Simple Modeling of Road Traffic.

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POLYTECH
NICE-SOPHIA

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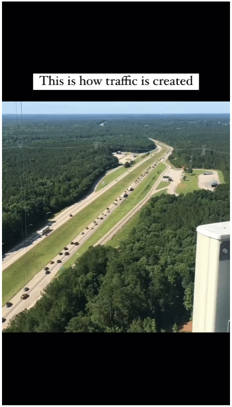
What is Road Traffic Modelling?

- Representing complex dynamics of vehicles moving along roads.
- Creating mathematical and computer models for:
  - Understanding vehicle flow.
  - Predicting movement patterns.
  - Analyzing interactions on roads and highways.

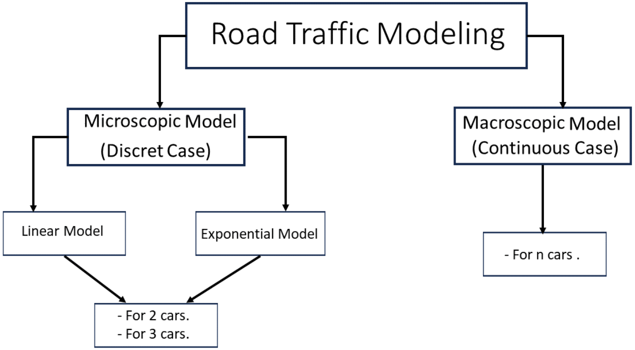


The Objective of SMRT

Key benefits of Road Traffic Modeling:

- **Avoiding traffic jams:** Helps find solutions to prevent traffic jams on roads.
- **Making Roads Better:** Finds ways to improve roads and make them work smoother.
- **Understanding how traffic works:** Helps figure out how different things affect traffic and predict what might happen.
- **Making transportation better:** Shows how well transportation works and helps make it even better.
- **Saving time and money:** Aims to reduce time spent waiting in traffic and the money spent on each trip.

Project organization overview and Useful Definition



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Ordinary Differential Equation (Theory):

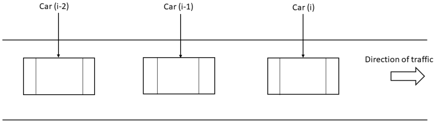

Figure 1: Discret Model.

ODE to solve

$$\boxed{\mathbf{y}'(\mathbf{t}) = \mathbf{f}(\mathbf{t}, \mathbf{y}(\mathbf{t}))}$$

Euler Explicit method to numerically solve the solutions:

- First step of the resolution: $\boxed{y_0 = y(t_0)}$.
- Recursive process to find the n-th solution of the ODE: $\boxed{y_{n+1} = y_n + hf(t_n, y_n)}$

The Linear Approach

Each car's movement is described by the equation

$$\boxed{\dot{x}_i(t) = V_i\left(1 - e^{-\frac{\lambda_i}{V_i}(x_{i-1}(t) - x_i(t) - d_i)}\right)}$$

Where

V_i: the maximum velocity of the ith car
λ_i: the capacity of acceleration/deceleration
d_i: safe following distance associated with the ith car

System of Equations for N cars

$$\begin{cases} \dot{x}_1 & = V_1 \\ & \vdots \\ \dot{x}_n(t) & = \alpha_n(x_{n-1} - x_n) \end{cases}$$

Systems for Positions of N cars

$$\begin{cases} x_1(t + \Delta t) & = x_1(t) + \Delta t \cdot V_1 \\ & \vdots \\ x_n(t + \Delta t) & = x_n(t) + \Delta t \cdot \alpha_n(x_{n-1} - x_n) \end{cases}$$

A good model for a first approach, but it lacks realism.

The Newell Approach

Each car's movement is described by the equation

$$\boxed{\dot{x}_i(t) = V_i\left(1 - e^{-\frac{\lambda_i}{V_i}(x_{i-1}(t) - x_i(t) - d_i)}\right)}$$

Where

V_i: the maximum velocity of the ith car
λ_i: the capacity of acceleration/deceleration
d_i: safe following distance associated with the ith car

System of Equations for N cars

$$\begin{cases} \dot{x}_1 &= V_1 \\ \dot{x}_2(t) &= V_2(1 - e^{-\frac{\lambda_2}{V_2}(x_1(t) - x_2(t) - d_2}) \\ & \vdots \\ \dot{x}_n(t) &= V_n(1 - e^{-\frac{\lambda_n}{V_n}(x_{n-1}(t) - x_n(t) - d_n}) \end{cases}$$

Systems for Positions of N cars

$$\begin{cases} x_1(t+\Delta t) &= x_1(t) + \Delta t \cdot V_1 \\ x_2(t+\Delta t) &= x_2(t) + \Delta t \cdot V_2(1 - e^{-\frac{\lambda_2}{V_2}(x_1(t) - x_2(t) - d_2}) \\ & \vdots \\ x_n(t+\Delta t) &= x_n(t) + \Delta t \cdot V_n(1 - e^{-\frac{\lambda_n}{V_n}(x_{n-1}(t) - x_n(t) - d_n}) \end{cases}$$

A simple model with an approach more realistic than the linear model, however, it is still too simplistic.

