^{2x}-1}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{a^{2x}-1}{x}=\left|\begin{array}{l}\text{Zëvendësojmë}:\\ a^{2x-1}=t\,\big|\ln\\ x=\dfrac{\ln(1+t)}{2\ln a}\\ x\to 0,\quad t\to 0\end{array}\right|=\lim_{t\to 0}\frac{t}{\dfrac{\ln(1+t)}{2\ln a}}=\lim_{t\to 0}\frac{t\cdot 2\ln a}{(1+t)}=\frac{2\ln a}{\ln e}=\frac{2\ln a}{1}=2\ln a$$

Detyra 119: $\lim\limits_{x\to 0}\dfrac{2^x-1}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{2^x-1}{x}=\left|\begin{array}{l}\textit{Zëvendësojmë}:\\ 2^x-1=t\\ 2^x=t+1\,\big|\ln\\ \ln 2^x=\ln(t+1)\\ x\cdot\ln 2=\ln(t+1)\\ x=\dfrac{\ln(t+1)}{\ln 2}\\ x\to 0, t\to 0\end{array}\right|=\lim_{t\to 0}\frac{t}{\dfrac{\ln(1+t)}{\ln 2}}=\lim_{t\to 0}\frac{t\cdot\ln 2}{\ln(1+t)}=\ln 2\cdot\lim_{t\to 0}\frac{1}{\dfrac{\ln(1+t)}{t}}$$

$$=\ln 2\cdot\frac{1}{\lim\limits_{t\to 0}\ln(1+t)^{\frac{1}{t}}}=\ln 2\cdot\frac{1}{\ln\lim\limits_{t\to 0}(1+t)^{\frac{1}{t}}}=\ln 2\cdot\frac{1}{\ln e}=\ln$$

Detyra 120: $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{e^x-1}{x}=\left|\begin{array}{l}\textit{Zëvendësojmë}:\\ e^x-1=t\\ e^x=1+t\,\big|\ln\\ x\ln e=\ln(1+t)\\ x=\dfrac{\ln(1+t)}{\ln e}\\ x\to 0, t\to 0\end{array}\right|=\lim_{t\to 0}\frac{t}{\dfrac{\ln(1+t)}{\ln e}}=\lim_{t\to 0}\frac{t\cdot\ln e}{\ln(1+t)}=\ln e=1$$

Detyra 121: $\lim\limits_{x\to\infty}\dfrac{3^{x+1}+5^{x+1}}{3^x+5^x}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{3^{x+1}+5^{x+1}}{3^x+5^x}=\lim_{x\to\infty}\frac{3^x\cdot 3+5^x\cdot 5}{3^x+5^x}=\lim_{x\to\infty}\frac{5^x\left(\dfrac{3^x\cdot 3}{5^x}+5\right)}{5^x\left(\dfrac{3^x}{5^x}+1\right)}=\lim_{x\to\infty}\frac{\left(\dfrac{3}{5}\right)^x\cdot 3+5}{\left(\dfrac{3}{5}\right)^x+1}=\frac{\left(\dfrac{3}{5}\right)^\infty\cdot 3+5}{\left(\dfrac{3}{5}\right)^\infty+1}=\frac{5}{1}=5$$

Detyra 122: $\lim\limits_{x\to\infty}\frac{2^{x+1}+3^{x+1}}{2^x-3^x}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{2^{x+1}+3^{x+1}}{2^x-3^x}=\lim_{x\to\infty}\frac{2^x\cdot 2+3^x\cdot 3}{2^x-3^x}=\lim_{x\to\infty}\frac{3^x\left(\frac{2^x\cdot 2}{3^x}+3\right)}{3^x\left(\frac{2^x}{3^x}-1\right)}=\lim_{x\to\infty}\frac{\left(\frac{2}{3}\right)^x\cdot 2+3}{\left(\frac{2}{3}\right)^x-1}=\frac{\left(\frac{2}{3}\right)^\infty\cdot 2+3}{\left(\frac{2}{3}\right)^\infty-1}=\frac{3}{-1}=-3$$

Detyra 123: $\lim\limits_{x\to\infty}\frac{1-5^{x+2}}{1-5^x}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{1-5^{x+2}}{1-5^x}=\lim_{x\to\infty}\frac{1-5^x\cdot 5^2}{1-5^x}=\lim_{x\to\infty}\frac{5^x\left(\frac{1}{5^x}-25\right)}{5^x\left(\frac{1}{5^x}-1\right)}=\lim_{x\to\infty}\frac{\frac{1}{5^x}-25}{\frac{1}{5^x}-1}=\lim_{x\to\infty}\frac{\left(\frac{1}{5}\right)^x-25}{\left(\frac{1}{5}\right)^x-1}=\frac{\left(\frac{1}{5}\right)^\infty-25}{\left(\frac{1}{5}\right)^\infty-1}=\frac{-25}{-1}=25$$

Detyra 124: $\lim\limits_{x\to\infty}\frac{2^x-1}{2^x+1}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{2^x-1}{2^x+1}=\lim_{x\to\infty}\frac{2^x\left(1-\frac{1}{2^x}\right)}{2^x\left(1+\frac{1}{2^x}\right)}=\lim_{x\to\infty}\frac{1-\left(\frac{1}{2}\right)^x}{1+\left(\frac{1}{2}\right)^x}=\frac{1-\left(\frac{1}{2}\right)^\infty}{1+\left(\frac{1}{2}\right)^\infty}=\frac{1}{1}=1$$

Detyra 125: $\lim\limits_{x\to\infty}\frac{7^x+1}{7^x-1}$

Zgjidhje: $\lim\limits_{x\to\infty}\frac{7^x+1}{7^x-1}=\lim\limits_{x\to\infty}\frac{7^x\left(1+\frac{1}{7^x}\right)}{7^x\left(1-\frac{1}{7^x}\right)}=\lim\limits_{x\to\infty}\frac{1+\left(\frac{1}{7}\right)^x}{1-\left(\frac{1}{7}\right)^x}=\frac{1+\left(\frac{1}{7}\right)^\infty}{1-\left(\frac{1}{7}\right)^\infty}=\frac{1}{1}=1$

Detyra 126: $\lim\limits_{x\to\infty}\frac{11^x-1}{11^x+1}$

Zgjidhje: $\lim\limits_{x\to\infty}\frac{11^x-1}{11^x+1}=\lim\limits_{x\to\infty}\frac{11^x\left(1-\frac{1}{11^x}\right)}{11^x\left(1+\frac{1}{11^x}\right)}=\lim\limits_{x\to\infty}\frac{1-\left(\frac{1}{11}\right)^x}{1+\left(\frac{1}{11}\right)^x}=\frac{1-\left(\frac{1}{11}\right)^\infty}{1+\left(\frac{1}{11}\right)^\infty}=\frac{1}{1}=1$

Detyra 127: $\lim\limits_{x\to 0} x\cot x$

Zgjidhje:

$$\lim_{x\to 0} x\cot x = \lim_{x\to 0}\frac{x}{\sin x}\cdot\cos x = \lim_{x\to 0}\frac{\cos x}{\frac{\sin x}{x}} = \frac{\lim\limits_{x\to 0}\cos x}{\lim\limits_{x\to 0}\frac{\sin x}{x}} = \frac{1}{1} = 1$$

Mund të veprojmë edhe kështu:

$$\lim_{x\to 0} x\cot x = \lim_{x\to 0}\frac{x}{\tan x} = \lim_{x\to 0}\frac{1}{\frac{\tan x}{x}} = \frac{\lim\limits_{x\to 0}1}{\lim\limits_{x\to 0}\frac{\tan x}{x}} = \frac{1}{1} =$$

Detyra 128: $\lim\limits_{x\to 0}\frac{1-\cos^2 x}{x\sin x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{1-\cos^2 x}{x\sin x} = \lim_{x\to 0}\frac{\sin^2 x}{x\sin x} = \lim_{x\to 0}\frac{\sin x}{x} = 1$$

Detyra 129: $\lim\limits_{x\to 0}\frac{1-\cos x}{\sin^2 x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{1-\cos x}{\sin^2 x} = \lim_{x\to 0}\frac{1-\cos x}{-\cos^2 x+1} = \lim_{x\to 0}\frac{1-\cos x}{(1-\cos x)(1+\cos x)} = \lim_{x\to 0}\frac{1}{1+\cos x} = \frac{1}{1+1} = \frac{1}{2}$$

Detyra 130: $\lim\limits_{x\to 0}\frac{\tan x}{\tan 3x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\tan x}{\tan 3x} = \lim_{x\to 0}\frac{\frac{\tan x}{x}}{\frac{\tan 3x}{x}} = \lim_{x\to 0}\frac{\frac{\tan x}{x}}{3\frac{\tan x}{x}} = \frac{1}{3\cdot 1} = \frac{1}{3}$$

Detyra 131: $\lim\limits_{x\to 0}\frac{\sin 3x}{\sin 4x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin 3x}{\sin 4x} = \lim_{x\to 0}\frac{\frac{\sin 3x}{3x}}{\frac{\sin 4x}{4x}} = \frac{3\lim\limits_{x\to 0}\frac{\sin 3x}{3x}}{4\lim\limits_{x\to 0}\frac{\sin 4x}{4x}} = \frac{3\cdot 1}{4\cdot 1} = \frac{3}{4}$$

Detyra 132: $\lim\limits_{x\to 0}\dfrac{\sin 10x}{\tan 5x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin 10x}{\tan 5x}=\lim_{x\to 0}\frac{\sin(2\cdot 5)x}{\dfrac{\sin 5x}{\cos 5x}}=\lim_{x\to 0}\frac{2\sin 5x\cdot\cos 5x}{\dfrac{\sin 5x}{\cos 5x}}=\lim_{x\to 0}\frac{2\sin 5x\cdot\cos^2 5x}{\sin 5x}=\lim_{x\to 0}2\cdot\cos^2 5x=2\cdot 1=2$$

Detyra 133: $\lim\limits_{x\to 0}\dfrac{\sin 4x}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin 4x}{x}=\lim_{x\to 0}\frac{\sin 4x|\cdot 4}{x|\cdot 4}=\lim_{x\to 0}\frac{4\cdot\sin 4x}{4x}=4\lim_{x\to 0}\frac{\sin 4x}{4x}=4\cdot 1=4$$

Detyra 134: $\lim\limits_{x\to 0}\dfrac{\sin kx}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin kx}{x}=\lim_{x\to 0}\frac{\sin 4x|\cdot k}{x|\cdot k}=\lim_{x\to 0}\frac{k\cdot\sin kx}{kx}=k\lim_{x\to 0}\frac{\sin kx}{kx}=k\cdot 1=k$$

Detyra 135: $\lim\limits_{x\to 0}\dfrac{\sin 5x}{\sin 7x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin 5x}{\sin 7x}=\lim_{x\to 0}\frac{\dfrac{\sin 5x}{x}}{\dfrac{\sin 7x}{x}}=\lim_{x\to 0}\frac{5\cdot\dfrac{\sin 5x}{x}}{7\cdot\dfrac{\sin 7x}{x}}=\frac{5\cdot 1}{7\cdot 1}=\frac{5}{7}$$

Detyra 136: $\lim\limits_{x\to 0}\dfrac{\tan x}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\tan x}{x}=\lim_{x\to 0}\frac{\dfrac{\sin x}{\cos x}}{x}=\lim_{x\to 0}\frac{\sin x}{x\cdot\cos x}=\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{1}{\cos x}=1\cdot 1=1$$

Detyra 137: $\lim\limits_{x\to 0}\dfrac{\sin\left(\dfrac{x}{x}\right)}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin\left(\frac{x}{x}\right)}{x}=\lim_{x\to 0}\frac{\sin\left(\frac{x}{x}\right)}{3\cdot\frac{x}{3}}=\frac{1}{3}\lim_{x\to 0}\frac{\sin\left(\frac{x}{x}\right)}{\frac{x}{3}}=\frac{1}{3}\cdot 1=\frac{1}{3}$$

Detyra 138: $\lim\limits_{x\to 0}\dfrac{\sin^2\dfrac{x}{2}}{x^2}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{x^2}=\left(\lim_{x\to 0}\frac{\sin\frac{x}{2}}{2\cdot\frac{x}{2}}\right)^2=\left(\frac{1}{2}\right)^2\cdot\underbrace{\left(\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2}_{1}=\frac{1}{4}\cdot 1=\frac{1}{4}$$

Mënyra e dytë

$$\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{x^2}=\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{\frac{x^2}{4}}\cdot\frac{1}{4}=\underbrace{\left(\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2}_{1}\cdot\frac{1}{4}=1\cdot\frac{1}{4}=\frac{1}{4}$$

Detyra 139: $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{1-\cos x}{x}\cdot\frac{1+\cos x}{1+\cos x}=\lim_{x\to 0}\frac{1-\cos^2 x}{x(1+\cos x)}=\frac{1}{2}\cdot\lim_{x\to 0}\frac{\sin^2 x}{x}=\frac{1}{2}\cdot\lim_{x\to 0}\frac{\sin x}{x}\cdot\sin x$$

$$=\frac{1}{2}\cdot\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\sin x=\frac{1}{2}\cdot 1\cdot 0=0$$

Detyra 140: $\lim\limits_{x\to 0}\dfrac{e^x-1}{\sin x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{e^x-1}{\sin x}=\lim_{x\to 0}\frac{\frac{e^x-1}{x}}{\frac{\sin x}{x}}=\frac{\lim\limits_{x\to 0}\frac{e^x-1}{x}}{\lim\limits_{x\to 0}\frac{\sin x}{x}}=\ln e=1$$

Detyra 141: $\lim\limits_{x\to 2}\dfrac{\sin(x-2)}{x-2}$

Zgjidhje:

$$\lim_{x\to 2}\frac{\sin(x-2)}{x-2}=\left|\begin{array}{l}\textit{Zëvendësojmë}\\ x-2=t\\ x\to 2\Rightarrow t\to 0\end{array}\right|=\lim_{t\to 0}\frac{\sin t}{t}=1$$

Detyra 142: $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\tan x}{\cot\left(x+\frac{\pi}{2}\right)}$

Zgjidhje:

$$\lim_{x\to\frac{\pi}{2}}\frac{\tan x}{\cot\left(x+\frac{\pi}{2}\right)}=\left|\begin{array}{l}\textit{Zëvendësojmë}\\ \cot\left(x+\frac{\pi}{2}\right)=-\tan x\end{array}\right|=\lim_{x\to\frac{\pi}{2}}\frac{\tan x}{-\tan x}=\lim_{x\to\frac{\pi}{2}}(-1)=-1$$

Detyra 143: $\lim\limits_{x\to 0}\dfrac{\sin\frac{x}{2}}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin\frac{x}{2}}{x}=\left|\begin{array}{l}\textit{Zëvendësojmë}:\\ \frac{x}{2}=t\Rightarrow x=2t\\ x\to 0,\quad t\to 0\end{array}\right|=\lim_{t\to 0}\frac{\sin t}{2t}=\frac{1}{2}\lim_{t\to 0}\frac{\sin t}{t}=\frac{1}{2}$$

Detyra 144: $\lim\limits_{x\to 0}\dfrac{\sin nx}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin nx}{x}=\lim_{x\to 0}\frac{\sin nx|\cdot n}{x|\cdot n}=\lim_{x\to 0}\frac{n\cdot\sin nx}{nx}=n\lim_{x\to 0}\frac{\sin nx}{nx}=n\cdot 1=n$$

Detyra 145: $\lim\limits_{x\to 0}\frac{\tan 2x}{\sin 5x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\tan 2x}{\sin 5x}=\lim_{x\to 0}\frac{\frac{\tan 2x}{x}}{\frac{\sin 5x}{x}\cdot\frac{5}{5}}=\lim_{x\to 0}\frac{\frac{\tan 2x}{x}}{5\cdot\lim\limits_{x\to 0}\frac{\sin 5x}{5x}}=\frac{1}{5}\cdot\lim_{x\to 0}\frac{\tan 2x}{x}=\frac{1}{5}\cdot\lim_{x\to 0}\frac{\frac{\sin 2x}{\cos 2x}}{x}=$$

$$=\frac{1}{5}\cdot\lim_{x\to 0}\frac{\sin 2x}{x\cdot\cos 2x}=\frac{1}{5}\cdot\lim_{x\to 0}\frac{\sin 2x}{x}\cdot\frac{2}{2}\cdot\lim_{x\to 0}\frac{1}{\cos 2x}=\frac{2}{5}\cdot\lim_{x\to 0}\frac{\sin 2x}{2x}\cdot\frac{1}{1}=\frac{2}{5}$$

Detyra 146: $\lim\limits_{x\to 0}\frac{\sin 4x-\sin 2x}{2x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin 4x-\sin 2x}{2x}=\lim_{x\to 0}\frac{2\sin x\cdot\cos 3x}{2x}=\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\cos 3x=1\cdot\cos 0=1\cdot 1=1$$

Detyra 147: $\lim\limits_{x\to 0}\frac{\tan -x}{x-\sin x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\tan x-x}{x-\sin x}=\lim_{x\to 0}\frac{\sin^2 -1}{1-\cos x}=\lim_{x\to 0}\frac{\tan^2 x}{1-\cos x}=\lim_{x\to 0}\frac{\sin^2 x\left(1+\cos x\right)}{\cos^2 x\left(1-\cos x\right)\left(1+\cos x\right)}=$$

$$=\lim_{x\to 0}\frac{\sin^2 x\left(1+\cos x\right)}{\cos^2 x-\sin^2 x}=\frac{1+1}{1}=2$$

Detyra 148: $\lim\limits_{x\to 0}\frac{\tan x-\sin x}{x^3}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}=\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}-\sin x}{x^3}=\lim_{x\to 0}\frac{\frac{\sin x-\sin x\cos x}{\cos x}}{x^3}=\lim_{x\to 0}\frac{1-\cos x}{x^2}\cdot\frac{\sin x}{x}\cdot\frac{1}{\cos x}=$$

$$=\frac{1}{2}\cdot 1\cdot 1=\frac{1}{2}$$

Detyra 149: $\lim\limits_{x\to 0}\frac{1-\cos 2x}{1-\cos x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{1-\cos 2x}{1-\cos x}=\lim_{x\to 0}\left(\frac{1-\cos 2x}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x}\right)=\lim_{x\to 0}\frac{(1-\cos 2x)(1+\cos x)}{1-\cos^2 x}=$$

$$=\lim_{x\to 0}\frac{2\sin^2 x(1+\cos x)}{\sin^2 x}=\lim_{x\to 0}2(1+\cos x)=2(1+1)=2\cdot 2=4$$

Detyra 150: $\lim\limits_{x\to 0}\frac{1-\cos^3 x}{x\sin x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin x}=\lim_{x\to 0}\frac{1^3-\cos^3 x}{x\sin x}=\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x+\cos^2 x)}{x\sin x}=$$

$$=\lim_{x\to 0}(1+\cos x+\cos^2 x)\cdot\lim_{x\to 0}\frac{1-\cos x}{x\sin x}\cdot\frac{1+\cos x}{1+\cos x}=3\lim_{x\to 0}\frac{\sin^2 x}{x\sin x}\cdot\lim_{x\to 0}\frac{1}{1+\cos x}=\frac{3}{2}\lim_{x\to 0}\frac{\sin x}{x}=\frac{3}{2}$$

Detyra 151: $\lim\limits_{x\to 0}\frac{2\sin x+3x\cdot\cos x}{3\sin x-2x\cdot\cos x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{2\sin x+3x\cdot\cos x}{3\sin x-2x\cdot\cos x}=\lim_{x\to 0}\frac{\frac{2\sin x}{x}+\frac{3x\cdot\cos x}{x}}{\frac{3\sin x}{x}-\frac{2x\cdot\cos x}{x}}=\frac{2\lim\limits_{x\to 0}\frac{\sin x}{x}+3\lim\limits_{x\to 0}\frac{x\cdot\cos x}{x}}{3\lim\limits_{x\to 0}\frac{\sin x}{x}-2\lim\limits_{x\to 0}\frac{x\cdot\cos x}{x}}=$$

$$=\frac{2\lim\limits_{x\to 0}\frac{\sin x}{x}+3\lim\limits_{x\to 0}\cos x}{3\lim\limits_{x\to 0}\frac{\sin x}{x}-2\lim\limits_{x\to 0}\cos x}=\frac{2\cdot 1+3\cdot 1}{3\cdot 1-2\cdot 1}=\frac{5}{1}=5$$

Detyra 152: $\lim\limits_{x\to 0}\frac{\sqrt{1-\cos 2x}}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{x}=\lim_{x\to 0}\frac{\sqrt{\sin^2 x+\cos^2 x-\cos^2 x+\sin^2 x}}{x}=\lim_{x\to 0}\frac{\sqrt{2\sin^2 x}}{x}=$$

$$=\sqrt{2}\cdot\lim_{x\to 0}\frac{\sin x}{x}=\sqrt{2}\cdot 1=\sqrt{2}$$

Detyra 153: $\lim\limits_{x\to 0}\dfrac{\sin x\cdot\tan x}{\sin^2 x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin x\cdot\tan x}{\sin^2 x}=\lim_{x\to 0}\frac{\sin x\cdot\frac{\sin x}{\cos x}}{\sin^2 x}=\lim_{x\to 0}\frac{\sin^2 x}{\sin^2 x}\cdot\frac{1}{\cos x}=\lim_{x\to 0}\frac{\left(\frac{\sin x}{x}\right)^2}{\frac{\sin^2 x}{x}}\cdot 1=\frac{1^2}{1}\cdot 1=1\cdot 1=1$$

Detyra 154: $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{1-\sin x}{\cos^2 x}$

Zgjidhje:

$$\lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{\cos^2 x}=\lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{(1-\sin x)(1+\sin x)}=\lim_{x\to\frac{\pi}{2}}\frac{1}{(1+\sin x)}=\frac{1}{1+\sin\frac{\pi}{2}}=\frac{1}{1+1}=\frac{1}{2}$$

Detyra 155: $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cos x-\sin x}{\cos 2x}$

Zgjidhje:

$$\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{\cos 2x}=\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{\cos^2 x-\sin^2 x}=\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{(\cos x-\sin x)(\cos x+\sin x)}=\lim_{x\to\frac{\pi}{4}}\frac{1}{\cos x+\sin x}=$$

$$=\frac{1}{\cos\frac{\pi}{4}+\sin\frac{\pi}{4}}=\frac{1}{\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

Detyra 156: $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{3\sin x\cos x}{\pi-2x}$

Zgjidhje:

$$\lim_{x\to\frac{\pi}{2}}\frac{3\sin x\cos x}{\pi-2x}=\lim_{x\to\frac{\pi}{2}}\frac{\frac{3}{2}(\sin 2x)}{\pi-2x}=\frac{3}{2}\lim_{x\to\frac{\pi}{2}}\frac{\sin 2x}{\pi-2x}=\frac{3}{2}\cdot 1=\frac{3}{2}$$

Detyra 157: $\lim\limits_{x\to 1}\dfrac{\tan\left(x^2-1\right)}{\left(x^2-1\right)}$

Zgjidhje: $$\lim_{x\to 1}\frac{\tan\left(x^2-1\right)}{\left(x^2-1\right)}=\left|\begin{array}{l}\textit{Zëvendësojmë}:\\ x^2-1=t\\ x\to 1\Rightarrow t\to 0\end{array}\right|=\lim_{t\to 0}\frac{\tan t}{t}=1$$

Detyra 158: $\lim\limits_{x\to 0}\frac{x-\sin 3x}{x+\sin 2x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{x-\sin 3x}{x+\sin 2x}=\lim_{x\to 0}\frac{x\left(1-\frac{\sin 3x}{x}\right)}{x\left(1+\frac{\sin 2x}{x}\right)}=\lim_{x\to 0}\frac{\left(1-\frac{\sin 3x}{x}\right)}{\left(1+\frac{\sin 2x}{x}\right)}=\frac{\lim\limits_{x\to 0}\left(1-\frac{\sin 3x}{x}\right)}{\lim\limits_{x\to 0}\left(1+\frac{\sin 2x}{x}\right)}=$$

$$=\frac{\lim\limits_{x\to 0}\left(1-\frac{3\sin 3x}{3x}\right)}{\lim\limits_{x\to 0}\left(1+\frac{2\sin 2x}{2x}\right)}=\frac{1-3}{1+3}=-\frac{2}{3}$$

Detyra 159: $\lim\limits_{x\to 1}\frac{\tan \pi x}{1-x}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\tan \pi x}{1-x}=\lim_{x\to 1}\frac{\frac{\sin \pi x}{\cos \pi x}}{1-x}=-1\lim_{x\to 1}\frac{\sin \pi x}{1-x}=-1\lim_{x\to 1}\frac{\sin\left(\pi-\pi x\right)}{1-x}=-\lim_{x\to 1}\frac{\frac{\sin\left(1-x\right)}{\pi}}{\frac{1-x}{\pi}}=$$

$$=-\lim_{x\to 1}\frac{\sin \pi\left(1-x\right)\pi}{\pi\left(1-x\right)}=-\pi$$

Detyra 160: $\lim\limits_{x\to 0}\frac{1-\cos 2x}{\cos 7x-\cos 3x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{1-\cos 2x}{\cos 7x-\cos 3x}=\lim_{x\to 0}\frac{2\sin^2 x}{-2\sin\frac{7x+3x}{2}\cdot\sin\frac{7x-3x}{2}}=-\lim_{x\to 0}\frac{\sin^2 x}{\sin 5x\cdot\sin 2x}=-\lim_{x\to 0}\frac{\sin x}{\sin 5x}\cdot\lim_{x\to 0}\frac{\sin x}{\sin 2x}=$$

$$=-\lim_{x\to 0}\frac{\frac{\sin x}{x}}{5\frac{\sin 5x}{x}}\cdot\lim_{x\to 0}\frac{\frac{\sin x}{x}}{2\frac{\sin 2x}{x}}=-\frac{1}{5}\cdot\frac{1}{2}=-\frac{1}{10}$$

Detyra 161: $\lim\limits_{x\to 0}\dfrac{1-\cos(1-\cos x)}{x^4}$

Zgjidhje:

$$\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}\cdot\frac{1+\cos(1-\cos x)}{1+\cos(1-\cos x)}=\lim_{x\to 0}\frac{1}{1+\cos(1-\cos x)}\cdot\lim_{x\to 0}\frac{\sin^2(1-\cos x)}{(1-\cos x)^2}\cdot\frac{(1-\cos x)^2}{x^4}=$$

$$=\frac{1}{2}\cdot 1\cdot\lim_{x\to 0}\frac{1}{(1-\cos x)^2}\cdot\lim_{x\to 0}\frac{\sin^4 x}{x^4}=\frac{1}{2}\cdot\frac{1}{4}=\frac{1}{8}$$

Detyra 162: $\lim\limits_{x\to\frac{\pi}{2}}\left(\dfrac{\sin x}{\cos^2 x}-\tan^2 x\right)$

Zgjidhje:

$$\lim_{x\to\frac{\pi}{2}}\left(\frac{\sin x}{\cos^2 x}-\tan^2 x\right)=\lim_{x\to\frac{\pi}{2}}\left(\frac{\sin x}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}\right)=\lim_{x\to\frac{\pi}{2}}\left(\frac{\sin x-\sin^2 x}{\cos^2 x}\right)=\lim_{x\to\frac{\pi}{2}}\left(\frac{\sin x(1-\sin x)}{\cos^2 x}\right)=$$

$$=\lim_{x\to\frac{\pi}{2}}\left(\frac{\sin x(1-\sin x)}{1-\sin^2 x}\right)=\lim_{x\to\frac{\pi}{2}}\left(\frac{\sin x(1-\sin x)}{(1-\sin x)(1+\sin x)}\right)=\lim_{x\to\frac{\pi}{2}}\frac{\sin x}{1+\sin x}=\frac{1}{2}$$

Detyra 163: $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\tan(4x-\pi)}{2x-\frac{\pi}{2}}$

Zgjidhje:

$$\lim_{x\to\frac{\pi}{4}}\frac{\tan(4x-\pi)}{2x-\frac{\pi}{2}}=\left|\begin{array}{l}\textit{Zëvendësojmë}:\\ 2x-\dfrac{\pi}{2}=t\\ x=\dfrac{t}{2}+\dfrac{\pi}{4}\\ x\to\dfrac{\pi}{4},t\to 0\end{array}\right|=\lim_{t\to 0}\frac{\tan\left[4\left(\frac{t}{2}+\frac{\pi}{4}\right)-\pi\right]}{t}=\lim_{t\to 0}\frac{\tan 2t}{t}=$$

$$=2\cdot\lim_{t\to 0}\frac{\tan 2t}{t}=2\cdot 1=2$$

Detyra 164: $\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{x-a}$

Zgjidhje:

$$\lim_{x\to a}\frac{\sin x-\sin a}{x-a}=\lim_{x\to a}\frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{x-a}=\lim_{x\to a}\frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}}\cdot\cos\frac{x+a}{2}=\lim_{x\to a}\frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}}\cdot\lim_{x\to a}\cos\frac{x+a}{2}=\cos a$$

Detyra 165: $\lim\limits_{x\to 2}\dfrac{\sin\left(\sqrt{x}-\sqrt{2}\right)}{x-2}$

Zgjidhje:

$$\lim_{x\to 2}\frac{\sin\left(\sqrt{x}-\sqrt{2}\right)}{x-2}=\lim_{x\to 2}\frac{\sin\left(\sqrt{x}-\sqrt{2}\right)}{\left(\sqrt{x}\right)^2-\left(\sqrt{2}\right)^2}=\lim_{x\to 2}\frac{\sin\left(\sqrt{x}-\sqrt{2}\right)}{\left(\sqrt{x}-\sqrt{2}\right)\left(\sqrt{x}+\sqrt{2}\right)}=\lim_{x\to 2}\frac{\sin\left(\sqrt{x}-\sqrt{2}\right)}{\left(\sqrt{x}-\sqrt{2}\right)}\cdot\frac{1}{\left(\sqrt{x}+\sqrt{2}\right)}=$$

$$=1\cdot\frac{1}{\sqrt{2}+\sqrt{2}}=\frac{1}{\sqrt{2}+\sqrt{2}}=\frac{1}{2\sqrt{2}}$$

Detyra 166: $\lim\limits_{x\to 0}\dfrac{\sin 4x}{\sqrt{x+1}-1}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin 4x}{\sqrt{x+1}-1}\cdot\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}=\lim_{x\to 0}\frac{\sin 4x\left(\sqrt{x+1}+1\right)}{x+1-1}=\lim_{x\to 0}\frac{\sin 4x\left(\sqrt{x+1}+1\right)}{x}=$$

$$=\lim_{x\to 0}\frac{4\sin 4x}{4x}\cdot\lim_{x\to 0}\left(\sqrt{x+1}+1\right)=4\lim_{x\to 0}\frac{\sin 4x}{4x}\cdot\lim_{x\to 0}\left(\sqrt{x+1}+1\right)=4\cdot 1\cdot\left(\sqrt{0+1}+1\right)=4\cdot 1\cdot 2=8$$

Detyra 167: $\lim\limits_{x\to 0}\dfrac{\sin x(1-\cos x)}{x^3\cdot\cos x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin x(1-\cos x)}{x^3\cdot\cos x}=\lim_{x\to 0}\frac{1}{\cos x}\cdot\frac{\sin x}{x}\cdot\frac{1-\cos x}{x^2}=\lim_{x\to 0}\frac{1}{\cos x}\cdot\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{1-\cos x}{x^2}=$$

$$=\lim_{x\to 0}\frac{1-\cos x}{x^2}\cdot\frac{1+\cos x}{1+\cos x}=\lim_{x\to 0}\frac{1-\cos^2 x}{x^2(1+\cos x)}=\lim_{x\to 0}\frac{\sin^2 x}{x^2}\cdot\lim_{x\to 0}\frac{1}{\cos x}=\frac{1}{2}$$

Detyra 168: $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin x-1}{6x-\pi}$

Zgjidhje:

$$\lim_{x\to\frac{\pi}{6}}\frac{2\sin x-1}{6x-\pi}=\lim_{x\to\frac{\pi}{6}}\frac{2\left(\sin x-\sin\dfrac{\pi}{6}\right)}{6\left(x-\dfrac{\pi}{6}\right)}=\lim_{x\to\frac{\pi}{6}}\frac{1}{3}\frac{2\sin\dfrac{x-\frac{\pi}{6}}{2}\cdot\cos\dfrac{x+\frac{\pi}{6}}{2}}{x-\dfrac{\pi}{6}}=\frac{1}{3}\cdot 2\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{6}$$

Detyra 169: $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin\left(x-\frac{\pi}{3}\right)}{1-2\cos x}$

Zgjidhje:

$$\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(x-\frac{\pi}{3}\right)}{1-2\cos x}=\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(\frac{x}{2}-\frac{\pi}{6}\right)\cdot\cos\left(\frac{x}{2}-\frac{\pi}{6}\right)}{2\left(\frac{1}{2}-\cos x\right)}=\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(\frac{x}{2}-\frac{\pi}{6}\right)\cdot\cos\left(\frac{x}{2}-\frac{\pi}{6}\right)}{\cos\frac{\pi}{3}-\cos x}=$$

$$=\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(\frac{x}{2}-\frac{\pi}{6}\right)\cdot\cos\left(\frac{x}{2}-\frac{\pi}{6}\right)}{-2\sin\left(\frac{x}{2}+\frac{\pi}{6}\right)\cdot\sin\left(\frac{\pi}{6}-\frac{x}{2}\right)}=\frac{\sqrt{3}}{3}$$

Detyra 170: $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$

Zgjidhje:

$$\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}=\lim_{x\to\frac{\pi}{6}}\frac{(2\sin x-1)(\sin x+1)}{(2\sin x-1)(\sin x+1)}=\lim_{x\to\frac{\pi}{6}}\frac{(\sin x+1)}{(\sin x+1)}=\frac{\sin\frac{\pi}{6}+1}{\sin\frac{\pi}{6}-1}=\frac{\frac{1}{2}+1}{\frac{1}{2}-1}=-3$$

Detyra 171: $\lim\limits_{x\to 0}\dfrac{\arcsin 5x}{\tan 2x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\arcsin 5x}{\tan 2x}=\lim_{x\to 0}\cos 2x\cdot\lim_{x\to 0}\frac{\arcsin 5x}{\sin 2x}=\lim_{x\to 0}\frac{\frac{\arcsin 5x}{x}}{\frac{\sin 2x}{x}}=\frac{1}{2}\lim_{x\to 0}\frac{\arcsin 5x}{x}=\begin{vmatrix}\arcsin x=t\\ 5x=\sin t\\ x\to 0, t\to 0\end{vmatrix}=$$

$$=\frac{1}{2}\lim_{t\to 0}\frac{1}{\frac{\sin t}{5}}=\frac{5}{2}\lim_{t\to 0}\frac{t}{\sin t}=\frac{5}{2}$$

Detyra 172: $\lim\limits_{x\to 0}\dfrac{\arctan x}{x}$

Zgjidhje: $\lim\limits_{x\to 0}\dfrac{\arctan x}{x}=\begin{vmatrix}\textit{Zëvendësojmë}:\\ \arctan=u\\ x=\tan u\\ x\to 0\Rightarrow u\to 0\end{vmatrix}=\lim\limits_{u\to 0}\dfrac{u}{\tan u}=1$

Detyra 173: $\lim\limits_{x\to 3}\sin\frac{x-3}{2}\cdot\tan\frac{\pi x}{6}$

Zgjidhje: $\lim\limits_{x\to 3}\sin\frac{x-3}{2}=0$ dhe $\lim\limits_{x\to 3}\tan\frac{\pi x}{6}=\infty$

$$\lim_{x\to 3}\sin\frac{x-3}{2}\cdot\tan\frac{\pi x}{6}=\lim_{x\to 3}\sin\frac{x-3}{2}\cdot\frac{\sin\frac{\pi x}{6}}{\cos\frac{\pi x}{6}}=\lim_{x\to 3}\sin\frac{\pi x}{6}\cdot\lim_{x\to 3}\frac{\sin\frac{x-3}{2}}{\cos\left(\frac{\pi}{2}-\frac{\pi x}{6}\right)}=$$

$$=1\cdot\lim_{x\to 3}\frac{\sin\frac{x-3}{2}}{\sin\left(\frac{\pi}{2}-\frac{\pi x}{6}\right)}\cdot\frac{\frac{x-3}{2}}{\frac{x-3}{2}}\cdot\frac{\frac{\pi(3-x)}{6}}{\frac{\pi(3-x)}{6}}=\lim_{x\to 3}\frac{\sin\frac{x-3}{2}}{\frac{x-3}{2}}\cdot\lim_{x\to 3}\frac{\frac{\pi(3-x)}{6}}{\frac{\pi(3-x)}{6}}\cdot\lim_{x\to 3}\frac{\frac{x-3}{2}}{\frac{\pi(3-x)}{6}}=$$

$$=1\cdot 1\cdot\left(-\frac{\frac{1}{2}}{\frac{\pi}{6}}\right)=1\cdot 1\cdot\left(-\frac{6}{2\pi}\right)=-\frac{3}{\pi}$$

Detyra 174: $\lim\limits_{x\to\infty}2x\cdot\sin\frac{1}{x}$

Zgjidhje:

$$\lim_{x\to\infty}2x\cdot\sin\frac{1}{x}=\left|\begin{array}{l}\textit{Zëvendësojmë}:\\ \frac{1}{x}=t\\ x\to\infty,\quad t\to 0\end{array}\right|=2\lim_{t\to 0}\frac{1}{t}\cdot\sin t=2\lim_{t\to 0}\frac{\sin t}{t}=2\cdot 1=2$$

Detyra 175: $\lim\limits_{x\to 1}(1-x)\cdot\tan\frac{\pi x}{2}$

Zgjidhje:

$$\lim_{x\to 1}(1-x)\cdot\tan\frac{\pi x}{2}=\left|\begin{array}{l}\textit{Zëvendësojmë}:\\ 1-x=t\\ x=1-t\\ x\to 1,\quad t\to 0\end{array}\right|=\lim_{t\to 0}t\cdot\tan\frac{\pi(1-t)}{2}=\lim_{t\to 0}t\cdot\tan\left(\frac{\pi}{2}-\frac{\pi t}{2}\right)=$$

$$=\lim_{t\to 0}t\cdot\cot\frac{\pi t}{2}=\lim_{t\to 0}t\cdot\frac{\cos\frac{\pi}{2}t}{\sin\frac{\pi}{2}t}=\lim_{t\to 0}\frac{t}{\sin\frac{\pi}{2}t}\cdot\lim_{t\to 0}\cos\frac{\pi}{2}t=\lim_{t\to 0}\frac{t}{\frac{\sin\frac{\pi}{2}}{t}}\cdot 1=\frac{1}{\lim\limits_{t\to 0}\left(\frac{\sin\frac{\pi}{2}t}{t}\right)}=\frac{2}{\pi}$$

Detyra 176: $\lim\limits_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-5\sin x+2}$

Zgjidhje:

$$\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-5\sin x+2}=\left|\sin x=t\right|=\lim_{x\to\frac{\pi}{6}}\frac{2t^2+t-1}{2t^2-5t+2}=\lim_{x\to\frac{\pi}{6}}\frac{2\left(t^2+\frac{1}{2}t-\frac{1}{2}\right)}{2\left(t^2-\frac{5}{2}t+1\right)}=$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{(t+1)\left(t-\frac{1}{2}\right)}{(t-2)\left(t-\frac{1}{2}\right)}=\lim_{x\to\frac{\pi}{6}}\frac{t+1}{t-2}=\lim_{x\to\frac{\pi}{6}}\frac{\sin x+1}{\sin x-2}=\frac{\sin\frac{\pi}{6}+1}{\text{in}\frac{\pi}{6}-2}=\frac{\frac{1}{2}-1}{\frac{1}{2}-2}=\frac{\frac{3}{2}}{-\frac{3}{2}}=-1$$

Detyra 177: $\lim\limits_{x\to-2}\frac{\arctan(x+2)}{4-x^2}$

Zgjidhje:

$$\lim_{x\to-2}\frac{\arctan(x+2)}{4-x^2}=\left|\begin{array}{l}\textit{Zëvendësojmë}\\ x+2=t\Rightarrow x=t-2\\ x\to-2,\quad t\to0\end{array}\right|=\lim_{t\to0}\frac{\arctan t}{t(4-t)}=\lim_{t\to0}\frac{\arctan t}{t}\cdot\lim_{t\to0}\frac{1}{4-t}=$$

$$=\frac{1}{4}\lim_{t\to0}\frac{\arctan t}{t}=\left|\begin{array}{l}\textit{Zëvendësojmë}\\ t=\tan u\Rightarrow x=t-2\\ t\to0,\quad u\to0\end{array}\right|=\frac{1}{4}\lim_{u\to0}\frac{u}{\tan u}=\frac{1}{4}\lim_{u\to0}\frac{u}{\sin u}\cos u=\frac{1}{4}$$

Detyra 178: $\lim\limits_{x\to-1}\frac{x^3+1}{\sin(x+1)}$

Zgjidhje:

$$\lim_{x\to-1}\frac{x^3+1}{\sin(x+1)}=\lim_{x\to-1}\frac{(x+1)(x^2-x+1)}{\sin(x+1)}=\lim_{x\to-1}\frac{x+1}{\sin(x+1)}\cdot\left(x^2-x+1\right)=3\lim_{x\to-1}\frac{x+1}{\sin(x+1)}=$$

$$=\left|\begin{array}{l}\textit{Zëvendësojmë}\\ \sin(x+1)=t\\ x+1=\arcsin t\\ ku\ x\to-1\Rightarrow t\to0\end{array}\right|=3\lim_{t\to0}\frac{\arcsin t}{t}=\left|\begin{array}{l}\arcsin t=u\\ t=\sin u\\ t\to0\Rightarrow u\to0\end{array}\right|=3\lim_{u\to0}\frac{u}{\sin u}=3\lim_{u\to0}\frac{\frac{u}{u}}{\frac{\sin u}{u}}=3\cdot1=3$$

Detyra 179: $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$

Zgjidhje:

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}=\frac{1}{2}\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}=\frac{1}{2}\lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2=\left|\begin{array}{l}\textit{Zëvendësojmë}\\ \frac{x}{2}=a\\ x\to 0,\ a\to 0\end{array}\right|=$$

$$=\frac{1}{2}\lim_{a\to 0}\left(\frac{\sin a}{a}\right)^2=\frac{1}{2}$$

Detyra 180: $\lim\limits_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}\cdot\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}=$$

$$=\lim_{x\to 0}\frac{1+\sin x-1+\sin x}{\frac{\sin x}{\cos x}}\cdot\lim_{x\to 0}\frac{1}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}=\lim_{x\to 0}\frac{2\sin x}{\frac{\sin x}{\cos x}}\cdot\frac{1}{\sqrt{1+\sin 0}+\sqrt{1-\sin 0}}=$$

$$\lim_{x\to 0}2\cos x\cdot\frac{1}{\sqrt{1+0}+\sqrt{1+0}}=2\cdot 1\cdot\frac{1}{2}=2\cdot\frac{1}{2}=1$$

Detyra 181: $\lim\limits_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}\cdot\frac{\sqrt{2}+\sqrt{1+\cos x}}{\sqrt{2}+\sqrt{1+\cos x}}=\lim_{x\to 0}\frac{2-(1+\cos x)}{\sin^2 x\left(\sqrt{2}+\sqrt{1+\cos x}\right)}=$$

$$=\lim_{x\to 0}\frac{1-\cos x}{\sin^2 x\left(\sqrt{2}+\sqrt{1+\cos x}\right)}=\lim_{x\to 0}\frac{1-\cos x}{\sin^2 x}\cdot\lim_{x\to 0}\frac{1}{\sqrt{2}+\sqrt{1+\cos x}}=$$

$$\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{\sin^2 x(1+\cos x)}\cdot\lim_{x\to 0}\frac{1}{\sqrt{2}+\sqrt{1+\cos x}}=\frac{1}{\sqrt{2}+\sqrt{2}}\cdot\lim_{x\to 0}\frac{1-\cos^2 x}{\sin^2 x(1+\cos x)}=$$

$$\frac{1}{2\sqrt{2}}\cdot\lim_{x\to 0}\frac{\sin^2 x}{\sin^2 x(1+\cos x)}=\frac{1}{2\sqrt{2}}\cdot\frac{1}{1+1}=\frac{1}{4\sqrt{2}}$$

Detyra 182: $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\tan x}-\sqrt{1-\tan x}}{\sin x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1-\tan x}}{\sin x}\cdot\frac{\sqrt{1+\tan x}+\sqrt{1-\tan x}}{\sqrt{1+\tan x}+\sqrt{1-\tan x}}=\lim_{x\to 0}\frac{1+\tan x-1+\tan x}{\sin x\left(\sqrt{1+\tan x}+\sqrt{1-\tan x}\right)}=$$

$$=\lim_{x\to 0}\frac{2\tan x}{\sin x\left(\sqrt{1+\tan x}+\sqrt{1-\tan x}\right)}=2\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{\sin x}\cdot\lim_{x\to 0}\frac{1}{\sqrt{1+\tan x}+\sqrt{1-\tan x}}=$$

$$=2\lim_{x\to 0}\frac{\sin x}{\sin x\cos x}\cdot\frac{1}{2}=\frac{2}{2}\lim_{x\to 0}\frac{1}{\cos x}=1$$

Detyra 183: $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x\sin x}-\sqrt{\cos 2x}}{\tan^2 x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sqrt{1+x\sin x}-\sqrt{\cos 2x}}{\tan^2 x}\cdot\frac{\sqrt{1+x\sin x}+\sqrt{\cos 2x}}{\underbrace{\sqrt{1+x\sin x}+\sqrt{\cos 2x}}_{A}}=\lim_{x\to 0}\frac{\left(\sqrt{1+x\sin x}\right)^2-\left(\sqrt{\cos 2x}\right)^2}{\tan^2 x(A)}=$$

$$=\lim_{x\to 0}\frac{1+x\sin x-\cos 2x}{\tan^2 x(A)}=\lim_{x\to 0}\frac{\left(2\sin^2 x+x\sin x\right)\cos^2 x}{\sin^2 x(A)}=\lim_{x\to 0}\left(\frac{2\sin^2 x}{\sin^2 x}+\frac{x\sin x}{\sin^2 x}\right)\cdot\frac{\cos^2 x}{(A)}=$$

$$=\lim_{x\to 0}\left(2+\frac{x}{\sin x}\right)\cdot\lim_{x\to 0}\frac{\cos^2 x}{\sqrt{1+x\sin x}+\sqrt{\cos 2x}}=(2+1)\cdot\frac{1}{\sqrt{1}+\sqrt{1}}=3\cdot\frac{1}{2}=\frac{3}{2}$$

Detyra 184: $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos x}$

*Zgjidhje***:**

$$\lim_{x\to\frac{\pi}{2}}\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos x}=\left|\cos x=\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right|=\lim_{x\to\frac{\pi}{2}}\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}=\lim_{x\to\frac{\pi}{2}}\frac{1}{\cos\frac{x}{2}+\sin\frac{x}{2}}=$$

$$=\frac{1}{\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}}=\frac{1}{\sqrt{2}}$$

Detyra 185: $\lim\limits_{x\to 0}\dfrac{1-\sqrt{\cos x}}{1-\cos\left(\sqrt{x}\right)}$

Zgjidhje:

$$\lim_{x\to 0}\frac{1-\sqrt{\cos x}}{1-\cos\left(\sqrt{x}\right)}\cdot\frac{1+\sqrt{\cos x}}{1+\sqrt{\cos x}}=\lim_{x\to 0}\frac{\left(1-\sqrt{\cos x}\right)\cdot\left(1+\sqrt{\cos x}\right)}{\left(1-\cos\sqrt{x}\right)\cdot\left(1+\sqrt{\cos x}\right)}=\lim_{x\to 0}\frac{1-\cos x}{2\sin^2\frac{\sqrt{x}}{2}\cdot\left(1+\sqrt{\cos x}\right)}=$$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{2\sin^2\frac{\sqrt{x}}{2}\cdot\left(1+\sqrt{\cos x}\right)}=\lim_{x\to 0}\frac{\frac{\sin\frac{x}{2}}{\frac{x}{2}}\cdot\frac{\sin\frac{x}{2}}{\frac{x}{2}}\cdot\frac{x^2}{4}}{\frac{\sin\frac{\sqrt{x}}{2}}{\frac{\sqrt{x}}{2}}\cdot\frac{\sin\frac{\sqrt{x}}{2}}{\frac{\sqrt{x}}{2}}\cdot\left(1+\sqrt{\cos x}\right)\cdot\frac{x}{4}}=$$

$$=\lim_{x\to 0}\frac{\frac{\sin\frac{x}{2}}{\frac{x}{2}}\cdot\frac{\sin\frac{x}{2}}{\frac{x}{2}}\cdot x}{\frac{\sin\frac{\sqrt{x}}{2}}{\frac{\sqrt{x}}{2}}\cdot\frac{\sin\frac{\sqrt{x}}{2}}{\frac{\sqrt{x}}{2}}\cdot\left(1+\sqrt{\cos x}\right)\cdot\frac{x}{4}}=\frac{1\cdot 1\cdot 0}{1\cdot 1\cdot 2}=0$$

