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U. S. DEPARTMENT OF AGRICULTURE,

DIVISION OF FORESTRY.

W. G. Brierley.

MEASURING THE FOREST CROP.

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of the graduated rule. Calipers from 4 to 5 feet long will answer in most cases.*

Since trees are rarely cylindrical, being often larger in one direction than in the other, it is advisable to make two measurements and take the average, or else take care to measure the estimated average diameter. Instead of measuring the diameter, the circumference may be measured by a tape and the diameter determined by dividing it by 3.14, which is the ratio of the circumference to the diameter.

MEASUREMENT OF VOLUME.

In determining the volume of a standing tree the stem or bole only is considered; the cubic contents of the branches may be estimated by themselves. It is rather difficult to determine the volume of a standing tree because geometrical forms which exactly correspond to the shape of a stem are not known. Moreover, the shapes of trunks differ with age, with species, and with the soil and forest conditions under which they grow; hence we can obtain the volume only approximately by comparing it to the mathematical form which it resembles most nearly. The form of a stem of a tree is neither a cone nor a cylinder, but resembles most closely the form known as a paraboloid. The volume of a paraboloid equals the product of its base by one-half of its height. The base of the tree is taken at a distance from the ground, usually breast-high, where the irregularities of the trunk caused by the root swellings terminate. Here the tree is calipered, and the area for the corresponding diameter (found in the area table, p. 37) is multiplied by one-half of the height of the tree.

Example: Let the height of the tree be 90 feet, the diameter, breast-high, 21 inches. The area corresponding to a circle of 21 inches diameter is 2.40 square feet. The volume of the tree then equals $\frac{2.40 \times 90}{2} = 108$ cubic feet.

Another method, devised by a German forester, Mr. Pressler, may be recommended for determining the volume of a standing tree: Find a place along the stem (fig. 6) where its diameter (d) is exactly one-

*The calipers should be so constructed that the arms work strictly parallel to each other and at right angles to the rule; it should, therefore, be made of wood which is not easily affected by moisture. Air-dry pear wood may be recommended as a material least subject to shrinkage. Swelling and shrinking of the wood makes the shifting of the arm either difficult or too easy, often throwing the arm out of the perpendicular, thus destroying the required parallelism between the arms. To avoid this various constructions of calipers have been adopted. The calipers of Gustav Heyer, a section of which is given in fig. 5, may be recommended. A represents the section of the movable arm; R is the cross section of the rule; S a spring fastened at A pressing on the rule and pushing it down; w is the cross section of a wedge made of brass and fastened to a screw which can be moved by the key K. By moving the wedge backward and forward the rule can be tightened or released, thus enabling the observer to regulate the shifting of the movable arm without throwing it out of the perpendicular.

half of that at breast height (D); this point is called the guide point. This point can be determined by estimate after some practice or else by use of a simple instrument (fig. 7) consisting of three hollow cylinders (A, B, and C), which fit one into the other. The instrument then can be lengthened and shortened in the same way as an ordinary telescope. The cylinders may be made of stiff manila or other similar paper. Into the outer cylinder (A) two pins (k and l) are thrust 1 inch from the end; they can be moved in and out, permitting a change of distance between their heads. Cylinders A and B are of the same length, 13 inches each, while that of C is 2 inches long. The end of cylinder C is closed by a paper cover, in the center of which a hole (y), of one-fourth inch in diameter, is made as an eyepiece. Looking through the eyepiece (y), arrange the heads of the pins so that the distance between them coincides exactly with the diameter of the tree at breast height. Without changing the distance between the heads of the pins, the observer draws out the cylinders so as to double the former length, allowing for that purpose the two inside cylinders to project into each other 1 inch; then range the telescope up the trunk until a point is found where the diameter of the tree again corresponds with the distance between the heads of the pins. At this point the diameter of the tree is one-half of that at breast height. To obtain the volume of the tree, estimate or measure the height of the guide point, add 2 feet, and multiply this sum by two-thirds of the area corresponding to the diameter (D) measured at breast height.

