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Computer Lab 3, max 3 credit points
Numerical Solution of a Boundary Value Problem

In this lab boundary value ODE-problems are solved numerically and the following problem is studied:

Stationary heat conduction in 1-D

Consider a long pipe of length L with with small cylindrical cross section. In the pipe there is a fluid heated by an electric coil. The heat is spreading along the pipe and the temperature $T(z)$ at steady state is determined by the diffusion-convection ODE:

$$-\frac{d}{dz}(\kappa\frac{dT}{dz}) + v\rho C\frac{dT}{dz} = Q(z) \qquad (*)$$

where all parameters are constant: κ is the heat conduction coefficient, v is the fluid velocity in the z-direction through the pipe, ρ is the fluid density and C is the heat capacity of the fluid. The driving function $Q(z)$, modeling the electric coil, is defined as

$$Q(z) = \begin{cases} 0, & \text{if } 0 \le z < a \\ Q_0 \cdot \sin\left(\frac{z-a}{b-a}\pi\right), & \text{if } a \le z \le b \\ 0, & \text{if } b < z \le L \end{cases}$$

At $z = 0$ the fluid has the inlet temperature T_0:

$$T(0) = T_0$$

At $z = L$ heat is leaking out to the exterior, having temperature T_{out}. This assumption gives the following BC:

$$-\kappa\frac{dT}{dz}(L) = k_v(T(L) - T_{out})$$

where k_v is a constant heat convection coefficient.

Discretize this BVP with the FD method using constant stepsize. Write a Matlab program that solves the linear system of equations.

Use the following values of the parameters in the problem: $L = 10$, $a = 1$, $b = 3$, $Q_0 = 50$, $\kappa = 0.5$, $k_v = 10$, $\rho = 1$, $C = 1$, $T_{out} = 300$, $T_0 = 400$ and $v = 0, 0.1, 0.5, 1, 10$. The case $v = 0$ corresponds to no convection, only diffusion.

Discretize the z-interval $[0, L]$ with constant stepsize and use a node-numbering where $z_0 = 0$ and $z_N = L$.

Discretize the ODE and the BC. An sparse system of linear equations is obtained.

1) The structure of the algebraic system of equations, i.e. show how the discretization of ODE and BC leads to a linear system of equations.

2) Plot of the solution $T(z)$ for $v = 0$, $N = 10, 20, 40, 80$ in the same graph. Note the convergence of the curves in the graph.

3) In 2) above, $N = 40$ gives a solution that is accurate enough for our purposes. Use this discretization to solve the problem for $v = 0.1, 0.5, 1, 10$ in the same graph. When $v = 10$ spurious oscillations occur! Make another plot when $v = 10$, showing the solution for $N = 10, 20, 40$ in the same graph. The oscillations become more pronounced when h is increasing. We have a spurious oscillation problem!