Comparison of Hossfeld's Method and Two Modern Methods for Volume Estimation of Standing Trees

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ABSTRACT

Two modern methods, centroid sampling and the paracone model, have been shown to be accurate approaches for obtaining the volumes of trees and logs when taper functions are unavailable or local volume estimates are desired. We show that the equation for whole-tree volume using Hossfeld's method, an older method that has all but disappeared from the North American literature, is nearly identical to that for centroid sampling and the paracone model. Hossfeld's method may be slightly simpler to implement in the field, and like the modern methods, it can be used either for whole trees or for unmerchantable tops. In comparison with detailed measurements taken on 186 ponderosa pine trees from the Black Hills of South Dakota, the paracone model was most accurate for whole trees, but Hossfeld's method was slightly more accurate than centroid sampling. Hossfeld's method was substantially more accurate than either modern method for estimating the volume in tops.

Keywords: importance sampling, tree volume, taper function, dendrometry

The potential development of forest-based biomass energy has renewed interest in quantifying the resource represented by trees and parts of trees that were previously considered unmerchantable (cf. Hall 2002). Unfortunately, for many species and/or locales, either whole-tree volume equations or suitable taper equations are unavailable. Alternatively, it may be desirable to obtain localized volume estimates. As Flewelling et al. (2000) point out, a volume or taper equation is unlikely to be correct for any particular stand, and bias can be substantial (in excess of 10%). Wiant et al. (1992a, p. 333) write, "Sometimes [volume tables or functions] are used by foresters without verification of their appropriateness. This practice can lead to errors in volume estimates of 30% or more, regardless of the sophistication of the sampling system used." Likewise, Iles (2003, p. 136) writes, "I believe that there is no substitute for checking a few trees to find out how accurate the volume table or taper equation will be." Whether for obtaining inventory estimates of volume directly or for checking the results of volume equations, simple techniques for estimating tree or log volume using a minimum number of diameter measurements are an important part of a forester's tool kit.

Centroid sampling (Wood et al. 1990) has been suggested as one alternative for obtaining local volume estimates without using a species-specific volume or taper equation. Originally derived from importance sampling (Gregoire et al. 1986), centroid sampling uses a single upper-stem diameter measurement, in conjunction with log or tree length, to obtain volumes. Several studies in a variety of species have shown that centroid sampling provides accurate estimates of log and tree volumes, with very little bias (Wood and Wiant 1990, Wood et al. 1990, Wiant et al., 1991, 1992a, 1992b, 1996, 2002, Patterson et al., 1993, Yavuz 1999, Coble and Lee 2003, Fraver et al. 2007, Özçelik 2008, Özçelik et al. 2008). The paracone model (Forslund 1982) has shown similar accuracy (Wiant et al. 1991, Özçelik 2008).

Hossfeld's method for obtaining log or tree volume is a very old equation: in North America it appears only in the earliest forest mensuration texts (Graves 1906, Schenck 1906, Chapman 1921). Here, we show that it is algebraically and practically nearly identical to centroid sampling and the paracone model for whole trees or for the tops of trees above a merchantable portion. It is slightly simpler, because certain coefficients are rational numbers (i.e., easily computed fractions). In comparison with a set of detailed measurements on ponderosa pine trees from South Dakota, it competitive with the modern methods for whole-tree volume and more accurate for estimating the volume in tops.

Centroid Sampling and the Paracone Model

Gregoire et al. (1986) developed the use of importance sampling for obtaining unbiased estimates of log volume. In importance sampling, a "proxy function" (usually a simple paraboloid) is used to conduct sampling with unequal probability along the bole, concentrating diameter measurements in the portions of the bole that have greater volume. Wood et al. (1990) noted that the variability of importance sampling was lowest when a single diameter measurement was taken at the point on the stem where half the predicted volume was below the point, and half the predicted volume was above it. Wood et al. (1990) called this point the "centroid point" and developed centroid sampling as a technique for estimating volume when the sample point was purposively located at the centroid point. Although the algebra for obtaining the volume of an arbitrary log can be somewhat involved (Wiant et al. 1992b), the equations simplify when the small end of the log is set to zero. In that case,

$$V = A_c L/\sqrt{2} \approx 0.707 A_c L, \tag{1}$$

where V is volume (ft^3 or m^3), L is log length, and A_c is log cross-sectional area (ft^2 or m^2) measured at the centroid height. The

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centroid height is located at a distance $(1 - 1/\sqrt{2})L \approx 0.293L$ from the large end of the log. Practical instructions for implementing centroid sampling for whole trees or logs are given by Wiant et al. (1992a).

