Simultaneous Localization And Mapping using Extended Kalman Filtering

Florian Breut

SLAM using EKF

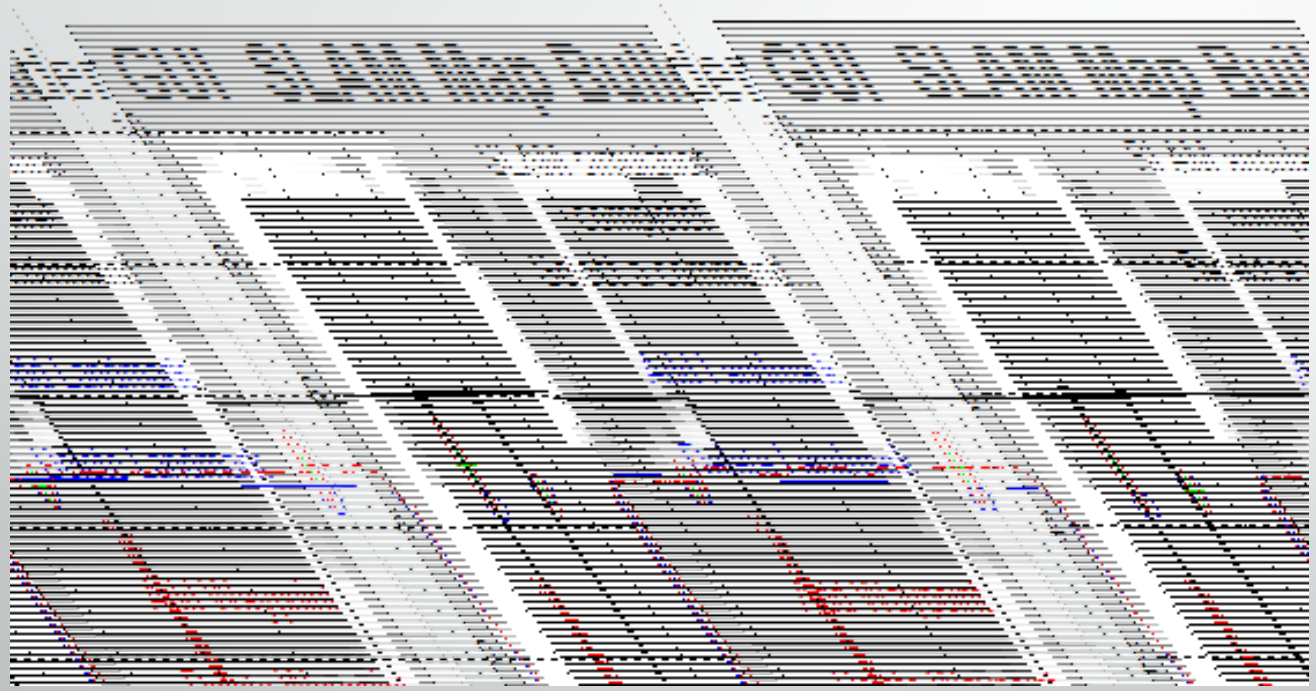
1. SLAM
    1. What is SLAM ?
    2. An easy task ?
2. EKF
    1. The Bayesian Approach
    2. Kalman Filter
    3. Extended KF
    4. SLAM EKF
3. Implementation & Results
    1. Matlab + V-rep
    2. Other Approaches

Simultaneous Localization and Mapping

PART 1

SLAM

What is SLAM ?



