Simple Modeling of Road Traffic.

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Presentation of the subject:

What is Road Traffic Modelling?

- Representing complex dynamics of vehicles moving along roads.
- Creating mathematical and computer models for:
  - Understanding vehicle flow.
  - Predicting movement patterns.
  - Analyzing interactions on roads and highways.

Fig. 1: Real-life Road Traffic
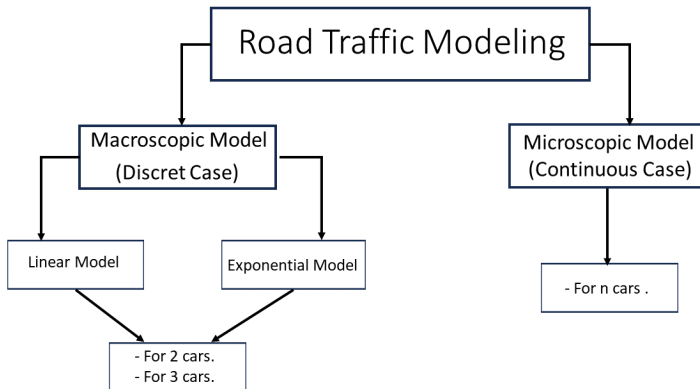
Presentation of the subject:

Key benefits of Road Traffic Modeling:

- **Avoiding traffic jams:** Helps find solutions to prevent traffic jams on roads.
- **Making Roads Better:** Finds ways to improve roads and make them work smoother.
- **Understanding how traffic works:** Helps figure out how different things affect traffic and predict what might happen.
- **Making transportation better:** Shows how well transportation works and helps make it even better.
- **Saving time and money:** Aims to reduce time spent waiting in traffic and the money spent on each trip.

Presentation of the subject

Animation à ajouter ??

Introduction:
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Study of the discret case:
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Study of the continuous model:
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Euler Explicit Method
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Conclusion
o

Project organization overview

Useful definitions:

Microscopic simulation:

Microscopic simulation is a computer-based modeling technique that simulates the behavior of individual entities, such as vehicles or pedestrians, within a system

Macroscopic simulation:

Macroscopic simulation models systems at a higher, aggregated level, considering overall behaviors like traffic flow without detailing individual movements.

Ordinary Differential Equation (ODE) :

An ODE is a mathematical equation that relates a function to its derivatives with respect to one or more independent variables.

$$F\left(x, y, y', \ldots, y^{(n-1)}\right) = y^{(n)}$$

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Ordinary Differential Equation (Theory):
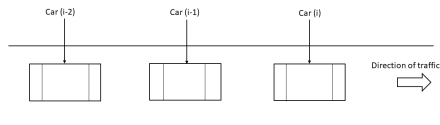
Figure 1: Discret Model.

ODE to solve

$$\mathbf{y'(t) = f(t, y(t))}$$

Euler Explicit method to numerically solve the solutions:

- First step of the resolution: $y_0 = y(t_0)$.
- Recursive process to find the n-th solution of the ODE:
  $y_{n+1} = y_n + h f(t_n, y_n)$

Velocity modeling: The linear approach

Each car's movement is governed by the basic equation

$$\dot{x}_i(t) = V_i = \alpha_i(x_{i-1} - x_i)$$

Where

$\dot{x}_i(t)$:	Instantaneous velocity of the i-th car at time t
V_i:	Current velocity of the i-th car
α_i:	Coefficient describing the behavior of the i-th car
x_{i-1}:	Previous position of the i-th car
x_i:	Current position of the i-th car

Velocity Modeling: The Linear Approach

System of Equations for N cars

$$\begin{cases} \dot{x}_1 & = V_1 \\ \dot{x}_2(t) & = \alpha_2(x_1 - x_2) \\ & \vdots \\ \dot{x}_n(t) & = \alpha_n(x_{n-1} - x_n) \end{cases}$$