The paracone model was developed by Forslund (1982) on the basis of simplifying assumptions about the taper rate of whole trees and taking a diameter measurement at the center of gravity rather than the centroid of the tree stem. Although its derivation differs from that of centroid sampling, the resulting equation for whole-tree volume is strikingly similar (Wiant et al. 1991):

$$V = 0.693A_{0.3L}L. \tag{2}$$

Wiant et al. (1991) found that the paracone model gave better overall volume estimates than centroid sampling for most of the tree species in their study. Lynch et al. (1994) provided further elaboration of center-of-gravity models, to which we shall return below.

Hossfeld's Method

Johann Hossfeld was a German forester in the early 19th century, and he is credited by Fernow (1907) with several major advances in forest mensuration. Graves (1906, p. 94) presented "Hossfeldt's method" for obtaining the cubic volume of a log and returned to it on page 155 for standing trees. Schenck (1905) provided a very brief treatment of Hossfeld's method, and Chapman (1921), who regarded it as impractical, relegated it to a dismissive footnote. Thereafter, Hossfeld's method seems to have disappeared from the North American literature. Hossfeld's method, in equation form, is

$$V = \frac{3A_{1/3L} + A_L}{4} L, \tag{3}$$

where V and L are as previously defined, and $A_{1/3L}$ and A_L are cross-sectional areas measured at distances $(1/3)L$ and L, respectively, from the large end. In other words, A_L is cross-sectional area at the small end of the log. When $A_L = 0$, Hossfeld's method reduces to

$$V = 0.75A_{1/3L}L. \tag{4}$$

Comparison of Equations 1, 2, and 4 shows that they are nearly identical, differing only in multipliers (ranging from 0.693 to 0.75) and the location of the cross-sectional area measurement (from $0.293L$ to $0.333L$). By analogy to the use of "centroid height" for $0.293L$, we will call $0.333L$ the "Hossfeld height." Given these very small differences, it is natural to ask whether the two equations yield volume estimates that would be appreciably different in practice.

Evaluation

Methods

To evaluate the differences in volume estimates, we used data from a previously published study on form and taper of ponderosa pine trees in the Black Hills of South Dakota (Williams et al. 1996). We used data from 186 felled trees that were measured on timber sales in 1993. For each tree, dbh and total height were recorded, and diameter inside bark (dib) was measured every 4 ft on the bole. For this study, we used linear interpolation to obtain dib every 0.1 ft for each tree and then summed the volumes of 0.1 ft thick "cookies" to obtain a "true" volume for each tree. Characteristics of the sample logs are summarized in Table 1.

Table 1. Summary of characteristics for 186 ponderosa pine trees from the Black Hills of South Dakota.

	Minimum	Median	Maximum
dbh (in.)	8.2	11.5	24.5
Total height (ft)	36	57	145
Merchantable height (to 8-in. top)	0	25	100
Total cubic volume (inside bark, ft^3)	5.0	16.8	157.1
Topwood volume (inside bark, ft^3)	1.4	4.5	10.6

We used linear interpolation to obtain dib and cross-sectional area for each tree at the centroid height, center of gravity, and Hossfeld height, assuming a negligible stump height consistent with modern mechanized harvesting. These values were used to compute whole-tree V using Equations 1, 2, and 4. Relative error was computed for each tree as

$$\mathrm{RE} = 100\,\frac{V - V_{\mathrm{true}}}{V_{\mathrm{true}}}$$

for each method, and relative bias was computed as the mean relative error. Statistical significance of bias and of differences in bias between the two methods was assessed using paired t tests. We also compared the methods in terms of their root mean squared error (RMSE) and mean absolute deviation (MAD). Finally, to explore whether errors depended in a more complex fashion on V_{true}, we fit a pair of regression equations,

$$V = \beta_1 V_{\mathrm{true}} + \varepsilon$$

$$V = \beta_1 V_{\mathrm{true}} + \beta_2 V_{\mathrm{true}}^2 + \varepsilon,$$

where the error term ε has zero mean and a variance that is a power function of V_{true} (to account for heteroscedasticity). Models were fit by maximum likelihood using the gls routine of the R statistical package (Pinheiro et al. 2009, R Development Core Team 2009) and the contribution of the parameter β_2 (which would indicate a change in bias with changing tree size) was evaluated using both t tests and changes in Akaike Information Criterion.