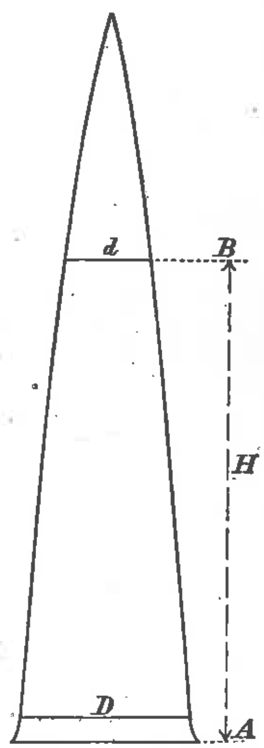

FIG. 6.—Pressler's method of determining the volume of a standing tree.

Example: A tree of 26 inches in diameter at breast height is 13

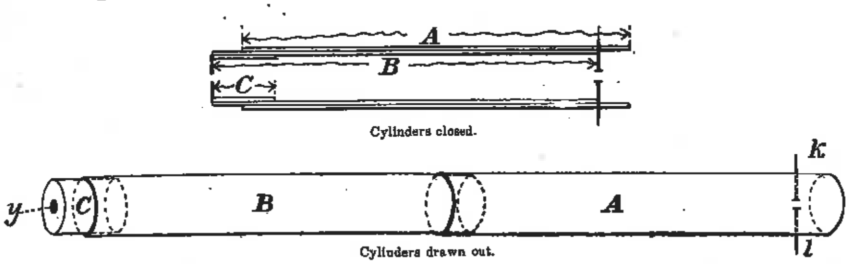

FIG. 7.—Instrument used for determining the guide point.

inches in diameter at a height of 60 feet from the ground—that is to say, the height of the tree to the guide point equals 60 feet. Adding 2 to 60 and multiplying by two-thirds of 3.69 (3.69 square feet represents the area of a circle with a diameter of 26 inches), we find the volume of the tree to be 152.5 cubic feet. The merit of this method lies in its being equally applicable to trees of various geometrical forms; it is correct for trees of parabolic and conical forms; for trees representing the form of a cone with a concave surface the difference is only 1.4 per cent.

MEASUREMENT OF VOLUME OF A STANDING TREE BY EMPLOYING THE FACTOR OF SHAPE.

The trunks of trees, as has been mentioned, differ in shape. The shape of the trunk of a cypress, a spruce, or a fir is totally different from that of a pine, hemlock, or oak. The cypress, spruce, and fir, tapering rapidly toward the top of the tree, form stems resembling either a cone, as in the spruce and fir, or a neloid or conical shape with a concave surface as in the cypress. The pine, the hemlock, and most of the hard-wood trees, tapering more gradually toward the top, form stems of a conical shape with a convex surface. An oak or a tulip tree, on the other hand, may nearly approach the shape of a cylinder. As we have stated before, trees never attain a mathematical form, but only approximate more or less closely one or the other form.

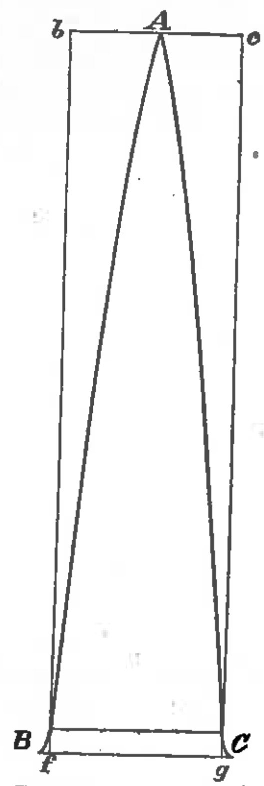

FIG. 8.—Determining the factor of shape.

The European foresters noticed long ago that there exists a relation between the actual volume of a tree and that of a regular geometrical body of corresponding dimensions. From actual calculation they learned further that this relation, varying with the kinds of trees, their dimensions, and conditions of growth, seems to be strikingly uniform. In Germany, for instance, there were measured more than forty thousand individual trees of various species and, all of them being felled, the forester was able to determine their volume in an accurate way. The actual volume of each individual tree thus obtained was compared with that of a cylinder of the same height and of the diameter at breast height. This comparison proved that the actual volume of the tree when divided by that of the cylinder of the corresponding dimensions gives a quotient which is constant for trees of the same species, approximately the same dimensions, and grown under the same forest conditions.

This quotient showing the taper of the tree, or the relation between the volume of a tree and of a cylinder of the same height and diameter breast high, is called the *factor of shape* or *form factor;* it is usually expressed in decimals and represents arithmetically the form of the stems.