Mapping from the robot pose

OK

Robot pose given the map

OK

Both from noisy sensors

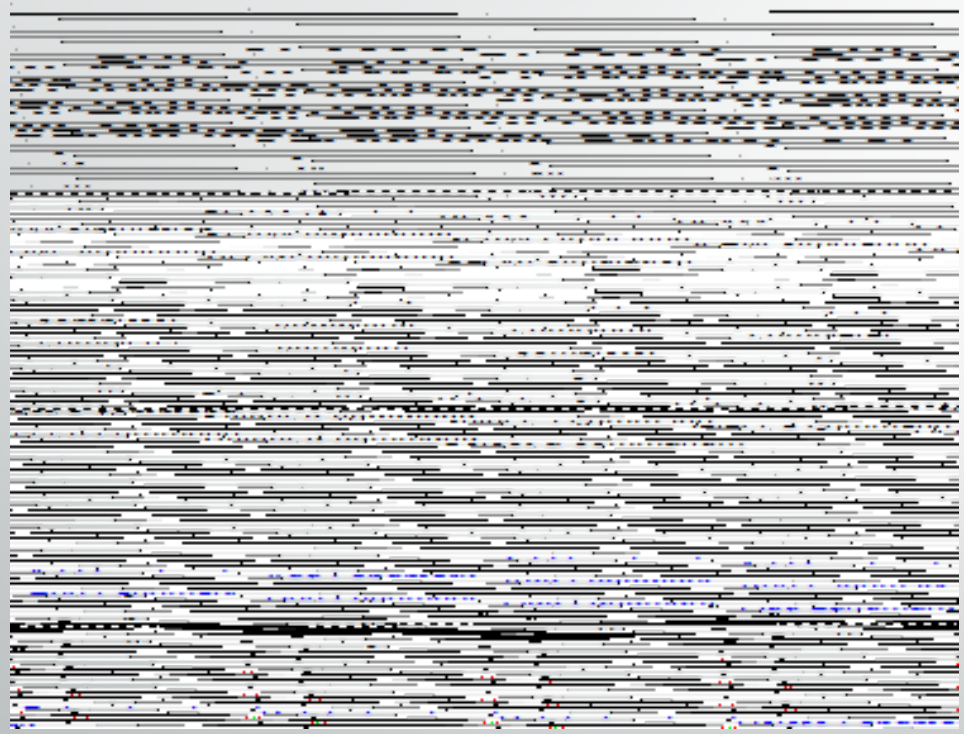
Not easy

Robot Pose

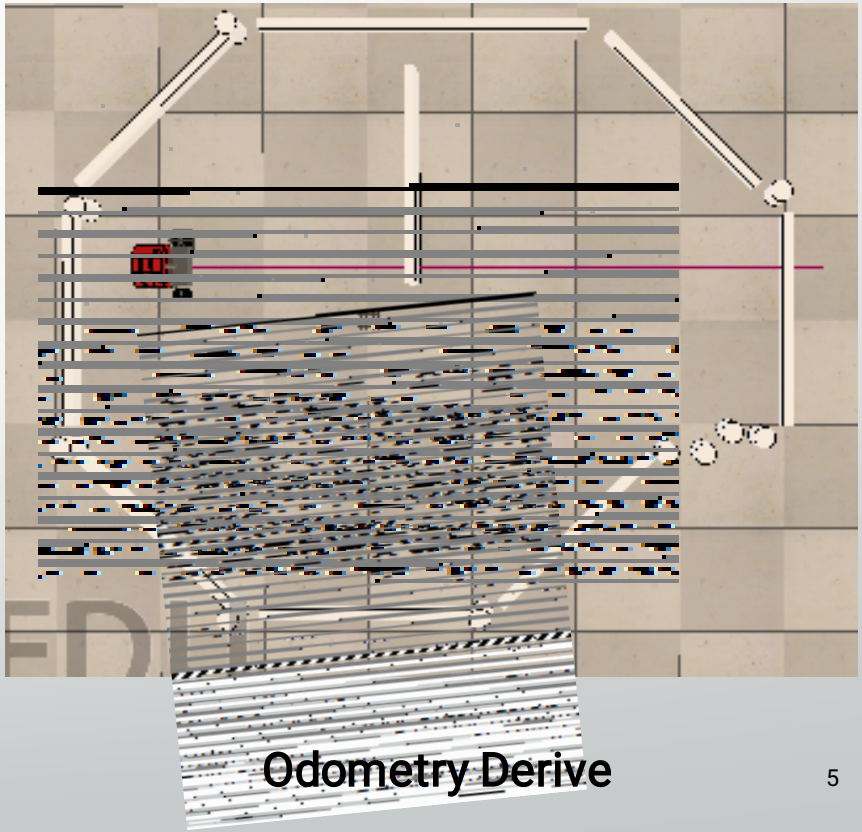
$x_t = (x, y, \theta)$

SLAM

An easy task ?