Detyra 186: $\lim\limits_{x\to 0}\dfrac{1-\cos x\sqrt{\cos 2x}}{x^2}$

Zgjidhje:

$$\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\cdot\frac{1+\cos x\sqrt{\cos 2x}}{1+\cos x\sqrt{\cos 2x}}=\lim_{x\to 0}\frac{1-\cos^2 x\cos 2x}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}=$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{1-\cos^2 x\left(\cos^2 x-\sin^2 x\right)}{x^2}=\frac{1}{2}\lim_{x\to 0}\frac{1-\cos^4 x+\cos^2 x\sin^2 x}{x^2}=$$

$$=\frac{1}{2}\left[\lim_{x\to 0}\frac{1-\cos^4 x}{x^2}+\frac{\cos^2 x\sin^2 x}{x^2}\right]=\frac{1}{2}\left[\lim_{x\to 0}\frac{\sin^4 x}{x^2}+\frac{\cos^2 x\sin^2 x}{x^2}\right]=$$

$$=\frac{1}{2}\left[\lim_{x\to 0}\frac{\sin^4 x}{x^2}\cdot\lim_{x\to 0}\sin^2 x+\lim_{x\to 0}\cos^2 x\cdot\lim_{x\to 0}\frac{\sin^2 x}{x^2}\right]=\frac{1}{2}\left[1\cdot 0+1\cdot 1\right]=\frac{1}{2}+1=\frac{3}{2}$$

Detyra 187: $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sqrt[3]{\tan x}-1}{2\sin^2 x-1}$

Zgjidhje:

$$\lim_{x\to\frac{\pi}{4}}\frac{\sqrt[3]{\tan x}-1}{2\sin^2 x-1}\cdot\frac{\sqrt[3]{\tan^2 x}+\sqrt[3]{\tan x}+1}{\sqrt[3]{\tan^2 x}+\sqrt[3]{\tan x}+1}=\lim_{x\to\frac{\pi}{4}}\frac{\tan x-1}{2\sin^2 x-1\left(\sqrt[3]{\tan^2 x}+\sqrt[3]{\tan x}+1\right)}=$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\frac{\sin x}{\cos x}-1}{2\sin^2 x-1\left(\sqrt[3]{\tan^2 x}+\sqrt[3]{\tan x}+1\right)}=\lim_{x\to\frac{\pi}{4}}\frac{\frac{\sin x-\cos x}{\cos x}}{2\sin^2 x-1\left(\sqrt[3]{\tan^2 x}+\sqrt[3]{\tan x}+1\right)}=$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{\cos x\left(\sin x-\cos x\right)\left(\sin x+\cos x\right)\left(\sqrt[3]{\tan^2 x}+\sqrt[3]{\tan x}+1\right)}=$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{1}{\cos x\left(\sin x+\cos x\right)\left(\sqrt[3]{\tan^2 x}+\sqrt[3]{\tan x}+1\right)}=$$

$$=\frac{\lim\limits_{x\to\frac{\pi}{4}}1}{\lim\limits_{x\to\frac{\pi}{4}}\cos x\left(\sin x+\cos x\right)\left(\sqrt[3]{\tan^2 x}+\sqrt[3]{\tan x}+1\right)}=$$

$$=\frac{1}{\cos\frac{\pi}{4}\left(\sin\frac{\pi}{4}+\cos\frac{\pi}{4}\right)\left(\sqrt[3]{\tan^2\frac{\pi}{4}}+\sqrt[3]{\tan\frac{\pi}{4}}+1\right)}=$$

$$=\frac{1}{\frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)\left(\sqrt[3]{1}+\sqrt[3]{1}+1\right)}=\frac{1}{3}$$

Detyra 188: $\lim\limits_{x\to 0}\frac{\sqrt{\cos x}-\sqrt[3]{\cos x}}{\sin^2 x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sqrt{\cos x}-\sqrt[3]{\cos x}}{\sin^2 x}=\lim_{x\to 0}\frac{\sqrt{\cos x}-1}{\sin^2 x}+\lim_{x\to 0}\frac{1-\sqrt[3]{\cos x}}{\sin^2 x}=$$

$$=\lim_{x\to 0}\frac{\cos x-1}{\sin^2 x}\cdot\frac{1}{\sqrt{\cos x}+1}+\lim_{x\to 0}\frac{1-\cos x}{\sin^2 x}\frac{1}{1+\sqrt[3]{\cos x}+\sqrt[3]{\cos^2 x}}=$$

$$=-\frac{1}{2}\lim_{x\to 0}\frac{x^2}{\sin^2 x}\frac{1-\cos x}{x^2}+\frac{1}{3}\lim_{x\to 0}\frac{x^2}{\sin^2 x}\frac{1-\cos x}{x^2}=-\frac{1}{2}\frac{1}{2}+\frac{1}{3}\frac{1}{2}=-\frac{1}{12}$$

Detyra 189: $\lim\limits_{x\to 0}\dfrac{\cos 5x-\cos 3x}{x^2}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\cos 5x-\cos 3x}{x^2}=\lim_{x\to 0}\frac{\cos 5x-1+1-\cos 3x}{x^2}=\lim_{x\to 0}\frac{\cos 5x-1}{x^2}+\lim_{x\to 0}\frac{1-\cos 3x}{x^2}=$$

$$=\lim_{x\to 0}\frac{\cos 5x-1}{x^2}\cdot\frac{\cos 5x+1}{\cos 5x+1}+\lim_{x\to 0}\frac{1-\cos 3x}{x^2}\cdot\frac{1+\cos 3x}{1+\cos 3x}=\frac{1}{2}\lim_{x\to 0}\frac{\cos^2 5x-1}{x^2}+\frac{1}{2}\lim_{x\to 0}\frac{1-\cos^2 3x}{x^2}=$$

$$=-\frac{1}{2}\lim_{x\to 0}\frac{1-\cos^2 5x}{x^2}+\frac{1}{2}\lim_{x\to 0}\frac{1-\cos^2 3x}{x^2}=-\frac{1}{2}\lim_{x\to 0}\frac{\sin^2 5x}{x^2}+\frac{1}{2}\lim_{x\to 0}\frac{\sin^2 3x}{x^2}=$$

$$=-\frac{1}{2}\lim_{x\to 0}\left(5\frac{\sin 5x}{5x}\right)^2+\frac{1}{2}\lim_{x\to 0}\left(3\frac{\sin 3x}{3x}\right)^2=-\frac{1}{2}5^2+\frac{1}{2}3^2=-\frac{25}{2}+\frac{9}{2}=-\frac{16}{2}=-8$$

Detyra 190: $\lim\limits_{x\to 0}\dfrac{1+\sin x-\cos x}{1+\sin px-\cos px}$

Zgjidhje:

$$\lim_{x\to 0}\frac{1+\sin x-\cos x}{1+\sin px-\cos px}=\lim_{x\to 0}\frac{\sin x+2\sin^2\frac{x}{2}}{\sin px+2\sin^2\frac{px}{2}}=\lim_{x\to 0}\frac{\dfrac{\sin x}{x}+\dfrac{x}{2}\left(\dfrac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2}{p\dfrac{\sin px}{px}+\dfrac{p^2x}{2}\left(\dfrac{\sin\frac{px}{2}}{\frac{px}{2}}\right)^2}=\frac{1}{p}$$

Detyra 191: $\lim\limits_{x\to 0}\dfrac{x^2}{\sqrt{1+x\sin x}-\sqrt{\cos x}}$

Zgjidhje:

$$\lim_{x\to 0}\frac{x^2}{\sqrt{1+x\sin x}-\sqrt{\cos x}}\cdot\frac{\sqrt{1+x\sin x}+\sqrt{\cos x}}{\sqrt{1+x\sin x}+\sqrt{\cos x}}=\lim_{x\to 0}\frac{x^2\left(\sqrt{1+x\sin x}+\sqrt{\cos x}\right)}{1+x\sin x-\cos x}=\lim_{x\to 0}\frac{\sqrt{1+x\sin x}+\sqrt{\cos x}}{\dfrac{1-\cos x}{x^2}+\dfrac{\sin x}{x}}=\frac{4}{3}$$

Detyra 192: $\lim\limits_{x\to \pi}\dfrac{\cos^2 x-3\cos x-4}{\cos^2 x-4\cos x-5}$

Zgjidhje:

$$\lim_{x\to \pi}\frac{\cos^2 x-3\cos x-4}{\cos^2 x-4\cos x-5}=\left|\begin{array}{l}\textit{Zëvendësojmë}:\\ \cos x=t\end{array}\right|=\lim_{x\to \pi}\frac{t^2-3t-4}{t^2-4t-5}=\lim_{x\to \pi}\frac{(t-4)(t+1)}{(t-5)(t+1)}=$$

$$=\lim_{x\to \pi}\frac{(t-4)}{(t-5)}=\lim_{x\to \pi}\frac{\cos x-4}{\cos x-5}=\frac{-1-4}{-1-5}=\frac{-5}{-6}=\frac{5}{6}$$

Detyra 193: $\lim_{x\to\pi}\frac{\pi^x-x^\pi}{x-\pi}$

Zgjidhje:

$$\lim_{x\to\pi}\frac{\pi^x-x^\pi}{x-\pi}=\underbrace{\lim_{x\to\pi}\frac{\pi^x-\pi^\pi}{x-\pi}}_{L_1}+\underbrace{\lim_{x\to\pi}\frac{\pi^x-x^\pi}{x-\pi}}_{L_2}$$

$$L_1=\lim_{x\to\pi}\frac{\pi^x-\pi^\pi}{x-\pi}=\lim_{x\to\pi}\frac{\pi^x\left[\pi^{x-\pi}-1\right]}{x-\pi}=\pi^\pi\cdot\ln\pi$$

$$L_2=\lim_{x\to\pi}\frac{\pi^x-x^\pi}{x-\pi}=\lim_{x\to\pi}\frac{-\pi^\pi\left[\left(\frac{x}{\pi}\right)^\pi-1\right]}{x-\pi}\lim_{x\to\pi}\frac{-\pi^\pi\left[e^{\ln\left(\frac{x}{\pi}\right)^\pi}-1\right]}{\ln\left(\frac{x}{\pi}\right)^\pi}\cdot\frac{\ln\left(\frac{x}{\pi}\right)^\pi}{x-\pi}=$$

$$=-\pi^\pi\lim_{x\to\pi}\frac{e^{\ln\left(\frac{x}{\pi}\right)^\pi}-1}{\ln\left(\frac{x}{\pi}\right)^\pi}\cdot\lim_{x\to\pi}\ln\left[1+\frac{x}{\pi}-1\right]^{\frac{\pi}{x-\pi}}=-\pi^\pi\left|\begin{array}{l}e^{\ln\left(\frac{x}{\pi}\right)^\pi}-1=t\\ \pi\ln\left(\frac{x}{\pi}\right)=\ln(1+t)\\ x\to\pi\Rightarrow t\to0\end{array}\right|\cdot\left|\begin{array}{l}\frac{x-\pi}{\pi}=t\\ \frac{\pi}{x-\pi}=\frac{1}{t}\\ x\to\pi\Rightarrow t\to0\end{array}\right|=$$

$$=-\pi^\pi\lim_{t\to0}\frac{t}{\ln(1+t)}\cdot\lim_{t\to0}\ln(1+t)^{\frac{1}{t}}=-\pi^\pi\cdot\frac{1}{\ln e}\cdot\ln e=-\pi^\pi\cdot\frac{1}{1}\cdot1=-\pi^\pi$$

$$L=L_1+L_2=\pi^\pi\ln\pi-\pi^\pi$$

Detyra 194: $\lim_{x\to0}\left(1+5\tan^2x\right)^{3\cot x}$

Zgjidhje:

$$\lim_{x\to0}\left(1+5\tan^2x\right)^{3\cot x}=\lim_{x\to0}\left(1+5\tan^2x\right)^{3\cdot\frac{1}{\tan x}}=\left|\begin{array}{l}\textit{Zëvendësojmë}:\\ \tan x=t\\ x\to0\Rightarrow t\to0\end{array}\right|=\lim_{t\to0}\left(1+5t^2\right)^{\frac{3}{t}}=\left(e^5\right)^3=e^{15}$$

Detyra 195: $\lim_{x\to0}(1+\tan x)^{\frac{3}{x}}$

Zgjidhje:

$$\lim_{x\to0}(1+\tan x)^{\frac{3}{x}}=\left|\begin{array}{l}\textit{Zëvendësojmë}:\\ (1+\tan x)^{\frac{3}{x}}=t\\ \ln t=\frac{3}{x}\ln(1+\tan x)\end{array}\right|=\lim_{x\to0}t=\lim_{x\to0}\frac{3}{x}\ln(1+\tan x)=3\lim_{x\to0}\frac{\ln(1+\tan x)}{\tan x}\cdot\frac{\tan x}{x}=e$$

Detyra 196: $\lim\limits_{x\to0}(\cos x)^{\frac{1}{\sin^2 x}}$

Zgjidhje:

$$\lim_{x\to0}(\cos x)^{\frac{1}{\sin^2 x}} = \lim_{x\to0}\left(\sqrt{1-\sin^2 x}\right)^{\frac{1}{\sin^2 x}} = \lim_{x\to0}\left(1-\sin^2 x\right)^{\frac{1}{2}\cdot\frac{1}{\sin^2 x}} = \lim_{x\to0}\left(1+\left(-\sin^2 x\right)\right)^{\frac{1}{\sin^2 x}\cdot\left(\frac{1}{2}\right)} =$$

$$= e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

Detyra 197: $\lim\limits_{x\to a}\left(\frac{\sin x}{\sin a}\right)^{\frac{1}{x-a}}$

Zgjidhje:

$$\lim_{x\to a}\left(\frac{\sin x}{\sin a}\right)^{\frac{1}{x-a}} = \lim_{x\to a}\left[1+\left(\frac{\sin x}{\sin a}-1\right)\right]^{\frac{1}{x-a}} = \lim_{x\to a}\left[1+\frac{2\cos\frac{x+a}{2}\cdot\sin\frac{x-a}{2}}{\sin a}\right]^{\frac{\sin a}{2\cos\frac{x+a}{2}\cdot\sin\frac{x-a}{2}}\cdot\frac{2\cos\frac{x+a}{2}\cdot\sin\frac{x-a}{2}}{\sin a}\cdot\frac{1}{x-a}} =$$

$$= e^{\lim\limits_{x\to a}\frac{2\cos\frac{x+a}{2}\cdot\sin\frac{x-a}{2}}{\sin a}\cdot\frac{1}{x-a}} = e^{\mathrm{ctg}x}$$

Detyra 198: $\lim\limits_{x\to0}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$

Zgjidhje:

$$\lim_{x\to0}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}} = \left|\begin{array}{l} \textit{Zëvendësojmë}: \\ \tan\sqrt{x}=t \\ x\to0, \tan\sqrt{0}=0 \\ x\to0, t\to0 \\ \sqrt{x}=\arctan t \text{ ose } x=(\arctan)^2 \end{array}\right| = \lim_{t\to0}\left(1+t^2\right)^{\frac{1}{2(\arctan)^2}} =$$

$$= \lim_{t\to0}\left(1+t^2\right)^{\frac{t^2}{t^2\cdot2(\arctan)^2}} = \lim_{t\to0}\left[\left(1+t^2\right)^{\frac{1}{t^2}}\right]^{\frac{t^2}{2(\arctan)^2}} = e^{\lim\limits_{t\to0}\frac{t^2}{2(\arctan)^2}} = e^{\frac{1}{2}\lim\limits_{t\to0}\frac{\tan\sqrt{x}}{\sqrt{x^2}}} = e^{\frac{1}{2}\lim\limits_{t\to0}\frac{\left(\frac{\sin\sqrt{x}}{\cos\sqrt{x}}\right)^2}{\sqrt{x^2}}} =$$

$$= e^{\frac{1}{2}\lim\limits_{t\to0}\left(\frac{\sin\sqrt{x}}{\sqrt{x}}\right)^2\cdot\frac{1}{\cos^2\sqrt{x}}} = e^{\frac{1}{2}\cdot1\cdot1} = e^{\frac{1}{2}} = \sqrt{e}$$

Detyra 199: $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{3x-\pi}{\cos\dfrac{9x}{2}}$

Zgjidhje:

$$\lim_{x\to\frac{\pi}{3}}\frac{3x-\pi}{\cos\frac{9x}{2}}=\left|\begin{array}{l} x-\dfrac{\pi}{3}=t \\ x=t+\dfrac{\pi}{3} \\ x\to\dfrac{\pi}{3}\Rightarrow t\to 0\end{array}\right|=\lim_{t\to 0}\frac{3\left(t+\frac{\pi}{3}\right)-\pi}{\cos\left[\frac{9}{2}\left(t+\frac{\pi}{3}\right)\right]}=\lim_{t\to 0}\frac{3t+\pi-\pi}{\cos\left(\frac{9t}{2}+\frac{3\pi}{2}\right)}=$$

$$=\lim_{t\to 0}\frac{3t}{\cos\frac{9\pi}{2}\cdot\cos\frac{3\pi}{2}-\sin\frac{9t}{2}\cdot\sin\frac{3\pi}{2}}=\lim_{t\to 0}\frac{3t}{0-\sin\frac{9t}{2}\cdot(-1)}=\lim_{t\to 0}\frac{3t}{\sin\frac{9t}{2}}=$$

$$=\lim_{t\to 0}\frac{3}{\frac{\sin\frac{9t}{2}}{t}}=\lim_{t\to 0}\frac{3}{\frac{\sin\frac{9t}{2}}{\frac{9t}{2}}\cdot\frac{9}{2}}=\frac{3}{1\cdot\frac{9}{2}}=\frac{6}{9}=\frac{2}{3}$$

Detyra 200: $\lim\limits_{x\to 0}\dfrac{e^{\alpha x}-e^{\beta x}}{\sin\alpha x-\sin\beta x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{e^{\alpha x}-e^{\beta x}}{\sin\alpha x-\sin\beta x}=\lim_{x\to 0}\frac{\frac{e^{\alpha x}-e^{\beta x}}{x}}{\frac{2\cos\frac{(\alpha+\beta)x}{2}\cdot\sin\frac{(\alpha-\beta)x}{2}}{x}}=\lim_{x\to 0}\frac{\frac{e^{\alpha x}-1}{x}-\frac{e^{\beta x}-1}{x}}{\frac{2(\alpha+\beta)}{2}\cdot\cos\frac{(\alpha+\beta)x}{2}\cdot\frac{\sin\frac{(\alpha+\beta)x}{2}}{\frac{\alpha-\beta}{2}}}=$$

$$=\frac{1}{\alpha-\beta}\left[\lim_{x\to 0}\frac{e^{\alpha x}-1}{x}-\lim_{x\to 0}\frac{e^{\alpha x}-1}{x}\right]=\left|\begin{array}{l} e^{\alpha x}-1=t \\ e^{\alpha x}=1+t\,/\ln \\ \alpha x=\ln(1+t) \\ x=\dfrac{\ln(1+t)}{\alpha} \\ x\to 0\Rightarrow t\to 0\end{array}\right|=\frac{1}{\alpha-\beta}\left[\lim_{x\to 0}\frac{t}{\frac{\ln(1+t)}{\alpha}}-\lim_{x\to 0}\frac{k}{\frac{\ln(1+t)}{\beta}}\right]=$$

$$=\frac{1}{\alpha-\beta}\left[\alpha\lim_{x\to 0}\frac{1}{\ln(1+t)^{\frac{1}{t}}}-\beta\lim_{x\to 0}\frac{1}{\ln(1+k)^{\frac{1}{k}}}\right]=\frac{1}{\alpha-\beta}[\alpha\cdot 1-\beta\cdot 1]=\frac{1}{\alpha-\beta}\cdot(\alpha-\beta)=1$$

2. DERIVATE

2.1. Derivati i rendit të parë

Le të jetë f funksion i përkufizuar në intervalin (a,b) dhe $x_0 \in (a,b)$. Shënojmë me Δx një shtesë të çfarëdoshme të x_0 të tillë që $x_0 + \Delta x \in (a,b)$. Shtesa përkatëse e funksionit f që i përgjigjet shtesës Δx është $\Delta y = f(x_0 + \Delta x) - f(x_0)$. Formojmë raportin

$$\frac{\Delta y}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}.$$

Vlera kufitare

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x},$$

nëse ekziston dhe është e fundme quhet **derivat i parë** ose shkurt **derivat** i funksionit f në pikën x_0 dhe e shënojmë me $f'(x_0)$. Pra:

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}.$$

Derivati i majtë $f'_-(x_0)$ dhe **i djathtë** $f'_+(x_0)$ i funksionit funksionit f në pikën x_0

përkufizohen me barazimet :

$$f'_-(x_0) = \lim_{\Delta x \to 0^-} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

$$f'_+(x_0) = \lim_{\Delta x \to 0^+} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}.$$

Funksioni i cili ka derivat në secilën pikë të intervalit (a,b) quhet funksion i derivueshëm në intervalin (a,b). Nëse nuk veçohet pika e intervalit në të cilën gjendet derivati, shkruajmë

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad \text{ose} \quad y' = f'(x) = \frac{dy}{dx}.$$

2.2. Rregullat themelore të derivatit:

Le të jenë f, g funksione të derivushme dhe c konstante. Atëherë:

1) $c' = 0$

2) $(f \pm g)' = f' \pm g'$

3) $(cf)' = cf'$

4) $(f \cdot g)' = f' \cdot g + f \cdot g'$ (rregulla e prodhimit)

5) $\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$ (rregulla e herësit)

2.3. Tabela e derivateve

$1^0 \ (c)' = 0, \quad c = \text{const}$

$2^0 \ (x_n)' = nx^{n-1}$

$3^0 \ (e^x)' = e^x$

$4^0 \ (\log_a x)' = \frac{1}{x \ln a} (a > 0, a \neq 1)$

$5^0 \ (\ln x)' = \frac{1}{x}$

$6^0 \ (\sin x)' = \cos x$

$7^0 \ (\cos x)' = -\sin x$

$8^0 \ (a^x)' = a^x \ln a$

$9^0 \ (\tan x)' = \frac{1}{\cos^2 x}.$

$10^0 \ (\cot x)' = -\frac{1}{\sin^2 x}.$

$11^0 \ (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}.$

$12^0 \ (\arccos x)' = -\frac{1}{\sqrt{1-x^2}}.$

$13^0 \ (\arctan x)' = \frac{1}{1+x^2}.$

$14^0 \ (\operatorname{arc}\cot x)' = -\frac{1}{1+x^2}.$

$15^0 \left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$

$16^0 \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$

2.4. Tabela e derivateve të funksioneve të përbëra

$1^0 \ \left(f^{\alpha}(x)\right)' = \alpha f^{\alpha-1}(x) f'(x)$

$2^0 \ \left(a^{f(x)}\right)' = a^{f(x)} \ln a \ f'(x) (a > 0, a \neq 1)$

$3^0 \ \left(e^{f(x)}\right)' = e^{f(x)} f'(x)$

$4^0 \ \left(\log_a f(x)\right)' = \dfrac{1}{f(x)\ln a} f'(x) (a > 0, a \neq 1)$

$5^0 \ \left(\ln f(x)\right)' = \dfrac{1}{f(x)} f'(x)$

$6^0 \ \left(\sin f(x)\right)' = \cos f(x) \cdot f'(x)$

$7^0 \ \left(\cos f(x)\right)' = -\sin f(x) \cdot f'(x)$

$8^0 \ \left(\tan f(x)\right)' = \dfrac{1}{\cos^2 f(x)} f'(x)$

$9^0 \ \left(\cot f(x)\right)' = -\dfrac{1}{\sin^2 f(x)} f'(x)$

$10^0 \ \left(\arcsin f(x)\right)' = \dfrac{1}{\sqrt{1 - f^2(x)}} f'(x)$

$11^0 \ \left(\arccos f(x)\right)' = -\dfrac{1}{\sqrt{1 - f^2(x)}} f'(x)$

$12^0 \ \left(\arctan f(x)\right)' = \dfrac{1}{1 + f^2(x)} f'(x)$

$13^0 \ \left(\operatorname{arccot} f(x)\right)' = -\dfrac{1}{1 + f^2(x)} f'(x)$

Derivatet e rendeve të larta:

$$f''(x) = \left[f'(x)\right]'$$

$$f'''(x) = \left[f''(x)\right] = \left[\left[f'(x)\right]'\right]'$$

Detyra të zgjidhura:

Detyra 1: $f(x) = ax + b \quad (a, b \in R)$

Zgjidhje:

$$\Delta y = f(x+\Delta x) - f(x) = a(x+\Delta x) - ax = a\Delta x$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{a\Delta x}{\Delta x} = a$$

Detyra 2: $f(x) = x^2$

Zgjidhje:

$$\Delta y = f(x+\Delta x) - f(x) = (x+\Delta x)^2 - x^2 = 2x\Delta x + (\Delta x)^2$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

Detyra 3: $f(x) = \sqrt{x}$

Zgjidhje:

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}} = \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)} =$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)} = \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Detyra 4: $f(x) = \frac{k}{x}$

Zgjidhje:

$$\Delta y = f(x+\Delta x) - f(x) = \frac{k}{x+\Delta x} - \frac{k}{x} = -\frac{k\Delta x}{(x+\Delta x)x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{-\frac{k\Delta x}{(x+\Delta x)x}}{\Delta x} = -\lim_{\Delta x \to 0} \frac{k}{(x+\Delta x)x} = -\frac{k}{x^2}$$

Detyra 5: $f(x)=ax^2+bx+c \quad (a,b,c\in R)$

Zgjidhje:

$$\Delta y = f(x+\Delta x)-f(x)=a(x+\Delta x)^2+b(x+\Delta x)+c-ax^2-bx-c=2ax\Delta x+b\Delta x+a(\Delta x)^2$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{2ax\Delta x+b\Delta x+a(\Delta x)^2}{\Delta x}=\lim_{\Delta x\to 0}(2ax+b+a\Delta x)=2ax+b$$

Detyra 6: $f(x)=\sqrt{3x+1}$

Zgjidhje:

$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}=\lim_{\Delta x\to 0}\frac{\sqrt{3(x+\Delta x)+1}-\sqrt{3x+1}}{\Delta x}\cdot\frac{\sqrt{3(x+\Delta x)+1}+\sqrt{3x+1}}{\sqrt{3(x+\Delta x)+1}+\sqrt{3x+1}}=$$

$$=\lim_{\Delta x\to 0}\frac{3(x+\Delta x)+1-(3x+1)}{\Delta x\left(\sqrt{3(x+\Delta x)}+\sqrt{3x+1}\right)}=\lim_{\Delta x\to 0}\frac{3x+3\Delta x+1-3x-1}{\Delta x\left(\sqrt{3(x+\Delta x)}+\sqrt{3x+1}\right)}=$$

$$=\lim_{\Delta x\to 0}\frac{3\Delta x}{\Delta x\left(\sqrt{3(x+\Delta x)}+\sqrt{3x+1}\right)}=\lim_{\Delta x\to 0}\frac{3}{\sqrt{3(x+\Delta x)}+\sqrt{3x+1}}=\frac{3}{2\sqrt{3x+1}}$$

Detyra 7: $y=3x^4-2x^3+x^2-x+1$

Zgjidhje:

$$y'=\left(3x^4-2x^3+x^2-x+1\right)'=\left(3x^4\right)'-\left(2x^3\right)'+\left(x^2\right)'-x'+1'=3\left(x^4\right)'-2\left(x^3\right)'+2x-1=$$

$$=3\cdot 4x^3-2\cdot 3x^2+2x-1=12x^3-6x^2+2x-1$$

Detyra 8: $y=2x^3-4x^2+3x-5$

Zgjidhje:

$$y'=\left(2x^3-4x^2+3x-5\right)'=\left(2x^3\right)'-\left(4x^2\right)'+(3x)'-(5)'=2\left(x^3\right)'-4\left(x^2\right)'+3(x)'-0=$$

$$=6x^2-8x+3$$

Detyra 9: $y=5x^3-12x^2+3x-14$

Zgjidhje:

$$y'=\left(5x^3-12x^2+3x-14\right)'=\left(5x^3\right)'-\left(12x^2\right)'+(3x)'-(14)'=5\left(x^3\right)'-12\left(x^2\right)'+3(x)'-0=$$

$$=15x^2-24x+3$$

Detyra 10: $y=x^5-3x^4+2x^3-4x^2-5x+3$

Zgjidhje:

$$y'=\left(x^5-3x^4+2x^3-4x^2-5x+3\right)'=\left(x^5\right)'-\left(3x^4\right)'+\left(2x^3\right)'-\left(4x^2\right)'-\left(5x\right)'+\left(3\right)'=$$
$$=\left(x^5\right)'-3\left(x^4\right)'+2\left(x^3\right)'-4\left(x^2\right)'-5\left(x\right)'+0=5x^4-12x^3+6x^2-8x-5$$

Detyra 11: $y=x^8+12x^5-4x^4+10x^3-5x+5$

Zgjidhje:

$$y'=\left(x^8+12x^5-4x^4+10x^3-5x+5\right)'=\left(x^8\right)'+\left(12x^5\right)'-\left(4x^4\right)'+\left(10x^3\right)'-\left(6x\right)'+\left(5\right)'=$$
$$\left(x^8\right)'+12\left(x^5\right)'-4\left(x^4\right)'+10\left(x^3\right)'-6\left(x\right)'+0=8x^7+60x^4-16x^3+30x^3-6$$

Detyra 12: $y=x^{\frac{4}{7}}$

Zgjidhje:

$$y'=\left(x^{\frac{4}{7}}\right)'=\frac{4}{7}x^{\frac{4}{7}-1}=\frac{4}{7}x^{-\frac{3}{7}}=\frac{4}{7}\cdot\frac{1}{x^{\frac{3}{7}}}=\frac{4}{7\sqrt[7]{x^3}}$$

Detyra 13: $y=(2x+1)(3x-2)$

Zgjidhje:

$$y'=\left[(2x+1)(3x-2)\right]'=(2x+1)'(3x-2)+(2x+1)(3x-2)'=$$
$$=(2\cdot1+0)(3x-2)+(2x+1)(3\cdot1-0)=2(3x-2)+(2x+1)3=6x-4+6x+3$$

Detyra 14: $y=(x+2)(x+5)$

Zgjidhje:

$$y'=\left[(x+2)(x+5)\right]'=(x+2)'(x+5)+(x+2)(x+5)'=$$
$$=(1+0)(x+5)+(x+2)(1+0)=1\cdot(x+5)+(x+2)\cdot1=x+5+x+2=2x+7$$

Detyra 15: $y=(2x-3)\left(x^2+5x\right)$

Zgjidhje:

$$y'=\left[(2x-3)\left(x^2+5x\right)\right]'=(2x-3)'\left(x^2+5x\right)+(2x-3)\left(x^2+5x\right)'=$$
$$=(2-0)\left(x^2+5x\right)+(2x-3)(2x+5)=2x^2+10x+4x^2-6x+10x-15=6x^2+14x-15$$

Detyra 16: $y=10(3x+1)(1-5x)$

Zgjidhje:

$$y'=\left[10(3x+1)(1-5x)\right]'=10\left[(3x+1)(1-5x)\right]'=10\left[(3x+1)'(1-5x)+(3x+1)(1-5x)'\right]=$$
$$=10\left[3(1-5x)+(3x+1)(-5)\right]=10\left[3-15x-15x-5\right]=10\left[-30x-2\right]=-300x-20$$

Detyra 17: $f(x)=\left(x^3-1\right)\left(x^2+x+1\right)$

Zgjidhje:

$$f'(x)=\left[\left(x^3-1\right)\left(x^2+x+1\right)\right]'=\left(x^3-1\right)'\left(x^2+x+1\right)+\left(x^3-1\right)\left(x^2+x+1\right)'=$$
$$=3x^2\left(x^2+x+1\right)+(2x+1)\left(x^3-1\right)=\left(x^2+x+1\right)\left(5x^2-x-1\right)$$

Detyra 18: $f(x)=\left(x^2-3x+3\right)\left(x^2+2x-1\right)$

Zgjidhje:

$$f'(x)=\left(x^2-3x+3\right)'\left(x^2+2x-1\right)+\left(x^2-3x+3\right)\left(x^2+2x-1\right)'=$$
$$=(2x-3)\left(x^2+2x-1\right)+\left(x^2-3x+3\right)(2x+2)=$$
$$=2x^3+4x^2-2x-3x^2-6x+3+2x^3-6x^2+6x+2x^2-6x+6=4x^3-3x^2-8x+9$$

Detyra 19: $y=\left(x^3-3x+2\right)\left(x^4+x^2-1\right)$

Zgjidhje:

$$y'=\left(x^3-3x+2\right)'\left(x^4+x^2-1\right)+\left(x^3-3x+2\right)\left(x^4+x^2-1\right)'=$$
$$=\left(3x^2-3\right)\left(x^4+x^2-1\right)+\left(x^3-3x+2\right)\left(4x^3+2x\right)=$$
$$=3x^6+x^4-5x^2+2+4x^6-10x^4-6x^2+8x^3+4x=7x^6-10x^4+8x^3-12x^2+4x+3$$

Detyra 20: $y=\left(x^2-4\right)\left(x^2-9\right)\left(x^2-16\right)$

Zgjidhje:

$$y'=\left(x^2-4\right)'\left(x^2-9\right)\left(x^2-16\right)+\left(x^2-4\right)\left(x^2-9\right)'\left(x^2-16\right)+\left(x^2-4\right)\left(x^2-9\right)\left(x^2-16\right)'=$$
$$=2x\left(x^2-9\right)\left(x^2-16\right)+2x\left(x^2-4\right)\left(x^2-16\right)+2x\left(x^2-4\right)\left(x^2-9\right)=$$
$$=2x\left(x^4-25x^2+144\right)+2x\left(x^4-20x^2+64\right)+2x\left(x^4-13x^2+36\right)=$$
$$=2x^5-50x^3+288x+2x^5-40x^3+128x+2x^5-26x^3+72x=$$
$$=6x^5-116x^3+488x$$

Detyra 21: $f\left(x\right)=2x\sqrt{x}$

Zgjidhje:

$$f\left(x\right)=2x\sqrt{x}=f\left(x\right)=2x^{\frac{3}{2}}$$

$$f'\left(x\right)=\left(2x^{\frac{3}{2}}\right)'=2\left(x^{\frac{3}{2}}\right)'=2\cdot\frac{3}{2}x^{\frac{3}{2}-1}=3x^{\frac{1}{2}}=3\sqrt{x}$$

Detyra 22: $y=\left(\sqrt{x}+1\right)\left(\frac{1}{\sqrt{x}}-1\right)$

Zgjidhje:

$$y'=\left(\frac{1}{\sqrt{x}}-1\right)'\left(\sqrt{x}+1\right)+\left(\frac{1}{\sqrt{x}}-1\right)\left(\sqrt{x}+1\right)'=\left(\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}\right)\left(\sqrt{x}+1\right)+\left(\frac{1}{\sqrt{x}}-1\right)\left(\frac{1}{2}x^{\frac{1}{2}-1}\right)=$$

$$=\frac{\frac{1}{\sqrt{x}}-1}{2\sqrt{x}}-\frac{\sqrt{x}+1}{2x^{\frac{3}{2}}}=\frac{-x^{\frac{1}{2}}+1}{2x^{\frac{1}{2}}}-\frac{x^{\frac{1}{2}}+1}{2x^{\frac{3}{2}}}=-\frac{x+1}{2x^{\frac{5}{2}}}$$

Detyra 23: $f\left(x\right)=\frac{1}{\sqrt{x}}$

Zgjidhje:

$$f\left(x\right)=\frac{1}{\sqrt{x}}=f\left(x\right)=x^{-\frac{1}{2}}$$

$$f'\left(x\right)=\left(x^{-\frac{1}{2}}\right)'=-\frac{1}{2}x^{-\frac{1}{2}-1}=-\frac{1}{2}x^{-\frac{3}{2}}=-\frac{1}{2\sqrt{x^3}}=-\frac{1}{2x\sqrt{x}}$$

Detyra 24: $f\left(x\right)=\sqrt[3]{x}-\frac{2}{\sqrt{x}}+\frac{3}{x^2}-\frac{1}{5x^3}+4$

Zgjidhje:

$$f'\left(x\right)=\left(\sqrt[3]{x}-\frac{2}{\sqrt{x}}+\frac{3}{x^2}-\frac{1}{5x^3}+4\right)'=\left(x^{\frac{1}{3}}-2x^{-\frac{1}{2}}+3x^{-2}-\frac{1}{5}x^{-3}+4\right)'=$$

$$=\frac{1}{3}x^{\frac{1}{3}-1}-2\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}+3(-2)x^{-2-1}-\frac{1}{5}(-3)x^{-3-1}=\frac{1}{3}x^{-\frac{2}{3}}+x^{-\frac{3}{2}}-6x^{-3}+\frac{3}{5}x^{-4}=$$

$$=\frac{1}{3\sqrt[3]{x^2}}+\frac{1}{x\sqrt{x}}-\frac{6}{x^3}+\frac{3}{5x^4}$$

Detyra 25: $y=\left(1+5x^3\right)\left(1-2x^2\right)$

Zgjidhje:

$$y'=\left(1+5x^3\right)'\left(1-2x^2\right)+\left(1+5x^3\right)\left(1-2x^2\right)'=15x^2\left(1-2x^2\right)+\left(1+5x^3\right)\left(-4x\right)=$$
$$=15x^2-30x^4-4x-20x^4=-50x^4+15x^2-4x$$

Detyra 26: $f\left(x\right)=x^2\cdot e^x\cdot\sin x$

Zgjidhje:

$$f'\left(x\right)=\left(x^2\cdot e^x\cdot\sin x\right)'=\left(x^2\right)'\cdot e^x\cdot\sin x+\left(e^x\right)'\cdot x^2\cdot\sin x+x^2\cdot e^x\left(\sin x\right)'=$$
$$=2x\cdot e^x\cdot\sin x+x^2\cdot e^x\cdot\sin x+x^2\cdot e^x\cdot\cos x=xe^x\left(2\sin x+x\sin x+x\cos x\right)$$

Detyra 27: $f\left(x\right)=5\sin x+3x^2-e^x$

Zgjidhje:

$$f'\left(x\right)=\left(5\sin x+3x^2-e^x\right)'=\left(5\sin x\right)'+\left(3x^2\right)'-\left(e^x\right)'=5\left(\sin x\right)'+3\left(x^2\right)'-e^x=$$
$$=5\cos x+6x-e^x$$

Detyra 28: $f\left(x\right)=\dfrac{3}{x^3}-8\dfrac{1}{x^4}-2\sqrt[3]{x^2}-\dfrac{1}{\sqrt[5]{x^3}}$

Zgjidhje:

$$f'\left(x\right)=3\left(\frac{1}{x^3}\right)'-8\left(\frac{1}{x^4}\right)'-2\left(\sqrt[3]{x^2}\right)'-\left(\frac{1}{\sqrt[5]{x^3}}\right)'=3\left(x^{-3}\right)'-8\left(x^{-4}\right)'-2\left(x^{\frac{2}{3}}\right)'-\left(x^{-\frac{3}{5}}\right)'=$$
$$=3\cdot\left(-3\right)x^{-3-1}-8\left(-4\right)x^{-4-1}-2\cdot\frac{2}{3}x^{\frac{2}{3}-1}-\left(-\frac{3}{5}\right)x^{-\frac{3}{5}-1}=-9x^{-4}+32x^{-5}-\frac{4}{3}x^{-\frac{1}{3}}+\frac{3}{5}x^{-\frac{8}{5}}=$$
$$=\frac{9}{x^4}+\frac{32}{x^5}-\frac{4}{3\sqrt[3]{x}}+\frac{3}{5\sqrt[5]{x^8}}$$

Detyra 29: $f\left(x\right)=3x+\sin x-4\cos x$

Zgjidhje:

$$f'\left(x\right)=\left(3x+\sin x-4\cos x\right)'=\left(3x\right)'+\left(\sin x\right)'-\left(4\cos x\right)'=3\left(x\right)'+\cos x-4\left(\cos x\right)'=$$
$$=3+\cos x-4\left(-\sin x\right)=3+\cos x+4\sin x$$

Detyra 30: $f(x) = x^2 - 3\sin x - 2\cos x + 4$

Zgjidhje:

$$f'(x) = \left(x^2 - 3\sin x - 2\cos x + 4\right)' = \left(x^2\right)' - \left(3\sin x\right)' - \left(2\cos x\right)' + \left(4\right)' =$$
$$= 2x - 3\cos x - 2\left(-\sin x\right) = 2x - 3\cos x + 2\sin x$$

Detyra 31: $y = \frac{1}{6}x^6 - 2x^5 + \frac{2}{3}x^3 - \frac{3}{2}x^2 - 7x + 8$

Zgjidhje:

$$y' = \left(\frac{1}{6}x^6 - 2x^5 + \frac{2}{3}x^3 - \frac{3}{2}x^2 - 7x + 8\right)' = 6\frac{1}{6}x^5 - 5 \cdot 2x^4 + 3\frac{2}{3}x^2 - 2\frac{3}{2}x - 7 =$$
$$= x^5 - 10x^4 + 2x^2 - 3x - 7$$

Detyra 32: $y = \sqrt{x^2 - 2x}$

Zgjidhje:

$$y = \sqrt{x^2 - 2x} = \left| \left(\sqrt{u}\right)' = \frac{u'}{2\sqrt{u}} \right|$$

$$y' = \left(\sqrt{x^2 - 2x}\right)' = \frac{\left(x^2 - 2x\right)'}{2\sqrt{x^2 - 2x}} = \frac{2x - 2}{2\sqrt{x^2 - 2x}} = \frac{2\left(x - 1\right)}{2\sqrt{x^2 - 2x}} = \frac{x - 1}{\sqrt{x^2 - 2x}}$$

Detyra 33: $y = \sqrt{x^2 - 2x + 4}$

Zgjidhje:

$$y = \sqrt{x^2 - 2x + 4} = \left(x^2 - 2x + 4\right)^{\frac{1}{2}}$$

$$y' = \left(\left(x^2 - 2x + 4\right)^{\frac{1}{2}}\right)' = \frac{1}{2}\left(x^2 - 2x + 4\right)^{\frac{1}{2} - 1} \cdot \left(x^2 - 2x\right)' = \left(2x - 2\right) \cdot \frac{1}{2}\left(x^2 - 2x + 4\right)^{-\frac{1}{2}} =$$
$$= \frac{2\left(x - 1\right)}{2\sqrt{x^2 - 2x + 4}} = \frac{x - 1}{\sqrt{x^2 - 2x + 4}}$$

Detyra 34: $y = \sqrt[3]{x^3 - 3}$

Zgjidhje: $y = \sqrt[3]{x^3 - 3} = \left(x^3 - 3\right)^{\frac{1}{3}}$

$$y' = \left(\left(x^3 - 3\right)^{\frac{1}{3}}\right)' \left(x^3 - 3\right)' = \frac{1}{3}\left(x^3 - 3\right)^{-\frac{2}{3}} \cdot 3x^2 = \frac{x^2}{\sqrt[3]{\left(x^3 - 3\right)^2}}$$

Detyra 35: $y=\frac{x^2}{2+x}$

Zgjidhje:

$$y'=\left(\frac{x^2}{2+x}\right)'=\frac{\left(x^2\right)'(2+x)-\left(x^2\right)(2+x)'}{(2+x)^2}=\frac{2x(2+x)-x^2(0+1)}{(2+x)^2}=\frac{4x+2x^2-x^2}{(2+x)^2}=\frac{x^2+4x}{(2+x)^2}$$

Detyra 36: $y=\frac{mx+n}{px+q}$

Zgjidhje:

$$y'=\left(\frac{mx+n}{px+q}\right)'=\frac{(mx+n)'(px+q)-(mx+n)(px+q)'}{(px+q)^2}=\frac{m(px+q)-(mx+n)p}{(px+q)^2}=$$

$$=\frac{mpx+mq-mpx-np}{(px+q)^2}=\frac{mq-np}{(px+q)^2}$$

Detyra 37: $f(x)=\frac{x^2+1}{x^2-1}$

Zgjidhje:

$$f'(x)=\left(\frac{x^2+1}{x^2-1}\right)'=\frac{\left(x^2+1\right)'\left(x^2-1\right)-\left(x^2+1\right)\left(x^2-1\right)'}{\left(x^2-1\right)^2}=\frac{2x\left(x^2-1\right)-2x\left(x^2+1\right)}{\left(x^2-1\right)^2}=$$

$$=\frac{2x^3-2x-2x^3-2x}{\left(x^2-1\right)^2}=-\frac{4x}{\left(x^2-1\right)^2}$$

Detyra 38: $f(x)=\frac{x+1}{x-3}$

Zgjidhje:

$$f'(x)=\left(\frac{x+1}{x-3}\right)'=\frac{(x+1)'(x-3)-(x+1)(x-3)'}{(x-3)^2}=\frac{(1+0)(x-3)-(x+1)(1-0)}{(x-3)^2}=$$

$$=\frac{1(x-3)-(x+1)1}{(x-3)^2}=\frac{x-3-x-1}{(x-3)^2}=-\frac{4}{(x-3)^2}$$

Detyra 39: $y=\frac{2x-1}{2x+1}$

Zgjidhje:

$$y'=\left(\frac{2x-1}{2x+1}\right)'=\frac{(2x-1)'(2x+1)-(2x-1)(2x+1)'}{(2x+1)^2}=\frac{2(2x+1)-2(2x-1)}{(2x+1)^2}=$$
$$=\frac{4x+2-4x+2}{(2x+1)^2}=\frac{4}{(2x+1)^2}$$

Detyra 40: $y=\frac{3x-2}{x^2+3}$

Zgjidhje:

$$y'=\left(\frac{3x-2}{x^2+3}\right)'=\frac{(3x-2)'(x^2+3)-(3x-2)(x^2+3)'}{(x^2+3)^2}=\frac{3(x^2+3)-2x(3x-2)}{(x^2+3)^2}=$$
$$=\frac{3x^2+9-6x^2+4x}{(x^2+3)^2}=\frac{-3x^2+4x+9}{(x^2+3)^2}$$

Detyra 41: $y=\frac{x-1}{x^2-2x+3}$

Zgjidhje:

$$y'=\left(\frac{x-1}{x^2-2x+3}\right)'=\frac{(x-1)'(x^2-2x+3)-(x-1)(x^2-2x+3)'}{(x^2-2x+3)^2}=\frac{x^2-2x-3-(2x-2)(x-1)}{(x^2-2x+3)^2}=$$
$$=\frac{x^2-2x-3-2x^2+2x+2x-2}{(x^2-2x+3)^2}=\frac{-x^2+2x+1}{(x^2-2x+3)^2}$$

Detyra 42: $y=\frac{3x^2-1}{2x+4}$

Zgjidhje:

$$y'=\left(\frac{3x^2-1}{2x+4}\right)'=\frac{(3x^2-1)'(2x+4)-(3x^2-1)(2x+4)'}{(2x+4)^2}=\frac{6x(2x+4)-(3x^2-1)3}{(2x+4)^2}=$$
$$=\frac{12x^2+24x-6x^2-2}{(2x+4)^2}=\frac{6x^2+24x-2}{(2x+4)^2}$$

Detyra 43: $f(x)=\dfrac{x^2-5}{x-2}$

Zgjidhje:

$$f'(x)=\left(\frac{x^2-5}{x-2}\right)'=\frac{\left(x^2-5\right)'(x-2)-\left(x^2-5\right)(x-2)'}{(x-2)^2}=\frac{2x(x-2)-\left(x^2-5\right)}{(x-2)^2}=$$

$$=\frac{2x^2-4x-x^2+5}{(x-2)^2}=\frac{x^2-4x+5}{(x-2)^2}$$

Detyra 44: $y=\dfrac{x^2+x-2}{x^3+6}$

Zgjidhje:

$$y'=\left(\frac{x^2+x-2}{x^3+6}\right)'=\frac{\left(x^2+x-2\right)'\left(x^3+6\right)-\left(x^2+x-2\right)\left(x^3+6\right)}{\left(x^3+6\right)^2}=$$

$$=\frac{\left(2x^4+x^3+12x+6\right)-\left(3x^4+3x^3-6x^2\right)}{\left(x^3+6\right)^2}=\frac{2x^4+x^3+12x+6-3x^4-3x^3+6x^2}{\left(x^3+6\right)^2}=$$

$$=\frac{-x^4-2x^3+6x^2+12x+6}{\left(x^3+6\right)^2}$$

Detyra 45: $f(x)=\dfrac{x+1}{x^2+1}$

Zgjidhje:

$$f'(x)=\left(\frac{x+1}{x^2+1}\right)'=\frac{(x+1)'\left(x^2+1\right)-(x+1)\left(x^2+1\right)'}{\left(x^2+1\right)^2}=\frac{\left(x^2+1\right)-2x(x+1)}{\left(x^2+1\right)^2}=$$

$$=\frac{x^2+1-2x^2-2x}{\left(x^2+1\right)^2}=\frac{-x^2-2x+1}{\left(x^2+1\right)^2}$$