For instance, if we take a tree of 22 inches diameter and 82 feet in height (fig. 8), whose volume by careful measurement we have found to be 93.1 cubic feet, we determine its form arithmetically or its factor of shape by dividing the volume of the tree by the volume of a cylinder of the same dimensions, which is 216.5 cubic feet. The factor of shape is, therefore, $\frac{93.1}{216.5}=0.43$. That means that the volume of the tree is forty-three hundredths of the volume of a cylinder of the same diameter and height. Applying this method when factors of shape have been determined by a number of previous measurements, the diameter and height of the tree are measured, the volume of the corresponding cylinder found, and that volume multiplied by the factor of shape in order to obtain the cubic contents of the tree. This method gives more accurate results than those obtained from calculations of geometrical forms which the stems of the trees are supposed to represent. The factors of shape of a species may be determined from a number of accurate measurements of the volume of felled trees.

Below we give the factors of shape for white pine when situated in a moderately dense forest. They are based upon 722 individual trees, which, being felled, were measured and the results collated in the Division of Forestry, with a view of determining the rate of growth of the species:

Diameter at breast height.	Corresponding factors of shape.	Diameter at breast height.	Corresponding factors of shape.	Diameter at breast height.	Corresponding factors of shape.	Diameter at breast height.	Corresponding factors of shape.
Inches.		*Inches.*		*Inches.*		*Inches.*	
6	0.51	17	0.46	28	0.42	39	
7	0.50	18	0.45	29		40	
8	0.50	19	0.44	30		41	
9	0.49	20	0.44	31		42	0.40
10	0.49	21	0.43	32		43	
11	0.48	22	0.43	33	0.41	44	
12	0.48	23	0.42	34		45	
13	0.48	24	0.42	35		46	0.39
14	0.47	25	0.42	36			
15	0.47	26	0.42	37	0.40		
16	0.46	27	0.42	38			

It is seen that for a pine from 29 to 36 inches in diameter the factor of shape is 0.41. Suppose we are to determine the volume of a standing white pine of 31 inches in diameter, breast high, and 130 feet in height. The volume of a cylinder of 31 inches in diameter and 130 feet high is equal to 681.4 cubic feet. Multiplying 681.4 by the factor of shape (0.41) we determine the volume of the tree to be 279.4 cubic feet.

MEASUREMENT OF FELLED TREES.

HEIGHT AND DIAMETER MEASURING.

The height of a felled tree is measured either by a tape (a steel tape measure being most accurate) or by a measuring pole from 4 to 8 feet long. The diameter of a felled tree at any given place is measured by

FOREST MENSURATION

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for example, diameter (breast high), 20 inches; contents 600 feet board measure.

4. The contents of a tree in feet Doyle approximate, assuming that the bole is cut into 16 foot logs, and that the tree tapers 2 inches per log:

$$N \times D\,(D - 12)$$

wherein N represents the number of logs obtainable; D the diameter of the butt log without bark at breast height.

5. The cordwood contained in a sound bole is:

$$\frac{D^2}{1000} \times C$$

wherein C amounts to:

1.5 in the case of trees 8" through;
2.0 in the case of trees 16" through;
2.5 in the case of trees 24" through.

PARAGRAPH XXVIII.

SCIENTIFIC METHODS OF ASCERTAINING THE CUBIC CONTENTS OF STANDING TREES BY MERE MEASUREMENT.

The cubic volume of the bole, on the basis of diameter measurement and height measurement, in the case of a standing tree, may (with the help of climbing iron, ladders, camera or instruments constructed for the purpose) be figured out:

1. According to the formulas of Hossfeldt, Riecke and Simony. In this case, the upper diameters must be measured indirectly.

2. According to Huber's and Smalian's formulas, the diameters of equal sections of the trees being indirectly measured.

3. According to Pressler's formula, which is, for the volume of the bole lying between chest height and top bud, 2/3 of sectional area "S" at chest height times "rectified" height of bole. The rectified height "r" is the distance of chest height from that point of the tree bole which has ½ of the chest height diameter (from the "guide point"). The equation 2/3 r x S holds good for paraboloid, cone and, at a slight mistake, for the neilloid.

The volume of that part of the tree bole which lies below chest height is ascertained (as a cylinder) as being equal to sectional area chest high times 4.5.

REMARK: 4.3' is the chest height usually recognized by the authors; Pinchot adopts 4.5'.

The Pressler formula does not hold good for truncated boles.

PARAGRAPH XXIX.

