Floretions associated with A115032

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1 Purpose

Since sequence A115032 was submitted in 2006, several interesting formulae and results involving inscribed circles have been shown by other contributors (see links on OEIS page). The notes here are only meant to show the context in which other sequences were generated alongside this one (some not listed in the OEIS). It is NOT meant as a proper introduction to floretions and assumes readers are already familiar with the general definition given here: A308496. Moreover, the notes given here are not exhaustive; simple variations such as reordering A, B, and C, below, changing signs, or applying binomial transformations (easily done with floretions- just take the floretion that generated the sequence and add the unit vector or subtract it for the inverse transform) can also produce related sequences.

2 Equilateral Triangle Representation of Floretions

Every floretion has a representation as a tiling of an equilateral triangle: if we divide an equilateral triangle into 4 smaller equilateral triangles by connecting the midpoints, we can associate i with the bottom left triangle, j with the top triangle, k with the bottom right triangle and e with the center triangle. Recall that i, j, k and e also have equivalent octal representations as 1, 2, 4, 7 respectively. These are the familiar quaternions (i.e. 1st order

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floretions). Triangle i can then be subdivided again into smaller triangles ii, ij, ik, ie. The original definition of A115032 has three floretions A, B, and C and states the sequence is generated by "tesseq" of $A * B * C$ (*tes* is the projection onto the unit vector, i.e. the real coefficient of the *ee* base vector (also known as "77" in octal or 63 in decimal) and "tesseq(A)" of some floretion "A" is short for the sequence $\mathrm{tes}(A), \mathrm{tes}(A^2), \mathrm{tes}(A^3), \ldots$.

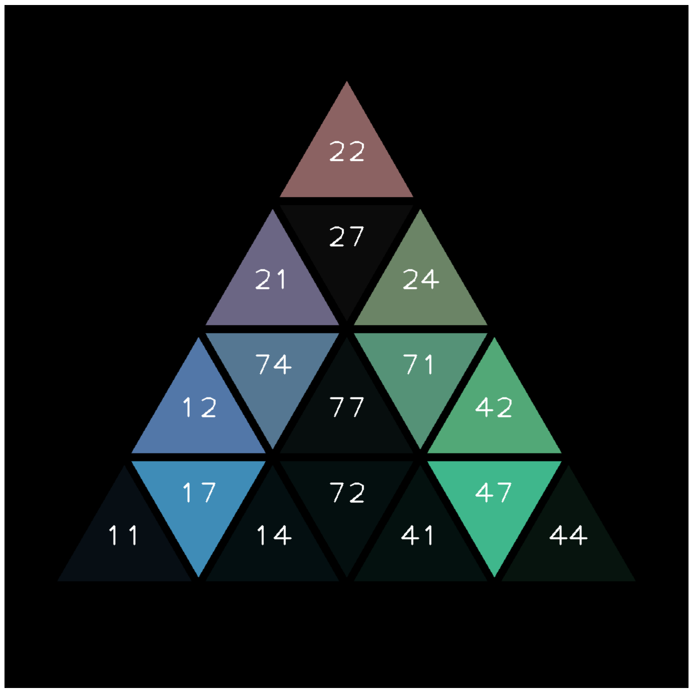

Figure 1: Triangles (base vectors in octal) associated with $A = -ij + ie - ji - jj - jk - kj - ke + ei - ek$

