Simple Modeling of Road Traffic.

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December 5, 2023

POLYTECH NICE-SOPHIA

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What is Road Traffic Modelling?

- Representing complex dynamics of vehicles moving along roads.
- Creating mathematical and computer models for:
  - Understanding vehicle flow.
  - Predicting movement patterns.
  - Analyzing interactions on roads and highways.

Fig. 1: Real-life Road Traffic

Presentation of the subject:

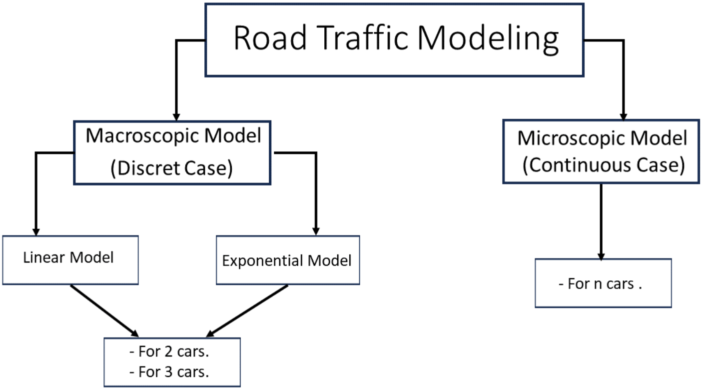
Key Benefits of Road Traffic Modeling

- **Avoiding Traffic Jams:** Helps find solutions to prevent traffic jams on roads.
- **Making Roads Better:** Finds ways to improve roads and make them work smoother.
- **Understanding How Traffic Works:** Helps figure out how different things affect traffic and predict what might happen.
- **Making Transportation Better:** Shows how well transportation works and helps make it even better.
- **Saving Time and Money:** Aims to reduce time spent waiting in traffic and the money spent on each trip.

Presentation of the subject

Animation à ajouter ??

Project Organization Overview



Useful Definitions

Microscopic simulation

Microscopic simulation is a computer-based modeling technique that simulates the behavior of individual entities, such as vehicles or pedestrians, within a system

Macroscopic simulation

Macroscopic simulation models systems at a higher, aggregated level, considering overall behaviors like traffic flow without detailing individual movements.

Ordinary Differential Equation

An ODE is a mathematical equation that relates a function to its derivatives with respect to one or more independent variables.

$$F\left(x, y, y', \ldots, y^{(n-1)}\right) = y^{(n)}$$

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Differential Ordinary Equation (Theory):

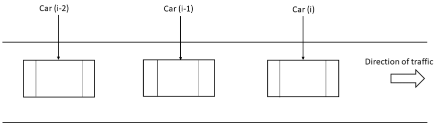

Figure: Discret Model.

- EDO to solve: $\boxed{y'(t) = f\left(t, y(t)\right)}$.
- First step of the resolution: $\boxed{y_0 = y(t_0)}$.
- Recursive process to find the n-th solution of the EDO:
  $\boxed{y_{n+1} = y_n + hf\left(t_n, y_n\right)}$

Linear Model to represent velocity:

Each car's movement is governed by a basic equation:

$$\dot{x}_i(t) = V_i = \alpha_i(x_{i-1} - x_i)$$

The system of equations for different cars is:

$$\begin{cases} \dot{x}_1 & = V_1 \\ \dot{x}_2(t) & = \alpha_2(x_1 - x_2) \\ & \vdots \\ \dot{x}_n(t) & = \alpha_n(x_{n-1} - x_n) \end{cases}$$

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Mathematical Theory

Conservation Law

- $\partial_t \rho + \partial_x \left[\rho \left(1 - \frac{\rho}{\rho_{\max}}\right) \cdot V_{\max}\right] = 0$

Initial and Boundary Conditions

- $\rho(x, 0) = \rho_0(x), \quad x \in \Omega,$
- $\rho(0, t) = \rho(L, t), \quad t \geq 0$
- $\Omega :=]0, L[\,,$
- $\rho(x, t)$ represents the traffic density at position x and time t,
- $F(\rho)$ denotes the traffic flux as a function of density,
- $F(\rho)$ is often represented by a function modeling the relationship between traffic density and traffic velocity,
- $F(\rho) = V(\rho) \cdot \rho$, where $V(\rho)$ is the traffic velocity as a function of density.

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Numerical Scheme for the resolution

Numerical Scheme

- $\rho_i^{n+1} = \rho_i^n - \frac{\Delta t}{\Delta x} \cdot \left(\rho_i^n \cdot v_i^n - \rho_{i-1}^n \cdot v_{i-1}^n\right) = 0$
- $v_i^n = \left(1 - \frac{\rho_i^n}{\rho_{max}}\right) \times V_{max}$

enrichir la slide. Peut etre ajouter le modèle dans la théorie et dire que quand on l'applique on obtient ca ou alors mettre une iage qui explique le shéma de Euler explicit ?

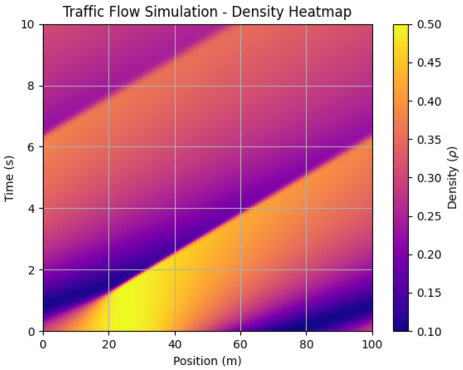

Figure: **Traffic Flow Simulation With Euler Explicit:** This figure illustrates the solution of the PDE at any time and position. Here is the link to the associated animation showing the movement of the traffic flow: **Traffic Flow Simulation**

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