FORM FACTOR METHOD.

The form factor or form figure method relies on the measurement of the sectional area—usually the one at breast height,—the measurement or the estimation of the total height and the estimation of the form figure.

The form factor is a fraction expressing the relation between the actual contents of a tree, in any unit, and the ideal contents which a tree would have if it were carrying its girth (like a cylinder) up to the top bud undiminished.

The form factor may be given in reference to the volume of the entire tree, inclusive of branches in cubic feet; or in reference to the volume of the bole only; or in reference to the merchantable part of the bole; in the latter case either in feet board measure or in standards or in cords.

HISTORIC REMARKS: Some of the older authors on mensuration saw in the cone and not in the cylinder the ideal form of the tree, basing their form factors on the ideal volume $\frac{s \times h}{3}$.

PARAGRAPH XXX.

KINDS OF FORM FACTORS MATHEMATICALLY.

Scientifically we distinguish between:

1. The absolute form factors which have reference only to the volume standing above chest height. They can be readily ascertained with the help of Pressler's formula. Generally speaking, V equals Sx H x F. After Pressler, V equals S x 2/3 x r; thus $\frac{\frac{2}{3} r}{H}$ equals F.

For the cone the absolute form factor is one-third; for the neilloid one-fourth; for the paraboloid one-half, whatever the height of the tree may be. Hans Rienicker, the author of these form factors, finds for trees up to 50 years old a form figure of 35% to 43% (in regular, dense German woods); in trees 50 to 100 years old, F increases up to 50%; thereafter occurs a slight decrease below 50%.

2. The normal form factors which were recommended by Smalian, Pressler and other old-time authors. They have reference to the entire volume and necessitate the measurement of the diameter at a given fraction (usually 1/20) of the total height of the tree. Frequently, in case of tall trees, the point of measurement cannot be reached from the ground. The bole form factor for diameters measured at 1/20 of the height is: For a paraboloid, 0.526; for a cone, 0.369; for a neilloid, 0.292. These form factors, like the absolute form factors, are independent of the height.

3. The so-called "common form factors" which do not express, as a matter of fact, the form of the tree, since they do not bear any direct ratio to the degree of the tree curve. They should be termed, more

properly, "reducing factors." These form factors alone are nowadays practically used. They are based on diameter measurements, chest high, and have reference not merely to the bole of the tree, but as well to any parts of the bole, to root and branch wood, to saw logs, etc. These form factors depend entirely on the height. If, for instance, a paraboloid is one rod high, the form factor is 0.673; and if it is 8 rods high, the form factor is 0.517.

PARAGRAPH XXXI.

KINDS OF COMMON FORM FACTORS IN EUROPEAN PRACTICE.

The following kinds of form factors may be distinguished:

1. Tree form factors. The tree is considered as bole plus branches.

2. Timber form factors. The term timber, in Europe, includes all parts of the tree having over 3 inches diameter at the small end.

3. Bole form factors. Bole is the central stem from soil to top bud. For America, form factors would be of great value ascertained by exact measurements and arranged according to diameter, height and smallest log diameter used.

Tables of form factors may be constructed, for instance, for shortleaf pine, on the basis of Olmsted's working plan, pages 17-33.

PINUS ECHINATA.

Diameter.	Merchantable length of bole.	Cubic feet Ideal cylinder.	Form fig.	Contents b. m. Doyle.
16″	36′	50.3	3.6	180′
18″	47′	83.1	3.6	300′
20″	51′	112.1	4.0	440′
22″	56′	147.8	4.0	600′
24″	59′	185.3	4.2	780′
26″	61′	224.9	4.4	980′
28″	62′	263.1	4.5	1190′
30″	62′ 6″	306.7	4.6	1420′
32″	63′	351.8	4.7	1680′
34″	63′ 6″	400.3	4.8	1930′
36″	64′	457.3	4.9	2200′

The influence of age, soil, density of stand, height, diameter and species on the various form factors, with cubic measure as a basis, has not been fully ascertained.

For the tree form factor, the most important influence, in the case of trees less than 150 years old and raised in a close stand, seems to be that of the height of the tree; with increasing height the tree form factor decreases—*e. g.*, for Yellow Pine:

One pole high	.93
Two poles high	.65
Four poles high	.53
Six poles high	.49

FOREST MENSURATION

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80. Estimate of the Contents of Standing Trees by Volume Tables and Form Factors.—Volume tables are used by foresters in this country more extensively than any other method of estimating the contents of standing trees. In Europe the method of form factors, as well as that of volume tables, is used. These methods are described in the next chapters.