Detyra 46: $y=\dfrac{x}{x-3}$

Zgjidhje:

$$y'=\left(\frac{x}{x-3}\right)'=\frac{(x)'(x-3)-(x)(x-3)'}{(x-3)^2}=\frac{1\cdot(x-3)-(x)\cdot 1}{(x-3)^2}=\frac{x-3-x}{(x-3)^2}=-\frac{3}{(x-3)^2}$$

Detyra 47: $f(x)=\dfrac{2x+1}{4x^2+3x-1}$

Zgjidhje:

$$f'(x)=\left(\frac{2x+1}{4x^2+3x-1}\right)'=\frac{(2x+1)'(4x^2+3x-1)-(2x+1)(4x^2+3x-1)'}{(4x^2+3x-1)^2}$$

$$=\frac{2(4x^2+3x-1)-(2x+1)(8x+3)}{(4x^2+3x-1)^2}=\frac{8x^2+6x-2-16x^2-6x-8x-3}{(4x^2+3x-1)^2}=-\frac{8x^2-8x-5}{(4x^2+3x-1)^2}$$

Detyra 48: $y=\dfrac{2x}{x^2+1}$

Zgjidhje:

$$y'=\left(\frac{2x}{x^2+1}\right)'=\frac{(2x)'(x^2+1)-(2x)(x^2+1)'}{(x^2+1)^2}=\frac{2(x^2+1)-2x(2x)}{(x^2+1)^2}=$$

$$=\frac{2x^2+2-4x^2}{(x^2+1)^2}=\frac{-2x^2+2}{(x^2+1)^2}=\frac{-2(x^2+1)}{(x^2+1)^2}=\frac{-2(x+1)(x-1)}{x^4+2x^2+1}$$

Detyra 49: $y=\dfrac{1-\sqrt{x}}{1+\sqrt{x}}=\dfrac{1-x^{\frac{1}{2}}}{1+x^{\frac{1}{2}}}$

Zgjidhje:

$$y'=\left(\frac{1-x^{\frac{1}{2}}}{1+x^{\frac{1}{2}}}\right)'=\frac{\left(1-x^{\frac{1}{2}}\right)'\left(1+x^{\frac{1}{2}}\right)-\left(1-x^{\frac{1}{2}}\right)\left(1+x^{\frac{1}{2}}\right)'}{\left(1+x^{\frac{1}{2}}\right)^2}=$$

$$=\frac{-\frac{1}{2}x^{-\frac{1}{2}}\left(1+x^{\frac{1}{2}}\right)-\left(1-x^{\frac{1}{2}}\right)\frac{1}{2}x^{-\frac{1}{2}}}{\left(1+\sqrt{x}\right)^2}=\frac{-2\cdot\frac{1}{2}x^{-\frac{1}{2}}}{\left(1+\sqrt{x}\right)^2}=-\frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}$$

Detyra 50: $y=\dfrac{1}{x-1}$

Zgjidhje: $y'=\left(\dfrac{1}{x-1}\right)'=\dfrac{(1)'(x-1)-(1)(x-1)'}{(x-1)^2}=\dfrac{0\cdot(x-1)-1\cdot(-1)}{(x-1)^2}=-\dfrac{1+0}{(x-1)^2}=-\dfrac{1}{(x-1)^2}$

Detyra 51: $y=\dfrac{4x+3}{x+4}$

Zgjidhje:

$$y'=\left(\frac{4x+3}{x+4}\right)'=\frac{(4x+3)'(x+4)-(4x+3)(x+4)'}{(x+4)^2}=\frac{4\cdot(x+4)-(4x+3)\cdot 1}{(x+4)^2}=$$

$$=\frac{4x+16-4x-3}{(x+4)^2}=\frac{13}{(x+4)^2}$$

Detyra 52: $y=\dfrac{ax+b}{cx+d}$

Zgjidhje:

$$y'=\left(\frac{ax+b}{cx+d}\right)'=\frac{(ax+b)'(cx+d)-(ax+b)(cx+d)'}{(cx+d)^2}=\frac{a\cdot(cx+d)-(ax+b)\cdot c}{(cx+d)^2}=$$

$$=\frac{acx+ad-acx-bc}{(cx+d)^2}=\frac{ad-bc}{(cx+d)^2}$$

Detyra 53: $y=\dfrac{1+x}{1+x^2}$

Zgjidhje:

$$y'=\left(\frac{1+x}{1+x^2}\right)'=\frac{(1+x)'(1+x^2)-(1+x)(1+x^2)'}{(1+x^2)^2}=\frac{1\cdot(1+x^2)-(1+x)\cdot 2x}{(1+x^2)^2}=$$

$$=\frac{1+x^2-2x-2x^2}{(1+x^2)^2}=-\frac{x^2+2x-1}{(1+x^2)^2}$$

Detyra 54: $y=\dfrac{x^2-x+1}{x^2+x+1}$

Zgjidhje:

$$y'=\frac{(x^2-x+1)'(x^2+x+1)-(x^2-x+1)(x^2+x+1)'}{(x^2+x+1)^2}=\frac{(2x-1)(x^2+x+1)-(x^2-x+1)(2x+1)}{(x^2+x+1)^2}=$$

$$=\frac{2x^3+2x^2+2x-x^2-x-1-2x^3-x^2+2x^2+x-2x-1}{(x^2+x+1)}=\frac{2x^2-2}{(x^2+x+1)}$$

Detyra 55: $y=\frac{1}{1+x^2}$

Zgjidhje:

$$y'=\left(\frac{1}{1+x^2}\right)'=\frac{(1)'\left(1+x^2\right)-(1)\left(1+x^2\right)'}{\left(1+x^2\right)^2}=\frac{0\cdot\left(1+x^2\right)-(1)\cdot 2x}{\left(1+x^2\right)^2}=-\frac{2x}{\left(1+x^2\right)^2}$$

Detyra 56: $y=\frac{(2x-3)(x+1)}{(x-1)(3x+1)}=\frac{2x^2-x-3}{3x^2-2x-1}$

Zgjidhje:

$$y'=\left(\frac{2x^2-x-3}{3x^2-2x-1}\right)'=\frac{\left[\left(2x^2-x-3\right)'\left(3x^2-2x-1\right)\right]-\left[\left(2x^2-x-3\right)\left(3x^2-2x-1\right)'\right]}{\left[(x-1)(3x+1)\right]^2}$$

$$=\frac{\left[(4x-1)\left(3x^2-2x-1\right)\right]-\left[\left(2x^2-x-3\right)(6x-2)\right]}{\left[(x-1)(3x+1)\right]^2}=$$

$$=\frac{\left[12x^3-8x^2-4x-3x^2+2x+1\right]-\left[12x^3-6x^2+8x-4x^2+2x+6\right]}{\left[(x-1)(3x+1)\right]^2}=$$

$$=\frac{\left[12x^3-11x^2-2x+1\right]-\left[12x^3-10x^2-16x+6\right]}{\left[(x-1)(3x+1)\right]^2}=$$

$$=\frac{12x^3-11x^2-2x+1-12x^3+10x^2+16x+6}{\left[(x-1)(3x+1)\right]^2}=$$

$$=\frac{\left(-x^2+14x-5\right)}{(x-1)^2(3x+1)^2}=\frac{-\left(x^2-14x+5\right)}{(x-1)^2(3x+1)^2}$$

Detyra 57: $y=\frac{x^2-4}{1-x^2}$

Zgjidhje:

$$y'=\left(\frac{x^2-4}{1-x^2}\right)'=\frac{\left(x^2-4\right)'\left(1-x^2\right)-\left(x^2-4\right)\left(1-x^2\right)'}{\left(1-x^2\right)^2}=\frac{2x\cdot\left(1-x^2\right)-\left(x^2-4\right)\cdot(-2x)}{\left(1-x^2\right)^2}=$$

$$=\frac{2x-2x^3-\left(-2x^3+8x\right)}{\left(1-x^2\right)^2}=\frac{2x-2x^3+2x^3-8x}{\left(1-x^2\right)^2}=-\frac{6x}{\left(1-x^2\right)^2}$$

Detyra 58: $y=\dfrac{x^2-3x}{x^2-3x+2}$

Zgjidhje:

$$y'=\left(\frac{x^2-3x}{x^2-3x+2}\right)'=\frac{\left(x^2-3x\right)'\left(x^2-3x+2\right)-\left(x^2-3x\right)\left(x^2-3x+2\right)'}{\left(x^2-3x+2\right)^2}=$$

$$=\frac{(2x-3)\cdot\left(x^2-3x+2\right)-\left(x^2-3x\right)\cdot(2x-3)}{\left(x^2-3x+2\right)^2}=\frac{2x^3-9x^2+13x-6-\left(2x^3-9x^2+9x\right)}{\left(x^2-3x+2\right)^2}=$$

$$=\frac{2x^3-9x^2+13x-6-2x^3+9x^2-9x}{\left(x^2-3x+2\right)^2}=\frac{4x-6}{\left(x^2-3x+2\right)^2}=\frac{2(2x-3)}{\left(x^2-3x+2\right)^2}$$

Detyra 59: $y=\dfrac{2x^2-3x+1}{x^2-4}$

Zgjidhje:

$$y'=\left(\frac{2x^2-3x+1}{x^2-4}\right)'=\frac{\left(2x^2-3x+1\right)'\left(x^2-4\right)-\left(2x^2-3x+1\right)\left(x^2-4\right)'}{\left(x^2-4\right)^2}=$$

$$=\frac{(4x-3)\cdot\left(x^2-4\right)-\left(2x^2-3x+1\right)\cdot(2x)}{\left(x^2-4\right)^2}=\frac{4x^3-16x-3x^2+12-\left(4x^3-6x^2+2x\right)}{\left(x^2-4\right)^2}=$$

$$=\frac{4x^3-3x^2-16x+12-4x^3+6x^2-2x}{\left(x^2-4\right)^2}=\frac{3x^2-18x+12}{\left(x^2-4\right)^2}=\frac{3\left(x^2-6x+4\right)}{\left(x^2-4\right)^2}$$

Detyra 60: $y=\dfrac{x^3+1}{x^2+3x-2}$

Zgjidhje:

$$y'=\left(\frac{x^3+1}{x^2+3x-2}\right)'=\frac{\left(x^3+1\right)'\left(x^2+3x-2\right)-\left(x^3+1\right)\left(x^2+3x-2\right)'}{\left(x^2+3x-2\right)^2}=$$

$$=\frac{3x^2\cdot\left(x^2+3x-2\right)-\left(x^3+1\right)\cdot(2x+3)}{\left(x^2+3x-2\right)^2}=\frac{3x^4+9x^3-6x^2-2x^4-3x^3-2x-3}{\left(x^2+3x-2\right)^2}=$$

$$=\frac{x^4+6x^3-6x^2-2x-3}{\left(x^2+3x-2\right)^2}$$

Detyra 61: $y=\frac{(x+4)^2}{x+3}$

Zgjidhje:

$$y'=\left[\frac{(x+4)^2}{x+3}\right]'=\frac{\left[(x+4)^2\right]'(x+3)-(x+4)^2(x+3)'}{(x+3)^2}=\frac{2(x+4)(x+4)'\cdot(x+3)-(x+4)^2\cdot 1}{(x+3)^2}=$$

$$=\frac{2\cdot(x+3)(x+4)-(x+4)^2}{(x+3)^2}=\frac{2\cdot\left(x^2+4x+3x+12\right)-(x+4)^2}{(x+3)^2}=\frac{2x^2+14x+24-\left(x^2+8x+16\right)}{(x+3)^2}=$$

$$=\frac{2x^2+14x+24-x^2-8x-16}{(x+3)^2}=\frac{x^2+6x+8}{(x+3)^2}=\frac{(x+2)(x+4)}{(x+3)^2}$$

Detyra 62: $y=\frac{x^3}{(1-x)^2}$

Zgjidhje:

$$y'=\left[\frac{x^3}{(1-x)^2}\right]'=\frac{\left(x^3\right)'(1-x)^2-x^3\left[(1-x)^2\right]'}{\left[(1-x)^2\right]^2}=\frac{3x^2\left(1-x^2\right)-2\cdot(1-x)(1-x)'\cdot x^3}{\left[(1-x)^2\right]^2}=$$

$$=\frac{3(1-x)^2\cdot x^2-2(1-x)\cdot x^3}{\left[(1-x)^2\right]^2}=\frac{3\cdot\left(x^2-2x^3+x^4\right)-2\left(x^3-x^4\right)}{\left[(1-x)^2\right]^2}=\frac{3x^2-6x^3+3x^4-2x^3+2x^4}{\left[(1-x)^2\right]^2}=$$

$$=\frac{6x^4-8x^3+3x^2}{\left[(1-x)^2\right]^2}=\frac{x^2\left(6x^2-8x+3\right)}{\left[(1-x)^2\right]^2}$$

Detyra 63: $y=\frac{x^2-2x+1}{x^2+1}$

Zgjidhje:

$$y'=\left(\frac{x^2-2x+1}{x^2+1}\right)'=\frac{\left(x^2-2x+1\right)'\left(x^2+1\right)-\left(x^2-2x+1\right)\left(x^2+1\right)'}{\left(x^2+1\right)^2}=$$

$$=\frac{(2x-2)\cdot\left(x^2+1\right)-\left(x^2-2x+1\right)\cdot 2x}{\left(x^2+1\right)^2}=\frac{2x^3+2x-2x^2-2-4x-2x^3+4x^2-2x}{\left(x^2+1\right)^2}=$$

$$=\frac{2x^2-2}{\left(x^2+1\right)^2}=\frac{2\left(x^2-1\right)}{\left(x^2+1\right)^2}$$

Detyra 64: $y=\dfrac{x^2-5x+7}{x-2}$

Zgjidhje:

$$y'=\left(\frac{x^2-5x+7}{x-2}\right)'=\frac{\left(x^2-5x+7\right)'(x-2)-\left(x^2-5x+7\right)(x-2)'}{(x-2)^2}=$$

$$=\frac{(2x-5)\cdot(x-2)-\left(x^2-5x+7\right)\cdot 1}{(x-2)^2}=\frac{2x^2-4x-5x+10-x^2+5x-7}{(x-2)^2}=\frac{x^2-4x+3}{(x-2)^2}$$

Detyra 65: $y=\dfrac{1+\sqrt{x}}{1+\sqrt{2x}}$

Zgjidhje:

$$y'=\left(\frac{1+\sqrt{x}}{1+\sqrt{2x}}\right)'=\frac{\left(1+\sqrt{x}\right)'\left(1+\sqrt{2x}\right)-\left(1+\sqrt{x}\right)\left(1+\sqrt{2x}\right)'}{\left(1+\sqrt{2x}\right)^2}=$$

$$=\frac{\frac{1}{2\sqrt{x}}\cdot\left(1+\sqrt{2x}\right)-\frac{2}{2\sqrt{2x}}\cdot\left(1+\sqrt{x}\right)}{\left(1+\sqrt{2x}\right)^2}=\frac{\frac{1+\sqrt{2x}}{2\sqrt{x}}-\frac{1+\sqrt{x}}{\sqrt{2x}}}{\left(1+\sqrt{2x}\right)^2}=\frac{1+\sqrt{2x}-\sqrt{2}\cdot\left(1+\sqrt{x}\right)}{\left(1+\sqrt{2x}\right)^2}=$$

$$=\frac{1-\sqrt{2}}{2\sqrt{x}\cdot\left(1+\sqrt{2x}\right)^2}$$

Detyra 66: $y=\dfrac{4x^2+x+1}{x}$

Zgjidhje:

$$y'=\left(\frac{4x^2+x+1}{x}\right)'=\frac{\left(4x^2+x+1\right)'(x)-\left(4x^2+x+1\right)(x)'}{x^2}=$$

$$=\frac{(8x+1)\cdot x-\left(4x^2+x+1\right)\cdot 1}{x^2}=\frac{8x^2+x-4x^2-x-1}{x^2}=\frac{4x^2-1}{x^2}$$

Detyra 67: $y=\dfrac{x^2+3x-3}{x-1}$

Zgjidhje:

$$y'=\left(\frac{x^2+3x-3}{x-1}\right)'=\frac{\left(x^2+3x-3\right)'(x-1)-\left(x^2+3x-3\right)(x-1)'}{(x-1)^2}=\frac{(2x+3)\cdot(x-1)-\left(x^2+3x-3\right)\cdot 1}{(x-1)^2}=$$

$$=\frac{2x^2-2x+3x-3-x^2-3x+3}{(x-1)^2}=\frac{x^2-2x}{(x-1)^2}=\frac{x(x-2)}{(x-1)^2}$$

Detyra 68: $y=\dfrac{x-1}{x^2(x-2)}$

Zgjidhje:

$$y'=\left[\frac{x-1}{x^2(x-2)}\right]'=\frac{(x-1)'\left[x^2(x-2)\right]-(x-1)\left[x^2(x-2)\right]'}{\left[x^2(x-2)\right]^2}=$$

$$=\frac{1\cdot\left[x^2(x-2)\right]-(x-1)\cdot\left(3x^2-4x\right)}{\left[x^2(x-2)\right]^2}=\frac{x^2(x-2)-\left(3x^2-4x\right)\cdot(x-1)}{\left[x^2(x-2)\right]^2}=$$

$$=\frac{x^3-2x^2-\left(3x^3-3x^2-4x^2+4x\right)}{x^4(x-2)^2}=\frac{x^3-2x^2-3x^3+7x^2-4x}{x^4(x-2)^2}=-\frac{2x^3+5x^2-4}{x^3(x-2)^2}$$

Detyra 69: $y=\dfrac{x^4+1}{x^2}$

Zgjidhje:

$$y'=\left(\frac{x^4+1}{x^2}\right)'=\frac{\left(x^4+1\right)'\left(x^2\right)-\left(x^4+1\right)\left(x^2\right)'}{\left(x^2\right)^2}=\frac{4x^3\cdot 2x-2x\cdot\left(x^4+1\right)}{\left(x^2\right)^2}=\frac{4x^5-2x^5-2x}{\left(x^2\right)^2}=$$

$$=\frac{2x^5-2x}{x^4}=\frac{2x\left(x^4-1\right)}{x^4}=\frac{2\left(x^4-1\right)}{x^3}$$

Detyra 70: $f(x)=\dfrac{3x+1}{2x-1}$

Zgjidhje:

$$f'(x)=\left(\frac{3x+1}{2x+1}\right)'=\frac{(3x+1)'(2x-1)-(3x+1)(2x-1)'}{(2x-1)^2}=\frac{3\cdot(2x-1)-(3x+1)\cdot 2}{(2x-1)^2}=$$

$$=\frac{6x+3-6x-2}{(2x-1)^2}=-\frac{5}{(2x+1)^2}$$

Detyra 71: $f(x)=\dfrac{x^2+x-21}{x-1}$

Zgjidhje:

$$f'(x)=\left(\frac{x^2+x-21}{x-1}\right)'=\frac{\left(x^2+x-21\right)'(x-1)-\left(x^2+x-21\right)(x-1)'}{(x-1)^2}=$$

$$=\frac{(2x+1)\cdot(x-1)-\left(x^2+x-21\right)\cdot 1}{(x-1)^2}=\frac{2x^2-x-1-x^2-x+21}{(x-1)^2}=\frac{x^2-2x+20}{(x+1)^2}$$

Detyra 72: $y=\dfrac{e^x-1}{e^x}$

Zgjidhje:

$$y'=\left(\frac{e^x-1}{e^x}\right)'=\frac{\left(e^x-1\right)'\left(e^x\right)-\left(e^x-1\right)\left(e^x\right)'}{\left(e^x\right)^2}=\frac{e^x\cdot e^x-e^x\cdot e^x+e^x}{e^{2x}}=\frac{e^x}{e^{2x}}=\frac{1}{e^x}=e^{-x}$$

Detyra 73: $y=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$

Zgjidhje:

$$y'=\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)'=\frac{\left(e^x-e^{-x}\right)'\left(e^x+e^{-x}\right)-\left(e^x-e^{-x}\right)\left(e^x+e^{-x}\right)'}{\left(e^x+e^{-x}\right)^2}=\frac{\left(e^x+e^{-x}\right)\left(e^x+e^{-x}\right)-\left(e^x-e^{-x}\right)\left(e^x-e^{-x}\right)}{\left(e^x+e^{-x}\right)^2}=$$

$$=\frac{\left(e^x+e^{-x}\right)^2-\left(e^x-e^{-x}\right)^2}{\left(e^x+e^{-x}\right)^2}=\frac{e^{2x}+2+e^{-2x}-e^{2x}+2-e^{-2x}}{\left(e^x+e^{-x}\right)^2}=\frac{4}{\left(e^x+e^{-x}\right)^2}$$

Detyra 74: $f(x)=\cos\left(x^3-x\right)$

Zgjidhje:

$$f'(x)=\left[\cos\left(x^3-x\right)\right]'=-\sin\left(x^3-x\right)\cdot\left(x^3-x\right)'=-\sin\left(x^3-x\right)\cdot\left(3x^2-1\right)$$

Detyra 75: $f(x)=x\cdot\sin x$

Zgjidhje:

$$f'(x)=(x\cdot\sin x)'=(x)'\cdot\sin x+x\cdot(\sin x)'=\sin x+x\cos x$$

Detyra 76: $f\left(x\right)=\sin^{3}x^{2}$

Zgjidhje:

$$f\left(x\right)=\sin^{3}x^{2}=\sin^{6}x$$

$$f'\left(x\right)=\left(\sin^{6}x\right)'=6\sin^{5}x\cdot\left(\sin x\right)'=6\cos x\cdot\sin^{5}x$$

Detyra 77: $f\left(x\right)=\sin x+\cos x$

Zgjidhje:

$$f'\left(x\right)=\left(\sin x+\cos x\right)'=\left(\sin x\right)'+\left(\cos x\right)'=\cos x-\sin x$$

Detyra 78: $f\left(x\right)=\sin x\cdot\cos x$

Zgjidhje:

$$f'\left(x\right)=\left(\sin x\cdot\cos x\right)'=\left(\sin x\right)'\cdot\left(\cos x\right)+\left(\sin x\right)\cdot\left(\cos x\right)'=\cos x\cdot\cos x+\sin x\cdot\left(-\sin x\right)=$$
$$=\cos^{2}x-\sin^{2}x=\cos 2x$$

Detyra 79: $f\left(x\right)=x-\sin x\cdot\cos x$

Zgjidhje:

$$f'\left(x\right)=\left(x-\sin x\cdot\cos x\right)'=\left(x\right)'-\left[\left(\sin x\right)\cdot\left(\cos x\right)\right]'=1+\left(\sin x\right)'\cdot\left(\cos x\right)+\left(\sin x\right)\cdot\left(\cos x\right)'=1-\cos 2x$$

Detyra 80: $y=x-3\sin x$

Zgjidhje:

$$y'=\left(x-3\sin x\right)'=\left(x\right)'-\left(3\sin x\right)'=1-3\cos x$$

Detyra 81: $y=x\cos x$

Zgjidhje:

$$y'=\left(x\cos x\right)'=\left(x\right)'\cdot\cos x+x\cdot\left(\cos x\right)'=1\cdot\cos x+x\cdot\left(-\sin x\right)=\cos x-x\sin x$$

Detyra 82: $y=\dfrac{e^x}{\cos x}$

Zgjidhje:

$$y'=\left(\frac{e^x}{\cos x}\right)'=\frac{\left(e^x\right)'\cdot(\cos x)-e^x\cdot(\cos x)'}{(\cos x)^2}=\frac{e^x\cdot\cos x-e^x\cdot\sin x}{(\cos x)^2}=\frac{e^x\left(\cos x+\sin x\right)}{(\cos x)^2}$$

Detyra 83: $y=\dfrac{e^x}{\sin x+\cos x}$

Zgjidhje:

$$y'=\left(\frac{e^x}{\sin x+\cos x}\right)'=\frac{\left(e^x\right)'\cdot(\sin x+\cos x)-e^x\cdot(\sin x+\cos x)'}{(\sin x+\cos x)^2}=$$

$$=\frac{e^x\cdot(\sin x+\cos x)-e^x\cdot\left[(\sin x)'+(\cos x)'\right]}{(\sin x+\cos x)^2}=\frac{e^x\cdot(\sin x+\cos x)-e^x\cdot\left[\cos x+(-\sin x)\right]}{(\sin x+\cos x)^2}=$$

$$=\frac{e^x\cdot(\sin x+\cos x)-e^x\cdot(\cos x-\sin x)}{(\sin x+\cos x)^2}=\frac{e^x\sin x+e^x\cos x-e^x\cos x+e^x\sin x}{(\sin x+\cos x)^2}=\frac{2e^x\sin x}{(\sin x+\cos x)^2}$$

Detyra 84: $f(x)=\dfrac{1-\sin x}{1+\sin x}$

Zgjidhje:

$$f'(x)=\left(\frac{1-\sin x}{1+\sin x}\right)'=\frac{(1-\sin x)'(1+\sin x)-(1-\sin x)(1+\sin x)'}{(1+\sin x)^2}$$

$$=\frac{-\cos x(1+\sin x)-\cos x(1-\sin x)}{(1+\sin x)^2}=\frac{-\cos x-\cos x\sin x-\cos x+\cos x\sin x}{(1+\sin x)^2}=-\frac{2\cos x}{(1+\sin x)^2}$$

Detyra 85: $y=\dfrac{\sin x}{1+\tan x}$

Zgjidhje:

$$y'=\left(\frac{\sin x}{1+\tan x}\right)'=\frac{(\sin x)'(1+\tan x)-(\sin x)(1+\tan x)'}{(1+\tan x)^2}=\frac{\cos x(1+\tan x)-\sin x\left(\dfrac{1}{\cos^2 x}\right)}{(1+\tan x)^2}=$$

$$=\cos^3 x+\cos^2 x\sin x-\cos x\sin x=\cos x\left(\cos^2 x+\sin x\cos x-\sin x\right)$$

Detyra 86: $y=\dfrac{\sin x+\cos x}{\cos x-\sin x}$

Zgjidhje:

$$y'=\left(\frac{\sin x+\cos x}{\cos x-\sin x}\right)'=\frac{(\sin x+\cos x)'(\cos x-\sin x)-(\sin x+\cos x)(\cos x-\sin x)'}{(\cos x-\sin x)^2}=$$

$$=\frac{(\cos x-\sin x)(\cos x-\sin x)-(\sin x+\cos x)(-\sin x-\cos x)}{(\cos x-\sin x)^2}=$$

$$=\frac{(\cos x-\sin x)^2+(\sin x+\cos x)^2}{(\cos x-\sin x)^2}=\frac{\cos^2 x-2\cos x\sin x-\sin^2 x+\sin^2 x+2\cos x\sin x}{(\cos x-\sin x)^2}=$$

$$=\frac{2}{(\cos x-\sin x)^2}$$

Detyra 87: $y=\dfrac{\sin x}{1+\cos x}$

Zgjidhje:

$$y'=\left(\frac{\sin x}{1+\cos x}\right)'=\frac{(\sin x)'(1+\cos x)-(\sin x)(1+\cos x)'}{(1+\cos x)^2}=$$

$$=\frac{\cos x(1+\cos x)-\sin x(-\sin x)}{(1+\cos x)^2}=\frac{\sin^2 x+\cos x\cdot(\cos x+1)}{(1+\cos x)^2}=\left|\sin^2 x=1-\cos^2 x\right|$$

$$=\frac{1-\cos^2 x+(1+\cos x)\cdot\cos x}{(1+\cos x)^2}=\frac{1-\cos^2 x+\cos x+\cos^2 x}{(1+\cos x)^2}=\frac{1+\cos x}{(1+\cos x)^2}$$

Detyra 88: $y=\dfrac{\sin x+1}{\sin x-1}$

Zgjidhje:

$$y'=\left(\frac{\sin x+1}{\sin x-1}\right)'=\frac{(\sin x+1)'(\sin x-1)-(\sin x+1)(\sin x-1)'}{(\sin x-1)^2}$$

$$=\frac{\cos x\cdot(\sin x-1)-(\sin x+1)\cdot\cos x}{(\sin x-1)^2}=\frac{-\cos x\cdot\sin x-\cos x-\cos x\sin x-\cos x+\cos x\sin x}{(\sin x-1)^2}=$$

$$=\left|-\cos x-\cos x=-2\cos x\right|=\frac{\cos x\cdot\sin x-2\cos x-\cos x\cdot\sin x}{(\sin x-1)^2}=-\frac{2\cos x}{(1+\sin x)^2}$$

Detyra 89: $y=\dfrac{x}{1-\cos x}$

Zgjidhje:

$$y'=\left(\frac{x}{1-\cos x}\right)'=\frac{(x)'(1-\cos x)-x(1-\cos x)'}{(1-\cos x)^2}=\frac{1-\cos x-x\sin x}{(1-\cos x)^2}$$

Detyra 90: $y=\dfrac{\sin x}{x}+\dfrac{x}{\sin x}$

Zgjidhje:

$$y'=\left(\frac{\sin x}{x}\right)'+\left(\frac{x}{\sin x}\right)'=\frac{(\sin x)'\cdot x-\sin x\cdot(x)'}{x^2}+\frac{(x)'\cdot\sin x-x\cdot(\sin x)'}{\sin^2 x}=$$

$$=\frac{(\cos x)\cdot x-(\sin x)\cdot 1}{x^2}+\frac{1\cdot(\sin x)-x\cdot(\cos x)}{\sin^2 x}=\frac{\cos x\cdot x-\sin x}{x^2}+\frac{\sin x-x\cdot\cos x}{\sin^2 x}$$

Detyra 91: $y=\dfrac{\sin x}{1-\cos x}$

Zgjidhje:

$$y'=\left(\frac{\sin x}{1-\cos x}\right)'=\frac{(\sin x)'\cdot(1-\cos x)-\sin x\cdot(1-\cos x)'}{(1-\cos x)^2}=\frac{(\cos x)\cdot(1-\cos x)-(\sin x)\cdot(\sin x)}{(1-\cos x)^2}=$$

$$=\frac{\cos x-\cos^2 x-\sin^2 x}{(1-\cos x)^2}=\frac{\cos x-\cos^2 x-1+\cos^2 x}{(1-\cos x)^2}=\frac{1}{1-\cos x}$$

Detyra 92: $y=\sqrt{x}\cos^2 x$

Zgjidhje:

$$y'=\left(\sqrt{x}\cos^2 x\right)'=\left(\sqrt{x}\right)'\cos^2 x+\sqrt{x}\left(\cos^2 x\right)=\frac{1}{2\sqrt{x}}\cdot\cos^2 x+\sqrt{x}\cdot 2\cos x(\cos x)'=$$

$$=\frac{\cos^2 x}{2\sqrt{x}}+\sqrt{x}\cdot 2\cos x\cdot(-\sin x)=\frac{\cos^2 x}{2\sqrt{x}}-\sqrt{x}\cdot\sin 2x$$

Detyra 93: $y=\ln\sin\sqrt{x}$

Zgjidhje:

$$y'=\left(\ln\sin\sqrt{x}\right)'=\frac{1}{\sin\sqrt{x}}\cdot\left(\sin\sqrt{x}\right)'=\frac{1}{\sin\sqrt{x}}\cdot\cos\sqrt{x}\cdot\left(\sqrt{x}\right)'=$$

$$=\frac{1}{\sin\sqrt{x}}\cdot\cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}=\frac{1}{2\sqrt{x}}\cdot\cot\sqrt{x}$$

Detyra 94: $y=\dfrac{1-\tan x}{1+\tan x}$

Zgjidhje:

$$y'=\left(\frac{1-\tan x}{1+\tan x}\right)'=\frac{(1-\tan x)'(1+\tan x)-(1-\tan x)(1+\tan x)'}{(1+\tan x)^2}=$$

$$=\frac{-\dfrac{1}{\cos^2 x}\cdot(1+\tan x)-(1-\tan x)\cdot\dfrac{1}{\cos^2 x}}{(1+\tan x)^2}=\frac{-\dfrac{1}{\cos^2 x}-\dfrac{1}{\cos^2 x}\cdot\tan x-\dfrac{1}{\cos^2 x}+\dfrac{1}{\cos^2 x}\cdot\tan x}{(1+\tan x)^2}=$$

$$=\frac{-\dfrac{2}{\cos^2 x}}{\left(1+\dfrac{\sin x}{\cos x}\right)^2}=\frac{-\dfrac{2}{\cos^2 x}}{\left(\dfrac{\cos x+\sin x}{\cos x}\right)^2}=\frac{-\dfrac{2}{\cos^2 x}}{\dfrac{(\cos x+\sin x)^2}{(\cos x)^2}}=\frac{-2}{(\cos x+\sin x)^2}$$

Detyra 95: $y=\dfrac{\cos x}{1-\sin x}$

Zgjidhje:

$$y'=\left(\frac{\cos x}{1-\sin x}\right)'=\frac{(\cos x)'\cdot(1-\sin x)-(\cos x)\cdot(1-\sin x)'}{(1-\sin x)^2}=\frac{-\sin x\cdot(1-\sin x)-(\cos x)\cdot(-\cos x)}{(1-\sin x)^2}=$$

$$=\frac{(\cos x)\cdot(\cos x)-\sin x\cdot(1-\sin x)}{(1-\sin x)^2}=\frac{\cos^2 x-\sin x(1-\sin x)}{(1-\sin x)^2}=\frac{1-\sin^2 x-(1-\sin x)\cdot\sin x}{(1-\sin x)^2}=$$

$$=\frac{1-\sin^2 x-\sin x+\sin^2 x}{(1-\sin x)^2}=\frac{-\sin x+1}{(1-\sin x)^2}=\frac{1}{1-\sin x}$$

Detyra 96: $y=\dfrac{1}{4}\tan^4 x$

Zgjidhje:

$$y'=\left(\frac{1}{4}\tan^4 x\right)'=\frac{1}{4}\cdot 4\tan^{4-1} x\cdot(\tan x)'=\tan^3 x\cdot\frac{1}{\cos^2 x}=\frac{\tan^3 x}{\cos^2 x}$$

Detyra 97: $y=x-\tan x$

Zgjidhje:

$$y'=(x-\tan x)'=(x)'-(\tan x)'=1-\frac{1}{\cos^2 x}=\frac{\cos^2 x-1}{\cos^2 x}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$$

Detyra 98: $y=1-2\sin^2 x+\sin^4 x$

Zgjidhje:

$$\left|\begin{matrix}\textit{Zëvendësojmë}:\\ \sin x=u\end{matrix}\right| \qquad y=1-2u^2+u^4$$

$$u'=\cos x$$
$$y'=\left(-4u+4u^3\right)\cdot u'$$
$$y'=4\sin x\left(-1+\sin^2 x\right)\cdot\cos x$$
$$y'=-4\sin x\cdot\cos^2 x\cdot\cos x$$
$$y'=-4\sin x\cdot\cos^3 x$$

Detyra 99: $y=\ln\sqrt{\frac{1+\cos x}{1-\cos x}}$

Zgjidhje:

$$y'=\frac{1}{\sqrt{\frac{1+\cos x}{1-\cos x}}}\cdot\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)'=\frac{1}{\sqrt{\frac{1+\cos x}{1-\cos x}}}\cdot\frac{1}{2\sqrt{\frac{1+\cos x}{1-\cos x}}}\cdot\left(\frac{1+\cos x}{1-\cos x}\right)'=$$

$$=\frac{1}{\sqrt{\frac{1+\cos x}{1-\cos x}}}\cdot\frac{1}{2\sqrt{\frac{1+\cos x}{1-\cos x}}}\cdot\frac{(1+\cos x)'(1-\cos x)-(1+\cos x)(1-\cos x)'}{(1-\cos x)^2}=$$

$$=\frac{1}{\sqrt{\frac{1+\cos x}{1-\cos x}}}\cdot\frac{\frac{-\sin x(1-\cos x)-\sin x(1+\cos x)}{(1-\cos x)^2}}{2\sqrt{\frac{1+\cos x}{1-\cos x}}}=\frac{1}{\sqrt{\frac{1+\cos x}{1-\cos x}}}\cdot\frac{-\sin x\left[(1+\cos x)+(1-\cos x)\right]}{2(1-\cos x)^2\cdot\sqrt{\frac{1+\cos x}{1-\cos x}}}=$$

$$=\frac{1}{\sqrt{\frac{1+\cos x}{1-\cos x}}}\cdot\frac{-2\sin x}{2(1-\cos x)^2\cdot\sqrt{\frac{1+\cos x}{1-\cos x}}}=\frac{1}{\sqrt{\frac{1+\cos x}{1-\cos x}}}\cdot\frac{-\sin x}{(1-\cos x)^2\cdot\sqrt{\frac{1+\cos x}{1-\cos x}}}=$$

$$=\frac{-\sin x}{\frac{1+\cos x}{1-\cos x}\cdot\sqrt{\frac{1+\cos x}{1-\cos x}\cdot(1-\cos x)^2}}=\frac{-\sin x}{\frac{1+\cos x}{1-\cos x}\sqrt{\left(1-\cos^2 x\right)(1-\cos x)^2}}=\frac{-\sin x}{\frac{1+\cos x}{1-\cos x}\sqrt{\sin^2 x(1-\cos x)^2}}=$$

$$=\frac{-\sin x}{\frac{1+\cos x}{1-\cos x}\cdot\sin x\cdot\sqrt{(1-\cos x)^2}}=\frac{-1}{\frac{1+\cos x}{1-\cos x}\cdot(1-\cos x)}=-\frac{1}{1+\cos x}$$

Detyra 100: $y=\dfrac{\ln 3\sin x+\cos x}{3^x}$

Zgjidhje:

$$y'=\left(\frac{\ln 3\sin x+\cos x}{3^x}\right)'=\frac{\left(\ln 3\sin x+\cos x\right)'\cdot 3^x-\left(\ln 3\sin x+\cos x\right)\cdot\left(3^x\right)'}{3^{2x}}=$$

$$=\frac{\left[\left(\ln 3\right)'\sin x+\ln\left(\sin x\right)'+\left(\cos x\right)'\right]\cdot 3^x-\left(\ln 3\sin x+\cos x\right)\cdot 3^x\ln 3}{3^{2x}}=$$

$$=\frac{\left(\ln 3\cdot\cos x-\sin x\right)\cdot 3^x-\left(\ln 3\sin x+\cos x\right)\cdot 3^x\ln 3}{3^{2x}}=\frac{3^x\left[\ln 3\cdot\cos x-\sin x-\left(\ln^2 3\sin x+\ln 3\cos x\right)\right]}{3^{2x}}=$$

$$=\frac{\ln 3\cos x-\sin x-\ln^2 3\sin x-\ln 3\cos x}{3^x}=\frac{-\sin x\left(1+\ln^2 3\right)}{3^x}$$

2.5. Derivatet e rendeve të larta

Në qoftë se për funksionin $y=f\left(x\right)$ konstatohet se në një interval të ndryshores x ekziston derivati i funksionit $f'\left(x\right)$, atëherë derivati i atij quhet derivati i dytë i funksionit dhe shënohet:

$$f''\left(x\right)=\left[f'\left(x\right)\right]'=\lim_{\Delta x\to 0}\frac{f'\left(x+\Delta x\right)-f'\left(x\right)}{\Delta x}$$

Analogjikisht fitohet edhe derivatet tjera. Pra derivati i funksionit të rendit n shënohet:

$$f^{(n)}\left(x\right)=\left[f^{(n-1)}\left(x\right)\right]'=\lim_{\Delta x\to 0}\frac{f^{(n-1)}\left(x+\Delta x\right)-f^{(n-1)}\left(x\right)}{\Delta x}$$

Detyra 1: Gjeni derivatin e tretë të funksionit: $y=4x^4+3x^3+2x-1$

Zgjidhje:

$$y=4x^4+3x^3+2x-1$$

$$y'=\left(4x^4+3x^3+2x-1\right)'=16x^3+9x^2+2$$

$$y''=48x^2+18x$$

$$y'''=96x+18$$

Detyra 2: Gjeni derivatin e katër të funksionit: $y = x^3 - 2x^2 + 1$

Zgjidhje:

$$y = x^3 - 2x^2 + 1$$
$$y' = \left(x^3 - 2x^2 + 1\right)' = 3x^2 - 4x$$
$$y'' = 6x - 4$$
$$y''' = 6$$
$$y^{(4)} = 0$$

Detyra 3: Gjeni derivatin e katër të funksionit: $y = \sin x$

Zgjidhje:

$$y = \sin x$$
$$y' = \left(\sin x\right)' = \cos x$$
$$y'' = -\sin x$$
$$y''' = -\cos x$$
$$y^{(4)} = \sin x$$

Detyra 4: Gjeni derivatin e dytë për funksioni: $y = \ln\left(1 - x\right)$

Zgjidhje:

$$y = \ln\left(1 - x\right)$$
$$y' = \left[\ln\left(1 - x\right)\right]' = \frac{1}{1-x} \cdot \left(1 - x\right)' = -\frac{1}{1-x}$$
$$y'' = \left(-\frac{1}{1-x}\right)' = \frac{1}{\left(1-x\right)^2}$$

Detyra 5: Gjeni derivatin e dytë për funksioni: $y = e^{x^2}$

Zgjidhje:

$$y = e^{x^2}$$
$$y' = \left(e^{x^2}\right)' = e^{x^2} \cdot \left(x^2\right)' = 2x \cdot e^{x^2}$$
$$y'' = \left(2x \cdot e^{x^2}\right)' = \left(2x\right)' \cdot e^{x^2} + 2x \cdot \left(e^{x^2}\right)' = 2 \cdot e^{x^2} + 2x \cdot \left(2x \cdot e^{x^2}\right) = 2 \cdot e^{x^2} + 4x^2 \cdot e^{x^2} = 2 \cdot e^{x^2} \cdot \left(1 + 2x^2\right)$$

Detyra 6: Gjeni derivatin e pestë për funksioni: $y=\cos x \quad (x\in \mathrm{R})$

Zgjidhje:

$$y=\cos x$$
$$y'=(\cos x)'=-\sin x$$
$$y''=\cos x$$
$$y'''=\sin x$$
$$y^{(4)}=\cos x$$
$$y^{(5)}=-\sin x$$

Detyra 7: Gjeni derivatin e rendit n për funksioni: $y=\sin x$

Zgjidhje:

$$y=\sin x$$
$$y'=(\sin x)'=\cos x=\sin\left(x+\frac{\pi}{2}\right)$$
$$y''=\cos x\left(x+\frac{\pi}{2}\right)=\sin\left(x+2\cdot\frac{\pi}{2}\right)$$
$$y'''=\cos x\left(x+2\cdot\frac{\pi}{2}\right)=\sin\left(x+3\cdot\frac{\pi}{2}\right)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$y^{(n)}=(\sin x)^{(n)}=\sin\left(x+n\cdot\frac{\pi}{2}\right)$$

Detyra 8: Gjeni derivatin e rendit n për funksioni: $f(x)=a^x$

Zgjidhje:

$$f(x)=a^x$$
$$f'(x)=(a^x)'=a^x\ln a$$
$$f''(x)=a^x\ln^2 a$$
$$f'''(x)=a^x\ln^3 a$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$f^{(n)}(x)=a^x\ln^n a$$

Detyra 9: Gjeni derivatin e rendit *n* për funksioni: $y=\cos x$

Zgjidhje:

$$y=\cos x$$

$$y'=(\cos x)'=-\sin x=\cos\left(x+\frac{\pi}{2}\right)$$

$$y''=-\sin x\left(x+\frac{\pi}{2}\right)=\cos\left(x+\frac{\pi}{2}+\frac{\pi}{2}\right)=\cos\left(x+\frac{2\pi}{2}\right)$$

$$y'''=-\sin x\left(x+\frac{2\pi}{2}\right)=\cos\left(x+\frac{2\pi}{2}+\frac{\pi}{2}\right)=\cos\left(x+\frac{3\pi}{2}\right)$$

$$y^{(4)}=-\sin x\left(x+\frac{3\pi}{2}\right)=\cos\left(x+\frac{3\pi}{2}+\frac{\pi}{2}\right)=\cos\left(x+\frac{4\pi}{2}\right)$$

....

Supozojmë:

$$y^{(n)}=(\cos x)^{(n)}=\sin\left(x+n\cdot\frac{\pi}{2}\right)$$

Vërtetojmë për $n+1$

$$y^{(n+1)}=-\sin x\left(x+n\cdot\frac{\pi}{2}\right)=\cos\left(x+n\cdot\frac{\pi}{2}+\frac{\pi}{2}\right)=\cos\left[x+(n+1)\cdot\frac{\pi}{2}\right]$$

Detyra 10: Gjeni derivatin e rendit *n* për funksioni: $f(x)=\ln(1+x)$

Zgjidhje:

$$f(x)=\ln(1+x)$$

$$f'(x)=\left[\ln(1+x)\right]'=\frac{1}{1+x}$$

$$f''(x)=\left(\frac{1}{1+x}\right)'=-\frac{1}{(1+x)^2}$$

$$f'''(x)=\left[-\frac{1}{(1+x)^2}\right]'=\frac{2\cdot(1+x)}{(1+x)^4}=\frac{2}{(1+x)^3}$$

....

Supozojmë:

$$f^{(n)}(x)=(-1)^{(n-1)}\cdot\frac{(n-1)}{(1+x)^n}$$

Vërtetojmë :

$$f^{(n+1)}(x)=\left[f^{(n+1)}(x)\right]'=\left[(-1)^{n-1}\cdot\frac{(n-1)!}{(1+x)^{n}}\right]$$

$$f^{(n+1)}(x)=\left[(-1)^{n-1}\cdot(n-1)!\cdot\frac{1}{(1+x)^{n}}\right]$$

$$f^{(n+1)}(x)=\left[(-1)^{n-1}\cdot(n-1)!\cdot\frac{n\cdot(1+x)^{n-1}}{(1+x)^{n}}\right]$$

$$f^{(n+1)}(x)=\left[(-1)^{n-1}\cdot(n-1)!\cdot n\cdot(1+x)^{-(n+1)}\right]$$

$$f^{(n+1)}(x)=\left[-(-1)^{n-1}\cdot\frac{n!}{(1+x)^{n}}\right]$$

$$f^{(n+1)}(x)=(-1)^{n-1}\cdot\frac{n!}{(1+x)^{n}}$$

2.6. Rregullat e L'hopitalit

1^0 **<u>Pacaktushmëria e formës</u>** $\left(\frac{0}{0}\right)$**.** Le të jenë f,g funksione të përkufizuar (a,b) dhe $x_0\in(a,b)$. Në qoftë se:

$i)\ f(x_0)=g(x_0)=0$

$ii)$ Funksionet f,g janë të derivueshme në (a,b) me përjashtim ndoshta në pikën x_0

$iii)$ Ekziston $\lim\limits_{x\to x_0}\frac{f'(x)}{g'(x)}$, atëherë ekziston edhe $\lim\limits_{x\to x_0}\frac{f(x)}{g(x)}$ dhe $\lim\limits_{x\to x_0}\frac{f(x)}{g(x)}=\lim\limits_{x\to x_0}\frac{f'(x)}{g'(x)}$.

2^0 **<u>Pacaktushmëria e formës</u>** $\left(\frac{\infty}{\infty}\right)$. Le të jenë f,g funksione të përkufizuar (a,b) dhe $x_0\in(a,b)$. Në qoftë se:

$i)\lim\limits_{x\to x_0}|f(x)|=\lim\limits_{x\to x_0}|g(x)|=\infty$

$ii)$ Funksionet f,g janë të derivueshme në (a,b) me përjashtim ndoshta në pikën x_0 dhe $g(x_0)\neq 0$

$iii)$ Ekziston $\lim\limits_{x\to x_0}\frac{f(x)}{g(x)}$ atëherë ekziston dhe $\lim\limits_{x\to x_0}\frac{f(x)}{g(x)}=\lim\limits_{x\to x_0}\frac{f'(x)}{g'(x)}$.

