

Simple Modeling of Road Traffic.

Florent Gerbaud
Fatima Ezzahra Rharrour
MAM4

Supervisor:

Didier Auroux

December 19, 2023



Table of contents

1 Introduction

2 Study of the discret case

- Ordinary Differential Equation
- Simulations and Results
- Study of Equilibrium and Stability

3 Study of the continuous case

- Euler's Explicit Method
- Lax-Friedrichs method

4 Conclusion

Table of Contents

1 Introduction

2 Study of the discret case

- Ordinary Differential Equation
- Simulations and Results
- Study of Equilibrium and Stability

3 Study of the continuous case

- Euler's Explicit Method
- Lax-Friedrichs method

4 Conclusion

What is Road Traffic Modelling?

- Representing complex dynamics of vehicles moving along roads.
- Creating mathematical and computer models for:
 - Understanding vehicle flow.
 - Predicting movement patterns.
 - Analyzing interactions on roads and highways.

The Objective of SMRT

Key benefits of Road Traffic Modeling:

- **Avoiding traffic jams:** Helps find solutions to prevent traffic jams on roads.
- **Making Roads Better:** Finds ways to improve roads and make them work smoother.
- **Understanding how traffic works:** Helps figure out how different things affect traffic and predict what might happen.
- **Making transportation better:** Shows how well transportation works and helps make it even better.
- **Saving time and money:** Aims to reduce time spent waiting in traffic and the money spent on each trip.

Project organization overview and Useful Definition

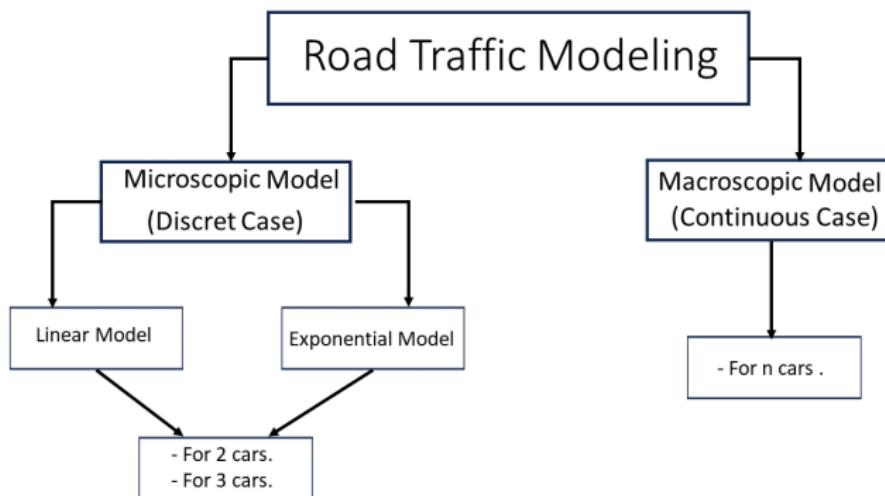


Table of Contents

1 Introduction

2 Study of the discret case

- Ordinary Differential Equation
- Simulations and Results
- Study of Equilibrium and Stability

3 Study of the continuous case

- Euler's Explicit Method
- Lax-Friedrichs method

4 Conclusion

Ordinary Differential Equation

Ordinary Differential Equation (Theory):

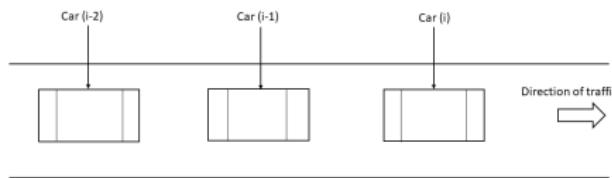


Figure 1: Discret Model.

ODE to solve

$$\mathbf{y}'(\mathbf{t}) = \mathbf{f}(\mathbf{t}, \mathbf{y}(\mathbf{t}))$$

Euler's Explicit method to numerically solve the solutions:

- First step of the resolution: $y_0 = y(t_0)$.
- Recursive process to find the n-th solution of the ODE:

$$y_{n+1} = y_n + h f(t_n, y_n)$$

Ordinary Differential Equation

The Linear Approach

Each car's movement is described by the equation

$$\dot{x}_i(t) = V_i = \alpha_i(x_{i-1} - x_i)$$

Where

- $\dot{x}_i(t)$: Instantaneous velocity of the i-th car at time t
- V_i : Current velocity of the i-th car
- α_i : Coefficient describing the behavior of the i-th car
- x_{i-1} : Previous position of the i-th car
- x_i : Current position of the i-th car

System of Velocity for N cars

$$\begin{cases} \dot{x}_1 &= V_1 \\ \vdots \\ \dot{x}_n(t) &= \alpha_n(x_{n-1} - x_n) \end{cases}$$

System of Positions for N cars

$$\begin{cases} x_1(t + \Delta t) &= x_1(t) + \Delta t \cdot V_1 \\ \vdots \\ x_n(t + \Delta t) &= x_n(t) + \Delta t \cdot \alpha_n(x_{n-1} - x_n) \end{cases}$$

A good model for a first approach, but it lacks realism.