To explore how the two equations would perform for estimating upper-stem volumes, we also used the interpolated dib values to find the height at which dib fell below 8 in. Previous work has suggested 8 in. as a merchantability standard for wood-based biomass energy (Biomass Energy Resource Center 2007). We used this height as merchantable height and did not force merchantable height to equal a round number of standard whole or half logs for this study. For nine trees that had dbh $\leq$8.7 in., the smallest recorded dib was $<$8 in., and these trees were assigned zero merchantable height. We then simulated centroid sampling, the paracone model, and Hossfeld's method treating the merchantable stem as an exceptionally tall stump. In other words, taking H as total tree height and H_{m} as merchantable height, $L = H - H_{\mathrm{m}}$ was used in Equations 1, 2, and 4, with the centroid height, center of gravity, and Hossfeld's height evaluated at $H_{\mathrm{m}} + 0.293L$, $H_{\mathrm{m}} + 0.3L$, and $H_{\mathrm{m}} + 0.333L$, respectively.

Results

The results of the whole-tree volume calculations for individual trees are shown in Figure 1. Actual trees do differ somewhat from the simplified forms underlying centroid sampling, the paracone model, and Hossfeld's method, and the differences in the performance of the three approaches reflects that reality. A statistical summary of the

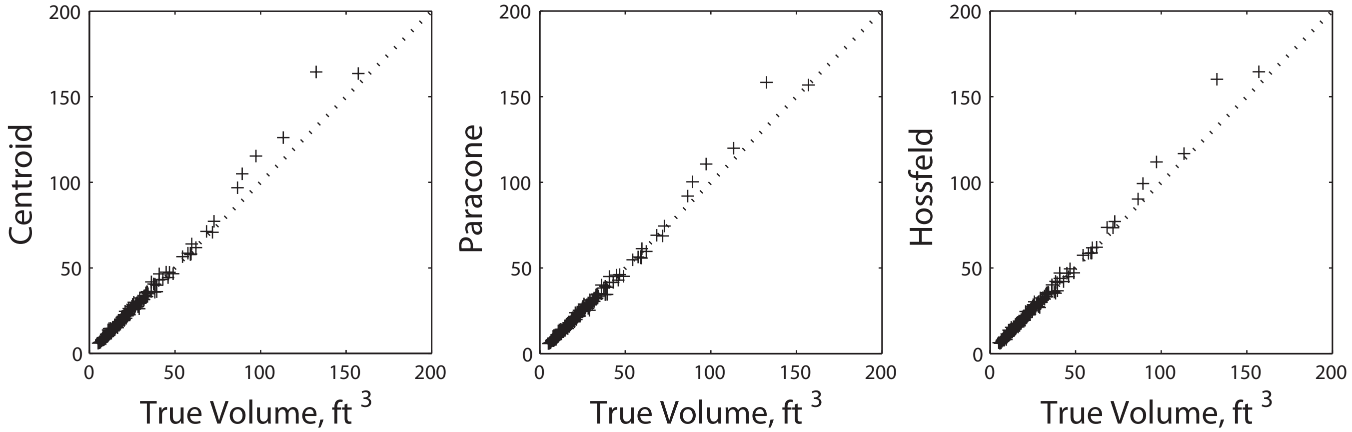

Figure 1. Estimates of whole-tree cubic volume for 186 ponderosa pine trees using centroid sampling, the paracone model, and Hossfeld's method.

Table 2. Statistical summary of results for whole-tree cubic volume for centroid sampling, the paracone model, and Hossfeld's method.

