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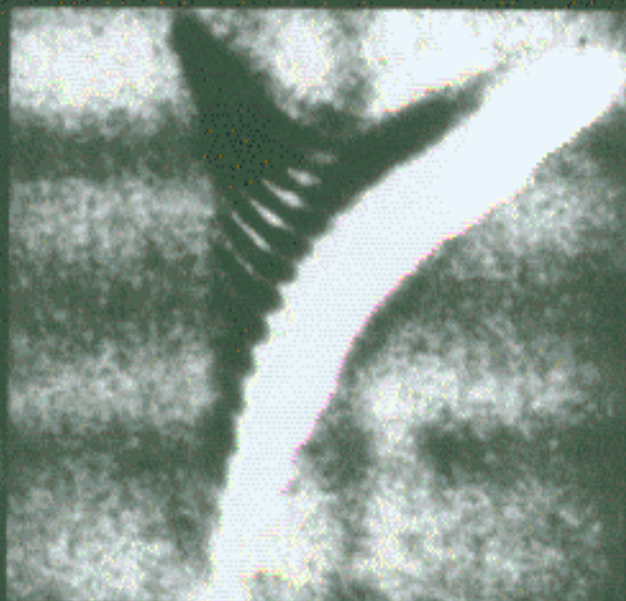


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Dynamical Theory of X-Ray Diffraction

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