Sensor Noise



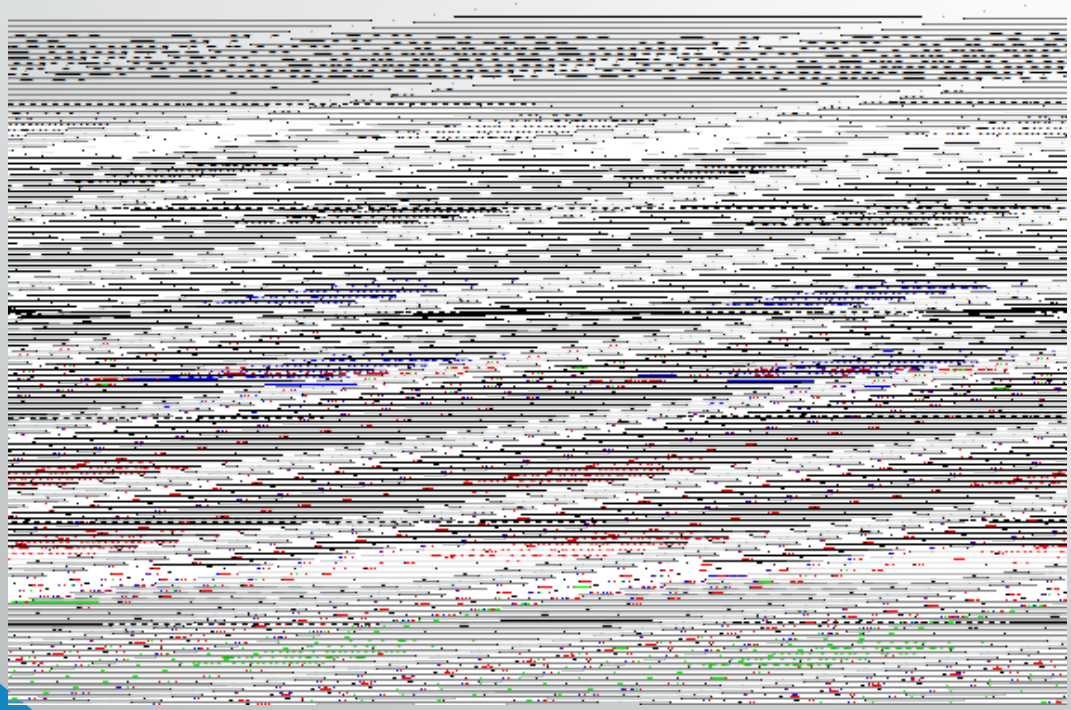
Odometry Derive

Extended Kalman Filter

PART 2

SLAM

Full SLAM Bayesian Network



Given

- The robot's Controls :

$$u_{1:T} = (u_1, u_2, \dots, u_T)$$

- Observations :

$$z_{1:T} = (z_1, z_2, \dots, z_T)$$

Wanted

- Map : M
- Robot pose :