	Centroid sampling	Paracone model	Hossfeld's method
Relative bias (%)	4.57	0.84	4.42
Standard error of relative bias	0.48	0.46	0.45
t	9.49	1.83	9.82
P	<0.0001	0.0692	<0.0001
RMSE (ft^3)	3.54	2.69	2.92
MAD (ft^3)	1.58	1.23	1.41

RMSE, root mean squared error; MAD, mean absolute deviation.

results is shown in Table 2. For whole-tree volume, the paracone model was the best performer, with lower relative bias than either centroid sampling or Hossfeld's method ($t = 81.7$ and 19.4, respectively; $P < 0.0001$ in both cases). The minor difference in relative bias between centroid sampling and Hossfeld's method was not statistically significant ($t = 0.6441$, $P = 0.52$). The paracone model was also the best in terms of RMSE and MAD, followed by Hossfeld's and then centroid sampling. Although the plots show clear evidence of heteroscedasticity, regression modeling did not support any departure from a uniform ratio with V_{true} for any of the three volume estimates ($P > 0.05$ for all t tests related to β_2, and models omitting β_2 also showed consistently lower AIC). The data do not support a change in bias with changing tree size.

The situation for topwood was quite different. Calculations of topwood volume for individual trees are shown in Figure 2, and statistical results are summarized in Table 3. The greater difference in performance between methods for topwood than for whole trees reflects the greater variability in form and taper between tops. In terms of relative bias, Hossfeld's method was the clear winner, followed by centroid sampling and then the paracone model; differences between all methods were statistically significant ($P < 0.0001$ for all comparisons). The ordering was not the same using RMSE as a criterion: centroid sampling edges out Hossfeld's method, with the paracone model again clearly the worst performer. However, MAD also shows Hossfeld as the best, followed by centroid and the paracone model. Once again, regression modeling failed to support any departure from a simple ratio with V_{true} for any of the three estimates.

Discussion and Conclusions

All three of these methods showed similar results (at least in comparison with the kinds of errors suggested by Wiant et al. 1992a and Flewelling et al. 2000), which should be unsurprising given their close algebraic resemblance. For topwood, the differences among methods might be great enough to have practical importance, highlighting the challenge of obtaining accurate volumes for that component. The bias of the centroid and paracone models reported here is similar in magnitude to that reported in other studies. For example, Wiant et al. (1991) report bias from −4.6% to +10.2% for total volume of several species using the centroid method and from −7.0% to +7.4% using the paracone method. One attractive feature of Hossfeld's method is that locating the measurement height requires merely a division by 3, once the height of the tree above the stump (or above merchantable height) is known. Of course, this may be a trivial advantage as weather-resistant handheld computers become more prevalent in timber cruising; the instructions given by Wiant et al. (1992a) for centroid sampling using a programmable calculator could probably be implemented on many foresters' cell phones. However, the simplicity of Hossfeld's method remains appealing, and its accuracy for estimating topwood volume in this study suggests that it might have broader applicability.

All three methods do require measuring an upper stem diameter. Upper stem diameter measurement is covered in current North American undergraduate mensuration texts (Avery and Burkhart 2002, p. 151–152; Husch et al. 2003, p. 93–94), as it has been to varying degrees by texts going back to Schenck (1905). Luckily technology has outpaced what was available to Schenck (1905), and a range of instruments, differing widely in their expense and accuracy, can be used for the task (Grosenbaugh 1963). Husch et al. (2003) point out that the most common of these is the relascope, but even the lowly wedge prism can be used (Beers and Miller 1973, p. E-8). Where inside bark diameters and volumes are desired, a variety of techniques can be used (cf. Husch et al. 2003, p. 132–136). Whether such measurements should be taken at all, how many trees should be selected and according to what subsampling protocol, and what dendrometer should be used are complex questions that depend on the needs, context, and resources available for a particular inventory.

The astute reader may object at this point that Hossfeld's method (like centroid sampling and the paracone model) fails to use one piece of information that is already known: dbh (for whole trees) or the diameter at the top of the merchantable bole (for topwood). For