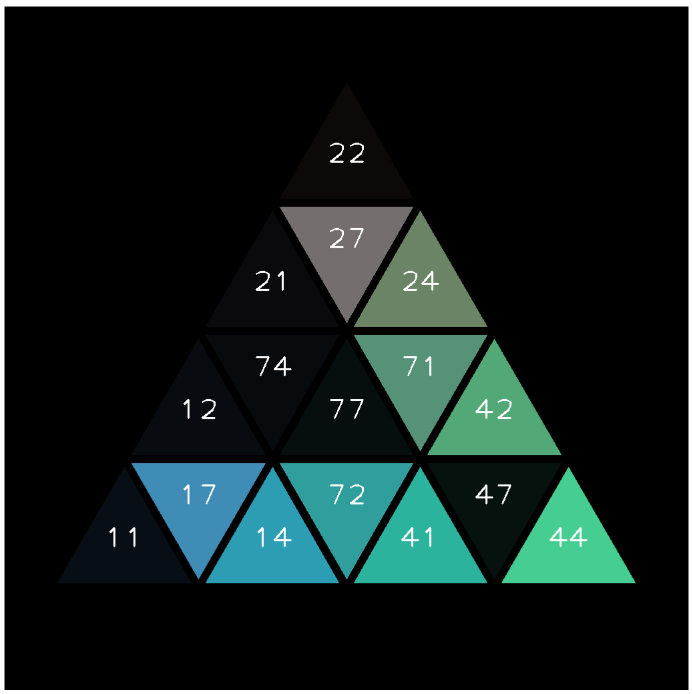

Figure 2: Triangles (base vectors in octal) associated with $B = -ik - ie - jk + je - ki - kj - kk - ei + ej$

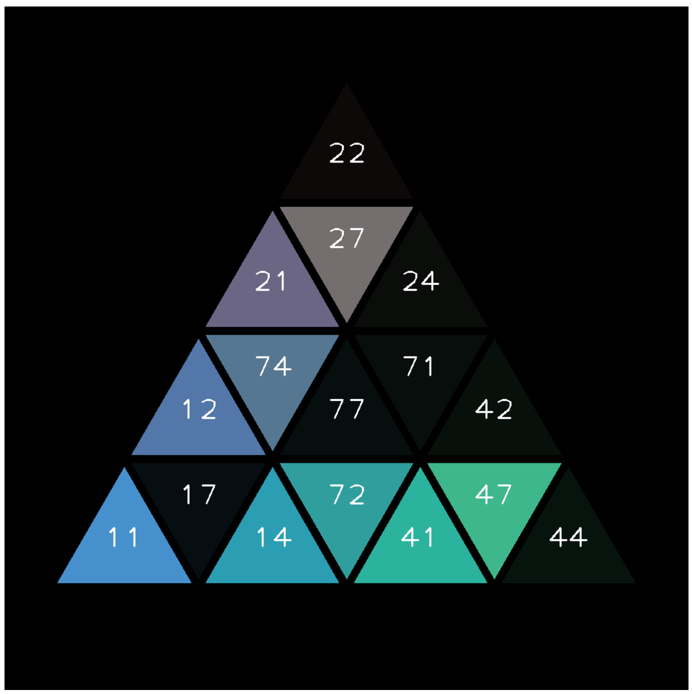

Figure 3: Triangles (base vectors in octal) associated with $C = -ii - ij - ik - ji - je - ki + ke - ej + ek$

$$
\begin{aligned}
A*B*C = &+4.0\,ii - 4.0\,ij + 14.0\,ie - 4.0\,ji - 12.0\,jj\\
&- 8.0\,jk - 2.0\,je - 8.0\,kj + 12.0\,kk - 2.0\,ke\\
&+ 14.0\,ei - 2.0\,ej - 2.0\,ek - 5.0\,ee,\\
(A*B*C)^2 = &-64.0\,ii + 80.0\,ij + 8.0\,ik - 268.0\,ie + 80.0\,ji\\
&+ 224.0\,jj + 152.0\,jk + 52.0\,je + 8.0\,ki + 152.0\,kj\\
&- 240.0\,kk + 36.0\,ke - 268.0\,ei + 52.0\,ej + 36.0\,ek\\
&+ 81.0\,ee,\\
(A*B*C)^3 = &+1140.0\,ii - 1444.0\,ij - 152.0\,ik + 4826.0\,ie\\
&- 1444.0\,ji - 4028.0\,jj - 2736.0\,jk - 950.0\,je\\
&- 152.0\,ki - 2736.0\,kj + 4332.0\,kk - 646.0\,ke\\
&+ 4826.0\,ei - 950.0\,ej - 646.0\,ek - 1445.0\,ee.
\end{aligned}
$$