$$x_{1:T} = (x_1, x_2, \dots, x_T)$$

EKF

The Bayesian Model

Dynamic Model

$$x_t = A_t x_{t-1} + B_t u_t + e_t$$

Observation Model

$$z_t = H_t x_{t-1} + v_t$$

Assumptions

Gaussian Noise

Linear Model

Full SLAM

$$P(x_{1:t}, M | z_{1:t}, u_{1:t})$$

Online SLAM

$$P(x_t, M | z_{1:t}, u_{1:t})$$

EKF

The Bayesian Model

$$bel(x_t) = P(x_t|z_{1:t}, u_{1:t})$$

$$bel(x_t) \propto P(z_t|x_t, z_{1:t-1}, u_{1:t})P(x_t|z_{1:t-1}, u_{1:t})$$

$$bel(x_t) \propto P(z_t|x_t)P(x_t|z_{1:t-1}, u_{1:t})$$

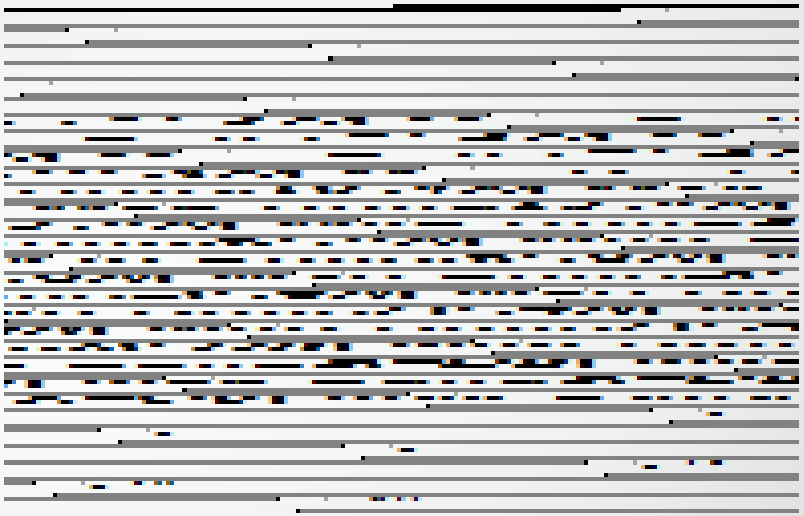
$$bel(x_t) \propto P(z_t|x_t) \int P(x_t|x_{t-1}, z_{1:t-1}, u_{1:t})P(x_{t-1}|z_{1:t-1}, u_{1:t})dx_{t-1}$$

$$bel(x_t) \propto P(z_t|x_t) \int P(x_t|x_{t-1}, u_t)P(x_{t-1}|z_{1:t-1}, u_{1:t-1})dx_{t-1}$$

$$bel(x_t) \propto P(z_t|x_t) \int P(x_t|x_{t-1}, u_t)bel(x_{t-1})dx_{t-1}$$

EKF

Gaussian Hypothesis



Observation Model

$$P(z_t|x_t) \Rightarrow \frac{1}{\sqrt{\det(2\pi Q_t)}} \exp\left(-\frac{1}{2}(z_t - C_t x_t)^T Q_t^{-1}(z_t - C_t x_t)\right)$$

Motion Model

$$P(x_t|x_{t-1}, u_t) \Rightarrow \frac{1}{\sqrt{\det(2\pi R_t)}} \exp\left(-\frac{1}{2}(x_t - A_t x_{t-1} - B_t u_t)^T R_t^{-1}(x_t - A_t x_{t-1} - B_t u_t)\right)$$

EKF
Motion Model

- **Odometry**

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} \delta_{trans} \cos(\theta + \delta_{rot1}) \\ \delta_{trans} \sin(\theta + \delta_{rot1}) \\ \theta + \delta_{rot1} + \delta_{rot2} \end{pmatrix}$$

- **Velocity Model**

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{\hat{v}}{\hat{w}} \sin\theta + \frac{\hat{v}}{\hat{w}} \sin(\theta + w\Delta t) \\ -\frac{\hat{v}}{\hat{w}} \cos\theta + \frac{\hat{v}}{\hat{w}} cos(\theta + w\Delta t) \\ \hat{w}\Delta t \end{pmatrix}$$