3^0 **<u>Pacaktushmëria e formës</u>** $(0\cdot\infty)$. Në qoftë f,g funksione të tilla që $\lim\limits_{x\to x_0} f(x)=0$ dhe $\lim\limits_{x\to x_0} g(x)=\infty,$ atëherë shprehja $\lim\limits_{x\to x_0} f(x)\cdot g(x)$ paraqet një pacaktushmëri të formës $0\cdot\infty$ e cila shndërrohet në pacaktushmëri të formës $\frac{0}{0}$ ose $\frac{\infty}{0}$ në këtë mënyrë:

$$\lim_{x\to x_0} f(x)\cdot g(x)=\lim_{x\to x_0}\frac{f(x)}{\frac{1}{g(x)}}=\left(\frac{0}{0}\right) \text{ ose } \lim_{x\to x_0} f(x)\cdot g(x)=\lim_{x\to x_0}\frac{g(x)}{\frac{1}{f(x)}}=\left(\frac{\infty}{\infty}\right).$$

4^0 **<u>Pacaktushmëria e formës</u>** $(\infty-\infty)$. Në qoftë f,g funksione të tilla që $\lim\limits_{x\to x_0} f(x)=\infty$ dhe $\lim\limits_{x\to x_0} g(x)=\infty$, atëherë shprehja $\lim\limits_{x\to x_0}\left(f(x)-g(x)\right)$ paraqet një pacaktushmëri të formës $\infty-\infty$ e cila shndërrohet në pacaktushmëri të formës $\frac{0}{0}$ në këtë mënyrë:

$$\lim_{x\to x_0}\left(f(x)-g(x)\right)=\lim_{x\to x_0}\frac{\frac{1}{g(x)}-\frac{1}{f(x)}}{\frac{1}{g(x)}\cdot\frac{1}{f(x)}}=\left(\frac{0}{0}\right)$$

5^0 **<u>Pacaktushmëria e formës</u>** (0^0). Në qoftë f,g funksione të tilla që $\lim\limits_{x\to x_0} f(x)=0$ dhe $\lim\limits_{x\to x_0} g(x)=0$, atëherë shprehja $\lim\limits_{x\to x_0}\left(f(x)\right)^{g(x)}$ paraqet një pacaktushmëri të formës 0^0 e cila shndërrohet në pacaktushmëri të formës $0\cdot\infty$ në këtë mënyrë: $\lim\limits_{x\to x_0}\left(f(x)\right)^{g(x)}=e^{\lim\limits_{x\to x_0} g(x)\ln f(x)}$

6^0 **<u>Pacaktushmëria e formës</u>** $\left(\infty^0\right)$. Në qoftë f, g funksione të tilla që $\lim\limits_{x\to x_0} f\left(x\right)=\infty$ dhe $\lim\limits_{x\to x_0} g\left(x\right)=0,$ atëherë shprehja $\lim\limits_{x\to x_0}\left(f\left(x\right)\right)^{g(x)}$ paraqet një pacaktushmëri të formës ∞^0 e cila shndërrohet në pacaktushmëri të formës $0\cdot\infty$ në ketë mënyrë: $\lim\limits_{x\to x_0}\left(f\left(x\right)\right)^{g(x)}=e^{\lim\limits_{x\to x_0} g(x)\ln f(x)}$

7^0 **<u>Pacaktushmëria e formës</u>** $\left(1^\infty\right)$. Në qoftë f, g funksione të tilla që $\lim\limits_{x\to x_0} f\left(x\right)=1$ dhe $\lim\limits_{x\to x_0} g\left(x\right)=0,$ atëherë shprehja $\lim\limits_{x\to x_0}\left(f\left(x\right)\right)^{g(x)}$ paraqet një pacaktushmëri të formës 1^∞ e cila shndërrohet në pacaktushmëri të formës $0\cdot\infty$ në ketë mënyrë: $\lim\limits_{x\to x_0}\left(f\left(x\right)\right)^{g(x)}=e^{\lim\limits_{x\to x_0} g(x)\ln f(x)}$

Detyra të zgjidhura:

Detyra 1: $\lim\limits_{x\to 1}\dfrac{x^3-3x+2}{x^3-x^2-x+1}$

Zgjidhje:

$$\lim_{x\to 1}\frac{x^3-3x+2}{x^3-x^2-x+1}=\left(\frac{0}{0}\right)=\lim_{x\to 1}\frac{\left(x^3-3x+2\right)'}{\left(x^3-x^2-x+1\right)'}=\lim_{x\to 1}\frac{3x^2-2}{3x^2-2x-1}=\lim_{x\to 1}\frac{6x}{6x-2}=$$

$$=\frac{6\cdot 1}{6\cdot 1-2}=\frac{6}{6-2}=\frac{6}{4}=\frac{3}{2}$$

Detyra 2: $\lim\limits_{x\to 1}\dfrac{x^2+x-2}{3x^2+x-4}$

Zgjidhje:

$$\lim_{x\to 1}\frac{x^2+x-2}{3x^2+x-4}=\left(\frac{0}{0}\right)=\lim_{x\to 1}\frac{\left(x^2+x-2\right)'}{\left(3x^2+x-4\right)'}=\lim_{x\to 1}\frac{2x+1}{6x+1}=\frac{2\cdot 1+1}{6\cdot 1+1}=\frac{2+1}{6+1}=\frac{3}{7}$$

Detyra 3: $\lim\limits_{x\to 1}\dfrac{x^2+x-2}{3x^2+x-4}$

Zgjidhje:

$$\lim_{x\to 1}\frac{x^2+x-2}{3x^2+x-4}=\left(\frac{0}{0}\right)=\lim_{x\to 1}\frac{\left(x^2+x-2\right)'}{\left(3x^2+x-4\right)'}=\lim_{x\to 1}\frac{2x+1}{6x+1}=\frac{2\cdot 1+1}{6\cdot 1+1}=\frac{2+1}{6+1}=\frac{3}{7}$$

Detyra 4: $\lim\limits_{x\to1}\dfrac{x^3-2x^2-x+2}{x^3-7x+6}$

Zgjidhje:

$$\lim_{x\to1}\frac{x^3-2x^2-x+2}{x^3-7x+6}=\left(\frac{0}{0}\right)=\lim_{x\to1}\frac{\left(x^3-2x^2-x+2\right)'}{\left(x^3-7x+6\right)'}=\lim_{x\to1}\frac{3x^2-4x-1}{3x^2-7}=$$

$$=\frac{3\cdot1-4\cdot1-1}{3\cdot1-7}=\frac{3-4-1}{3-7}=\frac{-2}{-4}=\frac{1}{2}$$

Detyra 5: $\lim\limits_{x\to-1}\dfrac{x^3+x+2}{x+1}$

Zgjidhje:

$$\lim_{x\to-1}\frac{x^3+x+2}{x+1}=\frac{(-1)^3+(-1)+2}{-1+1}=\frac{0}{0}$$

$$\lim_{x\to-1}\frac{\left(x^3+x+2\right)'}{\left(x+1\right)'}=\lim_{x\to-1}\frac{3x^2+1}{1}=3\cdot(-1)^2+1=4$$

Detyra 6: $\lim\limits_{t\to1}\dfrac{5t^4-4t^2-1}{10-t-9t^3}$

Zgjidhje:

$$\lim_{t\to1}\frac{5t^4-4t^2-1}{10-t-9t^3}=\left(\frac{0}{0}\right)=\lim_{t\to1}\frac{\left(5t^4-4t^2-1\right)'}{\left(10-t-9t^3\right)'}=\lim_{t\to1}\frac{20t^3-8t}{-1-27t^2}=\frac{20\cdot1^3-8\cdot1}{-1-27\cdot1^2}=\frac{20-8}{1-27}=\frac{12}{-28}=-\frac{3}{7}$$

Detyra 7: $\lim\limits_{x\to0}\dfrac{\sin x}{x}$

Zgjidhje:

$$\lim_{x\to0}\frac{\sin x}{x}=\left(\frac{0}{0}\right)=\lim_{x\to0}\frac{(\sin x)'}{(x)'}=\lim_{x\to0}\frac{\cos x}{1}=\cos0=1$$

Detyra 8: $\lim\limits_{x\to1}\dfrac{2\ln x}{x-1}$

Zgjidhje:

$$\lim_{x\to1}\frac{2\ln x}{x-1}=\left(\frac{0}{0}\right)=\lim_{x\to1}\frac{(2\ln x)'}{(x-1)'}=\lim_{x\to1}\frac{\frac{2}{x}}{1}=2$$

Detyra 9: $\lim\limits_{x\to 0}\dfrac{\tan x}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\tan x}{x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{(\tan x)'}{(x)'}=\lim_{x\to 0}\frac{\frac{1}{\cos^2 x}}{1}=\lim_{x\to 0}\frac{1}{\cos^2 x}=\frac{1}{\cos^2 0}=\frac{1}{1}=1$$

Detyra 10: $\lim\limits_{x\to 0}\dfrac{x}{\tan x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{x}{\tan x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{(x)'}{(\tan x)'}=\lim_{x\to 0}\frac{1}{\frac{1}{\cos^2 x}}=\frac{1}{\frac{1}{\cos^2 0}}=\frac{1}{1}=1$$

Detyra 11: $\lim\limits_{x\to 0}\dfrac{\ln x}{x^2-1}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\ln x}{x^2-1}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{(\ln x)'}{(x^2-1)'}=\lim_{x\to 0}\frac{\frac{1}{x}}{2x}=\frac{1}{2}$$

Detyra 12: $\lim\limits_{x\to 4}\dfrac{x\sqrt{x}-8}{4-x}$

Zgjidhje:

$$\lim_{x\to 4}\frac{x\sqrt{x}-8}{4-x}=\left(\frac{0}{0}\right)=\lim_{x\to 4}\frac{\left(x\sqrt{x}-8\right)'}{(4-x)'}=\lim_{x\to 4}\frac{\left(x^{\frac{3}{2}}-8\right)'}{(4-x)}=\lim_{x\to 4}\frac{\frac{3}{2}x^{\frac{1}{2}}}{-1}=$$

$$=\frac{3}{2}\lim_{x\to 4}\frac{\sqrt{x}}{-1}=\frac{3}{2}\cdot\frac{\sqrt{4}}{-1}=\frac{3}{2}\cdot\frac{2}{-1}=-3$$

Detyra 13: $\lim\limits_{x\to -1}\dfrac{\sqrt{2x+3}-1}{\sqrt{x+5}-2}$

Zgjidhje:

$$\lim_{x\to -1}\frac{\sqrt{2x+3}-1}{\sqrt{x+5}-2}=\left(\frac{0}{0}\right)=\lim_{x\to -1}\frac{\left(\sqrt{2x+3}-1\right)'}{\left(\sqrt{x+5}-2\right)'}=\lim_{x\to -1}\frac{\frac{2}{2\sqrt{2x+3}}}{\frac{1}{2\sqrt{x+5}}}=\lim_{x\to -1}\frac{2\sqrt{x+5}}{\sqrt{2x+3}}=\frac{2\cdot\sqrt{4}}{\sqrt{1}}=\frac{2\cdot 2}{1}=4$$

Detyra 14: $\lim\limits_{x\to 0}\dfrac{e^x-1}{\sin x}$

Zgjidhje: $\lim\limits_{x\to 0}\dfrac{e^x-1}{\sin x}=\left(\dfrac{0}{0}\right)=\lim\limits_{x\to 0}\dfrac{\left(e^x-1\right)'}{\left(\sin x\right)'}=\lim\limits_{x\to 0}\dfrac{e^x}{\cos x}=\dfrac{e^0}{\cos 0}=\dfrac{1}{1}=1$

Detyra 15: $\lim\limits_{x\to 0}\dfrac{x\cos x-\sin x}{x^3}$

Zgjidhje:

$$\lim_{x\to 0}\frac{x\cos x-\sin x}{x^3}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left(x\cos x-\sin x\right)'}{\left(x^3\right)'}=\lim_{x\to 0}\frac{\cos x-x\sin x-\cos x}{3x^2}=\lim_{x\to 0}\frac{-x\sin x}{3x^2}=\left(\frac{0}{0}\right)=$$

$$=\lim_{x\to 0}\frac{\left(-x\sin x\right)'}{\left(3x^2\right)'}=\lim_{x\to 0}\frac{-\sin x-x\cos x}{6x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left(-\sin x-x\cos x\right)'}{\left(6x\right)'}=\lim_{x\to 0}\frac{-\cos x-\cos x-x\sin x}{6}=$$

$$=\frac{-\cos 0-\cos 0-0\sin 0}{6}=\frac{-2}{6}=-\frac{1}{3}$$

Detyra 16: $\lim\limits_{x\to 0}\dfrac{x-\sin x}{x^2}$

Zgjidhje:

$$\lim_{x\to 0}\frac{x-\sin x}{x^2}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left(x-\sin x\right)'}{\left(x^2\right)'}=\lim_{x\to 0}\frac{1-\cos x}{2x}=\frac{1-\cos 0}{2\cdot 0}=\left(\frac{0}{0}\right)=$$

$$\lim_{x\to 0}\frac{1-\cos x}{2x}=\lim_{x\to 0}\frac{\left(1-\cos x\right)'}{\left(2x\right)'}=\lim_{x\to 0}\frac{\sin x}{2}=\frac{0}{2}=0$$

Detyra 17: $\lim\limits_{x\to 1}\dfrac{e^x-ex}{\left(x-1\right)^2}$

Zgjidhje:

$$\lim_{x\to 1}\frac{e^x-ex}{\left(x-1\right)^2}=\left(\frac{0}{0}\right)=\lim_{x\to 1}\frac{\left(e^x-ex\right)'}{\left[\left(x-1\right)^2\right]'}=\lim_{x\to 1}\frac{e^x-e}{2\left(x-1\right)}$$

$$\lim_{x\to 1}\frac{e^x-e}{2\left(x-1\right)}=\lim_{x\to 1}\frac{\left(e^x-e\right)'}{\left[2\left(x-1\right)\right]'}=\lim_{x\to 1}\frac{e^x}{2}=\frac{e}{2}$$

Detyra 18: $\lim\limits_{x\to 0}\dfrac{e^{3x}-1}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{e^{3x}-1}{x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left(e^{3x}-1\right)'}{\left(x\right)'}=\lim_{x\to 0}\frac{3e^{3x}}{1}=\frac{3e^{3\cdot 0}}{1}=\frac{3\cdot 1}{1}=3$$

Detyra 19: $\lim\limits_{x\to 0}\dfrac{\sin x-x\cos x}{\sin^3 x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin x-x\cos x}{\sin^3 x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left(\sin x-x\cos x\right)'}{\left(\sin^3 x\right)'}=\lim_{x\to 0}\frac{\cos x-\cos x+x\sin x}{3\sin^2 x\cos x}=\lim_{x\to 0}\frac{x}{3\sin x\cos x}=$$

$$\lim_{x\to 0}\frac{x}{\sin x}\cdot\lim_{x\to 0}\frac{1}{3\cos x}=\frac{1}{3}$$

Detyra 20: $\lim\limits_{x\to 0}\dfrac{\tan x-x}{x-\sin x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\tan x-x}{x-\sin x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left(\tan x-x\right)'}{\left(x-\sin x\right)'}=\lim_{x\to 0}\frac{\dfrac{1}{\cos^2 x}-1}{1-\cos x}=\lim_{x\to 0}\frac{\dfrac{2\sin x}{\cos^3 x}}{\sin x}=$$

$$=\lim_{x\to 0}\frac{2\sin x}{\sin x\cdot\cos^3 x}=\lim_{x\to 0}\frac{2}{\cos^3 x}=\frac{2}{1}=2$$

Detyra 21: $\lim\limits_{x\to 0}\dfrac{2\cos x\left(1-\cos x\right)}{\sin^2 x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{2\cos x\left(1-\cos x\right)}{\sin^2 x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left[2\cos x\left(1-\cos x\right)\right]'}{\left(\sin^2 x\right)'}=\lim_{x\to 0}\frac{\left(2\cos x-2\cos^2 x\right)'}{\left(\sin^2 x\right)'}=$$

$$=\lim_{x\to 0}\frac{-2\sin x-4\cos x\left(-\sin x\right)}{\sin x\cdot\cos x}=\lim_{x\to 0}\frac{2\sin x\left(-1+2\cos x\right)}{2\sin x\cos x}=\lim_{x\to 0}\frac{1-\cos x}{\cos x}=\frac{-1+2}{1}=\frac{1}{1}=1$$

Detyra 22: $\lim\limits_{x\to 0}\dfrac{\tan x + x}{x-\sin x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\tan x + x}{x-\sin x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{(\tan x + x)'}{(x-\sin x)'}=\lim_{x\to 0}\frac{\frac{1}{\cos^2 x}+1}{1-\cos x}=\lim_{x\to 0}\frac{\frac{1+\cos^2 x}{\cos^2 x}}{1-\cos x}=$$

$$=\lim_{x\to 0}\frac{1+\cos^2 x}{\cos^2 x(1-\cos x)}=\frac{1+1}{1(1-1)}=\frac{2}{0}=\infty$$

Detyra 23: $\lim\limits_{x\to 0}\dfrac{3\sin x-\sin 3x}{3\tan x-\tan 3x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{3\sin x-\sin 3x}{3\tan x-\tan 3x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{(3\sin x-\sin 3x)'}{(3\tan x-\tan 3x)'}=\lim_{x\to 0}\frac{3\cos x-3\cos 3x}{\frac{3}{\cos^2 x}-\frac{3}{\cos^2 3x}}=$$

$$=\lim_{x\to 0}\frac{\cos x-\cos 3x}{\cos^2 3x-\cos^2 x}\cdot\cos^2 3x\cdot\cos^2 x=\lim_{x\to 0}\frac{\cos x-\cos 3x}{(\cos 3x-\cos x)(\cos 3x+\cos x)}\cdot\lim_{x\to 0}\cos^2 x\cdot\cos^2 3x=$$

$$=\lim_{x\to 0}\frac{-1}{\cos 3x+\cos x}\cdot 1=-\frac{1}{2}$$

Detyra 24: $\lim\limits_{x\to\infty}\dfrac{\pi-2\arctan x}{\ln\left(1+\frac{1}{x}\right)}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{\pi-2\arctan x}{\ln\left(1+\frac{1}{x}\right)}=\left(\frac{0}{0}\right)=\lim_{x\to\infty}\frac{(\pi-2\arctan x)'}{\left[\ln\left(1+\frac{1}{x}\right)\right]'}=\lim_{x\to\infty}\frac{-\frac{2}{1+x^2}}{-\frac{1}{1+\frac{1}{x}}\cdot\frac{1}{x^2}}=2\lim_{x\to\infty}\frac{x^2+x}{1+x^2}=$$

$$=2\lim_{x\to\infty}\frac{\left(\frac{x^2}{x^2}+\frac{x}{x^2}\right)}{\left(\frac{1}{x^2}+\frac{x^2}{x^2}\right)}=2\lim_{x\to\infty}\frac{\left(1+\frac{1}{x}\right)}{\left(\frac{1}{x^2}+1\right)}=2\cdot 1=2$$

Detyra 25: $\lim\limits_{x\to 0}\dfrac{2-x^2-2\cos x}{x^4}$

Zgjidhje:

$$\lim_{x\to 0}\frac{2-x^2-2\cos x}{x^4}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left(2-x^2-2\cos x\right)'}{\left(x^4\right)'}=\lim_{x\to 0}\frac{2\sin x-2x}{4x^3}=\lim_{x\to 0}\frac{\sin x-x}{2x^3}=$$

$$=\lim_{x\to 0}\frac{\left(\sin x-x\right)'}{\left(2x^3\right)'}=\lim_{x\to 0}\frac{\cos x-1}{6x^2}=\lim_{x\to 0}\frac{\left(\cos x-1\right)'}{\left(6x^2\right)'}=\lim_{x\to 0}\frac{-\sin x}{12x}=$$

$$=\lim_{x\to 0}\frac{\left(-\sin x\right)'}{\left(12x\right)'}=\lim_{x\to 0}\frac{-\cos x}{12}=-\frac{1}{12}$$

Detyra 26: $\lim\limits_{x\to 0}\dfrac{e^x-e^{-x}-2x}{x-\sin x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{e^x-e^{-x}-2x}{x-\sin x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left(e^x-e^{-x}-2x\right)'}{\left(x-\sin x\right)'}=\lim_{x\to 0}\frac{e^x+e^{-x}-2}{1-\cos x}=\lim_{x\to 0}\frac{\left(e^x+e^{-x}-2\right)'}{\left(1-\cos x\right)'}=$$

$$\lim_{x\to 0}\frac{e^x-e^{-x}}{\sin x}=\lim_{x\to 0}\frac{\left(e^x-e^{-x}\right)'}{\left(\sin x\right)'}=\lim_{x\to 0}\frac{e^x+e^{-x}}{\cos x}=\frac{1+1}{1}=\frac{2}{1}=2$$

Detyra 27: $\lim\limits_{x\to 0}\dfrac{5^x-4^x}{x^2+x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{5^x-4^x}{x^2+x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left(5^x-4^x\right)'}{\left(x^2+x\right)'}=\lim_{x\to 0}\frac{\left(5^x\right)'-\left(4^x\right)'}{2x+1}=\left|\begin{array}{l}\left(5^x\right)'=5^x\ln 5\\ \left(4^x\right)'=4^x\ln 4\end{array}\right|=$$

$$\lim_{x\to 0}\frac{5^x\cdot\ln 5-4^x\cdot\ln 4}{2x+1}=\lim_{x\to 0}\frac{5^0\cdot\ln 5-4^0\cdot\ln 4}{2\cdot 0+1}=\frac{\ln 5-\ln 4}{1}=\ln 5-\ln 4$$

Detyra 28: $\lim\limits_{x\to 0}\dfrac{\sin 4x}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin 4x}{x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left(\sin 4x\right)'}{\left(x\right)'}=\lim_{x\to 0}\frac{4\cos 4x}{1}=\frac{4\left(\cos 4\cdot 0\right)}{1}=4\left(\cos 0\right)=4\cdot 1=4$$

Detyra 29: $\lim\limits_{x\to 0}\dfrac{1-\cos 3x}{x^2}$

Zgjidhje:

$$\lim_{x\to 0}\frac{1-\cos 3x}{x^2}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{(1-\cos 3x)'}{(x^2)'}=\lim_{x\to 0}\frac{\sin 3x}{2x}=\frac{1}{2}\lim_{x\to 0}\frac{\sin 3x}{x}=\left(\frac{0}{0}\right)=\frac{1}{2}\lim_{x\to 0}\frac{(\sin 3x)'}{(x)'}=$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{3\cos 3x}{1}=\frac{3}{2}\lim_{x\to 0}\cos 3x=\frac{3}{2}\cdot 1=\frac{3}{2}$$

Detyra 30: $\lim\limits_{x\to 1}\dfrac{x^x-1}{\ln x-x+1}$

Zgjidhje:

$$\lim_{x\to 1}\frac{x^x-1}{\ln x-x+1}=\left(\frac{0}{0}\right)=\lim_{x\to 1}\frac{(x^x-1)'}{(\ln x-x+1)'}=\lim_{x\to 1}\frac{x^x(\ln x+1)}{\frac{1}{x}-1}=\lim_{x\to 1}\frac{x^x(\ln x+1)}{\frac{1-x}{x}}=$$

$$=\lim_{x\to 1}\frac{x\left(x^x(\ln x+1)\right)}{1-x}=\lim_{x\to 1}\frac{\left(x\left(x^x(\ln x+1)\right)\right)'}{(1-x)'}=\lim_{x\to 1}\frac{x^x\left(x(\ln x+1)^2+\ln x+2\right)}{-1}=$$

$$=\frac{1^1\left(1(\ln 1+1)^2+\ln 1+2\right)}{-1}=-\left(1\cdot(1)^2+0+2\right)=-2$$

Detyra 31: $\lim\limits_{x\to 0}\dfrac{a^x-b^x}{x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{a^x-b^x}{x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{(a^x-b^x)'}{(x)'}=\lim_{x\to 0}\frac{(a^x\ln a-b^x\ln b)}{x}=\lim_{x\to 0}a^x\ln a-b^x\ln b=$$

$$=a^0\ln a-b^0\ln b=\ln a-\ln b=\ln\frac{a}{b}$$

Detyra 32: $\lim\limits_{x\to 0}\dfrac{\sin x-x}{x^3}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin x-x}{x^3}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{(\sin x-x)'}{(x^3)'}=\lim_{x\to 0}\frac{\cos x-1}{3x^2}=\lim_{x\to 0}\frac{(\cos x-1)'}{(3x^2)'}=\lim_{x\to 0}\frac{-\sin x}{6x}=$$

$$=-\frac{1}{6}\cdot\lim_{x\to 0}\frac{\sin x}{x}=-\frac{1}{6}\cdot 1=-\frac{1}{6}$$

Detyra 33: $\lim\limits_{x\to 0}\dfrac{x-\tan x}{x-\sin x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{x-\tan x}{x-\sin x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left(x-\tan x\right)'}{\left(x-\sin x\right)'}=\lim_{x\to 0}\frac{1-1-\tan^2 x}{1-\cos x}=\lim_{x\to 0}\frac{-\tan^2 x}{1-\cos x}=\lim_{x\to 0}\frac{-2\tan x}{\cos^2 x\cdot\sin x}$$

$$=\lim_{x\to 0}\frac{-2}{\cos^3 x}=-2$$

Detyra 34: $\lim\limits_{x\to 0}\dfrac{\sin 4x}{3x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\sin 4x}{3x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left(\sin 4x\right)'}{\left(3x\right)'}=\lim_{x\to 0}\frac{4\cos 4x}{3}=\frac{4\cos 4\cdot 0}{3}=\frac{4\cos 0}{3}=\frac{4\cdot 1}{3}=\frac{4}{3}$$

Detyra 35: $\lim\limits_{x\to 0}\dfrac{e^x-1}{\sin 5x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{e^x-1}{\sin 5x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left(e^x-1\right)'}{\left(\sin 5x\right)'}=\lim_{x\to 0}\frac{e^x}{5\cos 5x}=\frac{e^0}{5\cos 5\cdot 0}=\frac{1}{5\cos 0}=\frac{1}{5}$$

Detyra 36: $\lim\limits_{x\to 1}\dfrac{x-1}{x^n-1}$

Zgjidhje:

$$\lim_{x\to 1}\frac{x-1}{x^n-1}=\left(\frac{0}{0}\right)=\lim_{x\to 1}\frac{\left(x-1\right)'}{\left(x^n-1\right)'}=\lim_{x\to 1}\frac{1}{nx^{n-1}}=\frac{1}{n\cdot 1^{1-1}}=\frac{1}{n}$$

Detyra 37: $\lim\limits_{x\to 0}\dfrac{e^x-e^{-x}-4x}{3x-\sin x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{e^x-e^{-x}-4x}{3x-\sin x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left(e^x-e^{-x}-4x\right)'}{\left(3x-\sin x\right)'}=\lim_{x\to 0}\frac{e^x+e^{-x}-4}{3-\cos x}=\frac{e^0+e^0-4}{3-\cos 0}=\frac{1+1-4}{3-1}=-\frac{2}{2}=-1$$

Detyra 38: $\lim_{x\to 0}\frac{\tan x-\sin x}{x-\sin x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\tan x-\sin x}{x-\sin x}=\lim_{x\to 0}\frac{(\tan x-\sin x)'}{(x-\sin x)'}=\lim_{x\to 0}\frac{\frac{1}{\cos^2 x}-\cos x}{1-\cos x}=\lim_{x\to 0}\frac{\frac{1-\cos^2 x}{\cos^2 x}}{1-\cos x}=\lim_{x\to 0}\frac{1-\cos^2 x}{\cos^2 x(1-\cos x)}=$$

$$=\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x+\cos^2 x)}{\cos^2 x\cdot(1-\cos x)}=\lim_{x\to 0}\frac{\cos^2 x+\cos x+1}{\cos^2 x}=\frac{\cos^2 0+\cos 0+1}{\cos^2 0}=\frac{1+1+1}{1}=\frac{3}{1}=3$$

Detyra 39: $\lim_{x\to 1}\frac{\ln x}{x-1}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\ln x}{x-1}=\lim_{x\to 1}\frac{(\ln x)'}{(x-1)'}=\lim_{x\to 1}\frac{\frac{1}{x}}{1}=\lim_{x\to 1}\frac{1}{x}=\frac{1}{1}=1$$

Detyra 40: $\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x-1}$

Zgjidhje:

$$\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x-1}=\lim_{x\to\frac{\pi}{6}}\frac{(2\sin^2 x+\sin x-1)'}{(2\sin^2 x-3\sin x-1)'}=\lim_{x\to\frac{\pi}{6}}\frac{4\sin x\cos x+\cos x}{4\sin x\cos x-3\cos x}=$$

$$=\frac{4\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}}{4\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}-3\frac{\sqrt{3}}{2}}=\frac{\sqrt{3}+\frac{\sqrt{3}}{2}}{\sqrt{3}-\frac{3\sqrt{3}}{2}}=\frac{\frac{3\sqrt{3}}{2}}{-\frac{\sqrt{3}}{2}}=-3$$

Detyra 41: $\lim_{x\to 1}\frac{e^x-1}{x^2}$

Zgjidhje:

$$\lim_{x\to 1}\frac{e^x-1}{x^2}=\lim_{x\to 1}\frac{(e^x-1)'}{(x^2)'}=\lim_{x\to 1}\frac{e^x}{2x}=\infty$$

Detyra 42: $\lim\limits_{x\to+\infty}\dfrac{x}{\ln x}$

Zgjidhje:

$$\lim_{x\to+\infty}\frac{x}{\ln x}=\lim_{x\to+\infty}\frac{(x)'}{[\ln x]'}=\lim_{x\to+\infty}\frac{1}{\frac{1}{x}}=\lim_{x\to+\infty}x=+\infty$$

Detyra 43: $\lim\limits_{x\to1}\dfrac{x-e^{x-1}}{(x-1)^2}$

Zgjidhje:

$$\lim_{x\to1}\frac{\left(x-e^{x-1}\right)'}{\left[(x-1)^2\right]'}=\lim_{x\to1}\frac{1-e^{x-1}}{2(x-1)}=\lim_{x\to1}\frac{\left(1-e^{x-1}\right)'}{\left[2(x-1)\right]'}=\lim_{x\to1}\frac{-e^{x-1}}{2}=\frac{-e^{1-1}}{2}=-\frac{1}{2}$$

Detyra 44: $\lim\limits_{x\to0}\dfrac{1-e^x}{x}$

Zgjidhje:

$$\lim_{x\to0}\frac{\left(1-e^x\right)'}{(x)'}=\lim_{x\to0}\frac{-e^x}{1}=\lim_{x\to0}-e^x=-e^0=-1$$

Detyra 45: $\lim\limits_{x\to0}\left(\cot x-\dfrac{1}{x}\right)$

Zgjidhje:

$$\lim_{x\to0}\left(\cot x-\frac{1}{x}\right)=\lim_{x\to0}\left(\frac{1}{\tan x}-\frac{1}{x}\right)=\lim_{x\to0}\left(\frac{x-\tan x}{x\tan x}\right)$$

$$\lim_{x\to0}\left(\frac{x-\tan x}{x\tan x}\right)=\lim_{x\to0}\frac{(x-\tan x)'}{(x\tan x)'}=\lim_{x\to0}\frac{1-\frac{1}{\cos^2 x}}{\tan x+\frac{x}{\cos^2 x}}=\lim_{x\to0}\frac{\cos^2 x-1}{\sin x\cdot\cos x+x}=\lim_{x\to0}\frac{-\sin^2 x}{\frac{1}{2}\sin 2x+x}=$$

$$=\lim_{x\to0}\frac{\left(-\sin^2 x\right)'}{\left(\frac{1}{2}\sin 2x+x\right)'}=\lim_{x\to0}\frac{-2\sin x\cdot\cos x}{\frac{1}{2}2\cos 2x+1}=0$$

Detyra 46: $\lim\limits_{x\to+\infty}\dfrac{\ln^2 x}{x^3}$

Zgjidhje:

$$\lim_{x\to+\infty}\frac{\left(\ln^2 x\right)'}{\left(x^3\right)'}=\lim_{x\to+\infty}\frac{2\ln x\cdot\frac{1}{x}}{3x^2}=\frac{2}{3}\lim_{x\to+\infty}\frac{\ln x}{x^3}=\frac{2}{3}\lim_{x\to+\infty}\frac{\left(\ln x\right)'}{\left(x^3\right)'}=\frac{2}{3}\lim_{x\to+\infty}\frac{\frac{1}{x}}{3x^2}=\frac{2}{9}\lim_{x\to+\infty}\frac{1}{x^3}=0$$

Detyra 47: $\lim\limits_{x\to\infty}\dfrac{x^2-3x+5}{7+2x-3x^2}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{x^2-3x+5}{7+2x-3x^2}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{\left(x^2-3x+5\right)'}{\left(7+2x-3x^2\right)'}=\lim_{x\to\infty}\frac{2x-3}{2-6x}$$

$$\lim_{x\to\infty}\frac{2x-3}{2-6x}=\lim_{x\to\infty}\frac{\left(2x-3\right)'}{\left(2-6x\right)'}=\lim_{x\to\infty}\frac{2}{-6}=-\frac{1}{3}$$

Detyra 48: $\lim\limits_{x\to\infty}\dfrac{3x^2+x+3}{x^2+3}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{3x^2+x+3}{x^2+3}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{\left(3x^2+x+3\right)'}{\left(x^2+3\right)'}=\lim_{x\to\infty}\frac{6x+1+0}{2x+0}=\lim_{x\to\infty}\frac{6x+1}{2x}=\frac{6}{2}=3$$

Detyra 49: $\lim\limits_{x\to0}\dfrac{\ln x}{\cot x}$

Zgjidhje:

$$\lim_{x\to0}\frac{\ln x}{\cot x}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to0}\frac{\left(\ln x\right)'}{\left(\cot x\right)'}=\lim_{x\to0}\frac{\frac{1}{x}}{\frac{1}{\sin^2 x}}=\lim_{x\to0}-\frac{\sin^2 x}{x}=-\lim_{x\to0}\frac{\sin^2 x}{x}=$$

$$=-\lim_{x\to0}\frac{\sin x}{x}\cdot\lim_{x\to0}\sin x=-\lim_{x\to0}\sin x=-\sin 0=0$$

Detyra 50: $\lim\limits_{x\to\infty}\frac{3x^2}{e^x}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{3x^2}{e^x}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{\left(3x^2\right)'}{\left(e^x\right)'}=\lim_{x\to\infty}\frac{9x^2}{e^x}=\lim_{x\to\infty}\frac{\left(9x^2\right)'}{\left(e^x\right)'}=\lim_{x\to\infty}\frac{18x}{e^x}=\lim_{x\to\infty}\frac{\left(18x\right)'}{\left(e^x\right)'}=\lim_{x\to\infty}\frac{18}{e^x}=\frac{18}{e^\infty}=0$$

Detyra 51: $\lim\limits_{x\to\infty}\frac{5x-2}{7x+3}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{5x-2}{7x+3}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{\left(5x-2\right)'}{\left(7x+3\right)'}=\lim_{x\to\infty}\frac{5\cdot 1-0}{7\cdot 1+0}=\frac{5}{7}$$

Detyra 52: $\lim\limits_{x\to 0}\frac{\tan 2x}{\ln\left(x+1\right)}$

Zgjidhje:

$$\lim_{x\to 0}\frac{\tan 2x}{\ln\left(x+1\right)}=\lim_{x\to 0}\left[\frac{\tan 2x}{\ln\left(x+1\right)}\right]'=\lim_{x\to 0}\frac{\frac{2}{\cos^2 2x}}{\frac{1}{1+x}}=\lim_{x\to 0}\frac{2\left(1+x\right)}{\cos^2 2x}=\frac{2\left(1+0\right)}{\cos^2 2\cdot 0}=2$$

53: $\lim\limits_{x\to\infty}\frac{x^5+x^4+x^3+x^2+x+1}{2x^5+x^4+x^3+x^2+x+1}$

Zgjidhje:

$$\lim_{x\to\infty}\frac{x^5+x^4+x^3+x^2+x+1}{2x^5+x^4+x^3+x^2+x+1}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{\left(x^5+x^4+x^3+x^2+x+1\right)'}{\left(2x^5+x^4+x^3+x^2+x+1\right)'}=$$

$$=\lim_{x\to\infty}\frac{5x^4+4x^3+3x^2+2x+1}{10x^4+4x^3+3x^2+2x+1}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{\left(5x^4+4x^3+3x^2+2x+1\right)'}{\left(10x^4+4x^3+3x^2+2x+1\right)'}=$$

$$=\lim_{x\to\infty}\frac{20x^3+12x^2+6x+2}{40x^3+12x^2+6x+2}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{\left(20x^3+12x^2+6x+2\right)'}{\left(40x^3+12x^2+6x+2\right)'}=$$

$$=\lim_{x\to\infty}\frac{120x+24}{240x+24}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{\left(120x+24\right)'}{\left(240x+24\right)'}=\lim_{x\to\infty}\frac{120}{240}=\frac{120}{240}=\frac{1}{2}$$

Detyra 54: $\lim\limits_{x\to 0}\left(\dfrac{1}{x\sin x}-\dfrac{1}{x^2}\right)$

Zgjidhje:

$$\lim_{x\to 0}\left(\frac{1}{x\sin x}-\frac{1}{x^2}\right)=(\infty-\infty)=\lim_{x\to 0}\frac{x-\sin x}{x^2\cdot\sin x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{(x)'-(\sin x)'}{\left(x^2\right)'\sin x+(\sin x)'x^2}=$$

$$=\lim_{x\to 0}\frac{1-\cos x}{2x\sin x+x^2\cos x}=\lim_{x\to 0}\frac{(1-\cos x)'}{(2x)'\sin x+(\sin x)'2x+\left(x^2\right)'\cos x+(\cos x)'x^2}=$$

$$=\lim_{x\to 0}\frac{\sin x}{2\sin x+2x\cos x+2x\cos x-x^2\sin x}=\Big|:\sin x$$

$$=\lim_{x\to 0}\frac{\frac{\sin x}{\sin x}}{\frac{2\sin x}{\sin x}+\frac{2x\cos x}{\sin x}+\frac{2x\cos x}{\sin x}-\frac{x^2\sin x}{\sin x}}=\lim_{x\to 1}\frac{1}{2+\frac{4x\cos x}{\sin x}-x^2}=$$

$$=\lim_{x\to 1}\frac{1}{2+4x\cot x-x}=\frac{1}{2+4\cdot 0\cdot\cot 0\cdot 0^2}=\frac{1}{2}$$

Detyra 55: $\lim\limits_{x\to 0}\dfrac{x-\sin x}{2+2x+x^2-2e^x}$

Zgjidhje:

$$\lim_{x\to 0}\frac{x-\sin x}{2+2x+x^2-2e^x}=\lim_{x\to 0}\left(\frac{x-\sin x}{2+2x+x^2-2e^x}\right)'=\lim_{x\to 0}\frac{1-\cos x}{2+2x-2e^x}=\lim_{x\to 0}\left(\frac{1-\cos x}{2+2x-2e^x}\right)'=$$

$$=\lim_{x\to 0}\frac{\sin x}{2-2e^x}=\lim_{x\to 0}\left(\frac{\sin x}{2-2e^x}\right)'=\lim_{x\to 0}\frac{\cos x}{-2e^x}=\frac{\cos 0}{-2e^0}=-\frac{1}{2}$$

Detyra 56: $\lim\limits_{x\to 0}\left(\dfrac{1}{x}-\dfrac{1}{e^x-1}\right)$

Zgjidhje:

$$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)=(\infty-\infty)=\lim_{x\to 0}\left(\frac{e^x-1-x}{xe^x-x}\right)\lim_{x\to 0}\left[\left(\frac{e^x-1-x}{xe^x-x}\right)\right]'=\lim_{x\to 0}\frac{e^x-1}{e^x+xe^x-1}=$$

$$=\lim_{x\to 0}\left(\frac{e^x-1}{e^x+xe^x-1}\right)'=\lim_{x\to 0}\frac{e^x}{2e^x+xe^x}=\lim_{x\to 0}\left(\frac{e^x}{2e^x+xe^x}\right)'=\lim_{x\to 0}\frac{1}{2+x}=\frac{1}{2}$$

Detyra 57: $\lim\limits_{x\to 1}\frac{\sin \pi x}{\ln x}$

Zgjidhje:

$$\lim_{x\to 1}\frac{\sin \pi x}{\ln x}=\lim_{x\to 1}\frac{(\sin \pi x)'}{(\ln x)'}=\lim_{x\to 1}\frac{\pi\cos \pi x}{\frac{1}{x}}=-\pi$$

Detyra 58: $\lim\limits_{x\to 0}x^x$

Zgjidhje:

$$\lim_{x\to 0}x^x=\left(0^0\right)=\lim_{x\to 0}e^{\ln x^x}=e^{\lim\limits_{x\to 0}x\ln x}=e^{\lim\limits_{x\to 0}\frac{\ln x}{\frac{1}{x}}}=e^{\lim\limits_{x\to 0}\frac{(\ln x)'}{(x)'}}=e^{\lim\limits_{x\to 0}\frac{\frac{1}{x}}{-\frac{1}{x^2}}}=e^{-\lim\limits_{x\to 0}\frac{x^2}{x}}=$$

$$=e^{-\lim\limits_{x\to 0}x}=e^{-0}=\frac{1}{e^0}=\frac{1}{1}=1$$

Detyra 59: $\lim\limits_{x\to 0}(\sin x)^{\tan x}$

Zgjidhje: $\lim\limits_{x\to 0}(\sin x)^{\tan x}=e^{\lim\limits_{x\to 0}\tan\ln(\sin x)}=e^{\lim\limits_{x\to 0}\frac{\ln(\sin x)}{\tan x}}=e^{\lim\limits_{x\to 0}\frac{\frac{1}{\sin x}\cdot\cos x}{\cos\sin^2 x}}=e^{\lim\limits_{x\to 0}(-\sin x\cdot\cos x)}=e^0=1$

Detyra 60: $\lim\limits_{x\to 0}(\cot x)^x$

Zgjidhje:

$$\lim_{x\to x_0}(\cot x)^x=e^{\lim x\cdot\ln\cot x}=e^{\lim\limits_{x\to 0}\frac{\ln\cot x}{\frac{1}{x}}}=e^{\lim\limits_{x\to 0}\frac{\frac{1}{\cot x\cdot\sin^2 x}}{\frac{1}{x^2}}}=e^0=1$$

Detyra 61: $\lim\limits_{x\to 0}(\sin x)^{\sin x}$

Zgjidhje:

$$\lim_{x\to 0}(\sin x)^{\sin x}=e^{\lim\limits_{x\to 0}(\sin x)\ln\sin x}=e^{\lim\limits_{x\to 0}\frac{(\ln\sin x)'}{\left(\frac{1}{\sin x}\right)'}}=e^{\lim\limits_{x\to 0}\frac{\frac{1}{\sin x}\cdot(\sin x)'}{\frac{\cos x}{\sin^2 x}}}=e^{\lim\limits_{x\to 0}\frac{-\frac{\cos x}{\sin x}}{\frac{\cos x}{\sin x}}}=e^{\lim\limits_{x\to 0}(-\sin x)}=e^0=1$$

Detyra 62: $\lim\limits_{x\to\frac{\pi}{2}}(\cos x)^{\cos x}$

Zgjidhje:

$$\lim_{x\to\frac{\pi}{2}}(\cos x)^{\cos x}=e^{\lim\limits_{x\to\frac{\pi}{2}}\cos x\cdot\ln\cos x}=e^{\lim\limits_{x\to\frac{\pi}{2}}\frac{(\ln\cos x)'}{\left(\frac{1}{\cos x}\right)'}}=e^{\lim\limits_{x\to\frac{\pi}{2}}\frac{\frac{1}{\cos x}\cdot(\cos x)'}{\frac{\sin x}{\cos x}}}=e^{\lim\limits_{x\to\frac{\pi}{2}}\frac{\frac{-\sin x}{\cos x}}{\frac{\sin x}{\cos x}}}=e^{\lim\limits_{x\to\frac{\pi}{2}}(-\cos x)}=e^0=1$$

Detyra 63: $\lim\limits_{x\to 0}[x\cdot\cot x]$

Zgjidhje:

$$\lim_{x\to 0}[x\cdot\cot x]=\lim_{x\to 0}\frac{\cot x}{\frac{1}{x}}=\lim_{x\to 0}\frac{1}{-\frac{1}{x^2}\left(-\sin^2 x\right)}=\lim_{x\to 0}\frac{x^2}{\sin^2 x}=\lim_{x\to 0}\left(\frac{x^2}{\sin^2 x}\right)'=\lim_{x\to 0}\frac{2x}{2\sin x\cdot\cos x}=1$$

Detyra 64: $\lim\limits_{x\to -2}\left(x^2-4\right)^{x+2}$

Zgjidhje:

$$\lim_{x\to -2}\left(x^2-4\right)^{x+2}=e^{\lim\limits_{x\to -2}(x+2)\ln\left(x^2-4\right)}=e^{\lim\limits_{x\to -2}\frac{\ln\left(x^2-4\right)}{\left(\frac{1}{x+2}\right)}}=e^{\lim\limits_{x\to -2}\frac{\left(\frac{2x}{x^2-4}\right)}{\left(\frac{1}{x+2}\right)^2}}=e^{\lim\limits_{x\to -2}\frac{-2x(x+2)}{x-2}}=e^{\frac{-2(-2)(-2)\cdot 0}{-4}}=e^0=1$$

Detyra 65: $\lim\limits_{x\to\infty}x^{\frac{1}{x}}$

Zgjidhje:

Shënojmë $y=x^{\frac{1}{x}}\Rightarrow\ln y=\dfrac{\ln x}{x}$

$$\lim_{x\to\infty}x^{\frac{1}{x}}=\lim_{x\to\infty}\ln y=\lim_{x\to\infty}\frac{\ln x}{x}=\lim_{x\to\infty}\frac{\frac{1}{x}}{1}=\lim_{x\to\infty}\frac{1}{x}=0\Rightarrow\lim_{x\to\infty}y=\lim_{x\to\infty}x^{\frac{1}{x}}=e^0=1$$

Detyra 66: $\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)^{\frac{1}{1-\cos x}}$

Zgjidhje:

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{1-\cos x}}=e^{\lim\limits_{x\to 0}\frac{1}{1-\cos x}\cdot\ln\frac{\sin x}{x}}=e^{\lim\limits_{x\to 0}\frac{\ln\sin x-\ln x}{1-\cos x}}=e^{\lim\limits_{x\to 0}\frac{\frac{\cos x}{\sin x}-\frac{1}{x}}{\sin x}}=e^{\lim\limits_{x\to 0}\frac{x\cos x-\sin x}{x\sin^2 x}}=e^{\lim\limits_{x\to 0}\frac{\cos x-x\sin x-\cos x}{\sin^2 x+2x\sin x\cdot\cos x}}=$$

$$=e^{-\frac{2}{6}}=\frac{1}{\sqrt[3]{e}}$$

Detyra 67: $\lim\limits_{x\to 0}(\cos x)^{\frac{1}{x}}$

Zgjidhje:

$$\lim_{x\to 0}(\cos x)^{\frac{1}{x}}=\lim_{x\to 0}e^{\frac{1}{x}\ln\cos x}=e^{\lim\limits_{x\to 0}\frac{\ln\cos x}{x}}=e^{\lim\limits_{x\to 0}\frac{\frac{\sin x}{\cos x}}{1}}=e^{0}=1$$

Detyra 68: $\lim\limits_{x\to 0}\left(\cot^2 x-\dfrac{1}{x^2}\right)$

Zgjidhje:

$$\lim_{x\to 0}\left(\cot^2 x-\frac{1}{x^2}\right)=\lim_{x\to 0}\left(\frac{1}{\frac{1}{\cot^2 x}}-\frac{1}{x^2}\right)=\lim_{x\to 0}\left(\frac{x^2-\frac{1}{\cot^2 x}}{x^2\cdot\frac{1}{\cot^2 x}}\right)=\lim_{x\to 0}\frac{x^2\cos^2 x-\sin^2 x}{x^2\sin^2 x}=$$

$$=\lim_{x\to 0}\frac{(x\cos x-\sin x)(x\cos x+\sin x)}{x^2\sin^2 x}=\lim_{x\to 0}\frac{x\cos x+\sin x}{x}\cdot\lim_{x\to 0}\frac{x\cos x-\sin x}{x\sin^2 x}=$$

$$=2\lim_{x\to 0}\frac{\cos x-x\sin x-\cos x}{\sin^2 x+2x\sin x\cdot\cos x}=2\lim_{x\to 0}\frac{-x\sin x}{\sin^2 x+2x\sin x\cdot\cos x}=2\lim_{x\to 0}\frac{-1}{\frac{\sin x}{x}+2\cos x}=-\frac{2}{3}$$

Detyra 69: $\lim\limits_{x\to 0}x^{\sin x}$

Zgjidhje:

$$\lim_{x\to 0}x^{\sin x}=\begin{vmatrix}y=x^{\sin x}\\ \ln y=\sin x\cdot\ln x\end{vmatrix}=\lim_{x\to 0}\ln y=\lim_{x\to 0}\sin x\cdot\ln x=\lim_{x\to 0}\frac{\ln x}{\frac{1}{\sin x}}=$$

$$=\lim_{x\to 0}\frac{\frac{1}{x}}{\frac{\cos x}{\sin^2 x}}=\lim_{x\to 0}\frac{\sin^2 x}{x\cos x}=-\lim_{x\to 0}\frac{\sin 2x}{-x\sin x+\cos x}=0$$

$$\lim_{x\to 0}y=0\Rightarrow\ln\lim_{x\to 0}y=0\Rightarrow e^{0}=\lim_{x\to 0}y\Rightarrow\lim_{x\to 0}y=1$$

Detyra 70: $\lim\limits_{x\to 1}\dfrac{3\sin \pi x-\sin 3\pi x}{(x-1)^3}$

Zgjidhje:

$$\lim_{x\to 1}\frac{3\sin \pi x-\sin 3\pi x}{(x-1)^3}=\lim_{x\to 1}\frac{(3\sin \pi x-\sin 3\pi x)'}{\left[(x-1)^3\right]'}=\lim_{x\to 1}\frac{3\pi\cos \pi x-3\pi\cos 3\pi x}{3(x-1)^2}=$$

$$=\lim_{x\to 1}\frac{(3\pi\cos \pi x-3\pi\cos 3\pi x)'}{\left[3(x-1)^2\right]'}=\lim_{x\to 1}\frac{-3\pi^2\sin \pi x+9\pi^2\sin 3\pi x}{6(x-1)}=$$

$$=\lim_{x\to 1}\frac{(-3\pi^2\sin \pi x+9\pi^2\sin 3\pi x)'}{\left[6(x-1)\right]'}=\lim_{x\to 1}\frac{-3\pi^3\cos \pi x+27\pi^3\cos 3\pi x}{6}=-\frac{24\pi^3}{6}=-4\pi^3$$

3. INTEGRALE

3.1. Integralet e pacaktuar

Le të jetë f funksion i përkufizuar në një interval I. Funksioni i derivueshëm F quhet funksion primitiv i funksionit f nëse $F^{'}(x) = f(x)(x \in I)$.