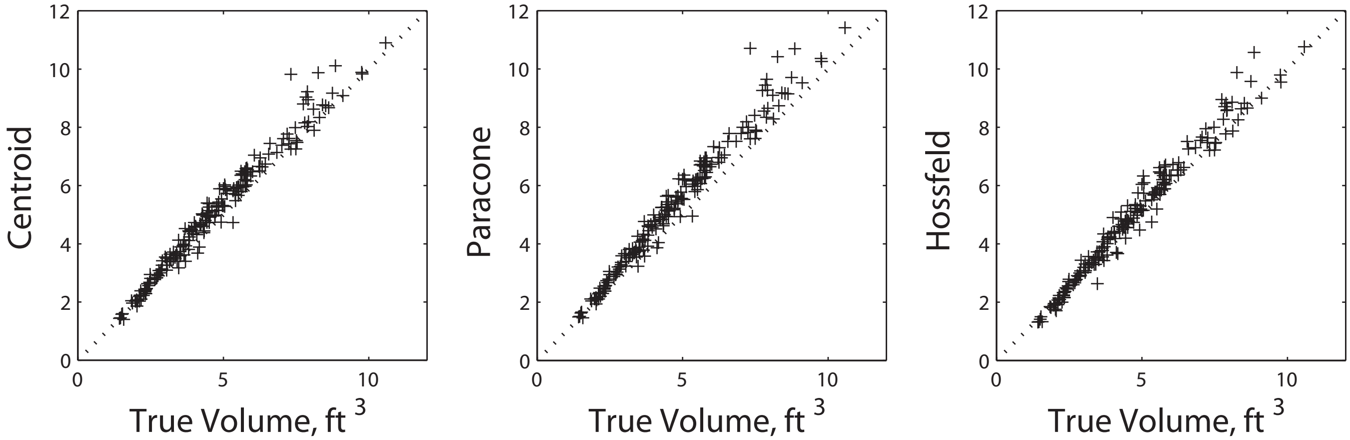

Figure 2. Estimates of topwood volume above an 8-in. merchantability limit for 186 ponderosa pine trees using centroid sampling, the paracone model, and Hossfeld's method.

Table 3. Statistical summary of results for topwood cubic volume for centroid sampling, the paracone model, and Hossfeld's method.

	Centroid sampling	Paracone model	Hossfeld's method
Relative bias (%)	7.51	12.36	4.45
Standard error of relative bias	0.51	0.55	0.65
t	14.87	22.52	6.82
P	<0.0001	<0.0001	<0.0001
RMSE (ft^3)	0.52	0.75	0.57
MAD (ft^3)	0.40	0.61	0.35

RMSE, root mean squared error; MAD, mean absolute deviation.

example, one might wish to use the equations of Wiant et al. (1992b), which use the cross-sectional areas at the large end of the log (A_0) and the small end (A_L) as well. Lynch et al. (1994) rewrite the original equations of Wiant et al. (1992b) so that the distance m to a cross-sectional measurement A_m taken somewhere in the middle of the log is measured from the large end. Using their formulation, the volume of a log is

$$V = A_0L + \tfrac{1}{2}b_1L + \tfrac{1}{3}b_2L^2, \tag{5}$$

where

$$b_1 = (A_0 - A_L - b_2L^2)/L, \tag{6}$$

$$b_2 = \frac{m(A_L - A_0) + L(A_0 - A_m)}{Lm(L - m)}. \tag{7}$$

However, if we substitute $m = L/3$ into Equations 5, 6, and 7, the equation collapses to

$$V = \frac{3A_{1/3L} + A_L}{4} L, \tag{8}$$

which is identical to Hossfeld's method (Equation 3); further substitution of $A_L = 0$ results in Equation 4, the whole-tree equation used in this study.

"Center of gravity" (as opposed to center of volume, or centroid) methods for estimating volume (Lynch et al. 1994) require measuring A_m at

$$m = \frac{L}{3}\left(\frac{1 + 2q^2}{1 + q^2}\right), \tag{9}$$

where q is the ratio of log diameter at the small end to log diameter at the large end. However, in the case of whole trees or tops, where diameter at the small end is zero, Equation 9 simply reduces to $m = L/3$, which forces the equations of Wiant et al. (1992b) to converge to Hossfeld's method. Center of gravity methods do not force such a convergence when the small-end diameter is not zero (as in, for example, the work of Özçelik et al. 2008). However, if the simplicity of Hossfeld's method is insufficient to persuade adoption of $m = L/3$, the center of gravity rationale may provide an additional reason.

Hossfeld's equation has appeared occasionally in the European literature; for example, Hossfeld's method appears in a list of log volume equations in the recent text by van Laar and Akça (2007), and Yavuz (1999) reports success estimating log volume using Hossfeld's method. However, its almost complete absence from the North American literature may be an oversight that is well worth remedying.

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