Systems for Positions of N cars

$$\begin{cases} x_1(t + \Delta t) & = x_1(t) + \Delta t \cdot V_1 \\ x_2(t + \Delta t) & = x_2(t) + \Delta t \cdot \alpha_2(x_1 - x_2) \\ & \vdots \\ x_n(t + \Delta t) & = x_n(t) + \Delta t \cdot \alpha_n(x_{n-1} - x_n) \end{cases}$$

Velocity Modeling: The Newell Approach

Each car's movement is described by the equation

$$\dot{x}_i(t) = V_i \left(1 - e^{-\frac{\lambda_i}{V_i}(x_{i-1}(t) - x_i(t) - d_i)} \right)$$

Where

V_i: the maximum velocity of the ith car
λ_i: the capacity of acceleration/deceleration
d_i: safe following distance associated with the ith car

Velocity Modeling: The Newell Approach
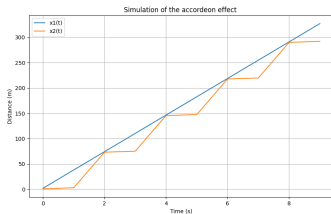
System of Equations for N cars

$$
\begin{cases}
\dot{x}_1 & = V_1 \\
\dot{x}_2(t) & = V_2(1 - e^{-\frac{\lambda_2}{V_2}(x_1(t) - x_2(t) - d_2)}) \\
& \vdots \\
\dot{x}_n(t) & = V_n(1 - e^{-\frac{\lambda_n}{V_n}(x_{n-1}(t) - x_n(t) - d_n)})
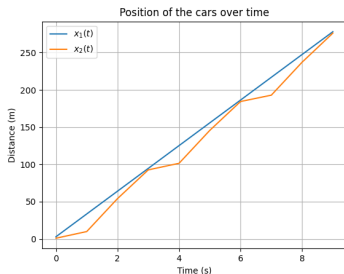\end{cases}
$$

Systems for Positions of N cars

$$
\begin{cases}
x_1(t + \Delta t) & = x_1(t) + \Delta t \cdot V_1 \\
x_2(t + \Delta t) & = x_2(t) + \Delta t \cdot V_2(1 - e^{-\frac{\lambda_2}{V_2}(x_1(t) - x_2(t) - d_2)}) \\
& \vdots \\
x_n(t + \Delta t) & = x_n(t) + \Delta t \cdot V_n(1 - e^{-\frac{\lambda_n}{V_n}(x_{n-1}(t) - x_n(t) - d_n)})
\end{cases}
$$

Accordion phenomenon

Figure 2: Modelisation of the accordion phenomenon with The Linear Model

Figure 3: Modelisation of the accordion phenomenon with The Newell's Model

We could see the difference of modelisation and realism between the Linear model (figure 2) and the Newell's model (figure 3)
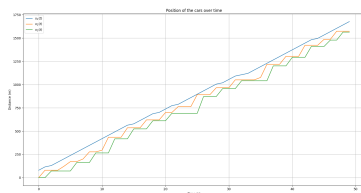
Drunk drivers

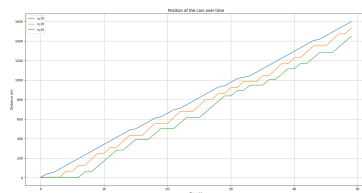Figure 4: Simulation of Traffic Flow with one drunk driver (Linear Model)

Figure 5: Simulation of Traffic Flow with one drunk driver (Newell's Model)

It is interesting to note that with Newell's Method (Figure 5), the variations are "smoothed and much less significant than in the case of the linear method (4)."