Nëse F është funksion primitiv i funksionit f në intervalin I, atëherë çdo funksion tjetër primitiv Φ i funksionit f në I ka formën $\Phi(x) = F(x) + C$. ku C është një konstantë e çfarëdoshme.

Bashkësia e të gjitha funksioneve primitive të funksionit f në intervalin I, quhet integral i pacaktuar i funksionit f dhe shënohet $\int f(x)dx$. Pra

$$\int f(x)dx = F(x) + C.$$

Funksioni f quhet funksion nënintegral, kurse shprehja $f(x)dx$ quhet shprehje nënintegrale.

Janë të vërteta barazimet :

1° $\int dF(x)dx = F(x) + C$

2° $\left(\int f(x)dx\right)' = f(x)$

3° $d\left(\int f(x)dx\right) = f(x)dx$

4° $\int cf(x)dx = c\int f(x)dx$

5° $\int\left(f(x) \pm g(x)\right)dx = \int f(x)dx \pm \int g(x)dx$

Tabela e integraleve. Duke pasur parasysh tabelën e derivateve të funksioneve elementare, në vazhdim po e japim tabelën e integraleve të pacaktuar të disa funksioneve elementare.

$1^\circ \int x^n dx = \frac{x^{n+1}}{n+1} + \mathrm{C}\left(n \neq -1\right)$

$2^\circ \int \frac{dx}{x} = \ln|x| + \mathrm{C}$

$3^\circ \int e^x dx = e^x + \mathrm{C}$

$4^\circ \int a^x dx = \frac{a^x}{\ln a} + \mathrm{C}$

$5^\circ \int \sin x dx = -\cos x + \mathrm{C}$

$6^\circ \int \cos x dx = \sin x + \mathrm{C}$

$$7^\circ \int e^{-x}dx = -e^{x} + C$$

$$8^\circ \int \frac{dx}{\cos^2 x} = \tan x + C$$

$$9^\circ \int \frac{dx}{\sin^2 x} = -\cot x + C$$

$$10^\circ \int \frac{dx}{1+x^2} = \begin{cases} \arctan x + C \\ -\operatorname{arc}\cot x + C \end{cases}$$

$$11^\circ \int \frac{dx}{\sqrt{1+x^2}} = \begin{cases} \arcsin x + C \\ -\operatorname{arc}\cos x + C \end{cases}$$

$$12^\circ \int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

$$13^\circ \int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} + C$$

$$14^\circ \int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right| + C$$

$$15^\circ \int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$$

Vetitë e integralit të pacaktuar

$$1^0 \int 0dx = C$$

$$2^0 \int dx = x + C$$

$$3^0 \int kf(x)dx = k\int f(x)dx$$

$$4^0 \int \left[f(x) \pm g(x)\right]dx = \int f(x)dx \pm \int g(x)dx$$

$$5^0 \int \left\{f_1(x) + f_2(x) + \ldots + f_n(x)\right\}dx = \int f_1(x)dx + \int f_2(x)dx + \ldots + \int f_n(x)dx$$

Metoda e zëvendësimit

Le të jetë $F(u)$ funksion primitiv i funksionit $f(u)$ dhe $u=\varphi(x)$ funksion i derivueshëm. Nëse ekziston funksioni i përbërë $F(\varphi(x))$, atëherë $F(\varphi(x))$ është funksion primitiv i funksionit $f(\varphi(x))\varphi^{'}(x)$, d.m.th. $\int f(\varphi(x))\varphi^{'}(x)dx = F(\varphi(x))+C.$

Integrimi i funksioneve racionale

Funksioni i formës $f(x)=\dfrac{p(x)}{q(x)}$ ku p, q janë polinome quhet funksion racional.

$1)\int \dfrac{A}{x-a}dx$

$2)\int \dfrac{A}{(x-a)^k}dx\left(k\geq 2\right)$

$3)\int \dfrac{Ax+B}{x^2+px+q}dx\left(p^2-4q<0\right)$

$4)\int \dfrac{Ax+B}{\left(x^2+px+q\right)^2}dx\left(k\geq 2\wedge p^2-4q<0\right)$

janë integralet e funksioneve elementare racionale

Integrimi i funksioneve trigonometrike

$\int R\left(\sin x, \cos x\right)dx,$ R- funksion racional i sin*x* dhe cos*x*. Zëvendësin universal për këto është

$\tan\dfrac{x}{2}=t$. Me që:

$$\sin x=\frac{2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}},\ \sin x=\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}},\ \sin x=\frac{2t}{1+t^2}$$

$$\cos x=\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}}=\frac{1-t^2}{1+t^2}$$

Nga zëvendësimi $\tan\dfrac{x}{2}=t\Rightarrow dx=\dfrac{2dt}{1+t^2}$

Integrim me pjesë

Integrimi me pjesë bëhet sipas kësaj formule: $\int udv=u\cdot v-\int vdu.$

Detyra të zgjidhura:

Detyra 1: $\int x^2 dx$

Zgjidhje:

$$\int x^2 dx = \frac{x^{2+1}}{2+1} + \mathrm{C} = \frac{x^3}{3} + \mathrm{C}$$

Detyra 2: $\int \frac{dx}{x^2}$

Zgjidhje:

$$\int \frac{dx}{x^2} = \int x^{-2} dx = \frac{x^{-2+1}}{-2+1} + \mathrm{C} = \frac{x^{-1}}{-1} + \mathrm{C} = -\frac{1}{x} + \mathrm{C}$$

Detyra 3: $\int \frac{dx}{\sqrt{x^3}}$

Zgjidhje:

$$\int \frac{dx}{\sqrt{x^3}} = \int x^{-\frac{3}{2}} dx = \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + \mathrm{C} = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + \mathrm{C} = -\frac{2}{\sqrt{x}} + \mathrm{C} = -\frac{2\sqrt{x}}{x} + \mathrm{C}$$

Detyra 4: $\int x^{\frac{1}{2}} dx$

Zgjidhje:

$$\int x^{\frac{1}{2}} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \mathrm{C} = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \mathrm{C} = \frac{2}{3}\sqrt{x^3} + \mathrm{C} = \frac{2x\sqrt{x}}{3} + \mathrm{C}$$

Detyra 5: $\int dx$

Zgjidhje:

$$\int dx = x + \mathrm{C} \;\left(\text{nga vetia } \int d\left(f\left(x\right)\right) = f\left(x\right) + \mathrm{C}\right)$$

Detyra 6: $\int \left(x^2 + x\right) dx$

Zgjidhje:

$$\int \left(x^2 + x\right) dx = \int x^2 dx + \int x dx = \frac{x^{2+1}}{2+1} + \frac{x^{1+1}}{1+1} + \mathrm{C} = \frac{x^3}{3} + \frac{x^2}{2} + \mathrm{C}$$

Detyra 7: $\int\left(-\frac{2}{5}\right)xdx$

Zgjidhje:

$$\int\left(-\frac{2}{5}\right)xdx=-\frac{2}{5}\int xdx=-\frac{2}{5}\cdot\frac{x^{1+1}}{1+1}+\mathrm{C}=-\frac{2}{5}\cdot\frac{x^{2}}{2}+\mathrm{C}=-\frac{x^{2}}{5}+\mathrm{C}$$

Detyra 8: $\int(-x)\,dx$

Zgjidhje:

$$\int(-x)\,dx=-\int xdx=-\frac{x^{1+1}}{1+1}+\mathrm{C}=\frac{x^{2}}{2}+\mathrm{C}$$

Detyra 9: $\int\sqrt{x}dx$

Zgjidhje:

$$\int\sqrt{x}dx=\int x^{\frac{1}{2}}dx=\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\mathrm{C}=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\mathrm{C}=\frac{2}{3}\sqrt{x^{3}}+\mathrm{C}$$

Detyra 10: $\int x\sqrt[3]{x}dx$

Zgjidhje:

$$\int x\sqrt[3]{x}dx=\int x\cdot x^{\frac{1}{3}}dx=\int x^{1+\frac{1}{3}}dx=\int x^{\frac{4}{3}}dx=\frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1}+\mathrm{C}=\frac{x^{\frac{7}{3}}}{\frac{7}{3}}+\mathrm{C}=\frac{3}{7}\sqrt[3]{x^{7}}+\mathrm{C}$$

Detyra 11: $\int\frac{x-3}{x^{4}}dx$

Zgjidhje:

$$\int\frac{x-3}{x^{4}}dx=\int\frac{x}{x^{4}}dx-\int\frac{3}{x^{4}}dx=\int\frac{dx}{x^{3}}-3\int\frac{dx}{x^{4}}=\int x^{-3}dx-3\int x^{-4}dx=\frac{x^{-3+1}}{-3+1}-3\frac{x^{-4+1}}{-4+1}+\mathrm{C}=$$
$$=-\frac{1}{2x^{2}}+\frac{1}{x^{3}}+\mathrm{C}$$

Detyra 12: $\int\left(3x^{4}+6x+\sqrt{34}\right)dx$

Zgjidhje:

$$\int\left(3x^{4}+6x+\sqrt{34}\right)dx=3\int x^{4}dx+6\int xdx+\sqrt{34}\int dx=\frac{3}{5}x^{5}+3x^{2}+\sqrt{34}x+\mathrm{C}$$

Detyra 13: $\int\left(x^2+3x+4\right)dx$

Zgjidhje:

$$\int\left(x^2+3x+4\right)dx=\int x^2dx+\int 3xdx+\int 4dx=\frac{x^{2+1}}{2+1}+3\int xdx+4\int dx=\frac{x^3}{3}+3\frac{x^{1+1}}{1+1}+4x+\mathrm{C}=$$

$$=\frac{x^3}{3}+3\frac{x^2}{2}+4x+\mathrm{C}$$

Detyra 14: $\int\left(2-3x-5x^2\right)dx$

Zgjidhje:

$$\int\left(2-3x-5x^2\right)dx=\int 2dx-\int 3xdx-\int 5x^2dx=2\int dx-2\int xdx-5\int x^2dx=$$

$$=2x-3\frac{x^2}{2}-5\frac{x^3}{3}+\mathrm{C}$$

Detyra 15: $\int\left(x^2+2x-3\right)dx$

Zgjidhje:

$$\int\left(x^2+2x-3\right)dx=\int x^2dx+\int 2xdx-\int 3dx=\int x^2dx+2\int xdx-3\int dx=\frac{x^3}{3}+2\frac{x^2}{2}-3x+\mathrm{C}$$

Detyra 16: $\int\left(2x^3-3x^2+2x-5\right)dx$

Zgjidhje:

$$\int\left(2x^3-3x^2+2x-5\right)dx=\int 2x^3dx-\int 3x^2dx+\int 2xdx-\int 5dx=$$

$$=2\frac{x^{3+1}}{3+1}-3\frac{x^{2+1}}{2+1}+2\frac{x^{1+1}}{1+1}-5x+\mathrm{C}=\frac{x^4}{2}-x^3+x^2-5x+\mathrm{C}$$

Detyra 17: $\int\left(5x^4-3x^3+x^2+7x+10\right)dx$

Zgjidhje:

$$\int\left(5x^4-3x^3+x^2+7x+10\right)dx=\int 5x^4dx-\int 3x^3+\int x^2+\int 7xdx+\int 10dx=$$

$$=5\int x^4dx-3\int x^3+\int x^2+7\int xdx+10\int dx=5\frac{x^{4+1}}{4+1}-3\frac{x^{3+1}}{3+1}+\frac{x^{2+1}}{2+1}+7\frac{x^{1+1}}{1+1}+10x+\mathrm{C}=$$

$$=5\frac{x^5}{5}-3\frac{x^4}{4}+\frac{x^3}{3}+7\frac{x^2}{2}+10x+\mathrm{C}=x^5-\frac{3}{4}x^4+\frac{1}{3}x^3+\frac{7}{2}x^2+10x+\mathrm{C}$$

Detyra 18: $\int\left(2x^3+3x^2-1\right)dx$

Zgjidhje:

$$\int\left(2x^3+3x^2-1\right)dx=\int 2x^3dx+\int 3x^2dx-\int 1dx=2\frac{x^{3+1}}{3+1}+3\frac{x^{2+1}}{2+1}-x+C=\frac{x^4}{2}+x^3-x+C$$

Detyra 19: $\int\frac{dx}{x\sqrt{x}}$

Zgjidhje:

$$\int\frac{dx}{x\sqrt{x}}=\int\frac{dx}{x\cdot x^{\frac{1}{2}}}=\int\frac{dx}{x^{\frac{3}{2}}}=\int x^{-\frac{3}{2}}dx=\frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}=\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}+C=-2x^{-\frac{1}{2}}+C=-2\cdot\frac{1}{x^{\frac{1}{2}}}+C=$$

$$=-2\frac{1}{\sqrt{x}}+C=-\frac{2}{\sqrt{x}}+C$$

Detyra 20: $\int(1-x)\sqrt{x}dx$

Zgjidhje:

$$\int(1-x)\sqrt{x}dx=\int\left(\sqrt{x}-x\sqrt{x}\right)dx=\int x^{\frac{1}{2}}dx-\int x^{\frac{3}{2}}dx=\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}-\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+C=$$

$$=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+C=\frac{2}{3}x^{\frac{3}{2}}-\frac{2}{5}x^{\frac{5}{2}}+C$$

Detyra 21: $\int\left(\frac{2}{\sqrt{x}}-\frac{1}{x^2}+\frac{4}{\sqrt[4]{x^3}}\right)dx$

Zgjidhje:

$$\int\left(\frac{2}{\sqrt{x}}-\frac{1}{x^2}+\frac{4}{\sqrt[4]{x^3}}\right)dx=\int\left(2x^{-\frac{1}{2}}-x^{-2}+4x^{-\frac{3}{4}}\right)dx=2\frac{1}{-\frac{1}{2}+1}x^{-\frac{1}{2}+1}-\frac{1}{-2+1}x^{-2+1}+4\frac{1}{-\frac{3}{4}+1}x^{-\frac{3}{4}+1}+C=$$

$$=4\sqrt{x}+\frac{1}{x}+16-\sqrt[4]{x}+C$$

Detyra 22: $\int \sqrt[3]{x}dx$

Zgjidhje:

$$\int \sqrt[3]{x}dx = \int x^{\frac{1}{3}}dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + \mathrm{C} = \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + \mathrm{C} = \frac{3}{4}x^{\frac{4}{3}} + \mathrm{C}$$

Detyra 23: $\int \left(x^2 - 3x + 4\right)dx$

Zgjidhje: $\left| Formula: \ \int x^n dx = \frac{x^{n+1}}{n+1} + \mathrm{C} \right|$

$$\int \left(x^2 - 3x + 4\right)dx = \int x^2 dx - \int 3xdx + \int 4dx = \int x^2 dx - 3\int xdx + 4\int dx = \frac{x^{2+1}}{2+1} - 3\frac{x^{1+1}}{1+1} + 4x + \mathrm{C} =$$
$$= \frac{x^3}{3} - 3\frac{x^2}{2} + 4x + \mathrm{C}$$

Detyra 24: $\int a^x \cdot e^x dx$

Zgjidhje:

$$\left| Formula: \ \int a^x dx = \frac{a^x}{\log a} + \mathrm{C} \right|$$

$$\int a^x \cdot e^x dx = \int (ae)^x dx = \frac{(ae)^x}{\log(ae)} + \mathrm{C} = \frac{a^x \cdot e^x}{\log(ae)} + \mathrm{C}$$

Detyra 25: $\int 2^{2x} \cdot 3^x dx$

Zgjidhje:

$$\int 2^{2x} \cdot 3^x dx = \int 4^x \cdot 3^x dx = \int (4 \cdot 3)^x dx = \int 12^x dx = \frac{12^x}{\log 12} + \mathrm{C}$$

Detyra 26: $\int 9^{2x} dx$

Zgjidhje:

$$\int 9^{2x} dx = \int 9^2 \cdot 9^x dx = \int 81 \cdot 9^x dx = 81\int 9^x dx = 81\left(\frac{9^x}{\log 9}\right) + \mathrm{C}$$

Detyra 27: $\int x^{\frac{5}{4}}dx$

Zgjidhje:

$$\int x^{\frac{5}{4}}dx=\frac{x^{\frac{5}{4}+1}}{\frac{5}{4}+1}+C=\frac{x^{\frac{9}{4}}}{\frac{9}{4}}+C=\frac{4}{9}x^{\frac{9}{4}}+C$$

Detyra 28: $\int\left(x^2-2x+4\right)^2 dx$

Zgjidhje:

$$\left|Formula:\left(a+b+c\right)^2=a^2+b^2+c^2+2ab+2bc+2ca\right|$$

$$\int\left(x^2-2x+4\right)^2 dx=\int\left(x^4+4x^2+16-4x^3-16x+8x^2\right)dx=\int\left(x^4-4x^3+12x^2-16x+16\right)dx=$$

$$=\int x^4dx-4\int x^3+12\int x^2dx-16\int xdx+16\int dx=\frac{x^{4+1}}{4+1}-4\frac{x^{3+1}}{3+1}+12\frac{x^{2+1}}{2+1}-16\frac{x^{1+1}}{1+1}+16x+C=$$

$$=\frac{x^5}{5}-4\frac{x^4}{4}+12\frac{x^3}{3}-16\frac{x^2}{2}+16x+C=\frac{1}{5}x^5-x^4+4x^3-8x^2+16x+C$$

Detyra 29: $\int 3^{2\log_3 x}dx$

Zgjidhje:

$$\left|\begin{array}{c}Formula:m\log x=\log n^m\\ a^{\log_a x}=x\end{array}\right|$$

$$\int 3^{2\log_3 x}dx=\int 3^{\log_3 x^2}dx=\int x^2dx=\frac{x^{2+1}}{2+1}+C=\frac{x^3}{3}+C=\frac{1}{3}x^3+C$$

Detyra 30: $\int\sqrt{x}\left(x^3+2x^2-x+3\right)dx$

Zgjidhje:

$$\int\sqrt{x}\left(x^3+2x^2-x+3\right)dx=\int x^{\frac{1}{2}}\left(x^3+2x^2-x+3\right)dx=\int\left(x^{\frac{7}{2}}+2x^{\frac{5}{2}}-x^{\frac{3}{2}}+3x^{\frac{1}{2}}\right)dx=$$

$$=\int x^{\frac{7}{2}}dx+2\int x^{\frac{5}{2}}dx-\int x^{\frac{3}{2}}dx+3\int x^{\frac{1}{2}}dx=\frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1}+2\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1}-\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+3\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=$$

$$=\frac{x^{\frac{9}{2}}}{\frac{9}{2}}+2\frac{x^{\frac{7}{2}}}{\frac{7}{2}}-\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+3\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+C=\frac{2}{9}x^{\frac{9}{2}}+\frac{4}{7}x^{\frac{7}{2}}-\frac{2}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}+C$$

Detyra 31: $\int \frac{1}{\sqrt[n]{x}}dx$

Zgjidhje:

$$\int \frac{1}{\sqrt[n]{x}}dx=\int \frac{1}{x^{\frac{1}{n}}}dx=\int x^{-\frac{1}{n}}dx=\frac{x^{\left(-\frac{1}{n}+1\right)}}{\left(-\frac{1}{n}+1\right)}+\mathrm{C}=\frac{x^{\left(\frac{n-1}{n}\right)}}{\left(\frac{n-1}{n}\right)}+\mathrm{C}=\frac{n}{n-1}x^{\frac{n-1}{n}}+\mathrm{C}$$

Detyra 32: $\int \sqrt{x\cdot\sqrt[3]{x}}dx$

Zgjidhje:

$$\int \sqrt{x\cdot\sqrt[3]{x}}dx=\int \sqrt{x\cdot x^{\frac{1}{2}}}dx=\int \sqrt{x^{\frac{4}{3}}}dx=\int x^{\frac{2}{3}}dx=\frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1}+\mathrm{C}=\frac{3}{5}\sqrt[3]{x^5}+\mathrm{C}$$

Detyra 33: $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$

Zgjidhje:

$$\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx=\int x^{\frac{1}{2}}dx+\int x^{\frac{-1}{2}}dx=\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+\mathrm{C}=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{-\frac{1}{2}}}{\frac{1}{2}}+\mathrm{C}=$$

$$=\frac{2}{3}\sqrt{x^3}+2\sqrt{x}+\mathrm{C}=\frac{2}{3}x\sqrt{x}+2\sqrt{x}+\mathrm{C}$$

Detyra 34: $\int\left(x-\frac{1}{\sqrt{x}}\right)^3dx$

Zgjidhje:

$$\int\left(x-\frac{1}{\sqrt{x}}\right)^3dx=\int\left(x^3-3x^{\frac{3}{2}}+3-x^{-\frac{3}{2}}\right)dx=\frac{x^4}{4}-\frac{3x^{\frac{5}{2}}}{\frac{5}{2}}+3x-\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}+\mathrm{C}=\frac{1}{4}x^4-\frac{6}{5}x^{\frac{5}{2}}+3x-2x^{-\frac{1}{2}}+\mathrm{C}$$

Detyra 35: $\int\frac{x^2-1}{x+1}dx$

Zgjidhje:

$$\int\frac{x^2-1}{x+1}dx=\int\frac{(x+1)(x-1)}{x+1}dx=\int(x-1)dx=\int xdx-\int dx=\frac{x^{1+1}}{1+1}-x+\mathrm{C}=\frac{x^2}{2}-x+\mathrm{C}$$

Detyra 36: $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^3 dx$

Zgjidhje:

$$\left|Formula:(a-b)^3=a^3-b^3-3ab(a-b)\right|$$

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^3 dx=\int\left(x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)^3 dx=\int\left[\left(x^{\frac{1}{2}}\right)^3-\left(x^{-\frac{1}{2}}\right)^3-3x^{\frac{1}{2}}\cdot x^{-\frac{1}{2}}\left(x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)\right]dx=$$

$$=\int\left[x^{\frac{3}{2}}-x^{-\frac{3}{2}}-3\left(x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)\right]dx=\left|x^{\frac{1}{2}}\cdot x^{-\frac{1}{2}}=x^0=1\right|=\int\left(x^{\frac{3}{2}}-x^{-\frac{3}{2}}-3x^{\frac{1}{2}}+3x^{-\frac{1}{2}}\right)dx=$$

$$=\int x^{\frac{3}{2}}dx-\int x^{-\frac{3}{2}}dx-3\int x^{\frac{1}{2}}dx+3\int x^{-\frac{1}{2}}dx=\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}-\frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}-3\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+3\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+\mathrm{C}=$$

$$=\frac{x^{\frac{5}{2}}}{\frac{5}{2}}-\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}-3\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+3\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+\mathrm{C}=\frac{2}{5}x^{\frac{5}{2}}+2x^{-\frac{1}{2}}-2x^{\frac{3}{2}}+6x^{\frac{1}{2}}+\mathrm{C}$$

Detyra 37: $\int\sqrt{x\sqrt{x\sqrt{x}}}\,dx$

Zgjidhje:

$$\int\sqrt{x\sqrt{x\sqrt{x}}}\,dx=\int\sqrt{x\sqrt{x\cdot x^{\frac{1}{2}}}}\,dx=\int\sqrt{x\sqrt{x^{3/2}}}\,dx=\int\sqrt{x\cdot\left(x^{3/2}\right)^{1/2}}\,dx=\int\sqrt{x\cdot x^{3/4}}\,dx=$$

$$=\int\sqrt{x^{7/4}}\,dx=\int\left(x^{7/4}\right)^{\frac{1}{2}}dx=\int x^{\frac{7}{8}}dx=\frac{x^{\frac{7}{8}+1}}{\frac{7}{8}+1}=\frac{x^{\frac{15}{8}}}{\frac{15}{8}}=\frac{8}{15}x^{\frac{15}{8}}=\frac{8}{15}\sqrt[8]{x^{15}}+\mathrm{C}$$

Detyra 38: $\int\left(1-\frac{1}{x}\right)\sqrt{x\sqrt{x}}\,dx$

Zgjidhje:

$$\int\left(1-\frac{1}{x}\right)\sqrt{x\sqrt{x}}\,dx=\int\left(1-\frac{1}{x}\right)x^{\frac{1}{2}}x^{\frac{1}{4}}dx=\int\left(1-\frac{1}{x}\right)x^{\frac{3}{4}}dx=\int x^{\frac{3}{4}}dx-\int x^{-1}\cdot x^{\frac{3}{4}}dx=$$

$$=\frac{x^{\frac{3}{4}+1}}{\frac{3}{4}+1}-\int x^{-1+\frac{3}{4}}dx=\frac{x^{\frac{7}{4}}}{\frac{7}{4}}-\frac{x^{\frac{3}{4}}}{\frac{3}{4}}+\mathrm{C}=\frac{4}{7}x^{\frac{7}{4}}-\frac{4}{3}x^{\frac{3}{4}}+\mathrm{C}$$

Detyra 39: $\int \frac{8x^2-2}{2x+1}dx$

Zgjidhje:

$$\int \frac{8x^2-2}{2x+1}dx = \int \frac{2\left(4x^2-1\right)}{2x+1}dx = 2\int \frac{(2x+1)(2x-1)}{2x+1}dx = 2\int (2x-1)\,dx = 2\left(\int 2xdx - \int dx\right) =$$
$$= 2\left(2\int xdx - \int dx\right) = 4\frac{x^{1+1}}{1+1} - 2x + \mathrm{C} = 4\frac{x^2}{2} - 2x + \mathrm{C} = 2x^2 - 2x + \mathrm{C}$$

Detyra 40: $\int \frac{3x^3-x-1}{x^3}dx$

Zgjidhje:

$$\int \frac{3x^3-x-1}{x^3}dx = \int 3dx - \int \frac{1}{x^2}dx - \int \frac{1}{x^3}dx = 3x - \frac{x^{-1}}{-1} - \frac{x^{-2}}{-2} + \mathrm{C} = 3x + \frac{1}{x} + \frac{1}{2x^2} + \mathrm{C}$$

Detyra 41: $\int \sqrt[7]{x^5}dx$

Zgjidhje:

$$\int \sqrt[7]{x^5}dx = \int x^{\frac{5}{7}}dx = \frac{x^{\frac{5}{7}+1}}{\frac{5}{7}+1} + \mathrm{C} = \frac{x^{\frac{12}{7}}}{\frac{12}{7}} + \mathrm{C} = \frac{7}{12}x^{\frac{12}{7}} + \mathrm{C} = \frac{7}{12}\sqrt[7]{x^{12}} + \mathrm{C} = \frac{7}{12}x\sqrt[7]{x^5} + \mathrm{C}$$

Detyra 42: $\int \frac{x^3-1}{x^2}dx$

Zgjidhje:

$$\int \frac{x^3-1}{x^2}dx = \int\left(\frac{x^3}{x^2} - \frac{1}{x^2}\right)dx = \int\left(x - x^{-2}\right)dx = \int xdx - \int x^{-2}dx = \frac{x^{1+1}}{1+1} - \frac{x^{-2+1}}{-2+1} + \mathrm{C} =$$
$$= \frac{x^2}{2} - \frac{x^{-1}}{-1} + \mathrm{C} = \frac{x^2}{2} - \frac{1}{x} + \mathrm{C}$$

Detyra 43: $\int \frac{x^4}{x-1}dx$

Zgjidhje:

$$\int \frac{x^4}{x-1}dx = \int\left(x^3 + x^2 + x + 1 + \frac{1}{x-1}\right)dx = \frac{x^{3+1}}{3+1} + \frac{x^{2+1}}{2+1} + \frac{x^{1+1}}{1+1} + x + \ln|x-1| + \mathrm{C} =$$
$$= \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + \mathrm{C}$$

Detyra 44: $\int \frac{\left(x^2+1\right)^2}{x^2}dx$

Zgjidhje:

$$\int \frac{\left(x^2+1\right)^2}{x^2}dx = \int \frac{x^4+2x^2+1}{x^2}dx = \int \frac{x^4}{x^2}dx + 2\int \frac{x^2}{x^2}dx + \int \frac{1}{x^2}dx =$$
$$= \int x^2 dx + 2\int dx + \int x^{-2}dx = \frac{x^{2+1}}{2+1} + 2x + \frac{x^{-2+1}}{-2+1} + \mathrm{C} = \frac{x^3}{3} + 2x + \frac{x^{-1}}{-1} + \mathrm{C} =$$
$$= \frac{x^3}{3} + 2x - 1\frac{1}{x} + \mathrm{C} = \frac{x^3}{3} + 2x - \frac{1}{x} + \mathrm{C}$$

Detyra 45: $\int \frac{(1+x)^2}{x\left(1+x^2\right)}dx$

Zgjidhje:

$$\int \frac{(1+x)^2}{x\left(1+x^2\right)}dx = \int \frac{1+2x+x^2}{x\left(1+x^2\right)}dx = \int \frac{1+x^2}{x\left(1+x^2\right)}dx + \int \frac{2x}{x\left(1+x^2\right)}dx = \int \frac{dx}{x} + 2\int \frac{dx}{1+x^2} =$$
$$= \ln|x| + 2\arctan x + \mathrm{C}$$

Detyra 46: $\int \frac{3+x}{3-x}dx$

Zgjidhje:

$$\int \frac{3+x}{3-x}dx = \int \frac{3}{3-x}dx + \int \frac{x}{3-x}dx = 3\int \frac{dx}{3-x} + \int \frac{x-3+3}{3-x}dx = -3\int \frac{d(3-x)}{3-x} - \int \frac{-x+3-3}{3-x}dx =$$
$$= 3 - \ln|3-x| - \int \frac{3-x}{3-x}dx + 3\int \frac{dx}{3-x} = -3\ln|3-x| - x - 3\ln|3-x| + \mathrm{C} = -6\ln|3-x| - x + \mathrm{C}$$

Detyra 47: $\int \left(3-x^2\right)^3 dx$

Zgjidhje:

$$\int \left(3-x^2\right)^3 dx = \int \left[3^3 - 3\cdot 3^2 x + 3\cdot 3x^4 - \left(x^2\right)^3\right]dx = 27\int dx - 27\int x^2 dx + 9\int x^4 dx - \int x^6 dx =$$
$$= 27x - 27\frac{x^3}{3} + 9\frac{x^5}{5} - \frac{x^7}{7} + \mathrm{C} = 27x - 9x^3 + \frac{9}{5}x^5 - \frac{1}{7}x^7 + \mathrm{C}$$

Detyra 48: $\int\left(x^4-\sqrt{x}+x\sqrt[3]{x}+\frac{1}{x^2}\right)dx$

Zgjidhje:

$$\int\left(x^4-\sqrt{x}+x\sqrt[3]{x}+\frac{1}{x^2}\right)dx=\int\left(x^4-x^{\frac{1}{2}}+x^{\frac{4}{3}}+x^{-2}\right)dx=\frac{x^5}{5}-\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{7}{3}}}{\frac{7}{3}}+\frac{x^{-1}}{-1}+\mathrm{C}=$$

$$=\frac{x^5}{5}-\frac{2}{3}\sqrt{x^3}+\frac{3}{7}\sqrt[3]{x^7}-\frac{1}{x}+\mathrm{C}$$

Detyra 49: $\int\frac{(x+1)^2}{\sqrt{x}}dx$

Zgjidhje:

$$\int\frac{(x+1)^2}{\sqrt{x}}dx=\int\frac{x^2+2x+1}{x^{\frac{1}{2}}}dx=\int\left(x^{\frac{3}{2}}+2x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)dx=\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+2\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+\mathrm{C}=$$

$$=\frac{2}{5}\sqrt{x^5}+\frac{4}{3}\sqrt{x^3}+2\sqrt{x}+\mathrm{C}$$

Detyra 50: $\int x(x+a)(x+b)dx$

Zgjidhje:

$$\int x(x+a)(x+b)dx=\int x\left(x^2+xb+ax+ba\right)dx=\int\left(x^3+bx^2+ax^2+xab\right)dx=$$
$$=\int x^3dx+\int bx^2dx+\int ax^2dx+\int xabdx=\int x^3dx+b\int x^2dx+a\int x^2dx+ab\int xdx=$$
$$=\frac{x^{3+1}}{3+1}+b\frac{x^{2+1}}{2+1}+a\frac{x^{2+1}}{2+1}+ab\frac{x^{1+1}}{1+1}+\mathrm{C}=\frac{x^4}{4}+b\frac{x^3}{3}+a\frac{x^3}{3}+ab\frac{x^2}{2}+\mathrm{C}=$$
$$=\frac{1}{4}x^4+\frac{1}{3}bx^3+\frac{1}{3}ax^3+\frac{1}{2}abx^2+\mathrm{C}$$

Detyra 51: $\int\frac{x^2-8x+1}{x}dx$

Zgjidhje:

$$\int\frac{x^2-8x+1}{x}dx=\int\left(\frac{x^2}{x}-\frac{8x}{x}+\frac{1}{x}\right)dx=\int\left(x-8+\frac{1}{x}\right)dx=\int xdx-8\int dx+\int\frac{dx}{x}=\frac{x^2}{2}-8x+\ln|x|+\mathrm{C}$$

Detyra 52: $\int \frac{\sqrt{x^2+2x+1}}{x}dx$

Zgjidhje:

$$\int \frac{\sqrt{x^2+2x+1}}{x}dx = \int \frac{\sqrt{(x+1)^2}}{x}dx = \int \frac{x+1}{x}dx = \int\left(1+\frac{1}{x}\right)dx = \int dx + \int \frac{1}{x}dx = x + \ln x + \mathrm{C}$$

Detyra 53: $\int\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)dx$

Zgjidhje:

$$\int\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)dx = \int\left[x\left(x^2+\frac{1}{x^2}\right)+\frac{1}{x}\left(x^2+\frac{1}{x^2}\right)\right]dx = \int\left(x^3+\frac{1}{x}+x+\frac{1}{x^3}\right)dx =$$

$$= \int\left(x^3+x+\frac{1}{x}+\frac{1}{x^3}\right)dx = \int x^3 dx + \int x dx + \int \frac{1}{x}dx + \int \frac{1}{x^3}dx = \frac{x^{3+1}}{3+1}+\frac{x^{1+1}}{1+1}+\log|x|+\frac{x^{-3+1}}{-3+1}+\mathrm{C} =$$

$$= \frac{x^4}{4}+\frac{x^2}{2}+\log|x|+\frac{x^{-2}}{-2}+\mathrm{C} = \frac{x^4}{4}+\frac{x^2}{2}+\log|x|+\frac{1}{2x^2}+\mathrm{C}$$

Detyra 54: $\int \frac{ax^2+bx+c}{x^2}dx$

Zgjidhje:

$$\int \frac{ax^2+bx+c}{x^2}dx = \int\left(\frac{ax^2}{x^2}+\frac{bx}{x^2}+\frac{c}{x^2}\right)dx = \int\left(a+\frac{b}{x}+\frac{c}{x^2}\right)dx = a\int dx + b\int \frac{1}{x}dx + c\int x^{-2}dx =$$

$$= ax + b\log|x| + \frac{cx^{-2+1}}{-2+1}+\mathrm{C} = ax + b\log|x| + \frac{cx^{-1}}{-1}+\mathrm{C} = ax + b\log|x| + \frac{c}{x}+\mathrm{C}$$

Detyra 55: $\int\left(\frac{3}{\sqrt{x}}+5x^4\right)dx$

Zgjidhje:

$$\int\left(\frac{3}{\sqrt{x}}+5x^4\right)dx = \int 3x^{-\frac{1}{2}}dx + \int 5x^4 dx = 3\int x^{-\frac{1}{2}}dx + 5\int x^4 dx = 3\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+5\frac{x^{4+1}}{4+1}+\mathrm{C} =$$

$$= 3\cdot\frac{2}{1}x^{\frac{1}{2}}+5\frac{x^5}{5}+\mathrm{C} = 6\sqrt{x}+x^5+\mathrm{C}$$

Detyra 56: $\int\left(\sqrt{x}+1\right)\left(x-\sqrt{x}+1\right)dx$

Zgjidhje: $\int\left(\sqrt{x}+1\right)\left(x-\sqrt{x}+1\right)dx = \int\left(x^{\frac{3}{2}}+1\right)dx = x+\frac{2}{5}x^{\frac{5}{2}}+\mathrm{C}$

Detyra 57: $\int\left(5x^3+2x^{-5}-7x+\frac{1}{\sqrt{x}}+\frac{5}{x}\right)dx$

Zgjidhje:

$$Formula\left|\int\frac{1}{x}dx=\log|x|+\text{C}\right|$$

$$\int\left(5x^3+2x^{-5}-7x+\frac{1}{\sqrt{x}}+\frac{5}{x}\right)dx=\int 5x^3dx+\int 2x^{-5}dx-\int 7xdx+\int\frac{1}{\sqrt{x}}dx+\int\frac{5}{x}dx=$$

$$=5\int x^3dx+2\int x^{-5}dx-7\int xdx+\int\frac{1}{\sqrt{x}}dx+5\int\frac{1}{x}dx=\frac{5x^{3+1}}{3+1}+\frac{2x^{-5+1}}{-5+1}-\frac{7x^{1+1}}{1+1}+\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+5\log|x|+\text{C}=$$

$$=\frac{5x^4}{4}+\frac{2x^{-4}}{-4}-\frac{7x^2}{2}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+5\log|x|+\text{C}=\frac{5x^4}{4}+\frac{1}{2x^4}-\frac{7x^2}{2}+2\sqrt{x}+5\log|x|+\text{C}$$

Detyra 58: $\int\frac{1+x+x^2}{x^2(1+x)}dx$

Zgjidhje:

$$\int\frac{1+x+x^2}{x^2(1+x)}dx=\int\frac{(1+x)+x^2}{x^2(1+x)}dx=\int\left[\frac{(1+x)}{x^2(1+x)}+\frac{x^2}{x^2(1+x)}\right]dx=\int\frac{1}{x^2}dx+\int\frac{1}{1+x}dx=$$

$$=\int x^{-2}dx+\int\frac{1}{1+x}dx=\frac{x^{-2+1}}{-2+1}+\log|1+x|+\text{C}=\frac{x^{-1}}{-1}+\log|1+x|+\text{C}=-\frac{1}{x}+\log|1+x|+\text{C}$$

Detyra 59: $\int\left(x^2-1\right)\sqrt{x}dx$

Zgjidhje:

$$\int\left(x^2-1\right)\sqrt{x}dx=\int\left(x^2-1\right)x^{\frac{1}{2}}dx=\int\left(x^{\frac{5}{2}}-x^{\frac{1}{2}}\right)dx=\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1}-\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\text{C}=\frac{2}{7}x^{\frac{7}{2}}-\frac{2}{3}x^{\frac{3}{2}}+\text{C}=$$

$$\frac{2}{7}\left(\sqrt{x}\right)^7-\frac{2}{3}\left(\sqrt{x}\right)^3+\text{C}=2\left(\sqrt{x}\right)^3\left(\frac{1}{7}x^2-\frac{1}{3}\right)+\text{C}$$

Detyra 60: $\int \frac{\left(x^2+1\right)\left(x^2-2\right)}{\sqrt[3]{x^2}}dx$

Zgjidhje:

$$\int \frac{\left(x^2+1\right)\left(x^2-2\right)}{\sqrt[3]{x^2}}dx=\int \frac{x^4-x^2-2}{x^{\frac{2}{3}}}dx=\int\left(x^{\frac{10}{3}}-x^{\frac{4}{3}}-2x^{-\frac{2}{3}}\right)dx=\frac{3}{13}x^{\frac{13}{3}}-\frac{3}{7}x^{\frac{7}{3}}-2\cdot 3x^{\frac{1}{3}}=$$

$$=\frac{3}{13}x^{\frac{13}{3}}-\frac{3}{7}x^{\frac{7}{3}}-6\sqrt[3]{x}+\mathrm{C}$$

Detyra 61: $\int \frac{2\sqrt{x}+\sqrt[3]{x^2}-1}{\sqrt{x}}dx$

Zgjidhje:

$$\int \frac{2\sqrt{x}+\sqrt[3]{x^2}-1}{\sqrt{x}}dx=\int\left(2+x^{\frac{1}{6}}-\frac{1}{\sqrt{x}}\right)dx=2\int dx+\int x^{\frac{1}{6}}dx-\int x^{-\frac{1}{2}}dx=2x+\frac{x^{\frac{7}{6}}}{\frac{7}{6}}-\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+\mathrm{C}$$

$$=2x+\frac{6\sqrt[6]{x^7}}{7}-2\sqrt{x}+\mathrm{C}$$

Detyra 62: $\int \sqrt{ax+b}dx$

Zgjidhje:

$$\int \sqrt{ax+b}dx=\int(ax+b)^{\frac{1}{2}}dx=\frac{(ax+b)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)a}+\mathrm{C}=\frac{(ax+b)^{\frac{3}{2}}}{\left(\frac{3}{2}a\right)}+\mathrm{C}=\frac{2}{3a}(ax+b)^{\frac{3}{2}}+\mathrm{C}$$

Detyra 63: $\int\left(1-\frac{1}{x^2}\right)\sqrt{x\sqrt{x}}dx$

Zgjidhje:

$$\int\left(1-\frac{1}{x^2}\right)\sqrt{x\sqrt{x}}dx=\int\left(1-x^{-2}\right)\left(x\cdot x^{\frac{1}{2}}\right)^{\frac{1}{2}}dx=\int\left(1-x^{-2}\right)\left(x^{\frac{3}{2}}\right)^{\frac{1}{2}}dx=\int\left(1-x^{-2}\right)x^{\frac{3}{4}}dx=$$

$$=\int\left(x^{\frac{3}{4}}-x^{-\frac{5}{4}}\right)dx=\frac{x^{\frac{7}{4}}}{\frac{7}{4}}-\frac{x^{-\frac{1}{4}}}{-\frac{1}{4}}+\mathrm{C}=\frac{4}{7}\sqrt[4]{x^7}+\frac{4}{\sqrt[4]{x}}+\mathrm{C}$$

Detyra 64: $\int x^2(3-x)^4\,dx$

Zgjidhje:

$$\int x^2(3-x)^4\,dx=\int x^2\left(81-108x+54x^2-12x^3+x^4\right)dx=\int\left(81x^2-108x^3+54x^4-12x^5+x^6\right)dx=$$
$$=81\frac{x^3}{3}-108\frac{x^4}{4}+54\frac{x^5}{5}-12\frac{x^6}{6}+\frac{x^7}{7}+\mathrm{C}=27x^3-27x^4+\frac{54}{5}x^5-2x^6+\frac{x^7}{7}+\mathrm{C}$$

Detyra 65: $\int\left(2+x^2\right)^3 dx$

Zgjidhje:

$$\int\left(2+x^2\right)^3 dx=\int\left(8+12x^2+6x^4+x^6\right)dx=\int 8dx+\int 12x^2dx+\int 6x^4dx+\int x^6dx=$$
$$=8\int dx+12\int x^2dx+6\int x^4dx+\int x^6dx=8x+12\frac{x^3}{3}+6\frac{x^5}{5}+\frac{x^7}{7}+\mathrm{C}=8x+4x^3+\frac{6}{5}x^5+\frac{x^7}{7}+\mathrm{C}$$

Detyra 66: $\int\left(4x^3-4x^{-5}\right)dx$

Zgjidhje:

$$\int\left(4x^3-4x^{-5}\right)dx=\int 4x^3dx-\int 4x^{-5}dx=4\int x^3dx-4\int x^{-5}dx=\frac{4x^{3+1}}{3+1}-\frac{4x^{-5+1}}{-5+1}+\mathrm{C}=$$
$$=\frac{4x^4}{4}-\frac{4x^{-4}}{-4}+\mathrm{C}=x^4+\frac{1}{x^4}+\mathrm{C}$$

Detyra 67: $\int\frac{x^3-x^2+x-1}{x-1}dx$

Zgjidhje:

$$\int\frac{x^3-x^2+x-1}{x-1}dx=\left|\begin{array}{l}x^3-x^2+x-1\\=x^2(x-1)+1(x-1)\\=(x-1)\left(x^2+1\right)\end{array}\right|=\int\frac{(x-1)\left(x^2+1\right)}{x-1}dx=$$
$$=\int\left(x^2+1\right)dx=\int x^2dx+1\int dx=\frac{x^3}{3}+x+\mathrm{C}$$

Detyra 68: $\int\frac{x^2}{1+x^2}dx$

Zgjidhje:

$$\int\frac{x^2}{1+x^2}dx=\int\frac{1+x^2-1}{1+x^2}dx=\int\left[\frac{1+x^2}{1+x^2}-\frac{1}{1+x^2}\right]dx=1\int dx-\int\frac{1}{1+x^2}dx=\left|\int\frac{1}{1+x^2}dx=\tan^{-1}x+\mathrm{C}\right|=$$
$$=x-\tan^{-1}x+\mathrm{C}$$

Detyra 69: $\int \frac{3x^2-7x+5}{\sqrt{x}}dx$

Zgjidhje:

$$\int \frac{3x^2-7x+5}{\sqrt{x}}dx = 3\int \frac{x^2}{\sqrt{x}}dx - 7\int \frac{x}{\sqrt{x}}dx + 5\int \frac{dx}{\sqrt{x}} = 3\int x^{2-\frac{1}{2}}dx - 7\int x^{1-\frac{1}{2}}dx + 5\int x^{-1}dx =$$

$$= 3\int x^{\frac{3}{4}}dx - 7\int x^{\frac{1}{2}}dx + 5\int x^{-1}dx = 3\frac{x^{\frac{3}{4}+1}}{\frac{3}{4}+1} - 7\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 5\ln|x| + \mathrm{C} = 3\frac{x^{\frac{7}{4}}}{\frac{7}{4}} - 7\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 5\ln|x| + \mathrm{C} =$$

$$= \frac{12x^{\frac{7}{4}}}{7} - \frac{14x^{\frac{3}{2}}}{3} + 5\ln|x| + \mathrm{C} = \frac{12}{7}x^{\frac{7}{4}} - \frac{14}{3}x^{\frac{3}{2}} + 5\ln|x| + \mathrm{C}$$

Detyra 70: $\int (x-3)^2 \cdot \sqrt{x}\ dx$

Zgjidhje:

$$\int (x-3)^2 \cdot \sqrt{x}\ dx = \int \left(x^2-6x+9\right) \cdot \sqrt{x}\ dx = \int \left(x^2-6x+9\right) \cdot x^{\frac{1}{2}}\ dx = \int \left(x^{\frac{5}{2}} - 6x^{\frac{3}{2}} + 9x^{\frac{1}{2}} \right) dx =$$

$$= \frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} - \frac{6x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{9x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \mathrm{C} = \frac{x^{\frac{7}{2}}}{\frac{7}{2}} - \frac{6x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{9x^{\frac{3}{2}}}{\frac{3}{2}} + \mathrm{C} = \frac{2}{7}x^{\frac{7}{2}} - \frac{12}{5}x^{\frac{5}{2}} + 6x^{\frac{3}{2}} + \mathrm{C}$$

Detyra 71: $\int \frac{dx}{a^2+x^2}$

Zgjidhje:

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a^2}\int \frac{dx}{1+\frac{x^2}{a^2}} = \frac{1}{a^2}\int \frac{dx}{1+\left(\frac{x}{a}\right)^2} = \frac{a}{a^2}\int \frac{d\left(\frac{x}{a}\right)}{1+\left(\frac{x}{a}\right)^2} = \frac{1}{a}\arctan\frac{x}{a} + \mathrm{C}$$

