Polytech Nice

Project report

---

numerical solution of the optimal orbit transfer problem in the plane

---

18th February 2024

Gerbaud Florent

Table des matières

List of Algorithms

1 Theory

1.1 Nomenclature

- r := satellite radial distance attracting body
- u := radial velocity
- v := tangential speed
- m := satellite mass
- $\frac{\partial m}{\partial t} = \dot{m} = -\frac{T}{g_0 \cdot Isp}$:= dm motor mass flow
- ϕ := angle of thrust direction with r
- μ := gravitational constant of the attracting body
- θ := the angle formed by r with r(0)

1.2 Conditions initiales et finales:

$$\boxed{\begin{array}{l} r(0) = r_0, \quad u(0) = 0, \quad v(0) = \sqrt{\dfrac{\mu}{r_0}} \\ r\left(t_f\right) = r_f, \quad u\left(t_f\right) = 0, \quad v\left(t_f\right) = \sqrt{\dfrac{\mu}{r_f}} \end{array}}$$

1.3 Hamiltonien et dynamique de l'état adioint:

$$\dot{x} = \begin{bmatrix} \dot{r} \\ \dot{u} \\ \dot{v} \end{bmatrix} = f(x, t, \phi) = \begin{bmatrix} u \\ \frac{v^2}{r} - \frac{\mu}{r^2} + \frac{T}{m_0 + \dot{m} \times t} \sin(\phi) \\ -\frac{uv}{r} + \frac{T}{m_0 + \dot{m} \times t} \cos(\phi) \end{bmatrix}$$

$$\lambda^T = \begin{bmatrix} \lambda_r & \lambda_u & \lambda_v \end{bmatrix}$$

$$H = 1 + \lambda^T f = 1 + \lambda_r u + \lambda_u \left(\frac{v^2}{r} - \frac{\mu}{r^2} + \frac{T}{m_0 + \dot{m} \times t} \sin(\phi) \right) + \lambda_v \left(-\frac{uv}{r} + \frac{T}{m_0 + \dot{m} \times t} \cos(\phi) \right)$$

$$\dot{\lambda} = \begin{bmatrix} \dot{\lambda}_r \\ \dot{\lambda}_u \\ \dot{\lambda}_v \end{bmatrix} = -\frac{\partial f^T}{\partial x} \lambda = \begin{bmatrix} -\lambda_u \left(-\frac{v^2}{r^2} + \frac{2\mu}{r^3} \right) - \lambda_v \left(\frac{uv}{r^2} \right) \\ -\lambda_r + \lambda_v \frac{v}{r} \\ -\lambda_u \frac{2v}{r} + \lambda_v \frac{u}{r} \end{bmatrix}$$

1.4 Equation de la commande

$$H = 1 + \lambda^T f = 1 + \lambda_r u + \lambda_u \left(\frac{v^2}{r} - \frac{\mu}{r^2} + \frac{T}{m_0 + \dot{m} \times t} \sin(\phi) \right) + \lambda_v \left(-\frac{uv}{r} + \frac{T}{m_0 + \dot{m} \times t} \cos(\phi) \right)$$

$$\frac{\partial H}{\partial \phi} = \lambda_u \left(\frac{T}{m_0 + \dot{m} \times t} \cos(\phi) \right) - \lambda_v \left(\frac{T}{m_0 + \dot{m} \times t} \sin(\phi) \right) = 0$$

$$\frac{\partial H}{\partial \phi} = 0 \Leftrightarrow \lambda_u \cos(\phi) - \lambda_v \sin(\phi) = 0 \Rightarrow \tan(\phi) = \frac{\lambda_u}{\lambda_v}$$

1.5 Condition d'optimalité sur t_f

$$\left[H = 1 + \lambda^T f = 1 + \lambda_r u + \lambda_u \left(\frac{v^2}{r} - \frac{\mu}{r^2} + \frac{T}{m_0 + \dot{m} \times t} \sin(\phi) \right) + \lambda_v \left(-\frac{uv}{r} + \frac{T}{m_0 + \dot{m} \times t} \cos(\phi) \right) \right]_{t_j} = 0$$

Cette condition peut être remplacée par la condition équivalente à $t_0 = 0$:

$$\left[\lambda^T \times \lambda \right]_{t_0} = 1$$

1.6 Formulation du problème aux deux bouts

Inconnues à $t = t_0 = 0$: $\lambda(t_0)^T = [\lambda_r \ \lambda_u \ \lambda_v]_{t_0}$
Inconnue à $t = t_f$: t_f Système d'équations à résoudre:

$$g(\lambda_{r0}, \lambda_{u0}, \lambda_{v0}, t_f) = \begin{bmatrix} r(t_f) - r_f \\ u(t_f) \\ v(t_f) - \sqrt{\frac{\mu}{r_f}} \\ \left[1 + \lambda^T f \right]_{t_f} \end{bmatrix} = 0$$

2 implementation

2.1 Problem data declaration

Algorithm 1 Problem data declaration

Data:

- **Initial Conditions:**
  - $AU = 149597870690$
  - $r_0 = AU$
  - $u_0 = 0$
  - $v_0 = \sqrt{\frac{\mu_{\text{body}}}{r_0}}$
  - $m_0 = 1000$
- **Final Conditions:**
  - $r_f = 1.5 \times AU$
  - $u_f = 0$
  - $f = \sqrt{\frac{\mu_{\text{body}}}{r_f}}$
- **Power**:
  - Set T to an array containing values from 0.1 to 0.6 with a step size of 0.1.
- **Earth gravity**
  - $g_0 = 9.80665$
- **Specific motor impulse**
  - $Isp = 3000$
- **sun's gravitational constant**
  - $\mu_{body} = 1.32712440018e + 20$