Accordion phenomenon

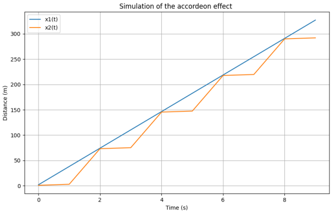

Figure 2: Modelisation of the accordion phenomenon with The Linear Model

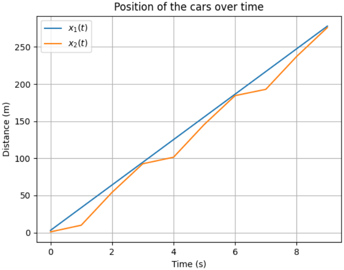

Figure 3: Modelisation of the accordion phenomenon with The Newell's Model

We could see the difference of modelisation and realism between the Linear model (figure 2) and the Newell's model (figure 3)

Drunk drivers

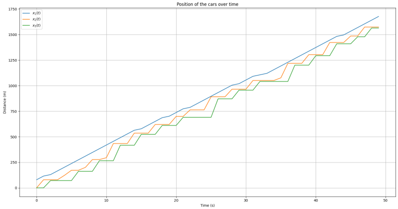

Figure 4: Simulation of Traffic Flow with one drunk driver (Linear Model)

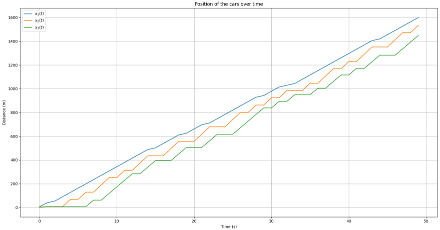

Figure 5: Simulation of Traffic Flow with one drunk driver (Newell's Model)

It is interesting to note that with Newell's Method (Figure 5), the variations are "smoothed and much less significant than in the case of the linear method (4)."

Accident phenomenon

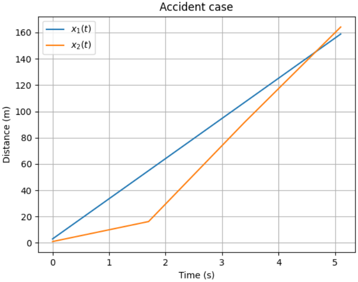

Figure 6: Modelisation of the accordion phenomenon with The Newell's Model

On the figure 6, you can see that when the curves intersect, there is an accident

Analytical Solutions for the Linear Model

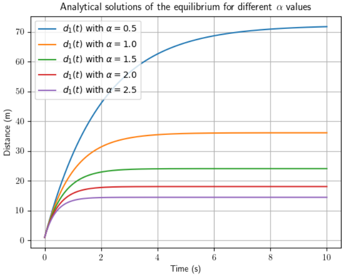

Figure 7: Analytical Solution for two Cars

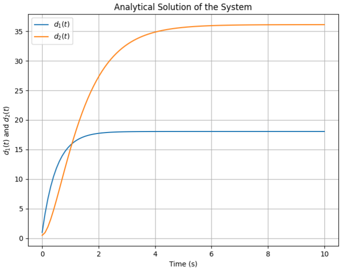

Figure 8: Analytical Solution for three Cars

Vector Field for Newell's model

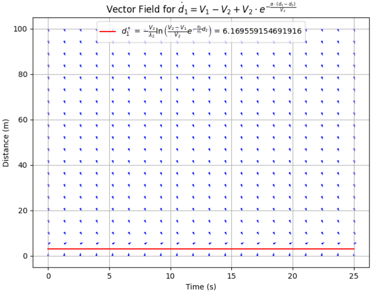

Figure 9: Stability for 2 cars

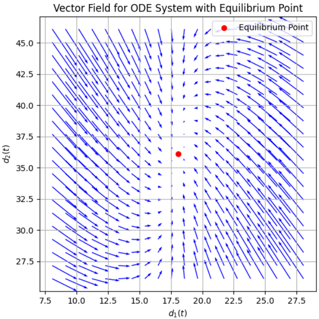

Figure 10: Stability for 3 cars

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Model used in the Macroscopic Model

