SF2520 - Lab 6

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Part 1

For the model hyperbolic problem

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0 \tag{1}$$

with $a > 0$, $0 < x < 2$ and $t > 0$. We consider the initial condition $u(x, 0) = 0$ for $0 < x \leq 2$. Since $a > 0$ the solution will travel towards the right and we need a boundary condition on the left-hand side of the domain. Take the square signal

$$u(0,t) = \begin{cases} 1 & -(n+1)T/2 < t \leq -nT/2 \quad n = 0, 2, 4 \dots \\ -1 & -(n+1)T/2 < t \leq -nT/2 \quad n = 1, 3, 5 \dots \end{cases}$$

where T is the period time.

The point of interest is to compare the behaviour of three methods 1) upwind, 2) Lax-Friedrich and 3) Lax-Wendroff. lets first define the following methods.

1) Upwind FTBS The upwind method uses forward time and backward space schemes. So we get the first order in space and time

$$u_{i,k+1} = (1-\sigma)u_{i,k} + \sigma u_{i-1,k}.$$

2) Lax-Friedrich method is defined by

$$u_{i,k+1} = \frac{u_{i-1,k} + u_{i+1,k}}{2} - \frac{\sigma}{2}(u_{i+1,k} - u_{i-1,k}).$$

3) Lax-Wendroff method is defined by

$$u_{i,k+1} = u_{i,k} - \frac{\sigma}{2}(u_{i+1,k} - u_{i-1,k}) + \frac{\sigma^2}{2}(u_{i+1,k} - 2u_{i,k} + u_{i-1,k}).$$

We look at the results on the test problem for the values $\sigma = \{0.8, 1, 1.1\}$. It is clear on figure 1 that $\sigma = 1.1$ gives an unstable scheme for each method. And the "magic" $\sigma = 1$ transports the solution as it should. The interesting points to analyse are in the case $\sigma = 0.8$ where different behaviour occur. The upwind and Lax-Friedrich methods introduce a smoothing. This is an illustration of an artificial dissipationpossibilité d'ajouter l'équation modifiée. Finally, the Lax-Wendroff generates oscillations, this is to be expected for a method that is second order in time and the errors come from the dispersion in the numerical method.cool mais je ne comprend pas trop ce que ça veut dire, c'est page 16 des notes

Part 2

Daviiiid

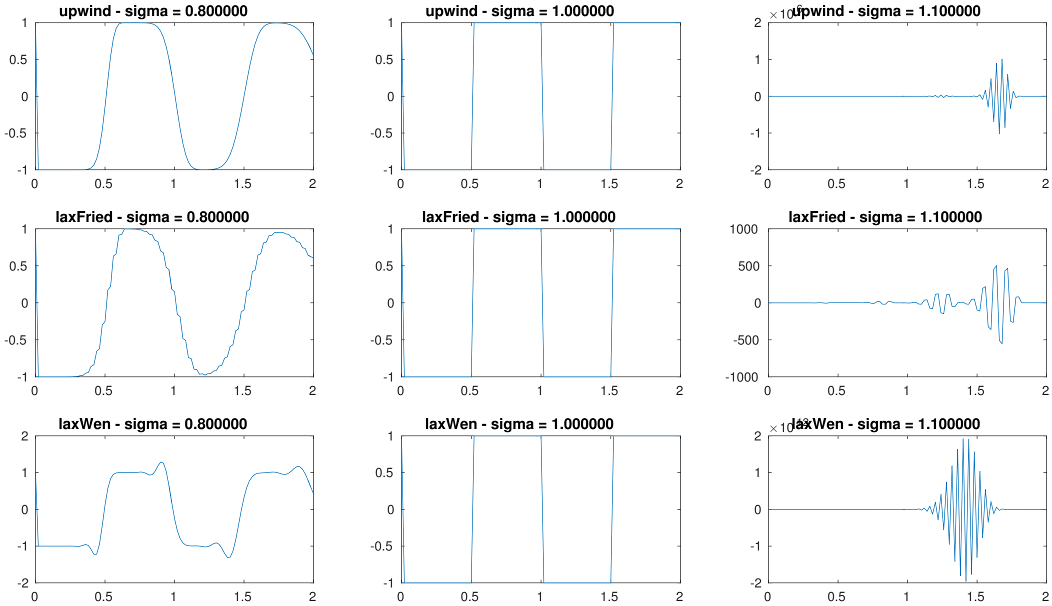

Figure 1: Results for the different methods