

$$\begin{aligned}
T^{\otimes n} \sum_{g \in \mathcal{G}} |g\rangle &= \sum_{g \in \mathcal{G}} \exp\left(\frac{\mathrm{i}\pi |g|}{4}\right) |g\rangle \\
&= \sum_{h \in \mathcal{G}_0} \sum_{x \in \mathbb{F}_2^{k_T}} \exp\left(\frac{\mathrm{i}\pi}{4} \sum_{a=1}^{k_T} x_a\right) \exp\left(\frac{\mathrm{i}\pi}{2} \left(\sum_{a=1}^m x_a \Gamma_a - 2 \sum_{a < b} x_a x_b \Gamma_{ab}\right)\right) \left| h + \sum_{a=1}^{k_T} x_a f^a \right\rangle,
\end{aligned}$$