81. Rough Method of Estimating the Cubic Contents of Standing Trees.—The cubic contents of the stem of a tree may roughly be obtained from the measured diameter at breast-height by the formula

$$V=\frac{BH}{2},$$

in which V is the volume, B the area at breast-height, and H the height. By reference to page 88, it will be seen that this is formula No. 3 for cubing a paraboloid.

82. Hossfeldt's Method.—On page 94 it was shown that a log may be cubed by the formula,

$$V=(3B_{\frac{1}{3}}+b)\frac{h}{4},$$

in which V is the volume, $B_{\frac{1}{3}}$ the sectional area at one-third the distance from the butt, b the sectional area at the top, and h the length of the log. In case of an entire stem b is 0, and the formula becomes

$$V=\tfrac{3}{4}B_{\frac{1}{3}}\times h.$$

To determine the cubic contents of a standing tree, the length of the stem above the probable stump is measured with a height measure, and the diameter at $\frac{1}{3}$ this length is estimated or is measured with a dendrometer. These measurements furnish data for the application of the Hossfeldt formula.

83. Pressler's Method.—In 1855, G. Pressler, a professor in in the Forest School in Tharandt, devised the following formula for cubing a standing tree:

$$V=\frac{2}{3}B\left(H+\frac{M}{2}\right),$$

in which V is the volume of the tree, B is the sectional area measured just above the butt swelling, H is the distance from the stump to the point on the stem where the diameter is exactly one-half that measured at the butt, and M is the distance from the stump to the point where B is measured. Ordinarily B is taken at breast-height. The stem of a tree is cubed as two sections, (1) the portion above the point where the diameter is taken, considered as a paraboloid or a cone; (2) the portion between the stump and the point of diameter measurement, considered as a cylinder with a diameter equal to that at the upper end of the section. The stump and branches are disregarded. The main part of the stem is cubed by the formula,

$$V=\tfrac{2}{3}B\times h,$$

in which h is the distance from B to the one-half diameter-point. This holds good for both the paraboloid and the cone, as may be seen in the following demonstration:

In a paraboloid the point at which the diameter is $\frac{1}{2}$ that at the base, is $\frac{3}{4}$ the altitude. If this distance is h, the total altitude H, the basal area B, then

$$h=\tfrac{3}{4}H, \quad \text{and} \quad H=\tfrac{4}{3}h$$

Substituting in the formula

$$V=\frac{BH}{2}, \quad \text{then} \quad V=\frac{B}{2}\times\frac{4}{3}h=\frac{2}{3}Bh.$$

The same process of reasoning will show the formula correct also for the cone.

The lower part of the tree is cubed as a cylinder by the formula,

$$V=B\ M.$$

The volume of the whole stem is then

$$V=\tfrac{2}{3}Bh+BM$$

$$=\frac{2}{3}B\left(H+\frac{M}{2}\right).$$

A dendrometer may be used to determine the point where the diameter is one-half that at the base. Pressler devised a special instrument for this purpose, consisting of a small pasteboard telescope with an eyepiece at one end and with two pins or screws at right angles to the axis of the instrument at the other end. In use the telescope is first closed and the tree is sighted where the base diameter is to be taken. The pins or screws are adjusted so that the stem appears to occupy the space between the points. The instrument is then drawn out to twice its former length, and sighted up and down the stem to find the point exactly fitting between the pins. The diameter of the tree at this point is one-half that at the point first measured

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8 – Volume of Individual Trees: Direct Method

This is very easy in theory, but rather laborious and often difficult in practice. The tree is imagined to consist of logs of e.g. 4 m length, and their mid-diameters are found at heights of 2 m, 6 m, 10 m, etc., taking these heights above the probable level of felling, rather than from ground level. The remaining top section above the last piece of 4 m has to be measured separately, for length and mid-diameter. Timber height may be taken at a diameter of 10 cm or in accordance with current practice. Research measurements are taken to 7 cm.

9 – Form Factors and Form-heights: Pressler's Formula

The following are more useful methods of getting at volumes, both of individual trees and of stands. It should be said in advance that the methods are remarkably simple. The explanations which lead up to them may seem hard to follow, although if they are taken step by step they are easy enough.