EKF
Observation Model

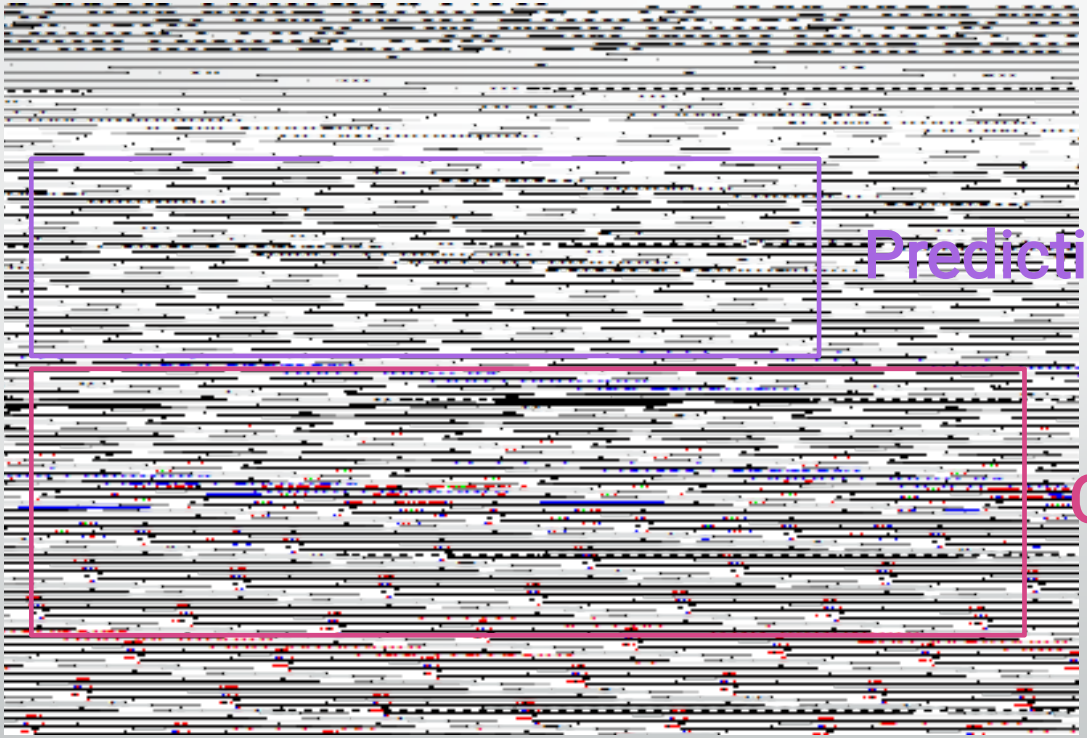
Range bearing observation

- Sonar
- Laser scan
- Triangulation with camera

$$\begin{pmatrix}\mu_{j,x}\\ \mu_{j,y}\end{pmatrix} = \begin{pmatrix}\mu_{t,x}\\ \mu_{t,y}\end{pmatrix} + \begin{pmatrix}r_t^i \cos(\varphi_t^i + \mu_{t,\theta})\\ r_t^i \sin(\varphi_t^i + \mu_{t,\theta})\end{pmatrix}$$

