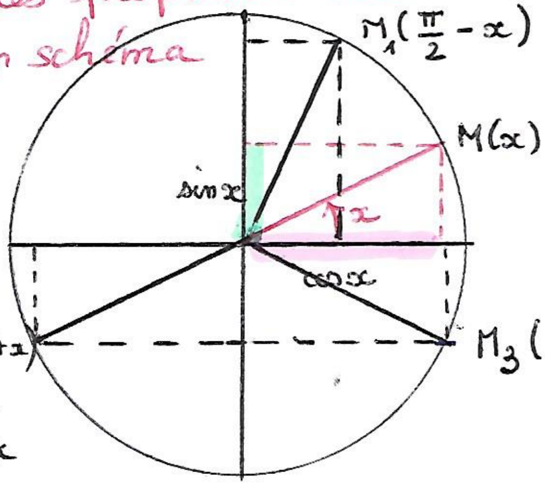
Méthode 5

1) a) $A = \cos\left(\frac{\pi}{2} - x\right) + \cos\left(2\pi + x\right) + 2\sin\left(\pi + x\right)$

On utilise les propriétés vues ci dessus, on les retrouve en faisant un schéma



$M_1\left(\frac{\pi}{2} - x\right) \longrightarrow \begin{cases} \cos\left(\frac{\pi}{2} - x\right) = \sin x \\ \sin\left(\frac{\pi}{2} - x\right) = \cos x \end{cases}$

$M_2(\pi + x)$: $\begin{cases} \cos(\pi + x) = -\cos x \\ \sin(\pi + x) = -\sin x \end{cases}$

$M_3(-x) \rightarrow \begin{cases} \cos(-x) = \cos x \\ \sin(-x) = -\sin x \end{cases}$

donc $A = \sin(x) + \cos(x) + 2(-\sin(x)) = \underline{\cos(x) - \sin(x)}$

b) $B = 3\cos(\pi + x) + 5\sin\left(\frac{\pi}{2} - x\right) + 2\sin(-x)$

$B = 3(-\cos(x)) + 5\cos(x) - 2\sin(x)$

$B = 2\cos(x) - 2\sin(x)$

2)

a) On sait que $\cos^2(x) + \sin^2(x) = 1$ ou $(\cos(x))^2 + (\sin(x))^2 = 1$

donc $\left(\cos\left(\frac{\pi}{5}\right)\right)^2 + \left(\sin\left(\frac{\pi}{5}\right)\right)^2 = 1$

$$\left(\frac{1+\sqrt{5}}{4}\right)^2 + \left(\sin\left(\frac{\pi}{5}\right)\right)^2 = 1$$

$$\left(\sin\left(\frac{\pi}{5}\right)\right)^2 = 1 - \left(\frac{1 + 2\sqrt{5} + 5}{16}\right)$$

$$\left(\sin\left(\frac{\pi}{5}\right)\right)^2 = \frac{10 - 2\sqrt{5}}{16}$$

comme $\frac{\pi}{5} \in \left[0 ; \frac{\pi}{2}\right]$ $\quad \sin\left(\frac{\pi}{5}\right) \geq 0$ donc $\sin\left(\frac{\pi}{5}\right) = \sqrt{\frac{10 - 2\sqrt{5}}{16}} = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$

b)
- $\cos\left(\frac{4\pi}{5}\right) = \cos\left(\pi - \frac{\pi}{5}\right) = -\cos\left(\frac{\pi}{5}\right) = -\frac{1+\sqrt{5}}{4}$

  $\sin\left(\frac{4\pi}{5}\right) = \sin\left(\pi - \frac{\pi}{5}\right) = \sin\left(\frac{\pi}{5}\right) = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$

- $\cos\left(-\frac{\pi}{5}\right) = \cos\left(\frac{\pi}{5}\right) = \frac{1+\sqrt{5}}{4}$

  $\sin\left(-\frac{\pi}{5}\right) = -\sin\left(\frac{\pi}{5}\right) = -\frac{\sqrt{10 - 2\sqrt{5}}}{4}$

- $\cos\left(\frac{6\pi}{5}\right) = \cos\left(\pi + \frac{\pi}{5}\right) = -\cos\frac{\pi}{5} = -\frac{1+\sqrt{5}}{4}$

  $\sin\left(\frac{6\pi}{5}\right) = \sin\left(\pi + \frac{\pi}{5}\right) = -\sin\frac{\pi}{5} = -\frac{\sqrt{10 - 2\sqrt{5}}}{4}$