Detyra 72: $\int \frac{x^2+3}{x^2-1}dx$

Zgjidhje:

$$\int \frac{x^2+3}{x^2-1}dx = \int \frac{x^2-1+4}{x^2-1}dx = \int \left(\frac{x^2-1}{x^2-1} + \frac{4}{x^2-1} \right) dx = \int \left(1 + \frac{4}{x^2-1} \right) dx =$$

$$= 1 - \frac{4}{2}\ln\frac{1+x}{1-x} + \mathrm{C} = x - 2\ln\frac{1+x}{1-x} + \mathrm{C} = x + 2\ln\frac{x-1}{x+1} + \mathrm{C}$$

Detyra 73: $\int \frac{x^3+x-2}{x^2+1}dx$

Zgjidhje:

$$\int \frac{x^3+x-2}{x^2+1}dx=\int \frac{x\left(x^2+1\right)-2}{x^2+1}dx=\int\left(\frac{x\left(x^2+1\right)}{x^2+1}-\frac{2}{x^2+1}\right)dx=\int xdx-2\int\frac{1}{x^2+1}dx=$$

$$=\frac{x^2}{2}-2\arctan x+\mathrm{C}$$

Detyra 74: $\int \frac{x^2-8x+1}{x}dx$

Zgjidhje:

$$\int \frac{x^2-8x+1}{x}dx=\int\left(\frac{x^2}{x}-\frac{8x}{x}+\frac{1}{x}\right)dx=\int\left(x-8+\frac{1}{x}\right)dx=\int xdx-8\int dx+\int\frac{dx}{x}=\frac{x^2}{2}-8x+\ln|x|+\mathrm{C}$$

Detyra 75: $\int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1-x^4}}dx$

Zgjidhje:

$$\int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1-x^4}}dx=\int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1-x^2}\sqrt{1-x^2}}dx=\int\left(\frac{\sqrt{1+x^2}}{\sqrt{1-x^2}\sqrt{1-x^2}}+\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}\sqrt{1-x^2}}\right)dx=$$

$$=\int\frac{1}{\sqrt{1-x^2}}dx+\int\frac{1}{\sqrt{1+x^2}}dx=\arcsin x+\ln\left|x+\sqrt{1+x^2}\right|+\mathrm{C}$$

Detyra 76: $\int\left(e^x+3\cos x-4x^3+2\right)dx$

Zgjidhje:

$$\int\left(e^x+3\cos x-4x^3+2\right)dx=\int e^xdx+3\int\cos xdx-4\int x^3dx+2\int dx=e^x+3\sin x-\frac{4x^4}{4}+2x+\mathrm{C}=$$

$$=e^x+3\sin x-x^4+2x+\mathrm{C}$$

Detyra 77: $\int \frac{e^{3x}+1}{e^x+1}dx$

Zgjidhje:

$$\int \frac{e^{3x}+1}{e^x+1}dx=\int\frac{\left(e^x+1\right)\left(e^{2x}-e^x+1\right)}{e^x+1}dx=\int\left(e^{2x}-e^x+1\right)dx=\int\left(e^2\right)^xdx-\int e^xdx+\int dx=$$

$$=\frac{\left(e^2\right)^x}{\ln e^2}-e^x+x+\mathrm{C}=\frac{e^{2x}}{2}-e^x+x+\mathrm{C}$$

Detyra 78: $\int\left(e^{3x}-2e^{x}+\frac{1}{x}\right)dx$

Zgjidhje:

$$\int\left(e^{3x}-2e^{x}+\frac{1}{x}\right)dx=\int e^{3x}dx-2\int e^{x}dx+\int\frac{1}{x}dx=\left|\begin{array}{l}\int e^{ax}dx=\frac{e^{ax}}{a}+\mathrm{C}\\ \int\frac{1}{x}dx=\log|x|+\mathrm{C}\end{array}\right|=$$

$$=\frac{e^{3x}}{3}-2e^{x}+\log|x|+\mathrm{C}=\frac{1}{3}e^{3x}-2e^{x}+\log|x|+\mathrm{C}$$

Detyra 79: $\int\left(3^{x}-4^{x}\right)dx$

Zgjidhje:

$$\int\left(3^{x}-4^{x}\right)dx=\int 3^{x}dx-\int 4^{x}dx=\frac{3^{x}}{\ln 3}-\frac{4^{x}}{\ln 4}+\mathrm{C}$$

Detyra 80: $\int\left(2^{x}+5^{x}\right)^{2}dx$

Zgjidhje:

$$\int\left(2^{x}+5^{x}\right)^{2}dx=\int\left(2^{2x}+2\cdot 2^{x}\cdot 5^{x}+5^{2x}\right)dx=\int\left(4^{24x}+2\cdot 10^{x}+25^{x}\right)dx=$$

$$=\int 4^{x}dx+2\int 10^{x}dx+\int 25^{x}dx=\frac{4^{x}}{\ln 4}+2\frac{10^{x}}{\ln 10}+\frac{25^{x}}{\ln 25}+\mathrm{C}$$

Detyra 81: $\int\frac{2^{x+1}-5^{x-1}}{10^{x}}dx$

Zgjidhje:

$$\int\frac{2^{x+1}-5^{x-1}}{10^{x}}dx=2\int\frac{2^{x}}{(2\cdot 5)^{x}}dx-\frac{1}{5}\int\frac{5^{x}}{(2\cdot 5)^{x}}dx=2\int 5^{-x}dx-\frac{1}{5}\int 2^{-x}dx=-2\int 5^{-x}d(-x)+\frac{1}{5}\int 2^{-x}d(-x)=$$

$$=-2\frac{5^{-x}}{\ln 5}+\frac{1}{5}\frac{2^{-x}}{\ln 2}+\mathrm{C}=\frac{1}{5\cdot 2^{x}\ln 2}-\frac{2}{5^{x}\ln 5}+\mathrm{C}$$

Detyra 82: $\int\frac{1-x}{1+x}dx$

Zgjidhje:

$$\int\frac{1-x}{1+x}dx=\int\frac{1}{1+x}dx-\int\frac{x}{1+x}dx=\int\frac{1}{1+x}dx-\int\frac{1+x-1}{1+x}dx=\int\frac{1}{x+1}dx-\int 1dx+\int\frac{1}{1+x}dx=$$
$$=\log|1+x|-x+\log|1+x|+\mathrm{C}=2\log|1+x|-x+\mathrm{C}$$

Detyra 83: $\int \frac{x+2}{(x+1)^2}dx$

Zgjidhje:

$$\int \frac{x+2}{(x+1)^2}dx = \int \frac{x+1+1}{(x+1)^2}dx = \int\left[\frac{x+1}{(x+1)^2}+\frac{1}{(x+1)^2}\right]dx = \int \frac{1}{x+1}dx + \int \frac{1}{(x+1)^2}dx =$$

$$= \int \frac{1}{x+1}dx + \int (x+1)^{-2}dx = \log|x+1| + \frac{(x+1)^{-2+1}}{-2+1} + \mathrm{C} = \log|x+1| - \frac{1}{1+x} + \mathrm{C}$$

Detyra 84: $\int \frac{2x}{(2x+1)^2}dx$

Zgjidhje:

$$\int \frac{2x}{(2x+1)^2}dx = \int \frac{2x+1-1}{(2x+1)^2}dx = \int\left[\frac{2x+1}{(2x+1)^2}-\frac{1}{(2x+1)^2}\right]dx = \int \frac{1}{(2x+1)}dx - \int (2x+1)^{-2}dx =$$

$$\left|\int \frac{1}{ax+b}dx = \frac{\log|ax+b|}{a}+\mathrm{C}\right|$$

$$= \frac{\log|2x+1|}{2} - \frac{(2x+1)^{-2+1}}{-2+1} + \mathrm{C} = \frac{1}{2}\log|2x+1| + \frac{1}{2(2x+1)} + \mathrm{C}$$

Detyra 85: $\int \frac{(x+1)^2}{x\sqrt{x}}dx$

Zgjidhje:

$$\int \frac{(x+1)^2}{x\sqrt{x}}dx = \left|(a+b)^2 = a^2+b^2+2ab\right| = \int \frac{x^2+2x+1}{x^{\frac{3}{2}}}dx = \int\left[\frac{x^2}{x^{\frac{3}{2}}}+\frac{2x}{x^{\frac{3}{2}}}+\frac{1}{x^{\frac{3}{2}}}\right]dx =$$

$$= \int x^{\frac{1}{2}}dx + 2\int x^{-\frac{1}{2}}dx + \int x^{-\frac{3}{2}}dx = = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{2x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + \mathrm{C} =$$

$$= \frac{2}{3}x^{\frac{3}{2}} + 4x^{\frac{1}{2}} - 2x^{\frac{1}{2}} + \mathrm{C} = \frac{2}{3}x\sqrt{x} + 4\sqrt{x} - \frac{2}{\sqrt{x}} + \mathrm{C}$$

Detyra 86: $\int 5^{3x+1}dx$

Zgjidhje:

$$\int 5^{3x+1}dx = \left|\int a^{mx+b}dx = \frac{a^{mx+b}}{m\log a}; a>0; a\neq 1\right| = \frac{5^{3x+1}}{\log(5)\cdot 3} + \mathrm{C} = \frac{5^{3x+1}}{3\log 5} + \mathrm{C}$$

Detyra 87: $\int\left(\frac{x}{a}+\frac{a}{x}+x^a+a^x+ax\right)dx$

Zgjidhje:

$$\int\left(\frac{x}{a}+\frac{a}{x}+x^a+a^x+ax\right)dx=\frac{1}{a}\int xdx+a\int\frac{1}{x}dx+\int x^a dx+\int a^x dx+a\int xdx=$$

$$=\frac{1}{a}\cdot\frac{x^2}{2}+a\log|x|+\frac{x^{a+1}}{a+1}+\frac{a^x}{\log a}+a\frac{x^2}{2}+\mathrm{C}=\frac{x^2}{2a}+a\log|x|+\frac{x^{a+1}}{\log a}+\frac{ax^2}{2}+\mathrm{C}$$

Detyra 88: $\int\left(1+\frac{1}{1+x^2}-\frac{2}{\sqrt{1-x^2}}\right)dx$

Zgjidhje

$$\int\left(1+\frac{1}{1+x^2}-\frac{2}{\sqrt{1-x^2}}\right)dx=\int 1dx+\int\frac{1}{1+x^2}dx-2\int\frac{1}{\sqrt{1-x^2}}dx-\int xdx=$$

$$=x+\tan^{-1}x-2\sin^{-1}x-\frac{x^2}{2}+\mathrm{C}$$

$$\left|\begin{array}{l}\int\frac{1}{1+x^2}dx=\tan^{-1}x+\mathrm{C}\\ \int\frac{1}{\sqrt{1-x^2}}dx=\sin^{-1}x+\mathrm{C}\end{array}\right|$$

Detyra 89: $\int\left(e^{x\log a}+e^{a\log x}+e^{a\log a}\right)dx$

Zgjidhje:

$$\int\left(e^{x\log a}+e^{a\log x}+e^{a\log a}\right)dx=\left|\begin{array}{l}m\log n=\log n^m\\ e^{\log f(x)}=f(x)\end{array}\right|=\int\left(e^{\log a^x}+e^{\log x^a}+e^{\log a^a}\right)dx=\int\left(a^x+x^a+a^a\right)dx=$$

$$=\int a^x dx+\int x^a dx+\int a^a dx=\frac{a^x}{\log a}+\frac{x^{a+1}}{a+1}+a^a\cdot x+\mathrm{C}$$

Detyra 90: $\int\left(x^a+a^x+e^x\cdot a^x+\sin\alpha\right)dx$

Zgjidhje:

$$\int\left(x^a+a^x+e^x\cdot a^x+\sin\alpha\right)dx=\int x^a dx+\int a^x dx+\int(ea)^x dx+\int\sin\alpha dx=$$

$$=\frac{x^{a+1}}{a+1}+\frac{a^x}{\log a}+\frac{(ea)^x}{\log ea}+(\sin\alpha)\cdot x+\mathrm{C}$$

Detyra 91: $\int \frac{\left(a^x+b^x\right)^2}{a^x \cdot b^x} dx$

Zgjidhje:

$$\int \frac{\left(a^x+b^x\right)^2}{a^x \cdot b^x} dx = \int \frac{\left(a^x\right)^2+\left(b^x\right)^2+2a^x \cdot b^x}{a^x \cdot b^x} dx = \int \left[\frac{\left(a^x\right)^2}{a^x \cdot b^x}+\frac{\left(b^x\right)^2}{a^x \cdot b^x}+\frac{2a^x \cdot b^x}{a^x \cdot b^x}\right] dx =$$

$$= \int \left(\frac{a^x}{b^x}+\frac{b^x}{a^x}+2\right) dx = \int \left(\frac{a}{b}\right)^x dx + \int \left(\frac{b}{a}\right)^x dx + 2\int dx = \left| \int a^x dx = \frac{a^x}{\log a} \right| =$$

$$= \frac{\left(\frac{a}{b}\right)^x}{\log\left(\frac{a}{b}\right)} + \frac{\left(\frac{b}{a}\right)^x}{\log\left(\frac{b}{a}\right)} + 2x + \mathrm{C}$$

Detyra 92: $\int \frac{x^4+x^2+1}{x^2-x+1} dx$

Zgjidhje:

$$\int \frac{x^4+x^2+1}{x^2-x+1} dx = \int \frac{\left(x^4+2x^2+1\right)-x^2}{x^2-x+1} dx = \int \frac{\left(x^2+1\right)^2-x^2}{x^2-x+1} dx = \int \frac{\left(x^2+1+x\right)\left(x^2+1-x\right)}{x^2-x+1} dx =$$

$$= \int \left(x^2+x+1\right) dx = \int x^2 dx + \int x dx + \int 1 dx = \frac{x^{2+1}}{2+1} + \frac{x^{1+1}}{1+1} + x + \mathrm{C} = \frac{x^3}{3} + \frac{x^2}{2} + x + \mathrm{C}$$

Detyra 93: $\int \frac{1}{\sqrt{2x+1}+\sqrt{2x+2}} dx$

Zgjidhje:

$$\int \frac{1}{\sqrt{2x+1}+\sqrt{2x+2}} dx = \int \frac{1}{\sqrt{2x+1}+\sqrt{2x+2}} \cdot \frac{\sqrt{2x+1}-\sqrt{2x+2}}{\sqrt{2x+1}-\sqrt{2x+2}} dx = \int \frac{\sqrt{2x+1}-\sqrt{2x+2}}{(2x+1)-(2x+2)} dx =$$

$$= \int \frac{\sqrt{2x+1}-\sqrt{2x+2}}{-1} dx = \int \sqrt{2x+2} dx - \int \sqrt{2x+1} dx = \int (2x+2)^{\frac{1}{2}} dx - \int (2x+1)^{\frac{1}{2}} dx =$$

$$= \left| \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + \mathrm{C} \right| = \frac{(2x+2)^{\frac{1}{2}+1}}{2\left(\frac{1}{2}+1\right)} - \frac{(2x+1)^{\frac{1}{2}+1}}{2\left(\frac{1}{2}+1\right)} + \mathrm{C} = \frac{1}{3}(2x+2)^{\frac{3}{2}} - \frac{1}{3}(2x+1)^{\frac{3}{2}} + \mathrm{C} =$$

$$= \frac{1}{3}\left[(2x+2)^{\frac{3}{2}} - (2x+1)^{\frac{3}{2}}\right] + \mathrm{C}$$

Detyra 94: $\int \frac{x}{\sqrt{1+x}}dx$

Zgjidhje:

$$\int \frac{x}{\sqrt{1+x}}dx = \int \frac{(1+x)-1}{\sqrt{1+x}}dx = \int \left(\frac{1+x}{\sqrt{1+x}} - \frac{1}{\sqrt{1+x}} \right) dx = \int (1+x)^{\frac{1}{2}} dx - \int (1+x)^{-\frac{1}{2}} dx =$$

$$= \frac{(1+x)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)\cdot 1} - \frac{(1+x)^{-\frac{1}{2}+1}}{\left(-\frac{1}{2}+1\right)\cdot 1} + \mathrm{C} = \frac{(1+x)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(1+x)^{\frac{1}{2}}}{\frac{1}{2}} + \mathrm{C} = \frac{2}{3}(1+x)^{\frac{3}{2}} - 2(1+x)^{\frac{1}{2}} + \mathrm{C}$$

Detyra 95: $\int \frac{2x}{\sqrt{a+x}+\sqrt{a-x}}dx$

Zgjidhje:

$$\int \frac{2x}{\sqrt{a+x}+\sqrt{a-x}}dx = \int \frac{2x}{\sqrt{a+x}+\sqrt{a-x}} \cdot \frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}dx = \int \frac{2x\left(\sqrt{a+x}-\sqrt{a-x}\right)}{(a+x)-(a-x)}dx =$$

$$= \int \frac{2x\left(\sqrt{a+x}-\sqrt{a-x}\right)}{2x}dx = \int \sqrt{a+x}dx - \int \sqrt{a-x}dx = \int (a+x)^{\frac{1}{2}} dx - \int (a-x)^{\frac{1}{2}} dx =$$

$$= \frac{(a+x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{(a-x)^{\frac{1}{2}+1}}{(-1)\cdot\left(\frac{1}{2}+1\right)} + \mathrm{C} = \frac{2}{3}(a+x)^{\frac{3}{2}} + \frac{2}{3}(a-x)^{\frac{3}{2}} + \mathrm{C} = \frac{2}{3}\left[(a+x)^{\frac{3}{2}} + (a-x)^{\frac{3}{2}}\right] + \mathrm{C}$$

Detyra 96: $\int x\sqrt{5x-2}dx$

Zgjidhje:

$$\int x\sqrt{5x-2}dx = \frac{1}{5}\int 5x\sqrt{5x-2}dx = \frac{1}{5}\int (5x-2+5)\sqrt{5x-2}dx =$$

$$= \frac{1}{5}\int (5x-2)\sqrt{5x-2}dx + \frac{2}{5}\int \sqrt{5x-2}dx = \frac{1}{5}\int (5x-2)^{\frac{3}{2}} dx + \frac{2}{5}\int (5x-2)^{\frac{1}{2}} dx =$$

$$= \frac{1}{5}\cdot\frac{(5x-2)^{\frac{3}{2}+1}}{5\cdot\left(\frac{3}{2}+1\right)} + \frac{2}{5}\cdot\frac{(5x-2)^{\frac{1}{2}+1}}{5\cdot\left(\frac{1}{2}+1\right)} + \mathrm{C} = \left| \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a\cdot(n+1)} + \mathrm{C} \right| =$$

$$= \frac{2}{125}(5x-2)^{\frac{5}{2}} + \frac{4}{75}(5x-2)^{\frac{3}{2}} + \mathrm{C}$$

Detyra 97: $\int\left(a^{5x-3}+e^{2x-3}\right)dx$

Zgjidhje:

$$\int\left(a^{5x-3}+e^{2x-3}\right)dx=\int a^{5x-3}dx+\int e^{2x-3}dx=\left|\begin{array}{l}\int a^{bx+c}dx=\dfrac{a^{bx+c}}{b\log a}+\mathrm{C}\\ \int e^{ax+b}dx=\dfrac{e^{ax+b}}{a}+\mathrm{C}\end{array}\right|=\frac{a^{5x-3}}{5\log a}+\frac{e^{2x-3}}{2}+\mathrm{C}$$

Detyra 98: $\int\frac{1}{\sqrt{x+a}+\sqrt{x+b}}dx$

Zgjidhje:

$$\int\frac{1}{\sqrt{x+a}+\sqrt{x+b}}\cdot\frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}}dx=\int\frac{\sqrt{x+a}-\sqrt{x+b}}{(x+a)-(x+b)}dx=\int\frac{\sqrt{x+a}-\sqrt{x+b}}{a-b}dx=$$

$$=\frac{1}{a-b}\int(x+a)^{\frac{1}{2}}dx-\frac{1}{a-b}\int(x+b)^{\frac{1}{2}}dx=\frac{1}{a-b}\cdot\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{1}{a-b}\cdot\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}}+\mathrm{C}=$$

$$=\frac{1}{a-b}\cdot\frac{2}{3}(x+a)^{\frac{3}{2}}-\frac{1}{a-b}\cdot\frac{2}{3}(x+a)^{\frac{3}{2}}+\mathrm{C}=\frac{2}{3}\cdot\frac{(x+a)^{\frac{3}{2}}}{a-b}-\frac{2}{3}\cdot\frac{(x+a)^{\frac{3}{2}}}{a-b}+\mathrm{C}$$

Detyra 99: $\int\frac{1}{\sqrt{5x+3}+\sqrt{5x-2}}dx$

Zgjidhje:

$$\int\frac{1}{\sqrt{5x+3}+\sqrt{5x-2}}\cdot\frac{\sqrt{5x+3}-\sqrt{5x-2}}{\sqrt{5x+3}-\sqrt{5x-2}}dx=\int\frac{\sqrt{5x+3}-\sqrt{5x-2}}{(5x+3)-(5x-2)}dx=\int\frac{\sqrt{5x+3}-\sqrt{5x-2}}{5}dx=$$

$$=\frac{1}{5}\left[\int\sqrt{5x+3}dx-\int\sqrt{5x-2}dx\right]=\frac{1}{5}\left[\int(5x+3)^{\frac{1}{2}}dx-\int(5x-2)^{\frac{1}{2}}dx\right]=\frac{1}{5}\left[\frac{(5x+3)^{\frac{3}{2}}}{\frac{3}{2}\cdot5}-\frac{(5x-2)^{\frac{3}{2}}}{\frac{3}{2}\cdot5}\right]+\mathrm{C}=$$

$$=\frac{1}{5}\left[\frac{2}{15}(5x+3)^{\frac{3}{2}}-\frac{2}{15}(5x-2)^{\frac{3}{2}}\right]+\mathrm{C}=\frac{2}{75}\left[(5x+3)^{\frac{3}{2}}-(5x-2)^{\frac{3}{2}}\right]+\mathrm{C}$$

Detyra 100: $\int \frac{x^4-3x^2-3x-2}{x^3-x^2-2x}dx$

Zgjidhje:

$$I=\int \frac{x^4-3x^2-3x-2}{x^3-x^2-2x}dx=\underbrace{\int (x+1)dx}_{A}-\underbrace{\int \frac{(x+2)dx}{x(x-2)(x+1)}}_{B}$$

$$A=\int (x+1)dx=\int xdx+\int dx=\frac{x^2}{2}+x+\mathrm{C}_1$$

$$B=\int \frac{(x+2)dx}{x(x-2)(x+1)}=-\int \frac{1}{x}dx+\frac{2}{3}\int \frac{1}{x-2}dx+\frac{1}{3}\int \frac{1}{x+1}dx=-\ln|x|+\frac{2}{3}\ln|x-2|+\frac{1}{3}\ln|x+1|+\mathrm{C}_2$$

$$I=\frac{x^2}{2}+x+\mathrm{C}_1-\ln|x|+\frac{2}{3}\ln|x-2|+\frac{1}{3}\ln|x+1|+\mathrm{C}_2=\frac{x^2}{2}+x+\frac{2}{3}\ln|x-2|+\frac{1}{3}\ln|x+1|-\ln|x|+\mathrm{C}$$

Detyra 101: $\int \frac{dx}{e^{2x}+e^x-2}$

Zgjidhje:

$$I=\int \frac{dx}{e^{2x}+e^x-2}=\int \frac{dx}{(e^x-1)(e^x+2)}=\int \left(\frac{\frac{1}{3}}{e^x-1}-\frac{\frac{1}{3}}{e^x+2}\right)dx=\frac{1}{3}\left[\underbrace{\int \frac{dx}{e^x-1}}_{A}-\underbrace{\int \frac{dx}{e^x+2}}_{B}\right]$$

$$A=\int \frac{dx}{e^x-1}=\int \frac{e^{-x}dx}{1-e^{-x}}=\int \frac{d\left(1-e^{-x}\right)}{1-e^{-x}}=\ln\left|1-e^{-x}\right|+\mathrm{C}_1$$

$$B=\int \frac{dx}{e^x+2}=\int \frac{e^{-x}dx}{1+2e^{-x}}=-\frac{1}{2}\int \frac{d\left(1+2e^{-x}\right)}{1+2e^{-x}}=-\frac{1}{2}\ln\left|1+2e^{-x}\right|+C_2$$

$$I=\frac{1}{3}[A-B]=\frac{1}{3}\ln\left|1-e^{-x}\right|+\frac{1}{6}\ln\left|1+2e^{-x}\right|+\mathrm{C}$$

Detyra 102: $\int \frac{e^{5\log_e x}-e^{4\log_e x}}{e^{3\log_e x}-e^{2\log_e x}}dx$

Zgjidhje:

$$\int \frac{e^{5\log_e x}-e^{4\log_e x}}{e^{3\log_e x}-e^{2\log_e x}}dx=\int \frac{e^{\log_e x^5}-e^{\log_e x^4}}{e^{\log_e x^3}-e^{\log_e x^2}}dx=\left|m\log_e n=\log_e n^m\right|=\int \frac{x^5-x^4}{x^3-x^2}dx=\left|e^{\log_e f(x)}=f(x)\right|=$$

$$=\int \frac{x^4(x-1)}{x^2(x-1)}dx=\int x^2dx=\frac{x^{2+1}}{2+1}+\mathrm{C}=\frac{x^3}{3}+\mathrm{C}$$

Detyra 103: $\int \frac{dx}{\cos^4 x}$

Zgjidhje:

$$\int \frac{dx}{\cos^4 x} = \int \frac{\sin^2 x + \cos^2 x}{\cos^4 x} dx = \int \frac{\sin^2 x}{\cos^2 x} \cdot \frac{dx}{\cos^2 x} + \int \frac{dx}{\cos^2 x} = \int \tan^2 x d\left(\tan x\right) + \int d\left(\tan x\right) =$$

$$= \frac{\tan^3 x}{3} + \tan x + \mathrm{C}$$

Detyra 104: $\int \sin x \sin 2x dx$

Zgjidhje:

$$\int \sin x \sin 2x dx = \frac{1}{2}\int \left(\cos\left(x-2x\right) - \cos\left(x+2x\right)\right) dx = \frac{1}{2}\left(\int \cos x dx - \int \cos 3x dx\right) =$$

$$= \frac{1}{2}\sin x - \frac{1}{6}\sin 3x + \mathrm{C}$$

Detyra 105: $\int \cos x \cos 2x \cos 3x dx$

Zgjidhje:

$$\int \cos x \cos 2x \cos 3x dx = \int \cos 2x \cdot \frac{1}{2}\left(\cos\left(x-3x\right) + \cos\left(x+3x\right)\right) dx = \frac{1}{2}\int \cos 2x \left(\cos\left(-2x\right) + \cos 4x\right) dx =$$

$$= \frac{1}{2}\left(\int \cos 2x \cos 2x dx + \int \cos 2x \cos 4x dx\right) = \frac{1}{4}\left(\int dx + \int \cos 4x + \int \cos 2x dx + \int \cos 6x dx\right) =$$

$$= \frac{1}{4}\left(x + \frac{1}{2}\sin 2x + \frac{1}{4}\sin 4x + \frac{1}{6}\sin 6x\right) + \mathrm{C}$$

Detyra 106: $\int \sin 3x \cdot \cos 5x dx$

Zgjidhje:

$$\int \sin 3x \cdot \cos 5x dx = \frac{1}{2}\int \left(\cos 8x + \cos 2x\right) dx = \frac{1}{2}\int \cos 8x dx + \frac{1}{2}\int \cos 2x dx =$$

$$= \frac{1}{16}\int \cos 8x \cdot d\left(8x\right) + \frac{1}{4}\int \cos 2x \cdot d\left(2x\right) = = \frac{1}{16}\sin 8x + \frac{1}{4}\sin 2x + \mathrm{C}$$

Detyra 107: $\int \sin 2x \cdot \sin 3x dx$

Zgjidhje:

$$\int \sin 2x \cdot \sin 3x dx = \frac{1}{2}\int \left(\cos(-x) - \cos 5x\right) dx = \frac{1}{2}\int \cos x dx - \frac{1}{2}\cos 5x dx = \frac{1}{2}\int \cos x dx - \frac{1}{10}\int \cos 5x d\left(5x\right) =$$
$$= \frac{1}{2}\sin x - \frac{1}{10}\sin 5x + \mathrm{C}$$

Detyra 108: $\int \frac{dx}{\sin^2 x \cdot \cos^2 x}$

Zgjidhje:

$$\int \frac{dx}{\sin^2 x \cdot \cos^2 x} = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x} dx = \int \frac{dx}{\cos^2 x} + \int \frac{dx}{\sin^2 x} = \tan x - \cot x + \mathrm{C}$$

Detyra 109: $\int \left(2x - 3\sin x + \cos x\right) dx$

Zgjidhje:

$$\int \left(2x - 3\sin x + \cos x\right) dx = 2\int x dx - 3\int \sin x dx + \int \cos x dx = \int \sqrt{\left(\sin x - \cos x\right)^2} dx =$$
$$= \int \pm\left(\cos x - \sin x\right) dx = \pm\left(\sin x + \cos x\right) + \mathrm{C}$$

Detyra 110: $\int \sqrt{1 - \sin 2x} dx$

Zgjidhje:

$$\int \sqrt{1 - \sin 2x} dx = \int \sqrt{\sin^2 x + \cos^2 x - 2\sin x \cos x} dx = \int \sqrt{\left(\sin x - \cos x\right)^2} dx =$$
$$= \int \pm\left(\cos x - \sin x\right) dx = \pm\left(\sin x + \cos x\right) + \mathrm{C}$$

Detyra 111: $\int \frac{1 - \cos 2x}{1 + \cos 2x} dx$

Zgjidhje:

$$\int \frac{1 - \cos 2x}{1 + \cos 2x} dx = \int \frac{1 - \cos^2 x + \sin^2 x}{1 + \cos^2 x - \sin^2 x} dx = \int \frac{2\sin^2 x}{2\cos^2 x} dx = \int \frac{1 - \cos^2 x}{2\cos^2 x} dx =$$
$$= \int \frac{1}{\cos^2 x} dx - \int dx = \tan x - x + \mathrm{C}$$

Detyra 112: $\int (\sin x + \cos x)\,dx$

Zgjidhje:

$$\int (\sin x + \cos x)\,dx = \int \sin x dx + \int \cos x dx = -\cos x + \sin x + \mathrm{C}$$

Detyra 113: $\int \sin^2 \frac{x}{2}\,dx$

Zgjidhje:

$$\int \sin^2 \frac{x}{2}\,dx = \frac{1}{2}\int 2\sin^2 \frac{x}{2}\,dx = \left| \begin{matrix} 1-\cos 2A = 2\sin^2 A \\ 1-\cos A = 2\sin^2 \frac{A}{2} \end{matrix} \right| = \frac{1}{2}\int (1-\cos x)\,dx =$$

$$= \frac{1}{2}\int dx - \frac{1}{2}\int \cos x dx = \frac{1}{2}x - \frac{1}{2}\sin x + \mathrm{C} = \frac{1}{2}(x - \sin x) + \mathrm{C}$$

Detyra 114: $\int \sin^3 (2x+1)\,dx$

Zgjidhje:

$$\int \sin^3 (2x+1)\,dx = \left| \begin{matrix} \sin 3A = 3\sin A - 4\sin^3 A \\ 4\sin^3 A = 3\sin A - \sin 3A \\ \sin^3 A = \frac{3}{4}\sin A - \frac{1}{4}\sin 3A \end{matrix} \right| = \int \left[\frac{3}{4}\sin(2x+1) - \frac{1}{4}3(2x+1) \right] dx =$$

$$= \frac{3}{4}\int \sin(2x+1)\,dx - \frac{1}{4}\int \sin(6x+3)\,dx = \frac{3}{4}\left[\frac{-\cos(2x+1)}{2} \right] - \frac{1}{4}\left[\frac{-\cos(6x+3)}{6} \right] + \mathrm{C} =$$

$$= -\frac{3}{8}\cos(2x+1) + \frac{1}{24}\cos(6x+3) + \mathrm{C}$$

Detyra 115: $\int \frac{\sin^2 x}{1+\cos x}\,dx$

Zgjidhje:

$$\int \frac{\sin^2 x}{1+\cos x}\,dx = \int \frac{1-\cos^2 x}{1+\cos x}\,dx = \int \frac{(1+\cos x)(1-\cos x)}{1+\cos x}\,dx = \int (1-\cos x)\,dx = \int dx - \int \cos x dx =$$

$$= x - \sin x + \mathrm{C}$$

Detyra 116: $\int\left(\sin^2 x-\cos^2 x\right)dx$

Zgjidhje:

$$\int\left(\sin^2 x-\cos^2 x\right)dx=\left|\cos 2A=\cos^2 A-\sin^2 A\right|=\int-\cos 2xdx=-\frac{\sin 2x}{2}+\mathrm{C}$$

Detyra 117: $\int\cos^4 2xdx$

Zgjidhje:

$$\int\cos^4 2xdx=\left|\begin{array}{l}1+\cos 2A=2\cos^2 A\\ 1+\cos 4A=2\cos^2 2A\\ \left(\dfrac{1+\cos 4A}{2}\right)=\cos^2 2A\end{array}\right|=\int\left(\frac{1+\cos 4x}{2}\right)^2 dx=\frac{1}{4}\int\left(1+2\cos 4x+\cos^2 4x\right)dx=$$

$$=\frac{1}{4}\int\left(1+2\cos 4x+\frac{1+\cos 8x}{2}\right)dx=\frac{1}{4}\int\left(1+2\cos 4x+\frac{1}{2}+\frac{1}{2}\cos 8x\right)dx=$$

$$=\frac{1}{4}\int\left(\frac{3}{2}+2\cos 4x+\frac{1}{2}\cos 8x\right)dx==\frac{3}{8}\int dx+\frac{1}{2}\int\cos 4xdx+\frac{1}{8}\int\cos 8xdx=$$

$$=\frac{3}{8}x+\frac{1}{2}\left(\frac{\sin 4x}{4}\right)+\frac{1}{8}\left(\frac{\sin 8x}{8}\right)+\mathrm{C}=\frac{3}{8}x+\frac{1}{8}\sin 4x+\frac{1}{64}\sin 8x+\mathrm{C}$$

Detyra 118: $\int\sqrt{1+\sin\frac{x}{2}}dx$

Zgjidhje:

$$\int\sqrt{1+\sin\frac{x}{2}}dx=\left|\begin{array}{l}\cos^2 A+\sin^2 A=1\\ \sin 2A=2\sin A\cos A\\ \sin A=2\sin\dfrac{A}{2}\cos\dfrac{A}{2}\end{array}\right|=\int\sqrt{\cos^2\frac{x}{4}+\sin^2\frac{x}{4}+2\sin\frac{x}{4}\cdot\cos\frac{x}{4}}dx=$$

$$=\int\sqrt{\left(\cos\frac{x}{4}+\sin\frac{x}{4}\right)^2}dx=\int\left(\cos\frac{x}{4}+\sin\frac{x}{4}\right)dx=\int\cos\frac{x}{4}dx+\int\sin\frac{x}{4}dx=$$

$$=\frac{\left(\sin\frac{x}{4}\right)}{\frac{1}{4}}+\frac{\left(-\cos\frac{x}{4}\right)}{\frac{1}{4}}+\mathrm{C}=4\sin\frac{x}{4}-4\cos\frac{x}{4}+\mathrm{C}=4\left(\sin\frac{x}{4}-\cos\frac{x}{4}\right)+\mathrm{C}$$

Detyra 119: $\int\left(\sqrt{x}-\cos^2\frac{x}{2}\right)dx$

Zgjidhje:

$$\int\left(\sqrt{x}-\cos^2\frac{x}{2}\right)dx=\int\sqrt{x}dx-\int\cos^2\frac{x}{2}dx=\left|\begin{matrix}\cos 2A=2\cos^2 A-1\\ \cos A=2\cos^2\frac{A}{2}-1\end{matrix}\right|=\int x^{\frac{1}{2}}dx-\int\frac{1+\cos x}{2}dx=$$

$$=\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}-\frac{1}{2}\left[x+\sin x\right]+\mathrm{C}=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{1}{2}x-\frac{1}{2}\sin x+\mathrm{C}=\frac{2}{3}x^{\frac{3}{2}}-\frac{1}{2}x-\frac{1}{2}\sin x+\mathrm{C}$$

Detyra 120: $\int\sin^4 xdx$

Zgjidhje:

$$\int\sin^4 xdx=\int\left(\sin^2 x\right)^2 dx=\int\left(\frac{1-\cos 2x}{2}\right)^2 dx=\frac{1}{4}\int\left(1+\cos^2 2x-2\cos 2x\right)dx=$$

$$=\frac{1}{4}\int\left(1+\frac{1+\cos 4x}{2}-2\cos 2x\right)dx=\frac{1}{4}\int\left(1+\frac{1}{2}+\frac{1}{2}\cos 4x-2\cos 2x\right)dx=$$

$$=\frac{1}{4}\int\left(\frac{3}{2}+\frac{1}{2}\cos 4x-2\cos 2x\right)dx==\int\left(\frac{3}{8}+\frac{1}{8}\cos 4x-\frac{1}{2}\cos 2x\right)dx=$$

$$=\frac{3}{8}\int dx+\frac{1}{8}\int\cos 4xdx-\frac{1}{2}\int\cos 2xdx=\frac{3x}{8}+\frac{1}{8}\cdot\frac{\sin 4x}{4}-\frac{1}{2}\cdot\frac{\sin 2x}{2}+\mathrm{C}=\frac{3x}{8}+\frac{1}{32}\sin 4x-\frac{1}{4}\sin 2x+\mathrm{C}$$

Detyra 121: $\int\cos^4 xdx$

Zgjidhje:

$$\int\cos^4 xdx=\int\left(\cos^2 x\right)^2 dx=\int\left(\frac{1+\cos 2x}{2}\right)^2 dx=\frac{1}{4}\int\left(1+\cos^2 2x+2\cos 2x\right)dx=$$

$$=\frac{1}{4}\int\left(1+\frac{1+\cos 4x}{2}+2\cos 2x\right)dx=\frac{1}{4}\int\left(1+\frac{1}{2}+\frac{1}{2}\cos 4x+2\cos 2x\right)dx=$$

$$=\frac{1}{4}\int\left(\frac{3}{2}+\frac{1}{2}\cos 4x+2\cos 2x\right)dx=\int\left(\frac{3}{8}+\frac{1}{8}\cos 4x+\frac{1}{2}\cos 2x\right)dx=$$

$$=\frac{3}{8}\int dx+\frac{1}{8}\int\cos 4xdx+\frac{1}{2}\int\cos 2xdx=\frac{3x}{8}+\frac{1}{8}\cdot\frac{\sin 4x}{4}+\frac{1}{2}\cdot\frac{\sin 2x}{2}+\mathrm{C}=\frac{3x}{8}+\frac{1}{32}\sin 4x+\frac{1}{4}\sin 2x+\mathrm{C}$$

Detyra 122: $\int \cos^2 nxdx$

Zgjidhje:

$$\int \cos^2 nxdx = \frac{1}{2}\int 2\cos^2 nxdx = \left|\cos 2A = 2\cos^2 A - 1\right| = \frac{1}{2}\int \left(1+\cos 2nx\right)dx =$$

$$= \frac{1}{2}\int dx + \frac{1}{2}\int \cos 2nxdx = \frac{x}{2} + \frac{1}{2}\cdot\frac{\sin 2nx}{2n} + \mathrm{C} = \frac{x}{2} + \frac{1}{4n}\cdot \sin 2nx + \mathrm{C}$$

Detyra 123: $\int \sin^3 xdx$

Zgjidhje:

$$\int \sin^3 xdx = \left|\begin{array}{l} \sin 3A = 3\sin A - 4\sin^3 A \\ 4\sin^3 A = 3\sin A - \sin 3A \\ \sin^3 A = \frac{3}{4}\sin A - \frac{1}{4}\sin 3A \end{array}\right| = \int \left(\frac{3}{4}\sin x - \frac{1}{4}\sin 3x\right)dx = \frac{3}{4}\int \sin xdx - \frac{1}{4}\int \sin 3xdx =$$

$$= \frac{3}{4}\left(-\cos x\right) - \frac{1}{4}\left(\frac{-\cos 3x}{3}\right) + \mathrm{C} = -\frac{3}{4}\cos x + \frac{1}{12}\cos 3x + \mathrm{C}$$

Detyra 124: $\int \sin^2\left(2x+5\right)dx$

Zgjidhje: $\int \sin^2\left(2x+5\right)dx = \left|\begin{array}{l} \cos 2A = 1 - 2\sin^2 A \\ 2\sin^2 A = 1 - \cos 2A \\ \sin^2 A = \frac{1-\cos 2A}{2} \end{array}\right| = \frac{1}{2}\int dx - \frac{1}{2}\int \cos\left(4x+10\right)dx =$

$$= \frac{x}{2} - \frac{1}{2}\cdot\frac{\sin\left(4x+10\right)}{4} + \mathrm{C} = \frac{x}{2} - \frac{1}{8}\sin\left(4x+10\right) + \mathrm{C}$$

Detyra 125: $\int \cos^3 xdx$

Zgjidhje: $\int \cos^3 xdx = \left|\begin{array}{l} \cos 3A = 4\cos^3 A - 3\cos A \\ 4\cos^2 A = \cos 3A + 3\cos A \\ \cos^3 A = \frac{\cos 3A + 3\cos A}{4} \end{array}\right| = \int \left(\frac{\cos 3x + 3\cos x}{4}\right)dx =$

$$= \frac{1}{4}\int \cos 3xdx + \int 3\cos xdx = \frac{1}{4}\left[\frac{\sin 3x}{x} + 3\sin x\right] + \mathrm{C}$$

Detyra 126: $\int \frac{dx}{\sin x}$

Zgjidhje:

$$\int \frac{dx}{\sin x} = \left| \begin{array}{l} \tan \frac{x}{2} = t \,/\, d \\ dx = \frac{2dt}{1+t^2} \\ \sin x = \frac{2t}{1+t^2} \end{array} \right| = \int \frac{\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2}} = \int \frac{dt}{t} = \ln|t| + \mathrm{C} = \ln\left|\tan \frac{x}{2}\right| + \mathrm{C}$$

Detyra 127: $\int \left(1 + \frac{1}{x} + \sin 2x\right) dx$

Zgjidhje:

$$\int \left(1 + \frac{1}{x} + \sin 2x\right) dx = \int dx + \int \frac{dx}{x} + \int \sin 2x dx = x + \ln|x| + \underbrace{\int \sin 2x dx}_{I_1} + \mathrm{C} =$$

$$I_1 = \int \sin 2x dx = \left| \begin{array}{l} Zëvendësojmë: \\ 2x = u \,/\, d \\ 2dx = du \\ dx = \frac{du}{2} \end{array} \right| = \int \frac{\sin u \cdot du}{2} = \frac{1}{2}\int \sin u du = \frac{1}{2}(-\cos u) + \mathrm{C}_1 = -\frac{\cos x}{2}$$

$$\int \left(1 + \frac{1}{x} + \sin 2x\right) dx = x + \ln|x| - \frac{\cos 2x}{2} + \mathrm{C}$$

Detyra 128: $\int \sin\left(\sqrt{x-1}\right) \frac{dx}{\sqrt{x-1}}$

Zgjidhje:

$$\int \sin\left(\sqrt{x-1}\right) \frac{dx}{\sqrt{x-1}} = \left| \begin{array}{l} Zëvendësojmë: \\ \sqrt{x-1} = t \,/^2 \\ x-1 = t^2 \,/\, d \\ dx = 2tdt \end{array} \right| = \int \frac{\sin t}{t} 2tdt = 2\int \sin tdt = -2\cos t + \mathrm{C} =$$

$$= -2\cos\left(\sqrt{x-1}\right) + \mathrm{C}$$

Detyra 129: $\int \cos(ax+b)\,dx$

Zgjidhje:

$$\int \cos(ax+b)\,dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ ax+b=u\ /\ d \\ adx=du \\ dx=\dfrac{du}{a} \end{array} \right| = \int \cos u \frac{du}{a} = \frac{1}{a}\int \cos u\,du = \frac{1}{a}\sin u + \mathrm{C} =$$

$$= \frac{1}{a}\sin(ax+b) + \mathrm{C}$$

Detyra 130: $\int \frac{\sin 2x}{1+\cos^2 x}\,dx$

Zgjidhje:

$$\int \frac{\sin 2x}{1+\cos^2 x}\,dx = \int \frac{2\sin x\cdot\cos x}{1+\cos^2 x}\,dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ 1+\cos^2 x=u\ /\ d \\ -2\sin x\cdot\cos x=du \\ 2\sin x\cdot\cos x=du \end{array} \right| = -\int \frac{du}{u} = -\ln|u| + \mathrm{C} =$$

$$= -\ln\left|1+\cos^2 x\right| + \mathrm{C}$$

Detyra 131: $\int \sin(5x-7)\,dx$

Zgjidhje:

$$\int \sin(5x-7)\,dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ 5x-7=u\ /\ d \\ 5dx=du \\ dx=\dfrac{du}{5} \end{array} \right| = \frac{1}{5}\int \sin u\,du = -\frac{1}{5}\cos u + \mathrm{C} = -\frac{1}{5}\cos(5x-7) + \mathrm{C}$$

Detyra 132: $\int \frac{\sin^3 x}{\cos^2 x}\,dx$

Zgjidhje:

$$\int \frac{\sin^3 x}{\cos^2 x}\,dx = \int \frac{\sin x\cdot\sin^2 x}{\cos^2 x}\,dx = \int \frac{\sin x\left(1-\cos^2 x\right)}{\cos^2 x}\,dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \cos x=u\ /\ d \\ \sin x dx=-du \end{array} \right| = -\int \frac{1-u^2}{u^2}\,du$$

$$= \frac{1}{u} + u + \mathrm{C} = \frac{1}{\cos x} + \cos x + \mathrm{C}$$

Detyra 133: $\int \cot^5 xdx$

Zgjidhje:

$$\int \cot^5 xdx = \int \frac{\cos^5 x}{\sin^5 x}dx = \int \frac{\cos x \cdot \cos^4 x}{\sin^5 x}dx = \int \frac{\cos x\left(1-\sin^2 x\right)^2}{\sin^5 x}dx = \begin{vmatrix} Zëvendësojmë: \\ \sin x = u / d \\ \cos xdx = du \end{vmatrix} =$$

$$= \int \frac{\left(1-u^2\right)^2}{u^5}du = \int \frac{1-2u^2+u^4}{u^5}du = \int u^{-5}du - 2\int u^{-5}du + \int \frac{du}{u} = -\frac{1}{4u^4} + \frac{1}{u^2} + \ln|u| + \mathrm{C} =$$

$$= -\frac{1}{4\sin^4 x} + \frac{1}{\sin^2 x} + \ln|\sin x| + \mathrm{C}$$

Detyra 134: $\int \frac{dx}{2\sin x - \cos x + 3}$

Zgjidhje:

$$\int \frac{dx}{2\sin x - \cos x + 3} = \begin{vmatrix} Zëvendësojmë: \\ \tan\frac{x}{2} = t / d; dx = \frac{2dt}{1+t^2} \\ \cos x = \frac{1-t^2}{1+t^2} \\ \sin x = \frac{2t}{1+t^3} \end{vmatrix} = \int \frac{\frac{2dt}{1+t^2}}{\frac{4t}{1+t^2} - \frac{1-t^2}{1+t^2} + 3} = 2\int \frac{dt}{4t^2+4t+2} = \frac{1}{2}\int \frac{dt}{t^2+t+\frac{1}{2}} =$$

$$= \frac{1}{2}\int \frac{dt}{\left(t+\frac{1}{2}\right)^2 + \frac{1}{4}} = \begin{vmatrix} Zëvendësojmë: \\ t+\frac{1}{2} = u / d; du = dt \\ a^2 = \frac{1}{4} \Rightarrow a = \frac{1}{2} \end{vmatrix} = \frac{1}{2}\int \frac{dt}{u^2+a^2} = \frac{1}{2a}\arctan\frac{u}{a} + \mathrm{C} = \frac{1}{2-\frac{1}{2}}\arctan\frac{t+\frac{1}{2}}{\frac{1}{2}} + \mathrm{C} =$$

$$= \arctan|2t+1| + \mathrm{C} = \arctan\left|2tg\frac{x}{2}+1\right| + \mathrm{C}$$

Detyra 135: $\int \frac{\sin x - \sin^3 x}{2\cos^2 x + \sin^2 x}dx$

Zgjidhje:

$$\int \frac{\sin x - \sin^3 x}{2\cos^2 x + \sin^2 x}dx = \int \frac{\sin x\left(1-\sin^2 x\right)}{\cos^2 x + 1}dx = \int \frac{\sin x \cos^2 x}{\cos^2 x + 1}dx = \begin{vmatrix} Zëvendësojmë: \\ \cos x = t / d \\ \sin xdx = -dt \end{vmatrix} = -\int \frac{t^2}{t^2+1}dt =$$

$$= -\int dt + \int \frac{dt}{t^2+1} = -t + \arctan t = -\cos x + \arctan\left(\cos x\right) + \mathrm{C}$$