EKF

Kalman Filter Algorithm



EKF

Extended Kalman Filter

Observation Model

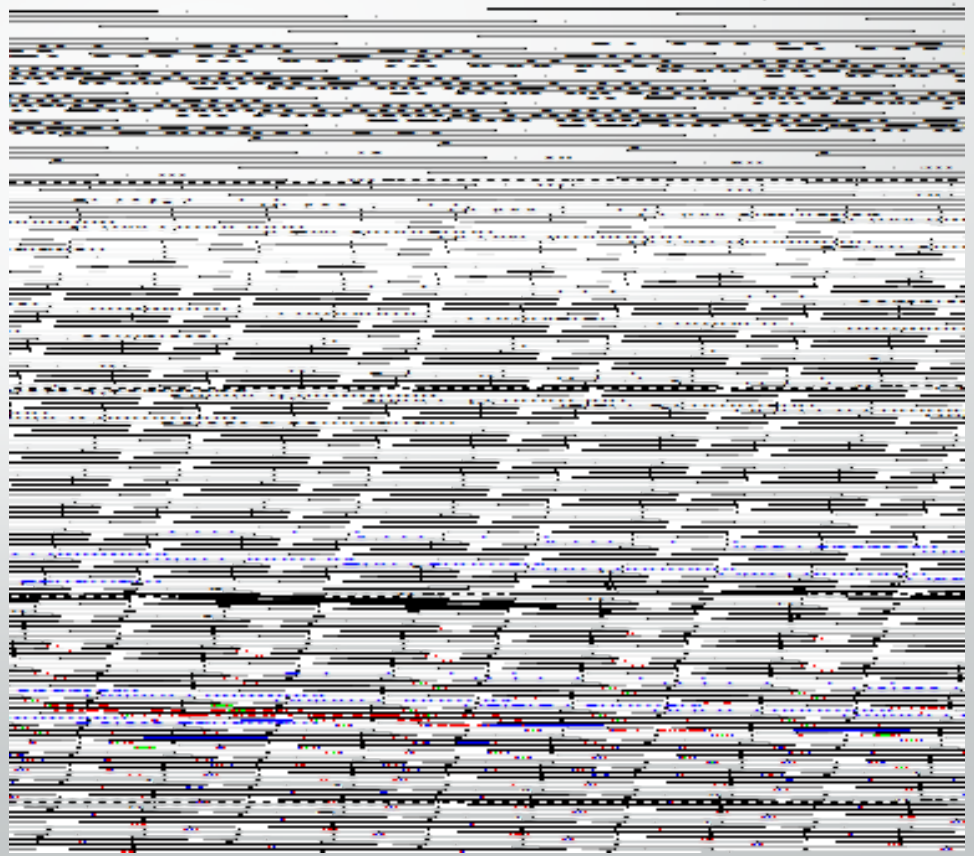
$$z_t = h(x_t) + v_t$$

$$h(x_t) \approx h(\mu_t) + \mathrm{H_t}(\mathrm{x_t} - \mu_t)$$

Dynamic Model

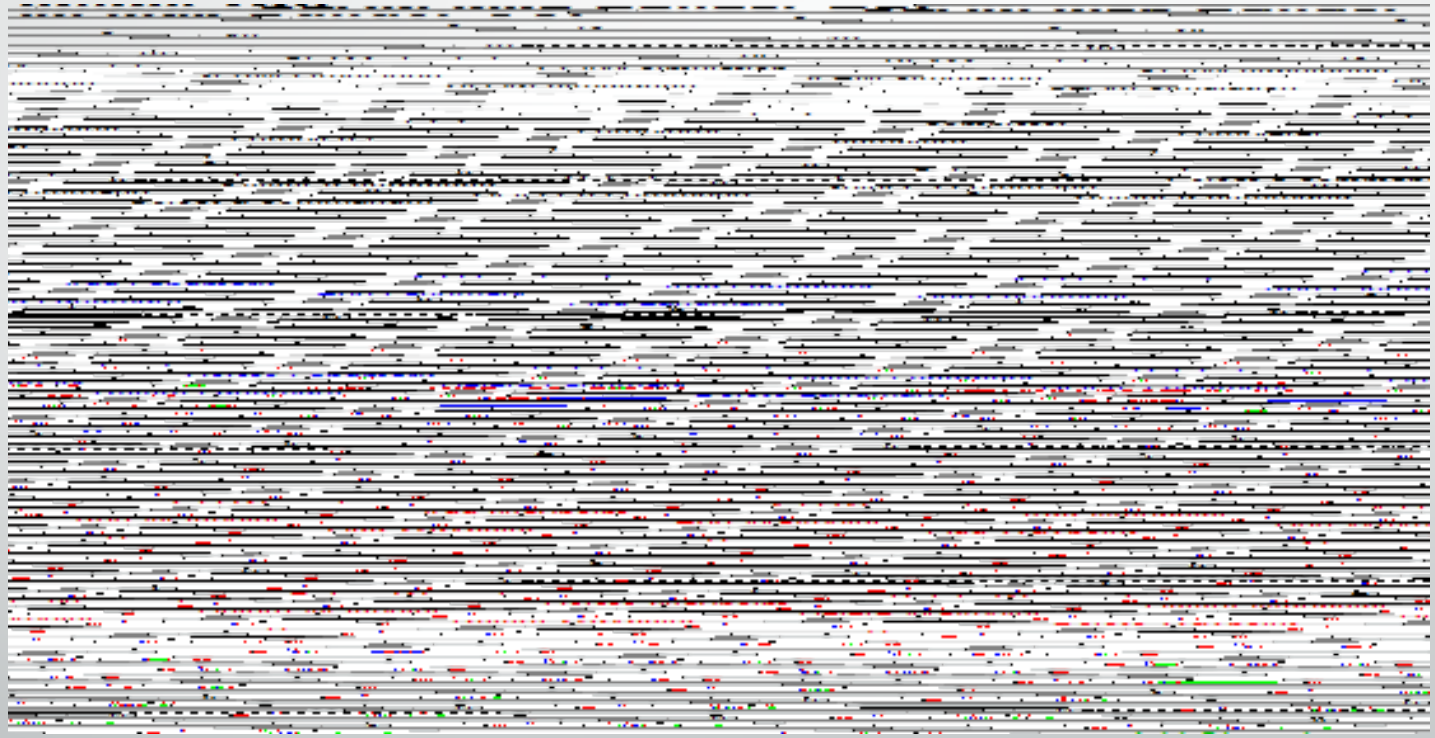
$$x_t = g(x_{t-1}, u_t) + e_t$$

$$g(x_{t-1}, u_t) \approx g(\mu_{t-1}, u_t) + \mathrm{G_t}(\mathrm{x_{t-1}} - \mu_{t-1})$$