Ordinary Differential Equation

The Newell Approach

Each car's movement is described by the equation

$$\dot{x}_i(t) = V_i \left(1 - e^{-\frac{\lambda_i}{V_i}(x_{i-1}(t) - x_i(t) - d_i)} \right)$$

Where

V_i : the maximum velocity of the i th car

λ_i : the capacity of acceleration/deceleration

d_i : safe following distance associated with the i th car

System of Equations for N cars

$$\begin{cases} \dot{x}_1 &= V_1 \\ \dot{x}_2(t) &= V_2(1 - e^{-\frac{\lambda_2}{V_2}(x_1(t) - x_2(t) - d_2)}) \\ \vdots & \\ \dot{x}_n(t) &= V_n(1 - e^{-\frac{\lambda_n}{V_n}(x_{n-1}(t) - x_n(t) - d_n)}) \end{cases}$$

System of Equations for N cars

$$\begin{cases} x_1(t + \Delta t) &= x_1(t) + \Delta t \cdot V_1 \\ x_2(t + \Delta t) &= x_2(t) + \Delta t \cdot V_2(1 - e^{-\frac{\lambda_2}{V_2}(x_1(t) - x_2(t) - d_2)}) \\ \vdots & \\ x_n(t + \Delta t) &= x_n(t) + \Delta t \cdot V_n(1 - e^{-\frac{\lambda_n}{V_n}(x_{n-1}(t) - x_n(t) - d_n)}) \end{cases}$$

A simple model with a more realistic approach than the linear model, however, it's still too simplistic

Simulations and Results

Accordion phenomenon

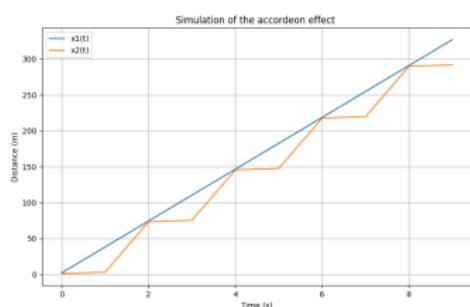


Figure 2: The Linear Model

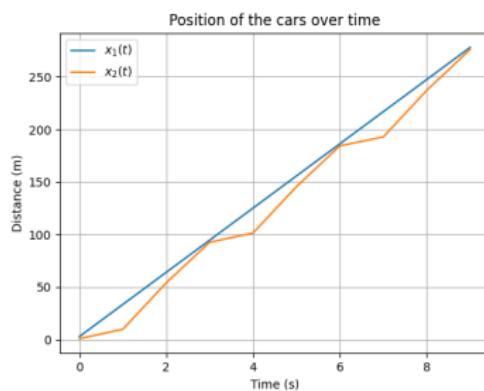


Figure 3: The Newell Model

We could see the difference of modelisation and realism between the Linear model (figure 2) and the Newell's model (figure 3)

Simulations and Results

One Drunk driver

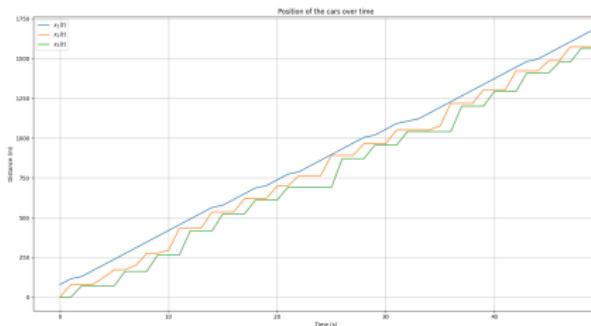


Figure 4: The Linear Model

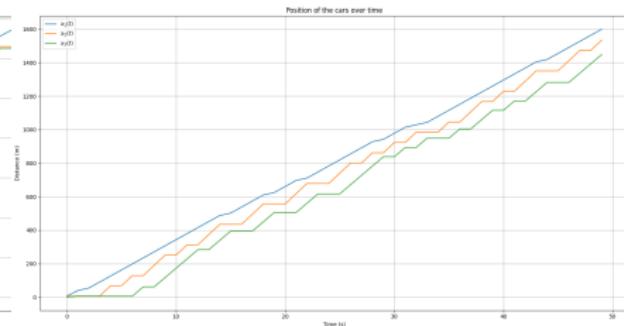


Figure 5: The Newell Model

On the Newell's Method (Figure 5), the variations are smoother and much less significant than in the case of the linear method (4).