Detyra 136: $\int \frac{2\arctan x}{1+x^2}dx$

Zgjidhje:

$$\int \frac{2\arctan x}{1+x^2}dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \arctan x = t \,/\, d \\ \dfrac{dx}{1+x^2} = dt \end{array} \right| = \int 2\cdot t dt = 2\frac{t^2}{2} + \mathrm{C} = t^2 + \mathrm{C} = \arctan x^2 + \mathrm{C}$$

Detyra 137: $\int \frac{(\arctan x)^2}{1+x^2}dx$

Zgjidhje:

$$\int \frac{(\arctan x)^2}{1+x^2}dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \arctan = u \,/\, d \\ \dfrac{dx}{1+x^2} = du \end{array} \right| = \int u^2 du = \frac{u^{2+1}}{2+1} + \mathrm{C} = \frac{u^3}{3} + \mathrm{C} = \frac{(\arctan x)^3}{3} + \mathrm{C}$$

Detyra 138: $\int \sin axdx$

Zgjidhje:

$$\int \sin axdx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ ax = t \,/\, d \\ dx = \dfrac{1}{a}dt \end{array} \right| = \frac{1}{a}\int \sin tdt = -\frac{1}{a}\cos t + \mathrm{C} = -\frac{\cos ax}{a}x + \mathrm{C}$$

Detyra 139: $\int \cos axdx$

Zgjidhje:

$$\int \cos axdx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ ax = t \,/\, d \\ dx = \dfrac{1}{a}dt \end{array} \right| = \frac{1}{a}\int \cos tdt = \frac{1}{a}\sin t + \mathrm{C} = \frac{\sin ax}{a}x + \mathrm{C}$$

Detyra 140: $\int \frac{\sqrt{\tan x}}{\cos^2 x} dx$

Zgjidhje:

$$\int \frac{\sqrt{\tan x}}{\cos^2 x} dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \cos x = t / d \\ \frac{1}{\cos^2 x} dx = dt \end{array} \right| = \int \sqrt{t} dt = \int t^{\frac{1}{2}} dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}\sqrt{t^3} + C = \frac{2}{3}\sqrt{\tan^3 x} + C$$

Detyra 141: $\int \frac{\sin x}{\sqrt{\cos^3 x}} dx$

Zgjidhje:

$$\int \frac{\sin x}{\sqrt{\cos^3 x}} dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \cos x = t / d \\ -\sin x dx = dt \end{array} \right| = -\int \frac{dt}{\sqrt{t^3}} = -\int t^{-\frac{3}{2}} dt = -\frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + C = \frac{2}{\sqrt{t}} + C = \frac{2}{\sqrt{\cos x}} + C$$

Detyra 142: $\int \frac{dx}{\sin^2 x \cdot \sqrt[4]{\cot x}}$

Zgjidhje:

$$\int \frac{dx}{\sin^2 x \cdot \sqrt[4]{\cot x}} = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \cot x = t / d \\ -\frac{1}{\sin^2 x} dx = dt \end{array} \right| = -\int \frac{dt}{\sqrt[4]{t}} = -\int t^{-\frac{1}{4}} dt = -\frac{t^{\frac{3}{4}}}{\frac{3}{4}} + C = -\frac{4}{3}\sqrt[4]{\cot^3 x} + C$$

Detyra 143: $\int \frac{\cos x + 1}{\sin x + x} dx$

Zgjidhje:

$$\int \frac{\cos x + 1}{\sin x + x} dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \sin x + x = t / d \\ (\cos x + 1) dx = dt \end{array} \right| = \int \frac{1}{t} dt = \ln|t| + C = \ln|\sin x + x| + C$$

Detyra 144: $\int \frac{\sin 2x}{\sin^2 x+3}dx$

Zgjidhje:

$$\int \frac{\sin 2x}{\sin^2 x+3}dx=\left|\begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \sin^2 x+3=t\,/\,d \\ (2\sin x\cos x)dx=dt \\ \sin 2xdx=dt \end{array}\right|=\int \frac{1}{t}dt=\ln|t|+\mathrm{C}=\ln\left|\sin^2 x+3\right|+\mathrm{C}$$

Detyra 145: $\int \sqrt{1+4\sin x}\cos xdx$

Zgjidhje:

$$\int \sqrt{1+4\sin x}\cos xdx=\left|\begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \sqrt{1+4\sin x}=t\,/^2 \\ 1+4\sin x=t^2\,/\,d \\ 4\cos dx=2tdt \\ \cos xdx=\frac{1}{2}tdt \end{array}\right|=\int t\cdot\frac{1}{2}tdt=\frac{1}{2}\int t^2dt=\frac{1}{2}\cdot\frac{t^3}{3}+\mathrm{C}=\frac{1}{6}t^3+\mathrm{C}=$$

$$=\frac{1}{6}\sqrt{(1+4\sin x)^3}+\mathrm{C}$$

Detyra 146: $\int \frac{\arctan\sqrt{x}}{\sqrt{x}}\cdot\frac{dx}{1+x}$

Zgjidhje:

$$\int \frac{\arctan\sqrt{x}}{\sqrt{x}}\cdot\frac{dx}{1+x}=\left|\begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \sqrt{x}=t\,/^2 \\ x=t^2\,/\,d \\ dx=2tdt \end{array}\right|=\int \frac{\arctan t}{t}\cdot\frac{2tdt}{1+t^2}tdt=2\int \arctan t\,\frac{dt}{1+t^2}=\left|\begin{array}{l} \arctan t=u\,/\,d \\ \frac{dt}{1+t^2}=du \end{array}\right|=$$

$$=2\int udu=2\cdot\frac{u^2}{2}+\mathrm{C}=u^2+\mathrm{C}=(\arctan t)^2+\mathrm{C}=\left(\arctan\sqrt{x}\right)^2+\mathrm{C}=\arctan^2\sqrt{x}+\mathrm{C}$$

Detyra 147: $\sqrt{1-x}dx$

Zgjidhje:

$$\int \sqrt{1-x}dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \sqrt{1-x} = t \,/^2 \\ 1-x = t^2 \,/\, d \\ -dx = 2tdt \\ dx = -2tdt \end{array} \right| = \int t(-2t)\,dt = -2\int t^2 dt = -\frac{2}{3}t^3 + \mathrm{C} = -\frac{2}{3}\sqrt{(1-x)^3} + \mathrm{C}$$

Detyra 148: $\int (3-2x)^6\, dx$

Zgjidhje:

$$\int (3-2x)^6\, dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ 3-2x = t \,/\, d \\ -2dx = dt \\ dx = -\frac{1}{2}dt \end{array} \right| = \int t^6 \left(-\frac{1}{2}\right) dt = -\frac{1}{2}\int t^6 dt = -\frac{1}{2}\cdot\frac{t^{6+1}}{6+1} + \mathrm{C} = -\frac{1}{2}\cdot\frac{t^7}{7} + \mathrm{C} =$$

$$= -\frac{t^7}{14} + \mathrm{C} = -\frac{(3-2x)^7}{14} + \mathrm{C}$$

Detyra 149: $\int (5-2x)^2\, dx$

Zgjidhje:

$$\int (5-2x)^2\, dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ 5-2x = t \,/\, d \\ -2dx = dt \\ dx = -\frac{1}{2}dt \end{array} \right| = \int t^2 \left(-\frac{1}{2}dt\right) = -\frac{1}{2}\int t^2 dt = -\frac{1}{2}\cdot\frac{t^{2+1}}{2+1} + \mathrm{C} =$$

$$= -\frac{1}{2}\cdot\frac{t^3}{3} + \mathrm{C} = -\frac{(5-2x)^3}{6} + \mathrm{C}$$

Detyra 150: $\int \frac{dx}{x+4}$

Zgjidhje:

$$\int \frac{dx}{x+4} = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ x+4 = t \,/\, d \\ dx = dt \end{array} \right| = \int \frac{dt}{t} = \ln|t| + \mathrm{C} = \ln|x+4| + \mathrm{C}$$

Detyra 151: $\int \frac{dx}{\sqrt{2x-5}}$

Zgjidhje:

$$\int \frac{dx}{\sqrt{2x-5}} = \left| \begin{array}{l} Zëvendësojmë \\ \sqrt{2x-5} = t \, /^2 \\ 2x-5 = t^2 \, / \, d \\ 2dx = 2tdt \\ dx = dt \end{array} \right| = \int \frac{tdt}{t} = \int dt = t + \mathrm{C} = \sqrt{2x-5} + \mathrm{C}$$

Detyra 152: $\int \frac{dx}{(x-2)^3}$

Zgjidhje:

$$\int \frac{dx}{(x-2)^3} = \left| \begin{array}{l} Zëvendësojmë: \\ x-2 = t \, / \, d \\ dx = dt \end{array} \right| = \int \frac{dt}{t^3} = \int t^{-3} dt = \frac{t^{-3+1}}{-3+1} + \mathrm{C} = \frac{t^{-2}}{-2} + \mathrm{C} =$$

$$= -\frac{1}{2t^2} + \mathrm{C} = -\frac{1}{2(x-2)^2} + \mathrm{C}$$

Detyra 153: $\int \frac{dx}{\sqrt{x}\left(1+\sqrt{x}\right)}$

Zgjidhje:

$$\int \frac{dx}{\sqrt{x}\left(1+\sqrt{x}\right)} = \left| \begin{array}{l} Zëvendësojmë: \\ 1+\sqrt{x} = t \, / \, d \\ \frac{1}{2\sqrt{x}} dx = dt \\ \frac{dx}{\sqrt{x}} = 2dt \end{array} \right| = \int \frac{2dt}{t} = 2\int \frac{dt}{t} = 2\ln|t| + \mathrm{C} = 2\ln\left|1+\sqrt{x}\right| + \mathrm{C}$$

Detyra 154: $\int \frac{dx}{(1-x)^4}$

Zgjidhje:

$$\int \frac{dx}{(1-x)^4} = \left| \begin{array}{l} Zëvendësojmë: \\ 1-x = t \, / \, d \\ dx = -dt \end{array} \right| = -\int \frac{dt}{t^4} = -\int t^{-4} dt = \frac{t^{-4+1}}{-4+1} + \mathrm{C} = \frac{t^{-3}}{-3} + \mathrm{C} = 3\frac{1}{t^3} = \frac{3}{(1-x)^3} + \mathrm{C}$$

Detyra 155: $\int \frac{dx}{\sqrt{x}-1}$

Zgjidhje:

$$\int \frac{dx}{\sqrt{x}-1} = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \sqrt{x} = t /^2 \\ x = t^2 / d \\ dx = 2tdt \end{array} \right| = \int \frac{2tdt}{t-1} = 2\int \frac{t}{t-1}dt = 2\int dt + 2\int \frac{dt}{t-1} = 2t + 2\ln|t-1| = 2\sqrt{x} + 2\ln\left|\sqrt{x}-1\right| + \mathrm{C}$$

Detyra 156: $\int \sqrt{2x-3}dx$

Zgjidhje:

$$\int \sqrt{2x-3}dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ 2x-3 = t / d \\ 2dx = dt \\ dx = \frac{1}{2}dt \end{array} \right| = \int \sqrt{t}\,\frac{1}{2}dt = \frac{1}{2}\int t^{\frac{1}{2}}dt = \frac{1}{2}\cdot\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \mathrm{C} = \frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \mathrm{C} =$$

$$= \frac{\sqrt{t^3}}{3} + \mathrm{C} = \frac{\sqrt{(2x-3)^3}}{3} + \mathrm{C}$$

Detyra 157: $\int \sqrt[3]{1-x}dx$

Zgjidhje:

$$\int \sqrt[3]{1-x}dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ 1-x = t / d \\ -dx = dt \\ dx = -dt \end{array} \right| = -\int \sqrt[3]{t}dt = -\int t^{\frac{1}{3}}dt = -\frac{t^{\frac{4}{3}}}{\frac{4}{3}} + \mathrm{C} = -\frac{3\sqrt[3]{t^4}}{3} + \mathrm{C} = -\frac{3\sqrt[3]{(1-x)^4}}{3} + \mathrm{C}$$

Detyra 158: $\int \frac{x}{x^2+1}dx$

Zgjidhje:

$$\int \frac{x}{x^2+1}dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ x^2+1 = t / d \\ 2xdx = dt \\ xdx = \frac{1}{2}dt \end{array} \right| = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t| = \frac{1}{2}\ln\left|x^2+1\right| + \mathrm{C}$$

Detyra 159: $\int \frac{x-2}{\sqrt{x-3}}dx$

Zgjidhje:

$$\int \frac{x-2}{\sqrt{x-3}}dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \sqrt{x-3} = t\ /^2 \\ x-3 = t^2 \Rightarrow x = t^2+3\ /\ d \\ dx = 2tdt \end{array} \right| = \int \frac{\left(t^2+3-2\right)\cdot 2tdt}{t} = 2\int\left(t^2+1\right)dt =$$

$$= 2\int t^2dt + 2\int dt = 2\frac{t^3}{3} + 2\cdot t = \frac{2}{3}\sqrt{\left(x-3\right)^3} + 2\sqrt{\left(x-3\right)} + \mathrm{C}$$

Detyra 160: $\int x\sqrt{2+3x^2}dx$

Zgjidhje:

$$\int x\sqrt{2+3x^2}dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \sqrt{2+3x^2} = t\ /^2 \\ 2+3x^2 = t^2\ /\ d \\ 6xdx = 2tdt \\ xdx = \frac{tdt}{3} \end{array} \right| = \int t\cdot\frac{tdt}{3} = \frac{1}{3}\int t^2dt = \frac{1}{3}\cdot\frac{t^3}{3} + C = \frac{t^3}{9} + \mathrm{C} = \frac{\sqrt{\left(2+3x^2\right)^3}}{9} + \mathrm{C}$$

Detyra 161: $\int \frac{x^5}{x^6-1}dx$

Zgjidhje:

$$\int \frac{x^5}{x^6-1}dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ x^6-1 = t\ /\ d \\ 6x^5dx = dt \\ x^5dx = \frac{dt}{6} \end{array} \right| = \frac{1}{6}\int\frac{dt}{t} = \frac{1}{6}\ln|t| = \frac{1}{6}\ln\left|x^6-1\right| + \mathrm{C}$$

Detyra 162: $\int \frac{dx}{5-3x}$

Zgjidhje:

$$\int \frac{dx}{5-3x}=\left|\begin{array}{l} \textit{Zëvendësojmë}: \\ 5-3x=t\,/\,d \\ -3dx=dt \\ dx=-\frac{1}{3}dt \end{array}\right|=-\frac{1}{3}\int\frac{dt}{t}=-\frac{1}{3}\ln|t|=-\frac{1}{3}\ln|5-3x|+\mathrm{C}$$

Detyra 163: $\int x\sqrt{x-1}dx$

Zgjidhje:

$$\int x\sqrt{x-1}dx=\left|\begin{array}{l} \textit{Zëvendësojmë}: \\ \sqrt{x-1}=t\,/^2 \\ x-1=t^2 \Rightarrow x=t^2+1\,/\,d \\ dx=2tdt \end{array}\right|=\int\left(t^2+1\right)t\cdot 2tdt=2\int\left(t^2+1\right)t^2dt=$$

$$=2\int\left(t^4+t^2\right)dt=2\int t^4dt+2\int t^2dt=2\frac{t^{4+1}}{4+1}+2\frac{t^{2+1}}{2+1}=2\frac{t^5}{5}+2\frac{t^3}{3}=\frac{2}{5}\sqrt{\left(x-1\right)^5}+\frac{2}{3}\sqrt{\left(x-1\right)^3}+\mathrm{C}$$

Detyra 164: $\int \sqrt[3]{1-3x}dx$

Zgjidhje:

$$\int \sqrt[3]{1-3x}dx=\left|\begin{array}{l} \textit{Zëvendësojmë}: \\ \sqrt[3]{1-3x}=t\,/^3 \\ 1-3x=t^3\,/\,d \\ -3dx=3t^2dt \\ dx=-t^2dt \end{array}\right|=-\int t\cdot t^2dt=-\int t^3dt=-\frac{t^{3+1}}{3+1}+\mathrm{C}=-\frac{t^4}{4}+\mathrm{C}=-\frac{1}{4}t^4+\mathrm{C}=-\frac{1}{4}\sqrt[3]{\left(1-3x\right)^4}+\mathrm{C}$$

Detyra 165: $\int\left(1+2x\right)^5dx$

Zgjidhje:

$$\int\left(1+2x\right)^5dx=\left|\begin{array}{l} \textit{Zëvendësojmë}: \\ 1+2x=t\,/\,d \\ 2dx=dt \\ dx=\frac{1}{2}dt \end{array}\right|=\frac{1}{2}\int t^5dt=\frac{1}{2}\cdot\frac{t^{5+1}}{5+1}+\mathrm{C}=\frac{1}{2}\cdot\frac{t^6}{6}+\mathrm{C}=\frac{t^6}{12}+\mathrm{C}=\frac{1}{12}t^6+\mathrm{C}=\frac{1}{12}\left(1+2x\right)^6+\mathrm{C}$$

Detyra 166: $\int\left(x^2-1\right)\sqrt{x}dx$

Zgjidhje:

Mënyra : I

$$\int\left(x^2-1\right)\sqrt{x}dx=\left|\begin{array}{l}\sqrt{x}=t\,|^2\\ x=t^2\,/\,d\\ dx=2tdt\end{array}\right|=\int\left(\left(t^2\right)^2-1\right)t\cdot 2tdt=2\int\left(t^4-1\right)t^2dt=2\int t^4\cdot t^2dt-2\int t^2dt=$$

$$=2\int t^6dt-2\int t^2dt=\frac{2}{7}t^7-\frac{2}{3}t^3=\frac{2}{7}\left(\sqrt{x}\right)^7-\frac{2}{3}\left(\sqrt{x}\right)^3+\mathrm{C}$$

Mënyra : II

$$\int\left(x^2-1\right)\sqrt{x}dx=\int\left(x^2-1\right)x^{\frac{1}{2}}dx=\int\left(x^2\cdot x^{\frac{1}{2}}\cdot x^{\frac{1}{2}}\right)dx=\int x^{\frac{5}{2}}dx-\int x^{\frac{1}{2}}dx=$$

$$=\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1}-\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}=\frac{2}{7}x^{\frac{7}{2}}-\frac{2}{3}x^{\frac{3}{2}}+\mathrm{C}=\frac{2}{7}\left(\sqrt{x}\right)^7-\frac{2}{3}\left(\sqrt{x}\right)^3+\mathrm{C}$$

Detyra 167: $\int\frac{x^2}{1+x^6}dx$

Zgjidhje:

$$\int\frac{x^2}{1+x^6}dx=\int\frac{x^2}{1+\left(x^3\right)^2}dx=\left|\begin{array}{l}Z\ddot{e}vend\ddot{e}sojm\ddot{e}:\\ x^3=u\,/\,d\\ 3x^2dx=du\\ x^2dx=\frac{du}{3}\end{array}\right|=\frac{1}{3}\int\frac{du}{1+u^2}=\frac{1}{3}\arctan u+\mathrm{C}=\frac{1}{3}\arctan x^3+\mathrm{C}$$

Detyra 168: $\int\frac{2x^3}{4x^4+1}dx$

Zgjidhje:

$$\int\frac{2x^3}{4x^4+1}dx=\left|\begin{array}{l}Z\ddot{e}vend\ddot{e}sojm\ddot{e}:\\ 4x^4+1=t\,/\,d\\ 16x^3dx=dt\\ 2\cdot 8x^3dx=dt\\ 2x^3dx=\frac{1}{8}dt\end{array}\right|=\frac{1}{8}\int\frac{dt}{t}=\frac{1}{8}\ln|t|=\frac{1}{8}\left|4x^4+1\right|+\mathrm{C}$$

Detyra 169: $\int \frac{e^x dx}{\sqrt{e^x+1}}$

Zgjidhje:

$$\int \frac{e^x dx}{\sqrt{e^x+1}} = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \sqrt{e^x+1} = t\ /^2 \\ e^x+1 = t^2\ /\ d \\ e^x dx = 2tdt \end{array} \right| = \int \frac{2tdt}{t} = 2\int dt = 2t + \mathrm{C} = 2\sqrt{e^x+1} + \mathrm{C}$$

Detyra 170: $\int \frac{dx}{\sqrt{3-4x}}$

Zgjidhje:

$$\int \frac{dx}{\sqrt{3-4x}} = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \sqrt{3-4x} = t\ |^2 \\ 3-4x = t^2\ /\ d \\ -4dx = 2tdt \\ dx = -\frac{2}{4}dt \\ dx = -\frac{1}{2}tdt \end{array} \right| = -\frac{1}{2}\int \frac{tdt}{t} = -\frac{1}{2}t = -\frac{1}{2}\sqrt{3-4x} + \mathrm{C}$$

Detyra 171: $\int \frac{dx}{\sqrt{e^x+1}}$

Zgjidhje:

$$\int \frac{dx}{\sqrt{e^x+1}} = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \sqrt{e^x+1} = u\ |^2 \\ e^x+1 = u^2\ /\ d \\ e^x = u^2-1 \\ e^x dx = 2udu \\ dx = \frac{2u\cdot du}{e^x} = \frac{2u\cdot du}{u^2-1} \end{array} \right| = \int \frac{2u\cdot du}{u\left(u^2-1\right)} = 2\int \frac{du}{u^2-1} = \int \frac{du}{u-1} + \int \frac{-du}{u+1} = \ln|u-1| - \ln|u+1| + \mathrm{C} =$$

$$= \ln\left|\frac{u-1}{u+1}\right| + \mathrm{C} = \ln\left|\frac{\sqrt{e^x+1}-1}{\sqrt{e^x+1}+1}\right| + \mathrm{C}$$

Detyra 172: $\int \frac{dx}{x(1+\ln x)}$

Zgjidhje:

$$\int \frac{dx}{x(1+\ln x)} = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ 1+\ln x = t \,/\, d \\ \frac{1}{x}dx = dt \\ dx = xdt \end{array} \right| = \int \frac{xdt}{x \cdot t} = \int \frac{dt}{t} = \ln|t| + \mathrm{C} = \ln|1+\ln x| + \mathrm{C}$$

Detyra 173: $\int \frac{\sqrt[3]{x}}{x\sqrt{x} - x\sqrt[3]{x}}dx$

Zgjidhje:

$$\int \frac{\sqrt[3]{x}}{x\sqrt{x} - x\sqrt[3]{x}}dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ x = t^6 \,/\, d \\ dx = 6t^5 dt \end{array} \right| = 6\int \frac{dt}{t(t-1)} = 6\ln\left|\frac{t-1}{t}\right| + \mathrm{C} = \ln\left|\frac{\sqrt[6]{x}-1}{\sqrt[x]{6}}\right| + \mathrm{C}$$

Detyra 174: $\int e^{\cos x} \sin x dx$

Zgjidhje: $$\int e^{\cos x} \sin x dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \cos x = t \,/\, d \\ \sin x dx = -dt \end{array} \right| = -\int e^t dt = -e^t + \mathrm{C} = -e^{\cos x} + \mathrm{C}$$

Detyra 175: $\int \frac{\sqrt{x+1}+2}{(x+1)^2 - \sqrt{x+1}}dx$

Zgjidhje:

$$\int \frac{\sqrt{x+1}+2}{(x+1)^2 - \sqrt{x+1}}dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \sqrt{x+1} = t \,/^2 \\ x+1 = t^2 \\ dx = 2tdt \end{array} \right| = \int \frac{t+2}{t^2 - t} \cdot 2tdt = 2\int \frac{t+2}{t-2} = 2\int \left[1 + \frac{3}{t-1}\right]dt =$$

$$= 2t + 6\ln|t-1| + \mathrm{C} = 2\sqrt{x+1} + 6\ln\left|\sqrt{x+1} - 1\right| + \mathrm{C}$$

Detyra 176: $\int \frac{e^x dx}{4+e^{2x}}$

Zgjidhje:

$$\int \frac{e^x dx}{4+e^{2x}} = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ e^x = t/d \\ e^x dx = dt \end{array} \right| = \int \frac{dt}{4+t^2} = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ a^2 = 4/\sqrt{\ } \\ a = 2 \end{array} \right| = \frac{1}{a}\arctan\frac{t}{a} + \mathrm{C} = \frac{1}{2}\arctan\frac{e^x}{2} + \mathrm{C}$$

Detyra 177: $\int \frac{\left(2^x+1\right)^3}{2^x}dx$

Zgjidhje:

$$\int \frac{\left(2^x+1\right)^3}{2^x}dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ 2^x = t/d \\ 2^x \ln 2 dx = dt \\ dx = \frac{dt}{\ln 2 \cdot 2^x} = \frac{dt}{\ln 2 \cdot t} \end{array} \right| = \int \frac{\left(t+1\right)^3}{t} \cdot \frac{dt}{\ln 2 \cdot t} = \frac{1}{\ln 2}\int \frac{t^3+3t^2+3t+1}{t^2}dt =$$

$$= \frac{1}{\ln 2}\left[\int\left(t+3+\frac{3}{t}+\frac{1}{t^2}\right)\right]dt = \frac{1}{\ln 2}\left(\frac{t^2}{2}+3t+3\ln|t|-\frac{1}{t}\right)+\mathrm{C} = \frac{1}{\ln 2}\left(\frac{2^{2x}}{2}+3\cdot 2^x+3x\ln 2-\frac{1}{2^x}\right)+\mathrm{C}$$

Detyra 178: $\int e^x\sqrt{1+e^x}dx$

Zgjidhje:

$$\int e^x\sqrt{1+e^x}dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ 1+e^x = t/d \\ e^x dx = dt \end{array} \right| = \int \sqrt{t}dt = \int t^{\frac{1}{2}}dt = \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\mathrm{C} = \frac{t^{\frac{3}{2}}}{\frac{3}{2}}+\mathrm{C} = \frac{2}{3}t^{\frac{3}{2}}+\mathrm{C} = \frac{2}{3}\sqrt{\left(1+e^x\right)^3}+\mathrm{C}$$

Detyra 179: $\int \frac{dx}{x^2+x+1}$

Zgjidhje:

$$\int \frac{dx}{x^2+x+1} = \int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}} = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ x+\frac{1}{2} = t/d; dx = dt \\ a^2 = \frac{3}{4} \Rightarrow a = \frac{\sqrt{3}}{2} \end{array} \right| = \int \frac{dt}{t^2+a^2} = \frac{1}{a}arctg\frac{t}{a}+\mathrm{C} = \frac{2}{\sqrt{3}}arctg\left(\frac{2x+1}{2\sqrt{3}}\right)+\mathrm{C}$$

Detyra 180: $\int \frac{dx}{\sqrt{2x^2+3x}}$

Zgjidhje:

$$\int \frac{dx}{\sqrt{2x^2+3x}} = \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{x^2+\frac{3}{2}x}} = \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\left(x+\frac{3}{4}\right)^2-\frac{9}{16}}} = \left| \begin{array}{l} Zëvendësojmë: \\ x+\frac{3}{4}=t; dx=dt \\ a^2=\frac{9}{16} \Rightarrow a=\frac{3}{4} \end{array} \right| = \frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{t^2-a^2}} =$$

$$= \frac{1}{\sqrt{2}}\ln\left(t+\sqrt{t^2-a^2}\right)+C = \frac{1}{\sqrt{2}}\ln\left(\frac{4x+3}{4}+\sqrt{x^2+\frac{3}{2}x}\right)+C$$

Detyra 181: $\int (4x+3)^{\frac{1}{3}}\,dx$

Zgjidhje:

$$\int (4x+3)^{\frac{1}{3}}\,dx = \left| \begin{array}{l} Zëvendësojmë: \\ 4x+3=u \\ 4dx=du \\ dx=\frac{du}{4} \end{array} \right| = \int u^{\frac{1}{3}}\frac{du}{4} = \frac{1}{4}\int u^{\frac{1}{3}}du = \frac{1}{4}\cdot\frac{u^{\frac{1}{3}+1}}{\frac{1}{3}+1}+C = \frac{1}{4}\cdot\frac{u^{\frac{4}{3}}}{\frac{4}{3}}+C =$$

$$= \frac{3}{16}u^{\frac{4}{3}}+C = \frac{3}{16}(4x+3)^{\frac{4}{3}}+C$$

Detyra 182: $\int \frac{xdx}{\left(1-x^2\right)^3}$

Zgjidhje:

$$\int \frac{xdx}{\left(1-x^2\right)^3} = \left| \begin{array}{l} Zëvendësojmë: \\ 1-x^2=u \\ -2xdx=du \\ xdx=\frac{du}{-2} \end{array} \right| = -\frac{1}{2}\int \frac{du}{u^3} = \frac{1}{4u^2}+C = \frac{1}{4\left(1-x^2\right)^2}+C$$

Detyra 183: $\int e^x \cdot \sqrt{1-e^x}\,dx$

Zgjidhje:

$$\int e^x \cdot \sqrt{1-e^x}\,dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ 1-e^x = u^2 \\ e^x dx = -2udu \\ -e^x dx = 2udu \end{array} \right| = -2\int u^2 du = -\frac{2}{3}u^3 + \mathrm{C} = -\frac{2}{3}\sqrt{\left(1-e^x\right)^3} + \mathrm{C}$$

Detyra 184: $\int e^{-x} dx$

Zgjidhje:

$$\int e^{-x} dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ -x = u \, / \, d \\ -dx = du \Rightarrow dx = \dfrac{du}{-1} \end{array} \right| = \int e^u \frac{du}{-1} = -\int e^u du = -e^u + \mathrm{C} = -e^{-x} + \mathrm{C}$$

Detyra 185: $\int \frac{dx}{3x^2-2x+5}$

Zgjidhje:

$$\int \frac{dx}{3x^2-2x+5} = \frac{1}{3}\int \frac{dx}{x^2-\frac{2}{3}x+\frac{5}{3}} = \left| \begin{array}{l} x^2 \pm p + q = \\ = \left(x \pm \dfrac{p}{2}\right)^2 - \left(\dfrac{p}{2}\right)^2 \pm q \end{array} \right| = \frac{1}{3}\int \frac{dx}{\left(x-\frac{1}{3}\right)^2 - \frac{1}{9} + \frac{5}{3}} =$$

$$= \frac{1}{3}\int \frac{dx}{\left(x-\frac{1}{3}\right)^2 + \frac{14}{9}} = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ x - \dfrac{1}{3} = t \, / \, d \\ dx = dt \\ a^2 = \dfrac{14}{9}, \quad a = \dfrac{\sqrt{14}}{3} \end{array} \right| = \frac{1}{3}\int \frac{dt}{t^2+a^2} = \frac{1}{3}\cdot\frac{1}{k}\arctan\frac{t}{k} = \frac{1}{3}\cdot\frac{3}{\sqrt{14}}\arctan\frac{\left(x-\frac{1}{3}\right)\cdot 3}{\sqrt{14}} =$$

$$= \frac{1}{\sqrt{14}}\arctan\frac{3x-1}{\sqrt{14}} + \mathrm{C}$$

Detyra 186: $\int \frac{dx}{ax+b}$

Zgjidhje:

$$\int \frac{dx}{ax+b} = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ ax+b=u\ /\ d \\ adx=du \\ dx=\frac{du}{a} \end{array} \right| = \int \frac{\frac{du}{a}}{u} = \frac{1}{a}\int \frac{du}{u} = \frac{1}{a}\ln|u| + \mathrm{C} = \frac{1}{a}\ln|ax+b| + \mathrm{C}$$

Detyra 187: $\int \sqrt{\frac{\ln\left(x+\sqrt{1+x^2}\right)}{1+x^2}}dx$

Zgjidhje:

$$\int \sqrt{\frac{\ln\left(x+\sqrt{1+x^2}\right)}{1+x^2}}dx = \left| \begin{array}{l} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: \\ \ln\left(x+\sqrt{1+x^2}\right) = t\ /\ d \\ \frac{1}{x+\sqrt{1+x^2}} \cdot \left(\sqrt{1+x^2}+x\right)dx = dt \\ \frac{1}{x+\sqrt{1+x^2}} \cdot \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}dx = dt \\ \frac{dx}{\sqrt{1+x^2}} = dt \end{array} \right| = \int \sqrt{t} \cdot \frac{dt}{\sqrt{1+x^2}} = \int \sqrt{t}dt = \int t^{\frac{1}{2}}dt =$$

$$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \mathrm{C} = \frac{2}{3}\sqrt{t^3} + \mathrm{C} = \frac{2}{3}\sqrt{\left[\ln\left(x+\sqrt{1+x^2}\right)\right]^3} + \mathrm{C}$$

Detyra 188: $\int x \cdot \sin xdx$

Zgjidhje:

$$\int x \cdot \sin xdx = \left| \begin{array}{ll} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: & \\ u = x, & dv = \sin xdx \\ du = dx, & v = \int \sin xdx = -\cos x \end{array} \right| = -x \cdot \cos x - \int \left(-\cos x\right)dx =$$

$$= -x \cdot \cos x + \int \cos xdx = = -x \cdot \cos x + \sin x + \mathrm{C}$$

Detyra 189: $\int \ln x dx$

Zgjidhje:

$$\int \ln x dx = \left| \begin{array}{ll} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: & \\ u = \ln x, & dv = dx \\ du = \dfrac{dx}{x}, & v = \int dx = x \end{array} \right| = x\ln x - \int x\frac{dx}{x} = x\ln x - \int dx = x\ln x - x + \text{C}$$

Detyra 190: $\int (x-1)\cdot e^x dx$

Zgjidhje:

$$\int (x-1)\cdot e^x dx = \left| \begin{array}{ll} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: & \\ u = x-1, & dv = e^x dx \\ du = dx, & v = \int e^x dx = e^x \end{array} \right| = (x-1)e^x - \int e^x \cdot \frac{1}{x} dx = (x-1)e^x - e^e + \text{C}$$

Detyra 191: $\int \left(x^2+2x\right)\cdot e^x dx$

Zgjidhje:

$$\int \left(x^2+2x\right)\cdot e^x dx = \left| \begin{array}{ll} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: & \\ u = x+1, & dv = e^x dx \\ du = dx, & v = \int e^x dx = e^x \end{array} \right| = \left(x^2+2x\right)e^x - \int (2x+2)\cdot e^x dx =$$

$$= \left(x^2+2x\right)e^x - 2\int (x+1)e^x dx = \left(x^2+2x\right)e^x - 2\left|(x+1)e^x - \int e^x dx\right| = \left(x^2+2x\right)e^x - 2(x+1)e^x + 2e^x + \text{C}$$

Detyra 192: $\int x\ln x dx$

Zgjidhje:

$$\int x\ln x dx = \left| \begin{array}{ll} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: & \\ u = \ln x, & dv = x dx \\ du = \dfrac{dx}{x}, & v = \int x dx = \dfrac{x^2}{2} \end{array} \right| = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{dx}{x} = \frac{x^2}{2}\ln x - \frac{1}{2}\int x dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + \text{C}$$

Detyra 193: $\int x \cdot e^x dx$

Zgjidhje:

$$\int x \cdot e^x dx = \left| \begin{array}{ll} \textit{Zëvendësojmë}: & \\ u = x, & dv = e^x dx \\ du = dx, & v = \int e^x dx = e^x \end{array} \right| = x \cdot e^x - \int e^x dx = xe^x - e^x + \mathrm{C}$$

Detyra 194: $\int \arctan x dx$

Zgjidhje:

$$\int \arctan x dx = \left| \begin{array}{ll} \textit{Zëvendësojmë}: & \\ u = \arctan x, & dv = dx \\ du = \dfrac{1}{1+x^2} dx, & v = \int dx = x \end{array} \right| = x \cdot \arctan x - \int \frac{x}{1+x^2} dx =$$

$$= x \cdot \arctan x - \frac{1}{2} \int \frac{d\left(1+x^2\right)}{1+x^2} = x \cdot \arctan x - \frac{1}{2} \ln\left|1+x^2\right| + \mathrm{C}$$

Detyra 195: $\int x \cdot \arctan x dx$

Zgjidhje:

$$\int x \cdot \arctan x dx = \left| \begin{array}{ll} \textit{Zëvendësojmë}: & \\ u = \arctan x, & dv = x dx \\ du = \dfrac{dx}{1+x^2}, & v = \int x dx = \dfrac{x^2}{2} \end{array} \right| = \frac{1}{2} x^2 \arctan x - \frac{1}{2} \int \frac{x^2 dx}{1+x^2} =$$

$$= \frac{1}{2} x^2 \arctan x - \frac{1}{2} \int \frac{x^2 + 1 - 1}{1+x^2} dx = \frac{1}{2} x^2 \arctan x - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2}\right) dx =$$

$$= \frac{1}{2} x^2 \arctan x - \frac{1}{2} x + \mathrm{C} = \frac{1}{2} \arctan x + \mathrm{C}$$

Detyra 196: $\int e^x \cos x dx$

Zgjidhje:

$$\int e^x \cos x dx = \left| \begin{array}{l} \textit{Zëvendësojmë}: \\ u = e^x \,/\, d; du = e^x \\ dv = \cos x dx \\ v = \sin x \end{array} \right| = e^x \sin x - \left[-e^x \cos x + \int e^x \cos x dx \right] =$$

$$= e^x \sin x + e^x \cos x - \int e^x \cos x dx = \frac{e^x}{2}(\sin x + \cos x) + \mathrm{C}$$

Detyra 197: $\int \frac{xdx}{\sin^2 x}$

Zgjidhje:

$$\int \frac{xdx}{\sin^2 x} = \left| \begin{array}{l} \textit{Zëvendësojmë}: \\ u = x \,|\, d; du = dx \\ dv = \dfrac{dx}{\sin^2 x} \\ v = \int \dfrac{dx}{\sin^2 x} = -\cot x + \mathrm{C} \end{array} \right| = \int \frac{xdx}{\sin^2 x} = u \cdot v \cdot \int udu = -x \cdot ctgx + \underbrace{\int \frac{\cos x}{\sin x} dx}_{I_1} =$$

$$\mathrm{I}_1 = \int \frac{\cos x}{\sin x} dx = \left| \begin{array}{l} \sin x = t \,/\, d \\ \cos x dx = dt \end{array} \right| = \int \frac{dt}{t} = \ln|t| + \mathrm{C} = \ln|\sin x| + \mathrm{C} = -x \cdot ctgx + \ln|\sin x| + \mathrm{C}$$

Detyra 198: $\int \frac{\ln x}{x^3} dx$

Zgjidhje:

$$\int \frac{\ln x}{x^3} dx = \left| \begin{array}{ll} \textit{Zëvendësojmë}: & \\ u = \ln x & du = \dfrac{1}{x} dx \\ dv = \dfrac{1}{x^3} dx & v = \int x^{-3} dx = \dfrac{x^{-2}}{-2} \end{array} \right| = -\frac{x^{-2}}{2} \ln x + \frac{1}{2} \int \frac{1}{x^2} \cdot \frac{1}{x} dx = -\frac{1}{2x^2} \ln x + \frac{1}{2} \int \frac{1}{x^3} dx =$$

$$= -\frac{1}{2x^2} \ln x + \frac{1}{2} \int x^{-3} dx = -\frac{1}{2x^2} \ln x + \frac{x^{-2}}{-4} + \mathrm{C} = -\frac{1}{2x^2} \left(\ln x + \frac{1}{2} \right) + \mathrm{C}$$

Detyra 199: $\int \ln\left(x+\sqrt{1+x^2}\right)dx$

Zgjidhje

$$\int \ln\left(x+\sqrt{1+x^2}\right)dx = \left| \begin{array}{ll} \text{Zëvendësojmë}: & \\ u=\ln\left(x+\sqrt{1+x^2}\right) & du=\dfrac{1}{\sqrt{1+x^2}} \\ dv=dx & v=x \end{array} \right| =$$

$$= x\ln\left(x+\sqrt{1+x^2}\right)-\int \frac{x}{\sqrt{1+x^2}}dx = x\ln\left(x+\sqrt{1+x^2}\right)-\left| \begin{array}{l} 1+x^2=t \\ xdx=\dfrac{1}{2}dx \end{array} \right| =$$

$$= x\ln\left(x+\sqrt{1+x^2}\right)-\frac{1}{2}\int t^{-\frac{1}{2}}dt = x\ln\left(x+\sqrt{1+x^2}\right)-\frac{1}{2}\cdot\frac{t^{\frac{1}{2}}}{\frac{1}{2}}+C = x\ln\left(x+\sqrt{1+x^2}\right)-\sqrt{1+x^2}+C$$

Detyra 200: $\int x^3(\ln x)^2\,dx$

Zgjidhje:

$$\int x^3(\ln x)^2\,dx = \left| \begin{array}{ll} \text{Zëvendësojmë}: & \\ u=(\ln x)^2 & du=2\ln x\cdot\dfrac{1}{x}dx \\ dv=x^3dx & v=\dfrac{x^4}{4} \end{array} \right| = \frac{x^4}{4}(\ln x)^2-\frac{1}{2}\int x^3\ln x dx =$$

$$= \frac{x^4}{4}(\ln x)^2-\frac{1}{2}\left| \begin{array}{ll} u=\ln x & du=\dfrac{1}{x}dx \\ dv=x^3dx & v=\dfrac{x^4}{4} \end{array} \right| = \frac{x^4}{4}(\ln x)^2-\frac{1}{2}\left(\frac{x^4}{4}\ln x-\frac{1}{4}\int x^3dx\right) =$$

$$= \frac{x^4}{4}(\ln x)^2-\frac{x^4}{8}\ln x+\frac{x^4}{32}+C = \frac{x^4}{4}\left((\ln x)^2-\frac{1}{2}\ln x+\frac{1}{8}\right)+C$$

Detyra 201: $\int x\ln\left(x^2-1\right)dx$

Zgjidhje:

$$\int x\ln\left(x^2-1\right)dx=\left|\begin{array}{l}\textit{Zëvendësojmë}:\\ x^2-1=t\,/\,d\\ 2xdx=dt\\ xdx=\frac{1}{2}dt\end{array}\right|=\frac{1}{2}\int \ln tdt=\left|\text{me integral parcial}\right|=$$

$$=\frac{1}{2}\left(t\ln t-t\right)+\mathrm{C}=\frac{1}{2}\left(x^2-1\right)\ln\left|x^2-1\right|-\frac{1}{2}\left(x^2-1\right)+\mathrm{C}=\frac{1}{2}\left(x^2-1\right)\ln\left|x^2-1\right|-\frac{1}{2}x^2+\mathrm{C}$$

Detyra 202: $\int x^4\cos xdx$

Zgjidhje:

$$\int x^4\cos xdx=\left|\begin{array}{ll}\textit{Zëvendësojmë}: & \\ u=x^4 & du=4x^3dx\\ dv=\cos xdx & v=\sin x\end{array}\right|=x^4\sin x-4\int x^3\sin xdx=$$

$$=x^4\sin x-4\left|\begin{array}{ll}u=x^3 & du=3x^2dx\\ dv=\sin xdx & v=-\cos x\end{array}\right|=x^4\sin x-4\left(-x^3\cos x+3\int x^2\cos xdx\right)=$$

$$=x^4\sin x+4x^3\cos x-12\int x^2\cos xdx=x^4\sin x+4x^3\cos x-12\left|\begin{array}{ll}u=x^2 & du=2xdx\\ dv=\cos xdx & v=\sin x\end{array}\right|=$$

$$=x^4\sin x+4x^3\cos x-12\left(x^2\sin x-2\int x\sin xdx\right)=x^4\sin x+4x^3\cos x-12x^2\sin x+24\int x\sin xdx=$$

$$=x^4\sin x+4x^3\cos x-12x^2\sin x+24\left|\begin{array}{ll}u=x & du=dx\\ dv=\sin xdx & v=-\cos x\end{array}\right|=$$

$$=x^4\sin x+4x^3\cos x-12x^2\sin x+24\left(-x\cos x+\int\cos xdx\right)=$$

$$=x^4\sin x+4x^3\cos x-12x^2\sin x-24x\cos x+24\sin x+\mathrm{C}=$$

$$=\sin x\left(x^4-12x^2+24\right)+\cos x\left(4x^3-24x\right)+\mathrm{C}$$

Detyra 203: $\int \frac{dx}{1-x^2}$

Zgjidhje:

$$\int \frac{dx}{1-x^2} = \left| \begin{array}{l} 1-x^2 = (1-x)(1+x) \\ \frac{1}{1-x^2} = \frac{A}{1-x} + \frac{B}{1+x} \Rightarrow 1 = A + Ax + B - Bx \Rightarrow \\ \Rightarrow 1 = (A-B)x + A + B \Rightarrow (A-B=0) \wedge (A+B=1) \Rightarrow A = B = \frac{1}{2} \end{array} \right| =$$

$$= \int \frac{dx}{1-x^2} = \frac{1}{2}\int \frac{dx}{1-x} + \frac{1}{2}\int \frac{dx}{1+x} = -\frac{1}{2}\ln(1-x) + \frac{1}{2}\ln(1+x) + \mathrm{C} = \ln\sqrt{\frac{1+x}{1-x}} + \mathrm{C}$$

Detyra 204: $\int \frac{5x-7}{x(2x^2-4x-6)}dx$

Zgjidhje:

$$\int \frac{5x-7}{x(2x^2-4x-6)}dx = \frac{1}{2}\int \frac{5x-7}{x(x^2-2x-3)}dx = \left| \begin{array}{l} x^2-2x-3 = (x-3)(x+1) \\ \frac{5x-7}{x(x-3)(x+1)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+1} \\ \Rightarrow 5x-7 = A(x-3)(x+1) + Bx(x+1) + Cx(x-3) \\ \Rightarrow 5x-7 = Ax^2 - 2Ax - 3A + Bx^2 + Bx + Cx^2 - 3Cx \\ \Rightarrow 5x-7 = (A+B+C)x^2 + (-2A+B-3C)x + (-3A) \\ \Rightarrow (A+B+C=0) \wedge (-2A+B-3C=5) \wedge (-3A=-7) \\ A = \frac{7}{3}, C = -3, B = \frac{2}{3}. \end{array} \right| =$$

$$= \frac{1}{2}\int \frac{5x-7}{x(x^2-2x-3)}dx = \frac{1}{2}\left(\frac{7}{3}\int \frac{dx}{x} + \frac{2}{3}\int \frac{dx}{x-3} - 3\int \frac{dx}{x+1}\right) = \frac{1}{2}\left(\frac{7}{3}\ln|x| + \frac{2}{3}\ln|x-3| - 3\ln|x+1|\right) + \mathrm{C} =$$

$$= \frac{7}{6}\ln|x| + \frac{1}{3}\ln|x-3| - \frac{3}{2}\ln|x+1| + \mathrm{C} = \ln\sqrt[6]{x^7} + \ln\sqrt[3]{x-3} - \ln\sqrt{(x+1)^3} + \mathrm{C} = \ln\sqrt[6]{\frac{x^7(x-3)^2}{(x+1)^9}} + \mathrm{C}$$

Detyra 205: $\int \frac{2x+1}{x^2-5x+4}dx$

Zgjidhje:

$$\int \frac{2x+1}{x^2-5x+4}dx = \left| \begin{array}{l} x^2-5x+4=(x-1)(x-4), \\ \frac{2x+1}{x^2-5x+4}=\frac{A}{x-1}+\frac{B}{x-4} \Rightarrow 2x+1=Ax-4A+Bx-B \\ \Rightarrow 2x-1=\left(A+B\right)x-\left(4A+B\right) \Rightarrow \left(A+B=2\right) \wedge \left(4A+B=-1\right) \\ \Rightarrow \left(A=-1\right) \wedge \left(B=3\right) \end{array} \right| =$$

$$\int \frac{2x+1}{x^2-5x+4}dx \ =-\int \frac{dx}{x-1}+3\int \frac{dx}{x-4}=-\ln\left(x-1\right)+3\ln\left(x-4\right)+\mathrm{C}=\ln\frac{\left(x-4\right)^3}{x-1}+\mathrm{C}$$

Detyra 206: $\int \frac{3x+4}{x^2+x-6}dx$

Zgjidhje:

$$\int \frac{3x+4}{x^2+x-6}dx = \left| \begin{array}{l} x^2+x-6=\left(x-2\right)\left(x+3\right) \\ \frac{3x+4}{x^2+x-6}=\frac{3x+4}{\left(x-2\right)\left(x+3\right)}=\frac{A}{x-2}+\frac{B}{x+3} \\ 3x+4=A\left(x+3\right)+B\left(x-2\right) \Rightarrow 3x+4=Ax+3A+Bx-2B \\ \Rightarrow 3x+4=\left(A+B\right)x+3A-2B \Rightarrow 3=A+B \ \wedge 3A-2B=4 \\ \Rightarrow \begin{cases} A+B=3 \\ 3A-2B=4 \end{cases} \Rightarrow A=2 \wedge \quad B=1 \end{array} \right| =$$

$$\int \frac{3x+4}{x^2+x-6}dx=\int \frac{2}{x-2}dx+\int \frac{dx}{x+3}=2\ln\left|x-2\right|+\ln\left|x+3\right|+\mathrm{C}$$

Detyra 207: $\int \frac{x^6+x^4-4x^2-2}{x^3\left(x^2+1\right)^2}dx$

Zgjidhje:

$$\int \frac{x^6+x^4-4x^2-2}{x^3\left(x^2+1\right)^2}dx = \left| \begin{array}{l} \frac{x^6+x^4-4x^2-2}{x^3\left(x^2+1\right)^2} = \frac{A}{x^3}+\frac{B}{x^2}+\frac{C}{x}+\frac{Dx+E}{\left(x^2+1\right)^2}+\frac{Fx+G}{x^2+1} \\ \Rightarrow x^6+x^4-4x^2-2 = A\left(x^2+1\right)^2+Bx\left(x^2+1\right)^2+Cx^2\left(x^2+1\right) \\ +\left(Dx+E\right)x^3+\left(Fx+G\right)\left(x^2+1\right)x^3 \\ \Rightarrow x^6+x^4-4x^2-2=\left(C+F\right)x^6+\left(B+G\right)x^5+\left(A+2C+D+F\right)x^4 \\ +\left(2B+E+G\right)x^3+\left(2A+C\right)x^2+Bx+A \\ \Rightarrow A=-2, B=0, C=0, D=2, E=0, F=1, G=0 \end{array} \right| =$$

$$\int \frac{x^6+x^4-4x^2-2}{x^3(x^2+1)^2}dx = -2\int\frac{dx}{x^3}+\int\frac{2xdx}{(x^2+1)^2}+\int\frac{xdx}{x^2+1} = \frac{1}{x^2}-\frac{1}{x^2+1}+\frac{1}{2}\ln\left(x^2+1\right)+\mathrm{C} = \frac{1}{x^2\left(x^2+1\right)}+\ln\sqrt{x^2+1}+\mathrm{C}$$

3.2 Integrali i caktuar

Le të jetë f funksion i përkufizuar në segmentin $[a,b]$ dhe le të jetë $P = \{a = x_0 < x_1 < \cdots < x_n = b\}$ një ndarje e çfarëdoshme e segmentit $[a,b]$ në n – pjesë dhe le të jenë $\xi_i \in [x_{i-1}, x_i](1 \le i \le n)$. Shënojmë me $\Delta x_i = x_i - x_{i-1}(1 \le i \le n)$.

