Project Report

Polytech Nice

Simple Road Traffic Modeling

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1 Presentation of the Subject

1.1 Useful Definition

Ordinary Differential Equation (ODE):

An ODE is a mathematical equation that relates a function to its derivatives with respect to one or more independent variables. ODEs are commonly represented given a function F of x, y, and derivatives of y. Then, an equation of the form

$$F\left(x, y, y', \ldots, y^{(n-1)}\right) = y^{(n)}$$

What is Road Traffic Modeling

The Road Traffic modeling is the study of how vehicles behave on road networks, aiming to simulate and analyze various aspects of traffic flow, congestion, and driver behavior. This field involves creating mathematical and computer models to understand and predict traffic patterns, particularly in scenarios such as congestion, erratic driving, and other relevant factors affecting road transportation. Road traffic modeling plays a crucial role in urban planning, traffic management, and the development of intelligent transportation systems (you could see an exemple on the 1).

Figure 1: **Road traffic :** In this picture, you can see an example of a road traffic phenomenon that we could study

1.1.1 Simple Road Traffic Modeling

Here explain what is the objective how we are going to do the study and so on

1.1.2 Using GitHub for Project Management

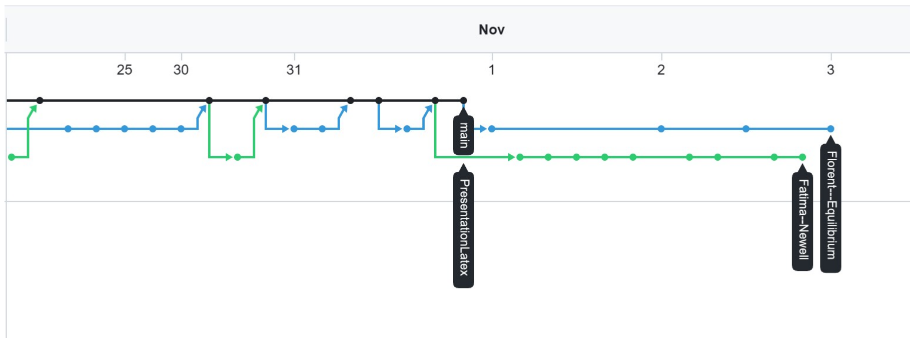

Figure 2: **Road traffic :** In this picture, you can see an example of a road traffic phenomenon that we could study

2 The equation for SMRT

2.1 Differential Ordinary Equation

In this part, the idea is to resolve two types of systems of Ordinary Differential Equations (ODEs) that allow us to simulate traffic flow. To achieve this, we will use the Euler Explicit method to numerically solve the solutions. The Euler Explicit method is given by the folowing eqution :

- EDO to solve: $\boxed{y'(t) = f\big(t, y(t)\big)}$.
- First step of the resolution: $\boxed{y_0 = y(t_0)}$.
- Recursive process to find the n-th solution of the EDO: $\boxed{y_{n+1} = y_n + hf(t_n, y_n)}$

2.1.1 Linear Model

Mathematical Theory:

In this section, we're going to explain the math behind our model for understanding how cars behave in traffic. To make things simple, we use a discrete model, which means we look at cars one at a time and how they interact on the road.

Each car's movement is governed by a basic equation:

$$\dot{x_i}(t) = V_i = \alpha_i(x_{i-1} - x_i)$$

In this equation, $x_i(t)$ represents where the i-th car is at a given time, V_i is how fast the i-th car is going, and α_i is a number that describes how that car behaves. The right side of the equation, $\alpha_i(x_{i-1} - x_i)$, tells us how the car's speed changes based on how close it is to the car in front.

When we put this equation to work for all the cars, we end up with a bunch of equations (one for each car), which helps us understand how they all move together in traffic. These equations give us a dynamic view of how cars influence each other as they drive.

The system of equations is written like this for each car, where i can be 1, 2, and so on, up to the number of cars:

$$\begin{cases} \dot{x}_1 & = V_1 \\ \dot{x}_2(t) & = \alpha_2(x_1 - x_2) \\ & \vdots \\ \dot{x}_n(t) & = \alpha_n(x_{n-1} - x_n) \end{cases}$$

These equations help us understand how traffic flows and how individual cars influence one another on the road.

In the first part of the study, we consider α_i as a constant. However, in the next part of the simulation, we define some functions $\alpha_i(t)$.

In fact, we have three types of simulations:

1. **Constant value:**

$$\alpha_i(t) = C_i$$

2. **Sinusoidal with noise:**

$$\alpha_i(t) = |W \cdot \sin(\omega t + \phi) + \mathcal{N}(0, 0.1)|$$

3. **Stochastic Driver Model:**

Consider a random driver model with an output labeled as $\alpha_i(t)$, which relies on various factors like the average, spread, and time t. This model produces a random noise part from a usual distribution with an average and spread of $\sqrt{\text{spread}}$. If a limit is given, the generated noise is confined within the interval $[-\text{limit}, \text{limit}]$.

Mathematically, we can express this as:

$$\alpha_i(t) = \begin{cases} \left|\mathcal{N}(\text{average}, \sqrt{\text{spread}})\right|, & \text{if } -\text{limit} \leq \alpha_i(t) \leq \text{limit}, \\ -\text{limit}, & \text{if } \alpha_i(t) < -\text{limit}, \\ \text{limit}, & \text{if } \alpha_i(t) > \text{limit}. \end{cases}$$

Here, average signifies the mean of the acceleration, and spread stands for the acceleration's spread. The limit is defined to prevent or create specific situations, like accidents, depending on the desired result.

Implementation :

Algorithm 1 $\dot{x_1}$

Output:

- $\dot{x_1} = V_1$

Algorithm:

- $\dot{x_1} = 130 \times \frac{1000}{3600}$
- **return** $\dot{x_1}$

Algorithm 2 $\dot{x_i}$

Input:

- $t :=$ "time step"
- $x_i :=$ Table of position for the i-th Car
- $x_{i-1} :=$ Table of position for the (i-1)-th Car

Output:

- $\dot{x_i} = V_i$

Algorithm:

- $\dot{x_i}(t) = V_i = \alpha_i(x_{i-1}[t] - x_i[t])$
- **return** $\dot{x_i}$

This algorithm allows to define an orthonormed local reference frame and we delete the lines to eliminate for reducing matrix

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