Recall that:-

$$v = fhg$$

where:-

v = volume

h = total height

f = form factor

g = basal area

and also fh = "form-height", which may be considered as a single factor. Then also:-

$$fh = v/g$$

Professor Bitterlich has revived an old and little used formula, <u>Pressler's formula</u>, which states that:-

$$v = g \times 2/3\ h_1$$

where h_1 is the <u>height at which the diameter is half the diameter at breast height</u>.

The mathematical basis of Pressler's formula is not dealt with here. It is enough to say that it can easily be shown to give a theoretically very sound approximation to the <u>total</u> volume of a tree, i.e. it tends to be a little higher than the volume of saleable timber. Strictly speaking, it applies to trees with a single main stem to the top, such as most conifers, poplars, eucalypts, etc., but it can also be used for other kinds.

The only way of locating the point corresponding to h_1 was to climb the tree, until Bitterlich's instrument made it very easy.

The procedure is to walk towards or away from a tree until its breast height point is exactly covered by band 4. Then tilt the instrument upwards until the point is found which is exactly covered by band 1. (Band 1 is half the width of band 4.)

Alternatively, use band 1 at breast height, and 2 narrow bands to locate h_1, or band 1 plus 2 narrow bands at breast height and then 3 narrow bands for h_1. Note that the use of these alternatives requires different figures in the calculations which follow.

It would be possible at this stage to measure the actual height of h_1 e.g. by moving to a distance of 20 m and reading it on the 20 m height band. The volume could then be found by measuring the basal area with callipers or tape, and using Pressler's formula.

The following method, however, is far easier, and gives more interesting results, although the reasoning behind it may be found difficult. The basic idea is that we express the height and the form-height not in units of metres, but in units of the diameter at breast height of the tree in question. Thus e.g. the expression fh/d is referred to as "the form-height in units of diameter at breast height".

The procedure is to locate h_1 as described above using band 4 and band 1, and then from the same observing position to find the value of h_1 on the 25 m band i.e. to take readings on this band to the point in question and to the foot of the tree. Obviously, the result is not the true value of h_1 in metres. It is in fact = h_1/d.

For an explanation of this, refer to the formula at the end of Section 3. Call the "false" reading h_1'. The true height is h_1. The actual distance, a', is 25 times the diameter, since we are using band 4, and a is 25 since we are using the 25 m height band. Therefore:-

$$h_1 = h_1' \times 25d/25 = h_1' \times d$$

$$\therefore h_1' = h_1/d$$

or in words, the reading is h_1 in units of the diameter at breast height.

This reading is then multiplied by 2/3 to get a value for $2/3 \times h_1/d$.

At this point return to Pressler's formula:-

$$v = g \times 2/3h_1$$

$$\therefore 2/3h_1 = v/g$$

$$\text{but } v/g = fh$$

$$\therefore 2/3h_1 = fh$$

$$\therefore 2/3 \times h_1/d = fh/d$$

So what we have found, $2/3 \times h_1/d$, is the same as the "form-height in units of diameter at breast height", fh/d.

Now d can be measured with callipers or tape, so it is easy to calculate fh.

An example will show how simple this is:-

Suppose the height as read is 48 m. This is h_1/d.
Then:-

$$fh/d = 2/3 \times h_1/d = 2/3 \times 48 = \underline{\underline{32}}$$

If $d = 30$ cm, then:-

$$fh = 32 \times 0.30 = \underline{\underline{9.60}}$$

This, the form-height, is a valuable figure in itself, which will be used in later calculations. However, fh/d also leads to a value for the volume of the tree:-

$$v = fh \times g = fh \times d^2 \times \pi/4$$

$$= fh/d \times d^3 \times \pi/4$$

In the above example, $fh/d = 32$ and $d = 0.30$.
Taking $\pi/4 = 0.785$ we have:-

$$v = 32 \times 0.30^3 \times 0.785$$

$$= \underline{\underline{0.678}}\ m^3$$

Values for $d^3 \times \pi/4$ are given in the Appendix. These make the calculation even easier.

The form factor, f, can be found by measuring h/d, i.e. the <u>total</u> height from the same observing position as the other measurements were taken from, using the 25 m band. Then:-

$$f = fh/d \div h/d$$

Suppose in the above example $h/d = 50$, then

$$f = 32/50 = \underline{\underline{0.64}}$$

If, instead of band 4 and band 1, we use band 1 + 2 narrow bands and then 3 narrow bands, we have to use 8/9 in the above calculations instead of 2/3. If we use band 1 and then 2 narrow bands, we have to use 4/3.