(1) Shuma

$$S_n(f) = \sum_{i=1}^{n} f(\xi_i)\Delta x_i,$$

quhet shumë integrale për funksionin f *në segmentin* $[a,b]$ që i përgjigjet ndarjes P.

(2) Numri $I = \lim_{n\to\infty} S_n(f)$ nëse ekziston dhe nëse nuk varet nga ndarja P *e as nga zgjedhja e pikave* $\xi_i(1 \le i \le n)$ quhet integral i caktuar i funksionit f në segmentin $[a,b]$. Do të shkruajmë

$$\int_a^b f(x)dx \lim_{n\to\infty} S_n(f) = \lim_{n\to\infty} \sum_{i=1}^{n} f(\xi_i)\Delta x_i.$$

Funksionin f e quajmë funksion nënintegral, kurse shprehjen $f(x)dx$ shprehje nënintegrale. Numri a quhet kufi i poshtëm i integralit, kurse b kufi i sipërm.

Funksioni f për të cilin ekziston $\int_a^b f(x)dx$ quhet funksion i integrueshëm.

Ekzistenca e integralit të caktuar. Janë të vërteta këto pohime:

(3) Çdo funksion i vazhdueshëm në segmentin $[a,b]$ është i integrueshëm në $[a,b]$.

(4) Çdo funksion monoton në segmentin $[a,b]$ $[a,b]$ është i integrueshëm në $[a,b]$.

Vetitë e integralit të caktuar

$$1^0 \int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$2^0 \int_a^b kf(x)dx = k\int_b^a f(x)dx$$

$$3^0 \int_a^b \left[f(x) \pm g(x) \right] dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

$$4^0 \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x) \qquad (a < c < b)$$

$5^0 \int_{-a}^{a} f(x)dx = 2\int_a^a f(x)dx$ për $f(x)$ është funksion çift

$6^0 \int_{-a}^{a} f(x)dx = 0$ për $f(x)$ është funksion tek

$7^0 \int_a^b UdV = U \cdot V\Big|_a^b - \int_a^b VdU$ integrimi në pjesë

Detyra të zgjidhura:

Detyra 1: $\int_1^3 x^3 dx$

Zgjidhje:

$$\int_1^3 x^3 dx = \frac{x^4}{4}\Bigg|_1^3 = \frac{3^4}{4} - \frac{1^4}{4} = \frac{81-1}{4} = \frac{80}{4} = 20$$

Detyra 2: $\int_1^2 4x^3 dx$

Zgjidhje:

$$\int_1^2 4x^3 dx = 4\int_1^2 x^3 dx = 4 \cdot \frac{x^4}{4}\Bigg|_1^2 = x^4\Big|_1^2 = 2^4 - 1^2 = 15$$

Detyra 3: $\int_1^4 x^2 dx$

Zgjidhje:

$$\int_1^4 x^2 dx = \left.\frac{x^{2+1}}{2+1}\right|_1^4 = \left.\frac{x^3}{3}\right|_1^4 = \frac{4^3}{3} - \frac{1^3}{3} = \frac{64}{3} - \frac{1}{3} = 21$$

Detyra 4: $\int_a^b xdx$

Zgjidhje:

$$\int_a^b xdx = \left.\frac{x^2}{2}\right|_a^b = \frac{b^2 - a^2}{2}$$

Detyra 5: $\int_0^1 xdx$

Zgjidhje:

$$\int_0^1 xdx = \left.\frac{x^{1+1}}{1+1}\right|_0^1 = \left.\frac{x^2}{2}\right|_0^1 = \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$$

Detyra 6: $\int_3^5 x^2 dx$

Zgjidhje:

$$\int_3^5 x^2 dx = \left.\frac{x^3}{3}\right|_3^5 = \frac{5^3}{3} - \frac{3^3}{3} = \frac{125-27}{3} = \frac{98}{3}$$

Detyra 7: $\int_1^5 x^3 dx$

Zgjidhje:

$$\int_1^5 x^3 dx = \left.\frac{x^4}{4}\right|_1^5 = \frac{5^4}{4} - \frac{1^4}{4} = \frac{625-1}{4} = \frac{624}{4} = 156$$

Detyra 8: $\int_{1}^{2} x^2 dx$

Zgjidhje:

$$\int_{1}^{2} x^2 dx = \frac{x^3}{3}\Bigg|_{1}^{2} = \frac{2^3}{3} - \frac{1^3}{3} = \frac{8-1}{3} = \frac{7}{3}$$

Detyra 9: $\int_{2}^{3} \frac{1}{x^2} dx$

Zgjidhje:

$$\int_{2}^{3} \frac{1}{x^2} dx = \left(-\frac{1}{x}\right)\Bigg|_{2}^{3} = \left(-\frac{1}{3}\right) - \left(-\frac{1}{2}\right) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

Detyra 10: $\int_{1}^{2} \frac{1}{x} dx$

Zgjidhje:

$$\int_{1}^{2} \frac{1}{x} dx = (\ln x)\Bigg|_{1}^{2} = \ln 2 - \ln 1 = \ln 2 - 0 = \ln 2$$

Detyra 11: $\int_{-2}^{3} 2x dx$

Zgjidhje:

$$\int_{-2}^{3} 2x dx = 2\frac{x^{1+1}}{1+1} = 2\frac{x^2}{2} = x^2\Bigg|_{-2}^{3} = 3^2 - (2)^2 = 9 - 4 = 5$$

Detyra 12: $\int_{1}^{3} e^{-x} dx$

Zgjidhje:

$$\int_{1}^{3} e^{-x} dx = -e^{-x}\Bigg|_{1}^{3} = -e^{-3} + e$$

Detyra 14: $\int_0^{\pi} \sin x dx$

Zgjidhje:

$$\int_0^{\pi} \sin x dx = -\cos x \Big|_0^{\pi} = -\cos \pi - (-\cos 0) = 1 + 1 = 2$$

Detyra 15: $\int_0^{\pi} \cos x dx$

Zgjidhje:

$$\int_0^{\pi} \cos x dx = \sin x \Big|_0^{\pi} = \sin \pi - \sin 0 = 0 - 0 = 0$$

Detyra 16: $\int_0^{1} e^{3x} dx$

Zgjidhje:

$$\int_0^{1} e^{3x} dx = \frac{1}{3} e^{3x} \Big|_0^{1} = \frac{e^3}{3} - \frac{e^0}{3} = \frac{e^3 - 1}{3}$$

Detyra 17: $\int_{-\pi}^{2\pi} \cos x dx$

Zgjidhje:

$$\int_{-\pi}^{2\pi} \cos x dx = \sin x \Big|_{-\pi}^{2\pi} = \sin(2\pi) - \sin(-\pi) = 0 + 0 = 0$$

Detyra 18: $\int_{-2\pi}^{\pi} \sin x dx$

Zgjidhje:

$$\int_{-2\pi}^{\pi} \sin x dx = -\cos x \Big|_{-2\pi}^{\pi} = -\cos(\pi) + \cos(-2\pi) = -(-1) + 1 = 1 + 1 = 2$$

Detyra 19: $\int_{-2\pi}^{\pi} \sin 2x dx$

Zgjidhje:

$$\int_{-2\pi}^{\pi} \sin 2x dx = -\frac{1}{2}\cos x \Big|_{-2\pi}^{\pi} = -\frac{1}{2}\cos(\pi) + \frac{1}{2}\cos(-2\pi) = -\frac{1}{2} + \frac{1}{2} = 0$$

Detyra 20: $\int_{-3\pi}^{0} \cos 3x dx$

Zgjidhje:

$$\int_{-3\pi}^{0} \cos 3x dx = \frac{1}{3}\sin 3x \Big|_{-3\pi}^{0} = \frac{1}{3}\sin(3\cdot 0) - \frac{1}{3}\sin(-9\pi) = 0 + 0 = 0$$

Detyra 21: $\int_{0}^{1} \left(x^2 - 3x + 2\right) dx$

Zgjidhje:

$$\int_{0}^{1} \left(x^2 - 3x + 2\right) dx = \left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right)\Bigg|_{0}^{1} = \frac{1}{3} - \frac{3}{2} + 2 = \frac{5}{6}$$

Detyra 22: $\int_{-1}^{1} 2^t dt$

Zgjidhje:

$$\int_{-1}^{1} 2^t dt = \frac{2^t}{\ln 2}\Bigg|_{-1}^{1} = \frac{1}{\ln 2}\left(2 - 2^{-1}\right) = \frac{3}{2\ln 2}$$

Detyra 23: $\int_{1}^{3} \left(2x^2 + 3x - 4\right) dx$

Zgjidhje:

$$\int_{1}^{3} \left(2x^2 + 3x - 4\right) dx = 2\int_{1}^{3} x^2 dx + 3\int_{1}^{3} x dx - 4\int_{1}^{3} dx = 2\cdot\frac{x^3}{3}\Bigg|_{1}^{3} + 3\cdot\frac{x^2}{2}\Bigg|_{1}^{3} + 4\cdot x\Big|_{1}^{3} =$$

$$= 2\cdot\left(9 - \frac{1}{3}\right) + 3\cdot\left(\frac{9}{2} - \frac{1}{2}\right) - 4\cdot(3-1) = \frac{52}{3} + 12 - 8 = \frac{64}{3}$$

Detyra 24: $\int_1^2 \left(3x^2 - \frac{2}{x} + 4\right) dx$

Zgjidhje:

$$\int_1^2 \left(3x^2 - \frac{2}{x} + 4\right) dx = 3\int_1^2 x^2 dx - 2\int_1^2 \frac{dx}{x} + 4\int_1^2 dx = 3\frac{x^3}{3}\Big|_1^2 - 2\ln x\Big|_1^2 + 4x\Big|_1^2 =$$
$$= \left(2^3 - 1^3\right) - 2\ln 2 + 4\left(2-1\right) = 11 - \ln 4$$

Detyra 25: $\int_1^2 \left(x-1\right)^3 dx$

Zgjidhje:

$$\int_1^2 \left(x-1\right)^3 dx = \int_1^2 \left(x-1\right)^3 d\left(x-1\right) = \frac{\left(x-1\right)^4}{4}\Bigg|_1^2 = \frac{1}{4}\left[\left(2-1\right)^4 - \left(1-1\right)^4\right] = \frac{1}{4}\left(1-0\right) = \frac{1}{4}$$

Detyra 26: $\int_1^2 \frac{2x^2+1}{x} dx$

Zgjidhje:

$$\int_1^2 \frac{2x^2+1}{x} dx = \int_1^2 2x dx + \int_1^2 \frac{dx}{x} = \left(2\frac{x^2}{2} + \ln|x|\right)\Bigg|_1^2 = \left(4+\ln 2\right) - \left(1+\ln 1\right) = 4 + \ln 2 - 1 - 0 = 3 + \ln 2$$

Detyra 27: $\int_1^2 \frac{x^2-9}{x^2-3x} dx$

Zgjidhje:

$$\int_1^2 \frac{x^2-9}{x^2-3x} dx = \int_1^2 \frac{\left(x-3\right)\left(x+3\right)}{x\left(x-3\right)} dx = \int_1^2 \frac{x+3}{x} dx = \int_1^2 \left(1+\frac{3}{x}\right) dx = \int_1^2 dx + 3\int_1^2 \frac{dx}{x} =$$
$$= \left(x + 3\ln|x|\right)\Big|_1^2 = \left(2 + 3\ln 2\right) - \left(1 + 3\cdot 0\right) = 2 + 3\ln 2 - 1 = 1 + 3\ln 2$$

Detyra 28: $\int_2^3 \frac{(x+3)(x-2)}{x^2}dx$

Zgjidhje:

$$\int_2^3 \frac{(x+3)(x-2)}{x^2}dx = \int_2^3 \frac{x^2+x-6}{x^2}dx = \int_2^3\left(1+\frac{1}{x}-\frac{6}{x^2}\right)dx = \int_2^3 dx + \int_2^3 \frac{dx}{x} - 6\int_2^3 x^{-2}dx =$$

$$=\left(x+\ln|x|-6\frac{x^{-1}}{-1}\right)\Bigg|_2^3 = \left(x+\ln|x|+\frac{6}{x}\Bigg|_2^3\right) = (3+\ln 3+2)-(2+\ln 2+3) = 5+\ln 3-\ln 2 = \ln\frac{3}{2}$$

Detyra 29: $\int_3^6 \frac{(2-x)^4}{x^2-4x+4}dx$

Zgjidhje:

$$\int_3^6 \frac{(2-x)^4}{x^2-4x+4}dx = \int_3^6 \frac{(2-x)^4}{(2-x)^2}dx = \int_3^6 (2-x)^2dx = -\int_3^6 (2-x)^2 d(2-x) = \left[-\frac{1}{3}(2-x)^3\right]\Bigg|_3^6 =$$

$$=-\frac{1}{3}\left[(2-x)^3\right]-\frac{1}{3}\left[-64-(-1)\right]\Big|_3^6 = -\frac{1}{3}\left[-64-(-1)\right] = -\frac{1}{3}(-63) = 21$$

Detyra 30: $\int_1^4 \frac{(x^4-x^2)^4}{x^2+x}dx$

Zgjidhje:

$$\int_1^4 \frac{(x^4-x^2)^4}{x^2+x}dx = \int_1^4 \frac{(x^2+x)(x^2-x)}{x^2+x}dx = \int_1^4 (x^2-x)dx = \left(\frac{x^3}{3}-\frac{x^2}{2}\right)\Bigg|_1^4 = \left(\frac{64}{3}-\frac{16}{2}\right)-\left(\frac{1}{3}-\frac{1}{2}\right)-\left(\frac{1}{3}-\frac{1}{2}\right)=$$

$$=\frac{128-48}{6}-\frac{2-3}{6}=\frac{80}{6}+\frac{1}{6}=\frac{81}{6}=\frac{27}{2}$$

Detyra 31: $\int_1^{16} \frac{x-1}{\sqrt{x}}dx$

Zgjidhje:

$$\int_1^{16} \frac{x-1}{\sqrt{x}}dx = \int_1^{16}\left(\frac{x}{\sqrt{x}}-\frac{1}{\sqrt{x}}\right)dx = \int_1^{16}\sqrt{x}dx - \int_1^{16}\frac{1}{\sqrt{x}}dx = \int_1^{16} x^{\frac{1}{2}}dx - \int_1^{16} x^{-\frac{1}{2}}dx =$$

$$=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_1^{16} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}}\Bigg|_1^{16} = \frac{2}{3}\left(16^{\frac{3}{2}}-1^{\frac{3}{2}}\right)-2\left(16^{\frac{1}{2}}-1^{\frac{1}{2}}\right) = \frac{2}{3}(64-1)-2(4-1) = \frac{2}{3}\cdot 63-2\cdot 3 = 42-6 = 36$$

Detyra 32: $\int_0^4 \sqrt{2x+1}\, dx$

Zgjidhje:

$$\int_0^4 \sqrt{2x+1}\, dx = \int_0^4 (2x+1)^{\frac{1}{2}} dx = \frac{1}{2}\int_0^4 (2x+1)^{\frac{1}{2}} d(2x+1) = \frac{1}{2}\cdot\frac{2}{3}(2x+1)^{\frac{3}{2}}\Big|_0^4 = \frac{1}{3}\sqrt{(2x+1)^3}\Big|_0^4 = \frac{26}{3}$$

Detyra 33: $\int_1^4 \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$

Zgjidhje:

$$\int_1^4 \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx = \int_1^4 x^{\frac{1}{2}}dx + \int_1^4 x^{-\frac{1}{2}}dx = \left(\frac{2}{3}x\sqrt{x}+2\sqrt{x}\right)\Big|_1^4 = \left(\frac{2}{4}\cdot 4\cdot 2+2\cdot 2\right)-\left(\frac{2}{3}\cdot 1\cdot 1+2\cdot 1\right)=$$

$$=\left(\frac{16}{3}+4\right)-\left(\frac{2}{3}+3\right)=\frac{28}{3}-\frac{8}{3}=\frac{20}{3}$$

Detyra 34: $\int_0^1 \sqrt[3]{(1-2x)^2}\,dx$

Zgjidhje:

$$\int_0^1 \sqrt[3]{(1-2x)^2}\,dx = -\frac{1}{2}\int_0^1 (1-2x)^{\frac{2}{3}} d(1-2x) = -\frac{1}{2}\cdot\frac{3}{5}(1-2x)^{\frac{5}{3}}\Big|_0^1 = -\frac{3}{10}(1-2x)\cdot\sqrt[3]{(1-2x)^2}\Big|_0^1 =$$

$$=-\frac{3}{10}\cdot(-1\cdot 1-1) = -\frac{3}{10}(-2) = \frac{3}{5}$$

Detyra 35: $\int_1^4 \left(\frac{3}{2}\sqrt{x}-3x^2+2x+3\right)dx$

Zgjidhje:

$$\int_1^4 \left(\frac{3}{2}\sqrt{x}-3x^2+2x+3\right)dx = \frac{3}{2}\int_1^4 x^{\frac{1}{2}}dx - 3\int_1^4 x^2dx + 2\int_1^4 xdx + 3\int_1^4 dx = \frac{3}{2}\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_1^4 - 3\cdot\frac{x^3}{3}\Big|_1^4 + 2\cdot\frac{x^2}{2}\Big|_1^4 + 3x\Big|_1^4 =$$

$$= \sqrt{x^3}\Big|_1^4 - x^3\Big|_1^4 + x^2\Big|_1^4 + 3x\Big|_1^4 = -3$$

Detyra 36: $\int\limits_{-1}^{1}\frac{2x^5-3x^3-2x^2-1}{x^4}dx$

Zgjidhje:

$$\int\limits_{-1}^{1}\frac{2x^5-3x^3-2x^2-1}{x^4}dx=\int\limits_{-1}^{1}\left(2x+\frac{3}{x}-\frac{2}{x^2}-\frac{1}{x^4}\right)dx=2\int\limits_{-1}^{1}xdx+3\int\limits_{-1}^{1}\frac{dx}{x}-2\int\limits_{-1}^{1}x^{-2}dx-\int\limits_{-1}^{1}x^{-4}dx=$$

$$=x^2\Big|_{-1}^{1}+3\ln|x|\Big|_{-1}^{1}+\frac{2}{x}\Big|_{-1}^{1}+\frac{1}{3x^2}\Big|_{-1}^{1}=\frac{14}{3}$$

Detyra 37: $\int\limits_{1}^{e}\left(\frac{x^2-5x+1}{x}\right)dx$

Zgjidhje:

$$\int\limits_{1}^{e}\left(\frac{x^2-5x+1}{x}\right)dx=\int\limits_{1}^{e}\left(x-5+\frac{1}{x}\right)dx=\int\limits_{1}^{e}xdx-5\int\limits_{1}^{e}dx+\int\limits_{1}^{e}\frac{1}{x}dx=\frac{x^2}{2}\Big|_{1}^{e}-5x\Big|_{1}^{e}+\ln x\Big|_{1}^{e}=$$

$$=\left(\frac{e^2}{2}-\frac{1}{2}\right)-5\cdot(e-1)+(\ln e-\ln 1)=\frac{e^2-10e+11}{2}$$

Detyra 38: $\int\limits_{1}^{2}\frac{5x-2}{\sqrt[3]{x}}dx$

Zgjidhje:

$$\int\limits_{1}^{2}\frac{5x-2}{\sqrt[3]{x}}dx=\int\limits_{1}^{2}\left(\frac{5x}{\sqrt[3]{x}}-\frac{2}{\sqrt[3]{x}}\right)dx=\int\limits_{1}^{2}\left(5\cdot x^1\cdot x^{-\frac{1}{3}}-2\cdot x^{-\frac{1}{3}}\right)dx=\int\limits_{1}^{2}\left(5x^{\frac{2}{3}}-2x^{-\frac{1}{3}}\right)dx=$$

$$=\int\limits_{1}^{2}5x^{\frac{2}{3}}dx-\int\limits_{1}^{2}2x^{-\frac{1}{3}}dx=5\cdot\frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1}\Bigg|_{1}^{2}-2\cdot\frac{x^{1-\frac{1}{3}}}{1-\frac{1}{3}}\Bigg|_{1}^{2}=5\cdot\frac{x^{\frac{5}{3}}}{\frac{5}{3}}\Bigg|_{1}^{2}-2\cdot\frac{x^{\frac{2}{3}}}{\frac{2}{3}}\Bigg|_{1}^{2}=5\cdot\frac{3}{5}x^{\frac{5}{3}}\Big|_{1}^{2}-2\cdot\frac{3}{2}x^{\frac{2}{3}}\Big|_{1}^{2}=$$

$$=3x^{\frac{5}{3}}\Big|_{1}^{2}-3x^{\frac{2}{3}}\Big|_{1}^{2}=3\cdot\left(2^{\frac{5}{3}}-1\right)-3\cdot\left(2^{\frac{2}{3}}-1\right)=3\cdot\left[2^{\frac{5}{3}}-1-2^{\frac{2}{3}}+1\right]=3\cdot\left[2^{\frac{5}{3}}-2^{\frac{2}{3}}\right]=3\cdot\left[2\cdot2^{\frac{2}{3}}-2^{\frac{2}{3}}\right]=$$

$$=3\cdot2^{\frac{2}{3}}=3\sqrt[3]{2^2}$$

Detyra 39: $\int_0^1 e^{3x+1}dx$

Zgjidhje:

$$\int_0^1 e^{3x+1}dx = \left.\frac{e^{3x+1}}{3}\right|_0^1 = \frac{e^4 - e}{3}$$

Detyra 40: $\int_a^b \sin x dx$

Zgjidhje:

$$\int_a^b \sin x dx = -\cos x\Big|_a^b = \cos a - \cos b$$

Detyra 41: $\int_a^b \cos x dx$

Zgjidhje:

$$\int_a^b \cos x dx = \sin x\Big|_a^b = \sin b - \sin a$$

Detyra 42: $\int_0^1 \frac{dx}{1+x^3}$

Zgjidhje:

$$\int_0^1 \frac{dx}{1+x^3} = \operatorname{arctan}\Big|_0^1 = \frac{\pi}{4}$$

Detyra 43: $\int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx$

Zgjidhje:

$$\int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx = (-\cos x + \sin x)\Big|_0^{\frac{\pi}{2}} = \left(-\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right) - (-\cos 0 + \sin 0) = (-0+1) - (-1+0) = 2$$

Detyra 44: $\int_{\frac{\pi}{2}}^{\pi}(4\cos x+2\sin x)\,dx$

Zgjidhje:

$$\int_{\frac{\pi}{2}}^{\pi}(4\cos x+2\sin x)\,dx=4\int_{\frac{\pi}{2}}^{\pi}\cos x dx+2\int_{\frac{\pi}{2}}^{\pi}\sin x dx=-4\sin x\Big|_{\frac{\pi}{2}}^{\pi}+2(-\cos x)\Big|_{\frac{\pi}{2}}^{\pi}=-4+2=-2$$

Detyra 45: $\int_{2}^{1}\sin \pi x dx$

Zgjidhje:

$$\int_{2}^{1}\sin \pi x dx=-\int_{2}^{1}\cos \pi x dx=-\frac{1}{\pi}\int_{1}^{2}\cos \pi x d(\pi x)=-\frac{1}{\pi}\sin \pi x\Big|_{1}^{2}=-\frac{1}{\pi}(\sin 2\pi-\sin \pi)=-\frac{1}{\pi}\cdot 0=0$$

Detyra 46: $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\sin\frac{x}{2}\cdot\cos\frac{x}{2}dx$

Zgjidhje:

$$\int_{-\pi/6}^{\pi/6}\sin\frac{x}{2}\cdot\cos\frac{x}{2}dx=\frac{1}{2}\int_{-\pi/6}^{\pi/6}\sin x dx=\frac{1}{2}(-\cos x)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{6}}=\frac{1}{2}\left[-\cos\frac{\pi}{6}-\left(-\cos\left(-\frac{\pi}{6}\right)\right)\right]=$$

$$=\frac{1}{2}\left(-\cos\frac{\pi}{6}+\cos\frac{\pi}{6}\right)=0$$

Detyra 47: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sin\left(3x-\frac{\pi}{4}\right)dx$

Zgjidhje:

$$\int_{\pi/6}^{\pi/3}\sin\left(3x-\frac{\pi}{4}\right)dx=\frac{1}{3}\int_{\pi/6}^{\pi/3}\sin\left(3x-\frac{\pi}{4}\right)d\left(3x-\frac{\pi}{4}\right)=-\frac{1}{3}\cos\left(3x-\frac{\pi}{4}\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}=-\frac{1}{3}\left(\cos\frac{3\pi}{4}-\cos\frac{\pi}{4}\right)=$$

$$=-\frac{1}{3}\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\right)=-\frac{1}{3}\left(-\sqrt{2}\right)=\frac{\sqrt{2}}{3}$$

Detyra 48: $\int\limits_0^{\frac{\pi}{2}} \cos^2 x dx$

Zgjidhje:

$$\int\limits_0^{\frac{\pi}{2}} \cos^2 x dx = \int\limits_0^{\frac{\pi}{2}} \frac{1+\cos 2x}{2} dx = \frac{1}{2}\int\limits_0^{\frac{\pi}{2}} dx + \frac{1}{2}\cdot\frac{1}{2}\int\limits_0^{\frac{\pi}{2}} \cos 2x d(2x) = \left(\frac{1}{2}x + \frac{1}{4}\sin 2x\right)\Bigg|_0^{\frac{\pi}{2}} =$$

$$= \frac{\pi}{4} + \left(\frac{1}{4}\sin\pi - \sin 0\right) = \frac{\pi}{4}$$

Detyra 49: $\int\limits_0^{\frac{\pi}{2}} \sin^3 x dx$

Zgjidhje:

$$\int\limits_0^{\pi/2} \sin^3 x dx = \int\limits_0^{\pi/2} \sin^2 \cdot \sin x dx = -\int\limits_0^{\pi/2} \left(1-\cos^2 x\right) d\left(\cos x\right) = -\int\limits_0^{\pi/2} d\left(\cos x\right) + \int\limits_0^{\pi/2} \cos^2 x d\left(\cos x\right) =$$

$$= \left(-\cos x + \frac{\cos^3 x}{3}\right)\Bigg|_0^{\frac{\pi}{2}} = \left(-0+\frac{0}{3}\right) - \left(-1+\frac{1}{3}\right) = 1 - \frac{1}{3} =$$

Detyra 50: $\int\limits_{-1}^{1} \sqrt{1-x^2} dx$

Zgjidhje:

$$\int\limits_{-1}^{1} \sqrt{1-x^2} dx = \left| \begin{array}{l} \textit{Zëvendësojmë}: \\ x = \sin t \\ dx = \cos t dt \\ x=-1, \quad t = -\dfrac{\pi}{2} \quad x = 1, \quad t = \dfrac{\pi}{2} \end{array} \right| = \int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\sin^2 t}\cos t dt = \int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 dt =$$

$$= \int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1-\cos 2t}{2} dt = \left(\frac{1}{2}t - \frac{1}{4}\sin 2t\right)\Bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

Detyra 51: $\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\cos^2 x} dx$

Zgjidhje:

$$\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\cos^2 x} dx = \left(\tan x\right)\Bigg|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \tan\frac{\pi}{3} - \tan\frac{\pi}{6} = \sqrt{3} - \frac{\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}$$

Detyra 52: $\int\limits_0^1 \sqrt{1-x}dx$

Zgjidhje:

$$\int\limits_0^1 \sqrt{1-x}dx = \left|\begin{array}{l} Zëvendësojmë: \\ \sqrt{1-x} = t^2 \\ 1-x = t^2 \\ x = 1-t^2 \,/\, d \\ dx = -2tdt \\ x\big|_0^1 = t\big|_1^0 \end{array}\right| = \int\limits_1^0 t\left(-2tdt\right) = -\int\limits_0^1 \left(-2t^2\right)dt = 2\int\limits_0^1 t^2 dt = 2\frac{t^3}{3}\Bigg|_0^1 = 2\frac{1^3}{3} - 0 = \frac{2}{3}$$

Detyra 53: $\int\limits_0^2 \frac{xdx}{\sqrt{1+4x}}$

Zgjidhje:

$$\int\limits_0^2 \frac{xdx}{\sqrt{1+4x}} = \left|\begin{array}{l} Zëvendësojmë: \\ \sqrt{1+4x} = t \,/^2 \Rightarrow 1+4x = t^2 \\ 4x = t^2 - 1 \Rightarrow x = \frac{1}{4}t^2 - \frac{1}{4} \,/\, d \\ dx = \frac{1}{2}tdt \\ x\big|_0^2 = t\big|_1^3 \end{array}\right| = \int\limits_1^3 \frac{\left(\frac{1}{4}t^2 - \frac{1}{4}\right)}{t} \cdot \frac{1}{2}tdt = \int\limits_1^3 \frac{1}{2} \cdot \frac{1}{4}\left(t^2 - 1\right)dt =$$

$$= \frac{1}{8}\int\limits_1^3 \left(t^2 - 1\right)dt = \frac{1}{8}\left(\frac{t^3}{3} - t\right)\Bigg|_1^3 = \frac{1}{8}\left[\left(\frac{27}{3} - 3\right) - \left(\frac{1}{3} - 1\right)\right] = \frac{1}{8}\left(6 + \frac{2}{3}\right) = \frac{1}{8} \cdot \frac{20}{3} = \frac{5}{6}$$

Detyra 54: $\int_0^1 \sqrt{3x+1}dx$

Zgjidhje:

$$\int_0^1 \sqrt{3x+1}dx = \left| \begin{array}{l} \text{Zëvendësojmë:} \\ 3x+1=u/d \\ 3dx=du \\ 3\cdot 0+1=1 \quad 3\cdot +1=4 \end{array} \right| = \frac{1}{3}\int_1^4 \sqrt{u}du = \frac{1}{3}\cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_1^4 = \frac{2u^{\frac{3}{2}}}{9}\Bigg|_1^4 = \frac{2\cdot 4^{\frac{3}{2}}}{9} - \frac{2\cdot 1^{\frac{3}{2}}}{9} = \frac{16-2}{9} = \frac{14}{2}$$

Detyra 55: $\int_0^3 x\sqrt{1+x}dx$

Zgjidhje:

$$\int_0^3 x\sqrt{1+x}dx = \left| \begin{array}{l} \text{Zëvendësojmë:} \\ \sqrt{1+x}=t \\ x=t^2-1 \\ dx=2tdt \\ x=0, \quad t=1 \\ x=3, \quad t=2 \end{array} \right| = \int_1^2 \left(t^2-1\right)\cdot t\cdot 2tdt = 2\int_1^2 \left(t^4-t^2\right)dt = 2\left(\frac{t^5}{5}-\frac{t^3}{3}\right)\Bigg|_1^2 =$$

$$= 2\left(\frac{32-1}{5}-\frac{8-1}{3}\right) = \frac{62}{5}-\frac{14}{3} = \frac{116}{15}$$

Detyra 56: $\int_0^1 \frac{e^x}{e^x+1}dx$

Zgjidhje:

$$\int_0^1 \frac{e^x}{e^x+1}dx = \left| \begin{array}{l} \text{Zëvendësojmë:} \\ e^x=t \\ e^x dx=dt \\ x\to 0, t\to 1 \\ x\to 1, t\to e \end{array} \right| = \int_1^e \frac{dt}{t+1} = \ln\left(t+1\right)\Big|_1^e = \ln\left(e+1\right)-\ln 2 = \ln\left(\frac{e+1}{2}\right)$$

Detyra 57: $\int_{1/2}^{\sqrt{3}/2} \frac{dx}{x^2\sqrt{1-x^2}}$

Zgjidhje:

$$\int_{1/2}^{\sqrt{3}/2} \frac{dx}{x^2\sqrt{1-x^2}} = \left| \begin{array}{l} Zëvendësojmë: \\ x=\sin t \\ dx=\cos t dt \\ x\Big|_{1/2}^{\sqrt{3}/2} = t\Big|_{\pi/6}^{\pi/3} \end{array} \right| = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos t dt}{\sin^2 t\sqrt{1-\sin^2 t}}\, dt = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos t}{\sin^2 t\sqrt{\cos^2 t}}\, dt =$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos t}{\sin^2 t\cdot\cos t}\, dt = = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sin^2 t}\, dt = -\operatorname{cotgt}\Big|_{\pi/6}^{\pi/3} == -\frac{\sqrt{3}}{3} - \left(-\sqrt{3}\right) = \sqrt{3} - \frac{\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}$$

Detyra 58: $\int_1^8 \frac{\sqrt{x+1}}{x}\, dx$

Zgjidhje:

$$\int_1^8 \frac{\sqrt{x+1}}{x}\, dx = \left| \begin{array}{l} Zëvendësojmë: \\ \sqrt{x+1} = t\ /^2 \Rightarrow x+1=t^2 \\ x=t^2-1\ /\ d \\ dx=2tdt \\ x\Big|_3^8 = t\Big|_2^3 \end{array} \right| = \int_2^3 \frac{t}{t^2-1} 2tdt = 2\int_2^3 \frac{t^2}{t^2-1}\, dt =$$

$$= 2\int_2^3 \frac{t^2-1+1}{t^2-1}\, dt = 2\int_2^3 \left(1+\frac{1}{t^2-1}\right) dt = 2\int_2^3 dt + 2\int_2^3 \frac{dt}{t^2-1} = 2\left(t+\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\right)\Bigg|_2^3 =$$

$$= 2\left[\left(3+\frac{1}{2}\ln\frac{2}{4}\right) - \left(2+\frac{1}{2}\ln\frac{1}{3}\right)\right] = 2\left[\left(3+\frac{1}{2}\ln\frac{1}{3}\right) - \left(2+\frac{1}{2}\ln\frac{1}{3}\right)\right] =$$

$$= 2\left(1+\frac{1}{2}\ln\frac{\frac{1}{2}}{\frac{1}{3}}\right) = 2+\ln\frac{3}{2}$$

Detyra 59: $\int_{0}^{1} \frac{e^x}{1+e^{2x}} dx$

Zgjidhje:

$$\int_{0}^{1} \frac{e^x}{1+e^{2x}} dx = \left| \begin{array}{l} Zëvendësojmë: \\ e^x = t \\ e^x dx = dt \\ x \to 0, t \to 1 \\ x \to 1, t \to e \end{array} \right| = \int_{1}^{e} \frac{dt}{1+t^2} = \arctan t \Big|_1^e = \arctan e - \frac{\pi}{4}$$

Detyra 60: $\int_{\ln 3}^{\ln 8} \sqrt{e^x + 1} dx$

Zgjidhje:

$$\int_{\ln 3}^{\ln 8} \sqrt{e^x+1} dx = \left| \begin{array}{l} Zëvendësojmë: \\ \sqrt{e^x+1} = t \,/^2 \Rightarrow e^x + 1 = t^2 \\ e^x = t^2 - 1 \,/\, d \\ e^x dx = 2dt \Rightarrow dx = \frac{2t}{e^x} dt \Rightarrow dx = \frac{2t}{t^2-1} dt \\ x\Big|_{\ln 3}^{\ln 8} = t\Big|_2^3 \end{array} \right| = \int_{2}^{3} t \cdot \frac{2t}{t^2-1} dt = 2\int_{2}^{3} \frac{t^2}{t^2-1} dt =$$

$$= 2\int_{2}^{3} \frac{\left(t^2-1\right)+1}{t^2-1} dt = 2\int_{2}^{3} \left(1 + \frac{1}{t^2-1}\right) dt = 2\left(t + \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\right)\Bigg|_2^3 =$$

$$= \left(6 + \ln\frac{1}{2}\right) - \left(4 + \ln\frac{1}{3}\right) = 6 + \ln\frac{\frac{1}{2}}{\frac{1}{3}} = 2 + \ln\frac{3}{2}$$

Detyra 61: $\int\limits_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}}$

Zgjidhje:

$$\int\limits_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}} = \left| \begin{array}{l} Zëvendësojmë: \\ x = \frac{1}{\sin t} \\ dx = \left(\frac{1}{\sin t}\right)' dt = \frac{1}{\sin^2 t}\cos t \\ x\Big|_{\sqrt{2}}^{2} = t\Big|_{\frac{\pi}{4}}^{\frac{\pi}{6}} \end{array} \right| = \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{6}} \frac{\left(\frac{1}{\sin^2 t}\right)\cos t}{\frac{1}{\sin t}\sqrt{\frac{1}{\sin^2 t}-1}} dt =$$

$$= \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{6}} \frac{\frac{\cos t}{\sin^2 t}}{\frac{1}{\sin t}\cdot\frac{\cos t}{\sin t}} dt = \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{6}} \frac{\cos t}{\sin^2 t}\cdot\frac{\sin^2 t}{\cos t} dt = = \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{4}} dt = t\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$$

Detyra 62: $\int\limits_{\frac{1}{2}}^{\frac{1}{4}} \frac{dx}{x^2\left(1-\frac{1}{x}\right)^3} dx$

Zgjidhje:

$$\int\limits_{\frac{1}{2}}^{\frac{1}{4}} \frac{dx}{x^2\left(1-\frac{1}{x}\right)^3} dx = -\int\limits_{\frac{1}{4}}^{\frac{1}{2}} \frac{dx}{x^2\left(1-\frac{1}{x}\right)^3} dx = -\int\limits_{\frac{1}{4}}^{\frac{1}{2}} \frac{d\left(1-\frac{1}{x}\right)}{\left(1-\frac{1}{x}\right)^3} dx = -\int\limits_{\frac{1}{4}}^{\frac{1}{2}} \left(1-\frac{1}{x}\right)^3 d\left(1-\frac{1}{x}\right) =$$

$$= \frac{\left(1-\frac{1}{x}\right)^{-3+1}}{-3+1}\Bigg|_{\frac{1}{4}}^{\frac{1}{2}} = \frac{1}{2}\cdot\frac{1}{\left(1-\frac{1}{x}\right)^2}\Bigg|_{\frac{1}{4}}^{\frac{1}{2}} = \frac{1}{2}\left(1-\frac{1}{9}\right) = \frac{1}{2}\cdot\frac{8}{9} = \frac{4}{9}$$

Detyra 63: $\int_1^e \ln x dx$

Zgjidhje:

$$\int_1^e \ln x dx = \left| \begin{array}{ll} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: & \\ u = \ln x, & dv = dx \\ du = \frac{dx}{x}, & v = \int dx = x \end{array} \right| = x \cdot \ln x \Big|_1^e - \int_1^e x \cdot \frac{dx}{x} = e \ln e - \ln 1 - \int_1^e dx =$$

$$= e \ln e - x\Big|_1^e = e \ln e - e + 1 = 1 \cdot e - e + 1 = 1$$

Detyra 64: $\int_1^e x^3 \ln x dx$

Zgjidhje:

$$\int_1^e x^3 \ln x dx = \left| \begin{array}{ll} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: & \\ u = \ln x, & dv = x^3 dx \\ du = \frac{1}{x} dx, & v = \int x^3 dx = \frac{x^4}{4} \end{array} \right| = \ln x \frac{x^4}{4}\Big|_1^e - \int_1^e \frac{1}{x} \cdot \frac{x^4}{4} dx = \left(\ln e \frac{e^4}{4} - \ln 1 \frac{1^4}{4} \right) - \frac{1}{4} \int_1^e x^3 dx =$$

$$= \frac{e^4}{4} - \frac{1}{4} \cdot \frac{x^4}{4}\Big|_1^e = \frac{e^4}{4} - \frac{1}{4}\left(\frac{e^4}{4} - \frac{1}{4} \right) = \frac{e^4}{4} - \frac{e^4}{16} + \frac{1}{16} = \frac{3e^4}{16} + \frac{1}{16} = \frac{3e^4 + 1}{16}$$

Detyra 65: $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x \cdot \cos x dx$

Zgjidhje:

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x \cdot \cos x dx = \left| \begin{array}{ll} Z\ddot{e}vend\ddot{e}sojm\ddot{e}: & \\ u = x, & dv = \cos dx \\ du = dx, & v = \int \cos dx = \sin x \end{array} \right| = \left(x \cdot \sin x - \int \sin x dx \right)\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \left(x \cdot \sin x + \cos x \right)\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} =$$

$$= \left(\frac{\pi}{4} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) - \left[\left(-\frac{\pi}{4} \cdot \left(-\frac{\sqrt{2}}{2} \right) + \frac{\sqrt{2}}{2} \right) \right] = \frac{\sqrt{2}\pi}{8} + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}\pi}{8} - \frac{\sqrt{2}}{2} = 0$$

Detyra 66: $\int\limits_0^1 x(2x-1)^5\,dx$

Zgjidhje:

$$\int\limits_0^1 x(2x-1)^5\,dx = \left| \begin{array}{ll} \textit{Zëvendësojmë}: & \\ u = x, & dv = (2x-1)^5 \\ du = dx, & v = \int (2x-1)^5\,dx = \dfrac{1}{2}\int (2x-1)^5\,d(2x-1) = \\ & \qquad = \dfrac{1}{2}\cdot\dfrac{(2x-1)^6}{6} = \dfrac{1}{12}(2x-1)^6 \end{array} \right| =$$

$$= \left[\frac{1}{12}x(2x-1)^6 - \int \frac{1}{12}(2x-1)^6\,dx\right]\Bigg|_0^1 = \left[\frac{1}{12}x(2x-1)^6 - \frac{1}{24}\int (2x-1)^6\,d(2x-1)\right]\Bigg|_0^1 =$$

$$= \left[\frac{1}{12}x(2x-1)^6 - \frac{1}{24}\cdot\frac{(2x-1)^7}{7}\right]\Bigg|_0^1 = \left[\frac{1}{12}x(2x-1)^6 - \frac{1}{168}\cdot(2x-1)^7\right]\Bigg|_0^1 =$$

$$= \left(\frac{1}{2}\cdot 1\cdot 1 - \frac{1}{168}\cdot 1\right) - \left(0 - \frac{1}{168}\cdot(-1)\right) = \frac{12}{168} = \frac{1}{14}$$

Detyra 67: $\int\limits_0^1 x(1-x)^{10}\,dx$

Zgjidhje:

$$\int\limits_0^1 x(1-x)^{10}\,dx = \left| \begin{array}{ll} \textit{Zëvendësojmë}: & \\ u = x, & dv = (1-x)^{10} \\ du = dx, & v = \int (1-x)^{10}\,dx = -\int (1-x)^{10}\,d(1-x) = -\dfrac{(1-x)^{11}}{11} \end{array} \right| =$$

$$= \left[-\frac{1}{11}x(1-x)^{11} - \int\left(-\frac{1}{11}\right)(1-x)^{11}\,dx\right]\Bigg|_0^1$$

$$= \left[-\frac{1}{11}x(1-x)^{11} - \frac{1}{11}\int (1-x)^{11}\,d(x-1)\right]\Bigg|_0^1 = -\frac{1}{11}x(1-x)^{11} - \frac{1}{11}\cdot\frac{(1-x)^{12}}{12} =$$

$$= (0-0) - \left(0 - \frac{1}{11}\cdot\frac{1}{12}\right) = \frac{1}{132}$$

Detyra 68: $\int_0^{\frac{\pi}{2}} x\cdot\sin x dx$

Zgjidhje:

$$\int_0^{\frac{\pi}{2}} x\cdot\sin x dx=\left|\begin{array}{ll}\textit{Zëvendësojmë}: & \\ u=x, & dv=\sin x dx \\ du=dx, & v=\int\sin x dx=-\cos x\end{array}\right|=\left(-x\cdot\cos x-\int\left(-\cos x\right)dx\right)\Big|_0^{\frac{\pi}{2}}=$$

$$=\left(-x\cdot\cos x+\int\cos x dx\right)\Big|_0^{\frac{\pi}{2}}=\left(\sin x-x\cdot\cos x\right)\Big|_0^{\frac{\pi}{2}}=\left(1-0\cdot\frac{\pi}{2}\right)-\left(0-0\cdot 1\right)=1$$

Detyra 69: $\int_1^{e} x\ln x dx$

Zgjidhje:

$$\int_1^e x\ln x dx=\left|\begin{array}{ll}\textit{Zëvendësojmë}: & \\ u=\ln x, & v'=x \\ u'=\dfrac{1}{x}, & v=\dfrac{x^2}{2}\end{array}\right|=\frac{x^2}{2}\ln x\Big|_1^e-\int_1^e\frac{1}{x}\cdot\frac{x^2}{2}dx=\left(\frac{x^2}{2}\ln x-\frac{x^2}{4}\right)\Big|_1^e=$$

$$=\left(\frac{e^2}{2}\ln e-\frac{e^2}{4}\right)-\left(\frac{1}{2}\ln 1-\frac{1}{4}\right)=\frac{e^2}{2}-\frac{e^2}{4}+\frac{1}{4}=\frac{e^2+1}{4}$$

Detyra 70: $\int_{-1}^{3}\frac{x+4}{x^2+2x}dx$

Zgjidhje:

$$\int_{-1}^{3}\frac{x+4}{x^2+2x}dx=\int_{-1}^{3}\frac{x+4}{x\left(x+2\right)}dx$$

$$\frac{x+4}{x\left(x+2\right)}=\frac{A}{x}+\frac{B}{x+2}=\frac{\left(A+B\right)x+2A}{x\left(x+2\right)}$$

$$\left.\begin{array}{l}A+B=1\\ 2A=4\end{array}\right\}\Rightarrow A=2,\quad B=-1$$

$$\int_{-1}^{3}\frac{2}{x}dx-\int_{-1}^{3}\frac{1}{x+2}dx=\left(2\ln|x|-\ln|x+2|\right)\Big|_{-1}^{3}=\ln\left|\frac{x^2}{x+2}\right|\Big|_{-1}^{3}=\ln\frac{9}{5}-\ln 1=\ln\frac{9}{5}$$

4. LITERATURA

[1] E. Hamiti, *Matematika II, Prishtinë, 2008*

[2] K. Bukuroshi, *Analiza matematike I, Tiranë, 1971*

[3] R. Zejnullahu, *Analiza matematike I, Prishtinë, 2010*

[4] Xh. Krasniqi, *Detyra të zgjedhura dhe të zgjidhura nga matematika 12, Prishtinë, 2017*

[5] M. Kadriu, *Përmbledhje detyrash të zgjidhura nga matematika IV, Prishtinë, 1995*

[6] M. Kadriu, *Përmbledhje detyrash të zgjidhura nga matematika III, Prishtinë, 1997*

[7] B. Apsen, *Repetitorij vise matematike II, Zagreb, 1983*

[8] B. Apsen, *Rijeseni zadaci vise matematike, Zagreb, 1971*

[9] V. Dajoviq, *Matematika për klasën e IV gjimnaz, Prishtinë, 1973*

[10] B. Vena, *Zbirka resenih zadataka iz matematike IV, Beograd, 1988*

[11] B. Vena, *Zbirka resenih zadataka iz matematike III, Beograd, 1996*