
SF2520 - Lab 6

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Part 1

For the model hyperbolic problem

$$\frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} = 0 \quad (1)$$

with $a > 0$, $0 < x < 2$ and $t > 0$. We consider the initial condition $u(x, 0) = 0$ for $0 < x \leq 2$. Since $a > 0$ the solution will travel towards the right and we need a boundary condition on the left-hand side of the domain. Take the square signal

$$u(0, t) = \begin{cases} 1 & -(n+1)T/2 < t \leq -nT/2 \quad n = 0, 2, 4, \dots \\ -1 & -(n+1)T/2 < t \leq -nT/2 \quad n = 1, 3, 5, \dots \end{cases}$$

where T is the period time.

The point of interest is to compare the behaviour of three methods 1) upwind, 2) Lax-Friedrich and 3) Lax-Wendroff. Let's first define the following methods.

1) Upwind FTBS The upwind method uses forward time and backward space schemes. So we get the first order in space and time

$$u_{i,k+1} = (1 - \sigma)u_{i,k} + \sigma u_{i-1,k}.$$

2) Lax-Friedrich method is defined by

$$u_{i,k+1} = \frac{u_{i-1,k} + u_{i+1,k}}{2} - \frac{\sigma}{2}(u_{i+1,k} - u_{i-1,k}).$$

3) Lax-Wendroff method is defined by

$$u_{i,k+1} = u_{i,k} - \frac{\sigma}{2}(u_{i+1,k} - u_{i-1,k}) + \frac{\sigma^2}{2}(u_{i+1,k} - 2u_{i,k} + u_{i-1,k}).$$

For the last two schemes, the value $u_{end,k+1}$ is not well defined. We use linear extrapolation $u_{end,k+1} = 2u_{end-1,k} + u_{end-2,k}$.

We look at the results on the test problem for the values $\sigma = \{0.8, 1, 1.1\}$. It is clear on figure 1 that $\sigma = 1.1$ gives an unstable scheme for each method. And the "magic" $\sigma = 1$ transports the solution as it should. The interesting points to analyse are in the case $\sigma = 0.8$ where different behaviour occur. The upwind and Lax-Friedrich methods introduce a smoothing. This is an illustration of an artificial dissipation. Finally, the Lax-Wendroff generates oscillations, this is to be expected for a method that is second order in time and the errors come from the dispersion in the numerical method.

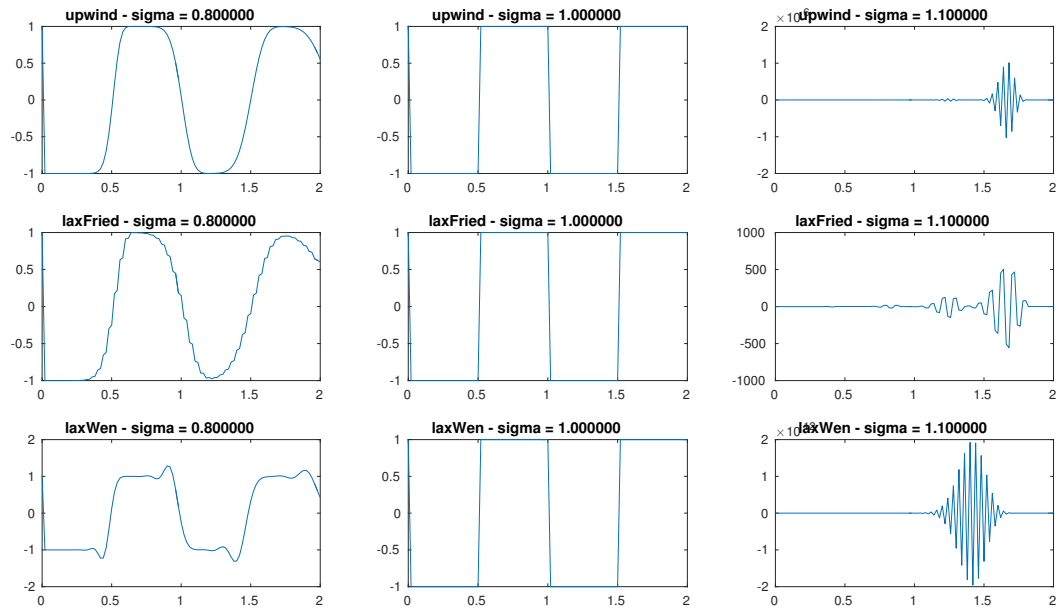


Figure 1: Results for the different methods

Part 2

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