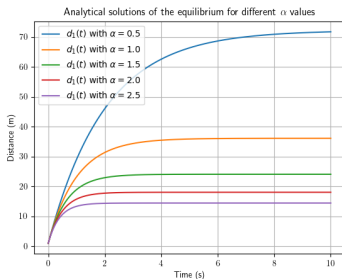## Simulations of both approaches:

Accident phenomenon:

Introduction: Study of the discret case: Study of the continuous model: Euler Explicit Method   Conclusion
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Study of Equilibrium and Stability

Analytical Solutions

Figure 6: Stability Analysis for Three Cars
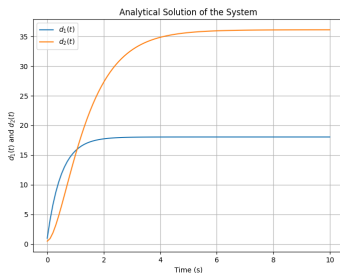
Figure 7: Analytical Solution for Three Cars

Introduction: Study of the discret case: Study of the continuous model: Euler Explicit Method   Conclusion
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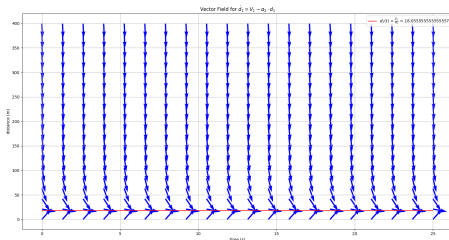
Study of Equilibrium and Stability

Equilibrium

Figure 8: Field Of Vector for the Newell's Model (Stability)
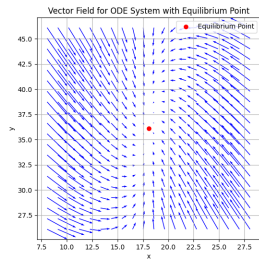
Figure 9: Field Of Vector for the Linear Model (Stability)

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Mathematical Theory

Conservation Law

- $\partial_t \rho + \partial_x \left[\rho \left(1 - \frac{\rho}{\rho_{\max}} \right) \cdot V_{\max} \right] = 0$

Initial and Boundary Conditions

- $\rho(x, 0) = \rho_0(x), \quad x \in \Omega$,
- $\rho(0, t) = \rho(L, t), \quad t \geq 0$
- $\Omega :=]0, L[$,
- $\rho(x, t)$ represents the traffic density at position x and time t,
- $F(\rho)$ denotes the traffic flux as a function of density,
- $F(\rho)$ is often represented by a function modeling the relationship between traffic density and traffic velocity,
- $F(\rho) = V(\rho) \cdot \rho$, where $V(\rho)$ is the traffic velocity as a function of density.
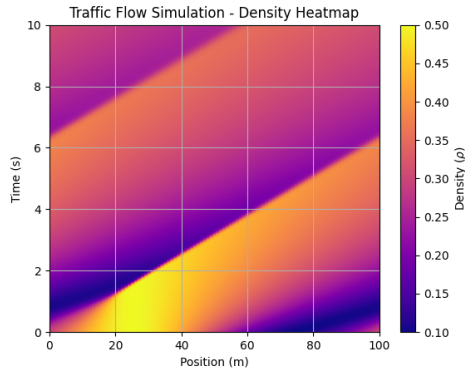
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Numerical Scheme for the resolution

Numerical Scheme

- $\rho_i^{n+1} = \rho_i^n - \frac{\Delta t}{\Delta x} \cdot \left(\rho_i^n \cdot v_i^n - \rho_{i-1}^n \cdot v_{i-1}^n \right) = 0$

- $v_i^n = \left(1 - \frac{\rho_i^n}{\rho_{max}} \right) \times V_{max}$

enrichir la slide. Peut etre ajouter le modèle dans la théorie et dire que quand on l'applique on obtient ca ou alors mettre une iage qui explique le shéma de Euler explicit ?

Figure 10: **Traffic Flow Simulation With Euler Explicit:** This figure illustrates the solution of the PDE at any time and position. Here is the link to the associated animation showing the movement of the traffic flow: **Traffic Flow Simulation**

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