Accident phenomenon

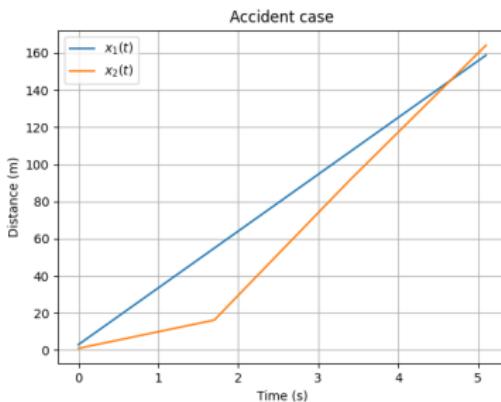


Figure 6: The Newell Model

On the figure 6, you can see that when the curves intersect, there is an accident

Study of Equilibrium and Stability

Analytical Solutions for the Linear Model

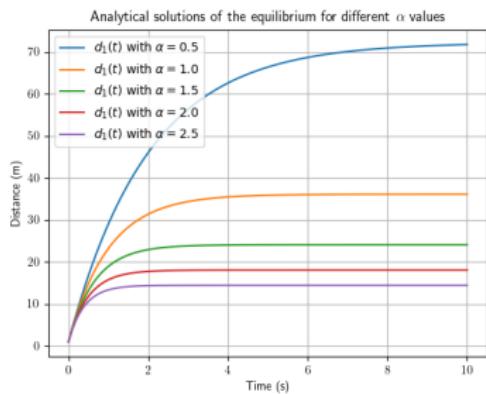


Figure 7: Analytical Solution for two Cars

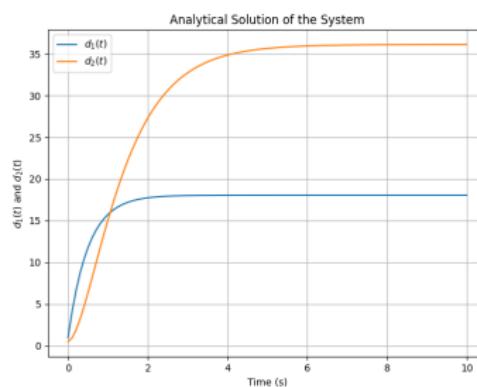


Figure 8: Analytical Solution for three Cars

Study of Equilibrium and Stability

Vector Field for Newell's model

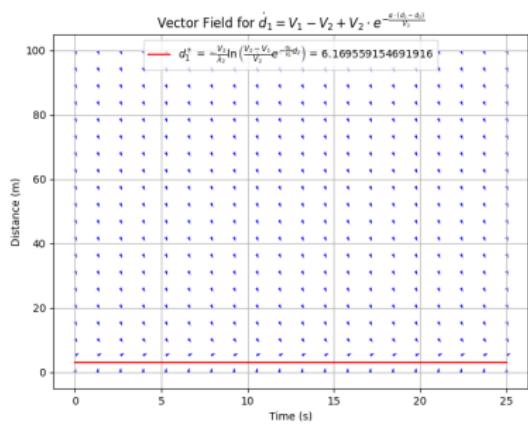


Figure 9: Stability for 2 cars

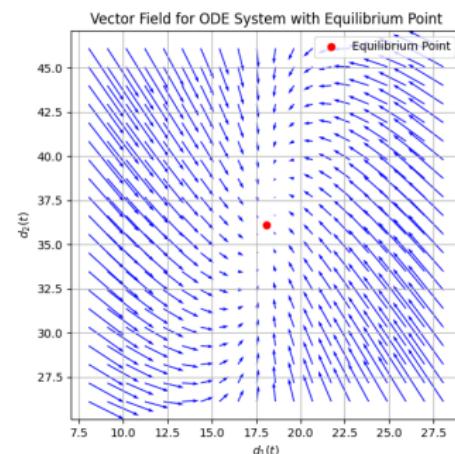


Figure 10: Stability for 3 cars

Table of Contents

- 1 Introduction
- 2 Study of the discret case
 - Ordinary Differential Equation
 - Simulations and Results
 - Study of Equilibrium and Stability
- 3 Study of the continuous case
 - Euler's Explicit Method
 - Lax-Friedrichs method
- 4 Conclusion

Model used in the Macroscopic Model

Conservation Law

- $\partial_t \rho + \partial_x \left[\rho \left(1 - \frac{\rho}{\rho_{\max}} \right) \cdot V_{\max} \right] = 0$

Initial and Boundary Conditions

- $\rho(x, t)$ represents the traffic density at position x and time t ,
- $\rho(x, 0) = \rho_0(x), \quad x \in \Omega,$
- $\rho(0, t) = \rho(L, t), \quad t \geq 0,$
- $\Omega :=]0, L[.$