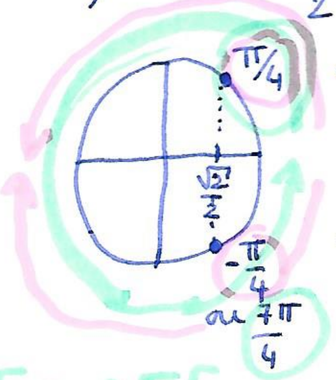
- $\cos\left(\frac{3\pi}{5}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{5}\right) = \sin\left(\frac{\pi}{5}\right) = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$

  $\sin\left(\frac{3\pi}{5}\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{5}\right) = \cos\left(\frac{\pi}{5}\right) = \frac{1+\sqrt{5}}{4}$

Méthode 6 Resolution equations trigonométriques

Pensez à faire des dessins

1) $\cos x = \frac{\sqrt{2}}{2}$



[0; 2π[

]−π; π]

- <u>Dans]−π; π]</u> → mesure principale

  Il y a 2 solutions $\mathcal{S} = \left\{ \frac{\pi}{4} ; -\frac{\pi}{4} \right\}$

- <u>Dans ℝ</u> → on donne toutes les solutions

  Il y a une infinité de solutions:

  $x = \frac{\pi}{4} + \underbrace{2k\pi}_{\text{tours complets}} \quad (k \in \mathbb{Z})$

  ou $x = -\frac{\pi}{4} + 2k\pi \quad (k \in \mathbb{Z})$

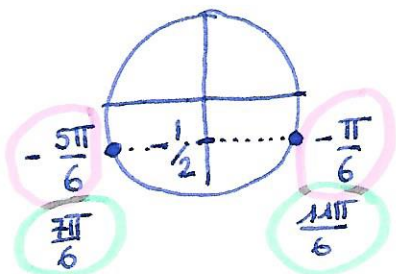
  Autre notation:
  $x \equiv \frac{\pi}{4} [2\pi]$
  ou $x \equiv -\frac{\pi}{4} [2\pi]$

- Dans [0; 2π[→ on fait un tour, valeur positive.

  Il y a 2 solutions $\mathcal{S} = \left\{ \frac{\pi}{4} ; \frac{7\pi}{4} \right\}$

2) $\sin x = -\frac{1}{2}$

<u>Dans]−π; π]</u> : $\mathcal{S} = \left\{ -\frac{5\pi}{6} ; -\frac{\pi}{6} \right\}$
2 solutions



<u>Dans ℝ</u>:
$x \equiv -\frac{5\pi}{6} [2\pi]$ ou $x \equiv -\frac{\pi}{6} [2\pi]$

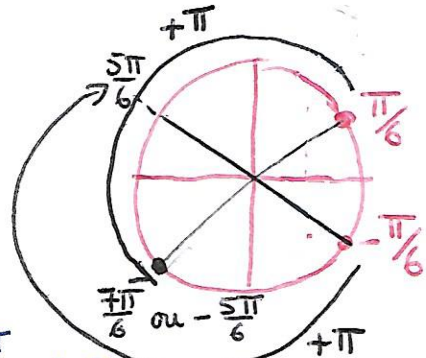
<u>Dans [0; 2π[</u> $\mathcal{S} = \left\{ \frac{7\pi}{6} ; \frac{11\pi}{6} \right\}$
2 solutions

3) $\cos x = \sin\left(\frac{3\pi}{4}\right)$

On modefie l'equation, on connait $\sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$

on a donc $\cos(x) = \frac{\sqrt{2}}{2}$ → on retrouve l'equation 1)

4) $\cos(2x) = \frac{1}{2}$ (Difficile)



On résout d'abord dans ℝ:

$2x = \frac{\pi}{3} + 2k\pi$ ou $2x = -\frac{\pi}{3} + 2k\pi$

$\Leftrightarrow x = \frac{\pi}{6} + k\pi$ $\quad x = -\frac{\pi}{6} + k\pi$.

l'on passe au $\frac{1}{2}$ tour

donc on a pour 4 solutions sur le cercle

$x = \frac{\pi}{6}$ ($k=0$), $x = \frac{\pi}{6} + \pi = \frac{7\pi}{6}$ ($k=1$), $x = -\frac{\pi}{6} + \pi = \frac{5\pi}{6}$ ($k=1$), $x = -\frac{\pi}{6}$

donc dans]−π; π] $\mathcal{S} = \left\{ \frac{\pi}{6} ; \frac{5\pi}{6} ; -\frac{5\pi}{6} ; -\frac{\pi}{6} \right\}$

dans]0; 2π] $\mathcal{S} = \left\{ \frac{\pi}{6} ; \frac{5\pi}{6} ; \frac{7\pi}{6} ; \frac{11\pi}{6} \right\}$