2.1 Table

n	ii	ij A132584	ik A05360	ie
$(A*B*C)$	4	−4	0	14
$(A*B*C)^2$	−64	80	8	−268
$(A*B*C)^3$	1140	−1444	−152	4826
$(A*B*C)^4$	−20448	25920	2736	−86616
$(A*B*C)^5$	366916	−465124	−49104	1554278
$(A*B*C)^6$	−6584032	8346320	881144	−27890404
$(A*B*C)^7$	118145652	−149768644	−15811496	500473010
$(A*B*C)^8$	−2120037696	2687489280	283725792	−8980623792
$(A*B*C)^9$	38042532868	−48225038404	−5091252768	161150755262
$(A*B*C)^{10}$	−682645553920	865363202000	91358824040	−2891732970940

n	ji A132584	jj	jk A05360	je
$(A*B*C)$	−4	−12	−8	−2
$(A*B*C)^2$	80	224	152	52
$(A*B*C)^3$	−1444	−4028	−2736	−950
$(A*B*C)^4$	25920	72288	49104	17064
$(A*B*C)^5$	−465124	−1297164	−881144	−306218

$(A*B*C)^6$	8346320	23276672	15811496	5494876
$(A*B*C)^7$	-149768644	-417682940	-283725792	-98601566
$(A*B*C)^8$	2687489280	7495016256	5091252768	1769333328
$(A*B*C)^9$	-48225038404	-134492609676	-91358824040	-31749398354
$(A*B*C)^{10}$	865363202000	2413371957920	1639367579960	569719837060

n	ki A05360	kj A05360	kk	ke A207832
$(A*B*C)$	0	-8	12	-2
$(A*B*C)^2$	8	152	-240	36
$(A*B*C)^3$	-152	-2736	4332	-646
$(A*B*C)^4$	2736	49104	-77760	11592
$(A*B*C)^5$	-49104	-881144	1395372	-208010
$(A*B*C)^6$	881144	15811496	-25038960	3732588
$(A*B*C)^7$	-15811496	-283725792	449305932	-66978574
$(A*B*C)^8$	283725792	5091252768	-8062467840	1201881744
$(A*B*C)^9$	-5091252768	-91358824040	144675115212	-21566892818
$(A*B*C)^{10}$	91358824040	1639367579960	-2596089606000	387002188980

n	ei	ej	ek A207832	ee A115032
$A*B*C$	14	-2	-2	-5
$(A*B*C)^2$	-268	52	36	81
$(A*B*C)^3$	4826	-950	-646	-1445
$(A*B*C)^4$	-86616	17064	11592	25921
$(A*B*C)^5$	1554278	-306218	-208010	-465125
$(A*B*C)^6$	-27890404	5494876	3732588	8346321
$(A*B*C)^7$	500473010	-98601566	-66978574	-149768645
$(A*B*C)^8$	-8980623792	1769333328	1201881744	2687489281
$(A*B*C)^9$	161150755262	-31749398354	-21566892818	-48225038405
$(A*B*C)^{10}$	-2891732970940	569719837060	387002188980	865363202001

- A207832: Numbers x such that $20*x^2+1$ is a perfect square
- A05360: (Fibonacci(6*n+3) - 2)/4
- A132584: a(n) = 18*a(n-1) - a(n-2) + 8

2.2 External Links

- Illustration of initial terms: `https://oeis.org/A115032/a115032_2.pdfl`
- GitHub repository: `https://github.com/Floretion-Inquisitor/floretions`
- Code example for A115032: `https://github.com/Floretion-Inquisitor/floretions/tree/main/examples/A115032`
- Wiki: `https://github.com/Floretion-Inquisitor/floretions/wiki`
- Explanation of 2nd order floretions at the OEIS: `http://www.mrob.com/pub/seq/floretion.html`