Conservation Law

- $\partial_t \rho + \partial_x \left[\rho \left(1 - \frac{\rho}{\rho_{\max}} \right) \cdot V_{\max} \right] = 0$

Initial and Boundary Conditions

- $\rho(x, 0) = \rho_0(x), \quad x \in \Omega,$
- $\rho(0, t) = \rho(L, t), \quad t \geq 0$
- $\Omega :=]0, L[\,,$
- $\rho(x, t)$ represents the traffic density at position x and time t,

First Numerical Scheme to perform the Solution of the Equation

Euler-Explicit Scheme

- $\rho_i^{n+1} = \rho_i^n - \frac{\Delta t}{\Delta x} \cdot \left(\rho_i^n \cdot v_i^n - \rho_{i-1}^n \cdot v_{i-1}^n\right) = 0$
- $v_i^n = \left(1 - \frac{\rho_i^n}{\rho_{max}}\right) \times V_{max}$

Initial Condition and Discretization

$$\boxed{\begin{aligned} &\rho_0(x) = 0.2 \cdot \sin\left(2 \cdot \pi \cdot \frac{x}{L}\right) + 0.3 \\ &\Delta t = 0.01 \\ &\Delta x = 1 \end{aligned}}$$

Results obtained with the Euler-Explicit Method

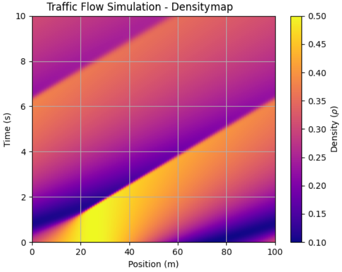


Figure 11: This figure illustrates the solution of the PDE at any time and position.

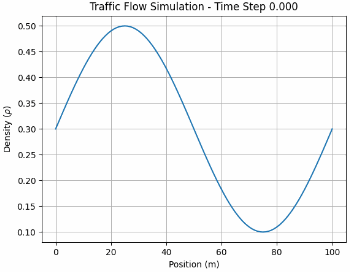


Figure 12: The Animation for the simulation with Euler-Explicit Method

- Scheme Diffusivity
- Periodic Boundary Conditions
- Recreation of new peaks of density
- Traffic moves forward

A new Scheme to Represent the High Density of the Traffic.

Euler-Explicit Scheme

- $\rho_i^{n+1} = \rho_i^n - \frac{\Delta t}{\Delta x} \cdot \left(\rho_i^n \cdot v_i^n - \rho_{i-1}^n \cdot v_{i-1}^n\right) = 0$
- $v_i^n = \left(1 - \frac{\rho_i^n}{\rho_{max}}\right) \times V_{max}$

$$\Downarrow$$

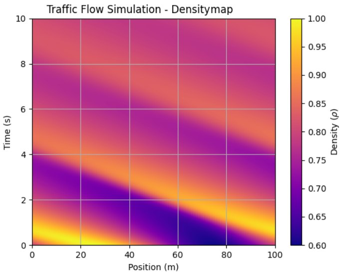
Lax-Friedrichs Scheme

$$\rho_j^t = \frac{1}{2}\left(\rho_{j+1}^{t-1} + \rho_{j-1}^{t-1}\right) - \frac{\Delta t}{2 \cdot \Delta x}\left(\rho_{j+1}^{t-1}\left(1 - \frac{\rho_{j+1}^{t-1}}{\rho_{max}}\right) \cdot V_{\max} - \rho_{j-1}^{t-1}\left(1 - \frac{\rho_{j-1}^{t-1}}{\rho_{max}}\right) \cdot V_{\max}\right)$$

Initial Condition and Discretization

$$\boxed{\begin{aligned} &\rho_0(x) = 0.2 \cdot \sin\left(2 \cdot \pi \cdot \frac{x}{L}\right) + 0.8 \\ &\Delta t = 0.01 \\ &\Delta x = 1 \end{aligned}}$$

Results obtained with the Lax-Friedrichs Method



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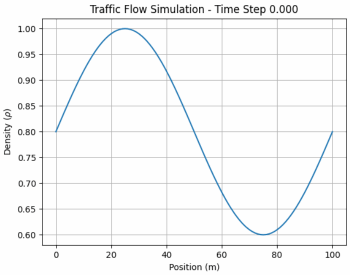

Figure 14: The Animation for the simulation with Lax-Friedrichs Method

- Scheme Diffusivity/Dispersivity
- Periodic Boundary Conditions
- Recreation of new peaks of density
- Traffic moves backward

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