5) $\sin\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ dans ℝ

$x - \frac{\pi}{6} = \frac{\pi}{3} + 2k\pi$ ou $x - \frac{\pi}{6} = \frac{2\pi}{3} + 2k$

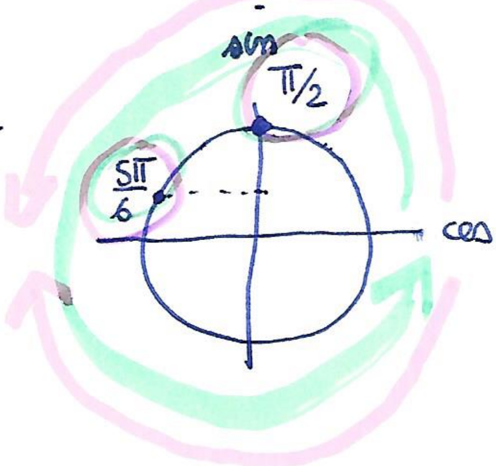
$x = \frac{\pi}{3} + \frac{\pi}{6} + 2k\pi$ ou $x = \frac{2\pi}{3} + \frac{\pi}{6} +$

$x = \frac{3\pi}{6} + 2k\pi$ ou $x = \frac{5\pi}{6} + 2$

Methode 6 (suite)

5 suite

on trouve $x = \frac{\pi}{2} + 2k\pi$ ou $x = \frac{5\pi}{6} + 2k\pi$



dans $]-\pi ; \pi]$

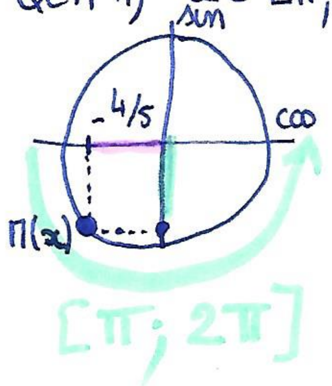
2 solutions $S = \left\{ \frac{\pi}{2} ; \frac{5\pi}{6} \right\}$

dans $[0 ; 2\pi[$

2 solutions $S = \left\{ \frac{\pi}{2} ; \frac{5\pi}{6} \right\}$

Methode 7

QCM 1) $x \in [\pi ; 2\pi]$



• si $\cos(x) = -\frac{4}{5}$ → $\boxed{\sin(x) \leqslant 0}$ → reponse 1 juste → réponse 3 fausse

• $(\cos(x))^2 + (\sin(x))^2 = 1$

$\left(-\frac{4}{5}\right)^2 + (\sin(x))^2 = 1$

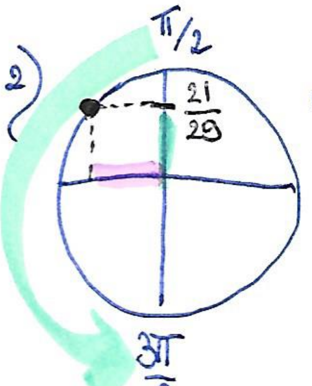
$\frac{16}{25} + (\sin(x))^2 = 1$

$(\sin(x))^2 = \frac{9}{25}$ réponse 2 fausse

donc $\cancel{\sin(x) = \frac{3}{5}}$ ou $\boxed{\sin(x) = -\frac{3}{5}}$ reponse 4 juste

rejeté car > 0

2)



$x \in \left[\frac{\pi}{2} ; \frac{3\pi}{2}\right]$

$\sin x = \frac{21}{29}$ → $\cos x \leqslant 0$ reponse 1 juste

$\cos(x) = \sqrt{1 - \sin^2(x)}$ faux reponse 2 fausse

car $\cos(x) < 0$ donc

$\cos(x) = -\sqrt{1 - \sin^2(x)}$

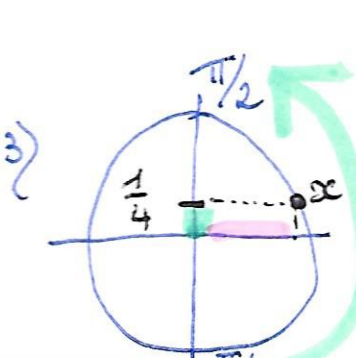
$\cos(x) = -\sqrt{1 - \left(\frac{21}{29}\right)^2} = -\sqrt{\frac{400}{841}}$

$= -\frac{20}{29}$

reponse 3 juste

reponse 4 fausse.

3)



$\cos(x) > 0$ → reponse 1 fausse

$\cos(x) = \sqrt{1 - \sin^2(x)} = \sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{4}$ → réponse 2 fausse

réponse 3 fausse

réponse 4 juste