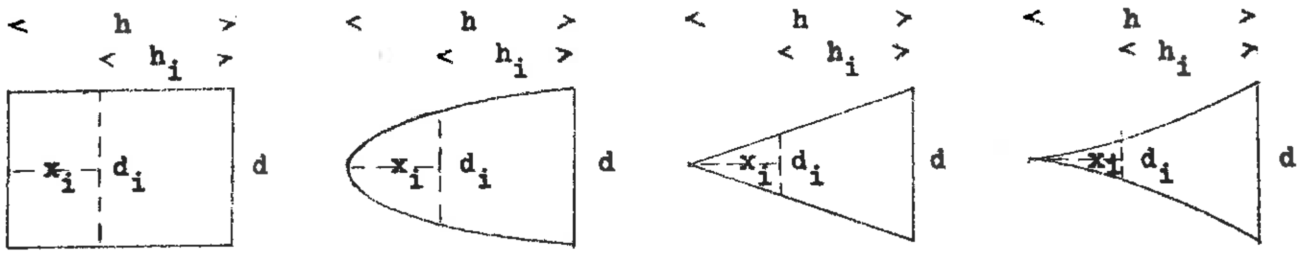
<u>Note</u>.

Strictly speaking, there is an error in the above applications of Pressler's formula, because the formula really applies to a geometrical shape measured <u>at its base</u> and not at 1.30 m. It is not practical to measure trees <u>at their bases</u>, as these are always irregular. Various ways of correcting this error have been suggested. It is probably more practical, however, to use the method given, and from time to time to compare the volumes so found with volumes measured after felling.

ROTATIONSKÖRPER der allgemeinen Form: $\left(\frac{d_i}{d}\right)^2 = \left(\frac{x_i}{h}\right)^r$; $x_i = h - h_i$

und ihre Inhaltsformeln

Walze	apollonisches Paraboloid	Kegel	Neiloid
r=0	r=1	r=2	r=3



A l l g e m e i n :

$$d_i^2 \frac{\pi}{4} = p_r \cdot x_i^r = g_i \; ; \; d^2 \frac{\pi}{4} = p_r \cdot h^r = g \qquad x)$$

$$v = \int_{x=0}^{x=h} g_i \, dx = \int_0^h p_r x_i^r \cdot dx = \left| \frac{p_r}{r+1} \cdot x^{r+1} \right|_0^h = \frac{p_r}{r+1} \cdot h^{r+1} = \qquad \boxed{h \times h^r = h^{r+1}}$$

$$= \frac{1}{r+1} \cdot p_r h^r \cdot h = \frac{1}{r+1} g \cdot h$$

$v = g \cdot h$	$= \frac{1}{2} g \cdot h$	$= \frac{1}{3} g \cdot h$	$= \frac{1}{4} g \cdot h$

FORM-HEIGHT

Lage des Richtpunktes nach PRESSLER :

$$* \quad \frac{g}{4} = \frac{p_r \cdot h^r}{4} = p_r \cdot x_p^r \; ; \quad x_p^r = \frac{h^r}{4} \; ; \quad x_p = \sqrt[r]{\frac{h^r}{4}} = h \sqrt[r]{\frac{1}{4}}$$

$x_p =$	$= \frac{1}{4} h$	$= \frac{1}{2} h$	$= \sqrt[3]{\frac{1}{4}}\, h^{\times\times)}$
$h_p = (\frac{3}{2}) h$	$= \frac{3}{4} h$	$= \frac{1}{2} h$	$= 0{,}375\, h$
$\frac{2}{3} = h$	$= \frac{1}{2} h$	$= \frac{1}{3} h$	$= 0{,}250\, h$
$v = \frac{2}{3} h_p \cdot g$	$= \frac{2}{3} h_p \cdot g$	$= \frac{2}{3} h_p \cdot g$	$\doteq \frac{2}{3} h_p \cdot g$

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x) $p_r = \frac{\pi}{4} \cdot \frac{d^2}{h^r}$;

xx) $\sqrt[3]{\frac{1}{4}} \doteq 0{,}625$

* $\frac{1}{2}$ DIAMETER [illegible] $\frac{1}{4}$ [illegible]