Simple Modeling of Road Traffic.

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POLYTECH NICE-SOPHIA

Table of contents

Table of Contents

Presentation of the subject:

What is Road Traffic Modelling?

- Representing complex dynamics of vehicles moving along roads.
- Creating mathematical and computer models for:
    - Understanding vehicle flow.
    - Predicting movement patterns.
    - Analyzing interactions on roads and highways.

Fig. 1: Real-life Road Traffic

Presentation of the subject:

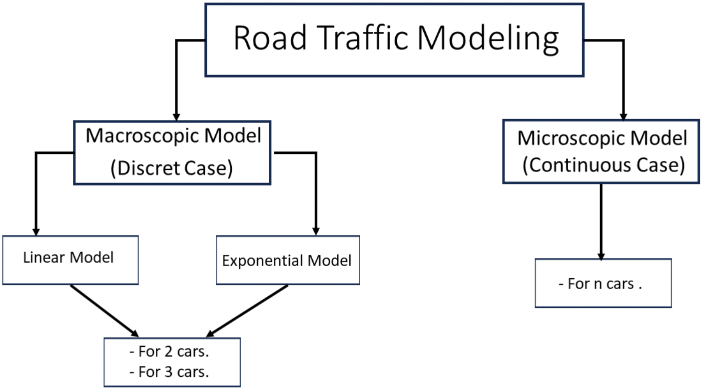
Key benefits of Road Traffic Modeling:

- **Avoiding traffic jams:** Helps find solutions to prevent traffic jams on roads.
- **Making Roads Better:** Finds ways to improve roads and make them work smoother.
- **Understanding how traffic works:** Helps figure out how different things affect traffic and predict what might happen.
- **Making transportation better:** Shows how well transportation works and helps make it even better.
- **Saving time and money:** Aims to reduce time spent waiting in traffic and the money spent on each trip.

Presentation of the subject

Animation à ajouter ??

Project organization overview



Useful definitions:

Microscopic simulation:

Microscopic simulation is a computer-based modeling technique that simulates the behavior of individual entities, such as vehicles or pedestrians, within a system

Macroscopic simulation:

Macroscopic simulation models systems at a higher, aggregated level, considering overall behaviors like traffic flow without detailing individual movements.

Ordinary Differential Equation (ODE) :

An ODE is a mathematical equation that relates a function to its derivatives with respect to one or more independent variables.

$$F\left(x, y, y', \ldots, y^{(n-1)}\right) = y^{(n)}$$

Table of Contents

Ordinary Differential Equation (Theory):

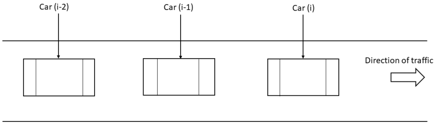


Figure: Discret Model.

ODE to solve: $\boxed{\mathbf{y}'(\mathbf{t}) = \mathbf{f}(\mathbf{t}, \mathbf{y}(\mathbf{t}))}$

Euler Explicit method to numerically solve the solutions:

- First step of the resolution: $\boxed{y_0 = y(t_0)}$.
- Recursive process to find the n-th solution of the ODE: $\boxed{y_{n+1} = y_n + hf(t_n, y_n)}$

Velocity modeling: The linear approach

Each car's movement is governed by the basic equation:

$$\boxed{\dot{x}_i(t) = V_i = \alpha_i(x_{i-1} - x_i)}$$

Where:

$\dot{x}_i(t)$:	Instantaneous velocity of the i-th car at time t
V_i:	Current velocity of the i-th car
α_i:	Coefficient describing the behavior of the i-th car
x_{i-1}:	Previous position of the i-th car
x_i:	Current position of the i-th car

Modeling Velocity: The Linear Approach

The system of equations for different cars is:

$$\begin{cases} \dot{x}_1 & = V_1 \\ \dot{x}_2(t) & = \alpha_2(x_1 - x_2) \\ & \vdots \\ \dot{x}_n(t) & = \alpha_n(x_{n-1} - x_n) \end{cases}$$

Velocity Modeling: The Newell Approach

Each car's movement is described by the equation:

$$\boxed{\dot{x}_i(t) = V_i(1 - e^{-\frac{\lambda_i}{V_i}(x_{i-1}(t) - x_i(t) - d_i)})}$$

Where:

V_i:	the maximum velocity of the ith car
λ_i:	the capacity of acceleration/deceleration
d_i:	safe following distance associated with the ith car

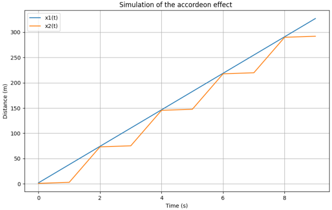
Velocity Modeling: The Newell Approach

The system of equations for different cars is:

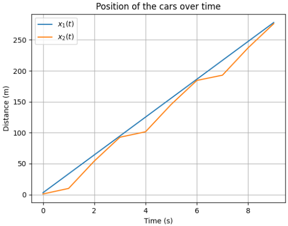
$$\begin{cases} x_1(t+\Delta t) & = x_1(t) + \Delta t V_1 \\ x_2(t+\Delta t) & = x_2(t) + \Delta t V_2(1 - e^{-\frac{\lambda_2}{V_2}(x_1(t)-x_2(t)-d_2}) \\ & \vdots \\ x_n(t+\Delta t) & = x_n(t) + \Delta t V_n(1 - e^{-\frac{\lambda_n}{V_n}(x_{n-1}(t)-x_n(t)-d_n}) \end{cases}$$

Simulations of both approaches:

Drivers reacting similary(Accordion phenomenon):



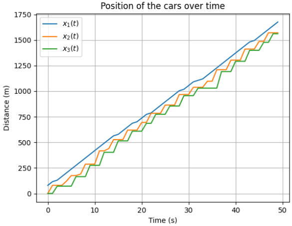
With Linear Model
Interpretation à ajouter



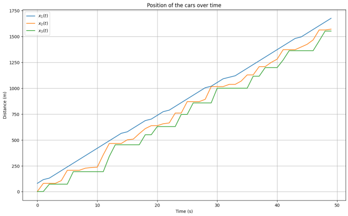
With Newell's Model

Simulations of both approaches:

Drunk drivers(Real-life case):



One drunk driver



Unpredictable Drivers

Simulations of both approaches:

Accident phenomenon:

Table of Contents

Mathematical Theory

Conservation Law

- $\partial_t \rho + \partial_x \left[\rho \left(1 - \frac{\rho}{\rho_{\max}} \right) \cdot V_{\max} \right] = 0$

Initial and Boundary Conditions

- $\rho(x, 0) = \rho_0(x), \quad x \in \Omega,$
- $\rho(0, t) = \rho(L, t), \quad t \geq 0$
- $\Omega :=]0, L[\,,$
- $\rho(x, t)$ represents the traffic density at position x and time t,
- $F(\rho)$ denotes the traffic flux as a function of density,
- $F(\rho)$ is often represented by a function modeling the relationship between traffic density and traffic velocity,
- $F(\rho) = V(\rho) \cdot \rho$, where $V(\rho)$ is the traffic velocity as a function of density.

Table of Contents

Numerical Scheme for the resolution

Numerical Scheme

- $\rho_i^{n+1} = \rho_i^n - \frac{\Delta t}{\Delta x} \cdot \left(\rho_i^n \cdot v_i^n - \rho_{i-1}^n \cdot v_{i-1}^n\right) = 0$
- $v_i^n = \left(1 - \frac{\rho_i^n}{\rho_{max}}\right) \times V_{max}$

enrichir la slide. Peut etre ajouter le modèle dans la théorie et dire que quand on l'applique on obtient ca ou alors mettre une iage qui explique le shéma de Euler explicit ?

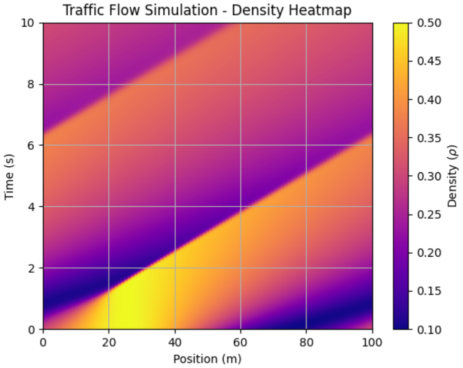


Figure: **Traffic Flow Simulation With Euler Explicit:** This figure illustrates the solution of the PDE at any time and position. Here is the link to the associated animation showing the movement of the traffic flow: **Traffic Flow Simulation**

Table of Contents