Applied Numerical Methods - Computer Lab 1

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Part 1. Solution of ODE-systems with constant coefficients

In the electrical circuit that follows, we define the variable $\dot{q} = i$. dessin du circuit to come, je sais pas si on a vraiment besoin... The RLC circuit is described by the equation

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = E, \qquad q(0) = 0, \qquad \dot{q}(0) = 0.$$

We easily rewrite this as a first order system of linear equations. Setting $y_1 = q$ and $y_2 = \dot{q}$, this gives

$$\begin{aligned} \dot{y_1} &= y_2 \\ \dot{y_2} &= -\tfrac{R}{L}y_2 - \tfrac{1}{LC}y_1 + E \end{aligned}$$

$$\begin{pmatrix} \dot{y_1} \\ \dot{y_2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{pmatrix} + \begin{pmatrix} 0 \\ E \end{pmatrix}$$

Part 2. Stability of ODE-systems and equilibrium points

a. Stability of the solutions of an ODE-system of LCC-type

The purpose of this section is to investigate the stability of an ODE while a parameter changes continuously.

The given third-order equation is the following :

$$\begin{aligned} y''' + 3y'' + 2y' + Ky &= 0 \\ y(0) &= 1 \\ y'(0) &= 1 \\ y''(0) &= 1 \end{aligned}$$

It is possible to rewrite this as a system of ODE's, introducing new variables.

$$\mathbf{u'} = \begin{pmatrix} y' \\ y'' \\ y''' \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K & -2 & -3 \end{pmatrix} \begin{pmatrix} y \\ y' \\ y'' \end{pmatrix} = \mathbf{Au}$$

The initial conditions are rewritten by :

$$\mathbf{u(0)} = \begin{pmatrix} y(0) \\ y'(0) \\ y''(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

The solutions, computed analytically for different values of K, are given in figure 1. It is possbile to see that for the first three values of K, the maximal amplitude of the blue curve (that

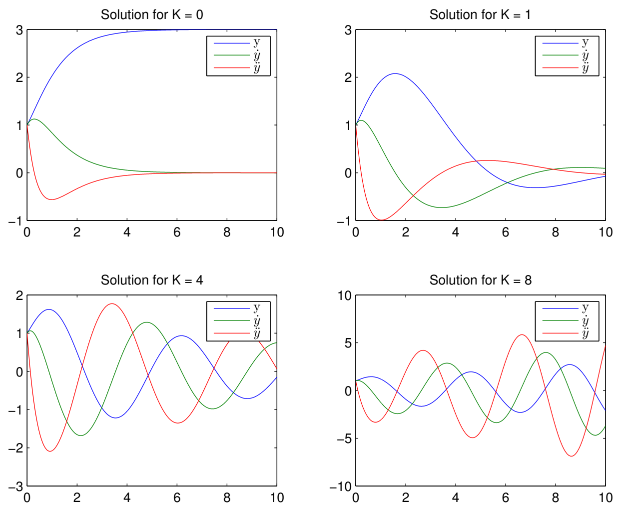

Figure 1: Solutions of the system in part 2a for various values of K

is the function y) tends to decrease. This being an LCC system, we know that a perturbation will follow the same ODE so we can conclude that the system is stable for those values of K. On the other hand, for the last value ($K = 8$), this amplitude increases.

```
function [] = LAB1_21()
%LAB1_21
K = [0 1 4 8];
k = {'0','1','4','8'};
u0 = [1 1 1]';
N = 1000;
u = [u0 zeros(3,N)];
tfinal = 10;
h = tfinal/N;
t=0:h:tfinal;

for i=1:4
    A=[0 1 0;0 0 1;-K(i) -2 -3];
    for j=2:N+1
        u(:,j) = expm(A*t(j))*u0;
    end
    subplot(2,2,i);
    string = sprintf('Solution for K = %d',K(i));
    plot(t,u);hl=legend('y','$\dot{y}$','$\ddot{y}$');title(string);
    set(hl, 'Interpreter', 'latex');
end

%root locus
sys = tf(1,[1 3 2 0]);
figure;
rlocus(sys,0:0.001:10);
end
```

b. Stability of the critical points of a nonlinear ODE-system