Euler's Explicit Method

First Numerical Scheme to perform the Solution of the Equation

Euler's Explicit Scheme

- $\rho_i^{n+1} = \rho_i^n - \frac{\Delta t}{\Delta x} \cdot (\rho_i^n \cdot v_i^n - \rho_{i-1}^n \cdot v_{i-1}^n) = 0$
- $v_i^n = \left(1 - \frac{\rho_i^n}{\rho_{max}}\right) \times V_{max}$

Initial Condition and Discretization

$$\rho_0(x) = 0.2 \cdot \sin \left(2 \cdot \pi \cdot \frac{x}{L}\right) + 0.3$$

$$\Delta t = 0.01s$$

$$\Delta x = 1m$$

Euler's Explicit Method

Results obtained with the Euler's Explicit Method

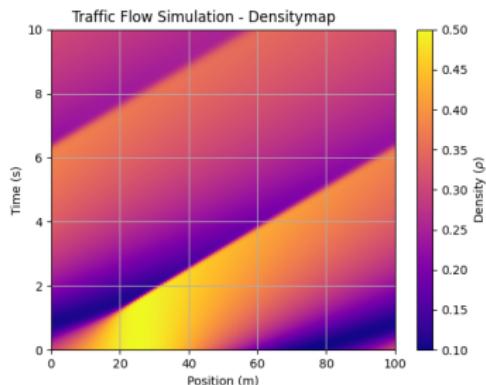


Figure 11: The solution of the PDE at any time and position.

- Scheme Diffusivity
- Periodic Boundary Conditions

Figure 12: The Animation for the simulation with Euler's Explicit Method

- Recreation of new peaks of density
- Traffic moves forward



Lax-Friedrichs method

A new Scheme to Represent the High Density of the Traffic.

Euler-Explicit Scheme

- $\rho_i^{n+1} = \rho_i^n - \frac{\Delta t}{\Delta x} \cdot (\rho_i^n \cdot v_i^n - \rho_{i-1}^n \cdot v_{i-1}^n) = 0$
- $v_i^n = \left(1 - \frac{\rho_i^n}{\rho_{max}}\right) \times V_{max}$



Lax-Friedrichs Scheme

$$\begin{aligned}\rho_j^t = & \frac{1}{2} \left(\rho_{j+1}^{t-1} + \rho_{j-1}^{t-1} \right) - \frac{\Delta t}{2 \cdot \Delta x} \left(\rho_{j+1}^{t-1} \left(1 - \frac{\rho_{j+1}^{t-1}}{\rho_{max}} \right) \cdot V_{max} \right. \\ & \left. - \rho_{j-1}^{t-1} \left(1 - \frac{\rho_{j-1}^{t-1}}{\rho_{max}} \right) \cdot V_{max} \right)\end{aligned}$$

Initial Condition and Discretization

$$\begin{aligned}\rho_0(x) &= 0.2 \cdot \sin \left(2 \cdot \pi \cdot \frac{x}{L} \right) + 0.8 \\ \Delta t &= 0.01 \\ \Delta x &= 1\end{aligned}$$

Lax-Friedrichs method

Results obtained with the Lax-Friedrichs Method

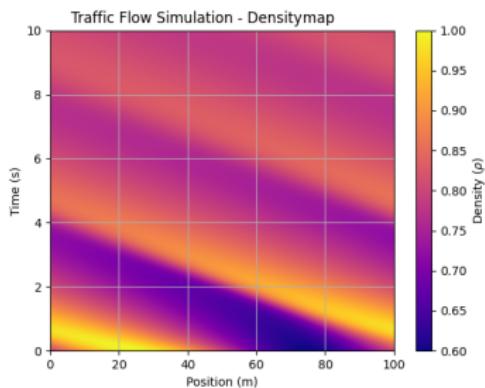


Figure 13: The solution of the PDE at any time and position.

Figure 14: The Animation for the simulation with Lax-Friedrichs Method

- Scheme
- Diffusivity/Dispersivity
- Periodic Boundary Conditions

- Recreation of new peaks of density
- Traffic moves backward



Table of Contents

- 1 Introduction
- 2 Study of the discret case
 - Ordinary Differential Equation
 - Simulations and Results
 - Study of Equilibrium and Stability
- 3 Study of the continuous case
 - Euler's Explicit Method
 - Lax-Friedrichs method
- 4 Conclusion

Conclusion

Main challenges

- Implementing Macroscopic model
- Managing time
- Achieving equilibrium

New abilities that we learned

- Modelizing fluid equations
- Understanding the behavior of traffic flow
- Applying our theoretical knowledge to concrete cases
- Strengthening teamworking abilities