EKF

Extended Kalman Filter



SLAM with EKF
EKF SLAM algorithm

- N landmarks
- New landmark proposal at each t
- ML function for landmark rejection
- Extension of the state representation

SLAM with EKF

EKF SLAM algorithm

- **Extension of the state vector**

$$x_t = (x, y, \theta) \quad => \quad x_t = (x, y, \theta, m_{x1}, m_{y1}, \ldots, m_{xn}, m_{yn})$$

- **Extension of the belief representation**

$$\begin{pmatrix} \Sigma_{xx} & \Sigma_{xy} & \Sigma_{x\theta} \\ \Sigma_{yx} & \Sigma_{yy} & \Sigma_{y\theta} \\ \Sigma_{\theta x} & \Sigma_{\theta y} & \Sigma_{\theta\theta} \end{pmatrix} =>$$

$$\begin{pmatrix} \Sigma_{xx} & \Sigma_{xy} & \Sigma_{x\theta} & \Sigma_{xmx1} & \Sigma_{xmy1} & \cdots & \Sigma_{xmxn} & \Sigma_{xmyn} \\ \Sigma_{yx} & \Sigma_{yy} & \Sigma_{y\theta} & \Sigma_{ymx1} & \Sigma_{ymy1} & \cdots & \Sigma_{ymxn} & \Sigma_{ymyn} \\ \Sigma_{\theta x} & \Sigma_{\theta y} & \Sigma_{\theta\theta} & \Sigma_{\theta xm1} & \Sigma_{\theta my1} & \cdots & \Sigma_{\theta mxn} & \Sigma_{\theta myn} \\ \Sigma_{mx1x} & \Sigma_{mx1y} & \Sigma_{mx1\theta} & \Sigma_{mx1mx1} & \Sigma_{mx1my1} & \cdots & \Sigma_{mx1mxn} & \Sigma_{mx1myn} \\ \Sigma_{my1x} & \Sigma_{my1y} & \Sigma_{my1\theta} & \Sigma_{my1mx1} & \Sigma_{my1my1} & \cdots & \Sigma_{my1mxn} & \Sigma_{my1myn} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \Sigma_{mxnx} & \Sigma_{mxny} & \Sigma_{mxn\theta} & \Sigma_{mxnmx1} & \Sigma_{mxnmy1} & \cdots & \Sigma_{mxnmxn} & \Sigma_{mxnmyn} \\ \Sigma_{mynx} & \Sigma_{myny} & \Sigma_{myn\theta} & \Sigma_{mynmx1} & \Sigma_{mynmy1} & \cdots & \Sigma_{mynmxn} & \Sigma_{mynmyn} \end{pmatrix}$$

SLAM with EKF

EKF SLAM algorithm

- **Motion Model** $x_t = x_{t-1} + F_x^T \begin{pmatrix} -\frac{\hat{v}}{\hat{w}}\sin\theta + \frac{\hat{v}}{\hat{w}}\sin(\theta + w\Delta t) \\ -\frac{\hat{v}}{\hat{w}}\cos\theta + \frac{\hat{v}}{\hat{w}}cos(\theta + w\Delta t) \\ \hat{w}\Delta t \end{pmatrix} + N(0, F_x^T R_t F_x)$

With $F_x = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \end{pmatrix}$

- **Observation Model** $z_t = \begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ atan2(m_{j,y} - y, m_{j,x} - x) \\ m_{j,s} \end{pmatrix} + \mathrm{N}(0, \begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\varphi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix})$

Implementation and Results

PART 3

Implementation & Simulation tools

Matlab SLAM toolbox

V-rep

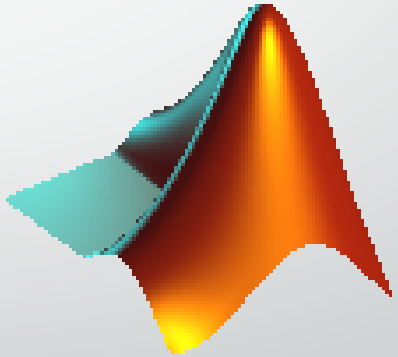
Robot model

simulation

Matlab

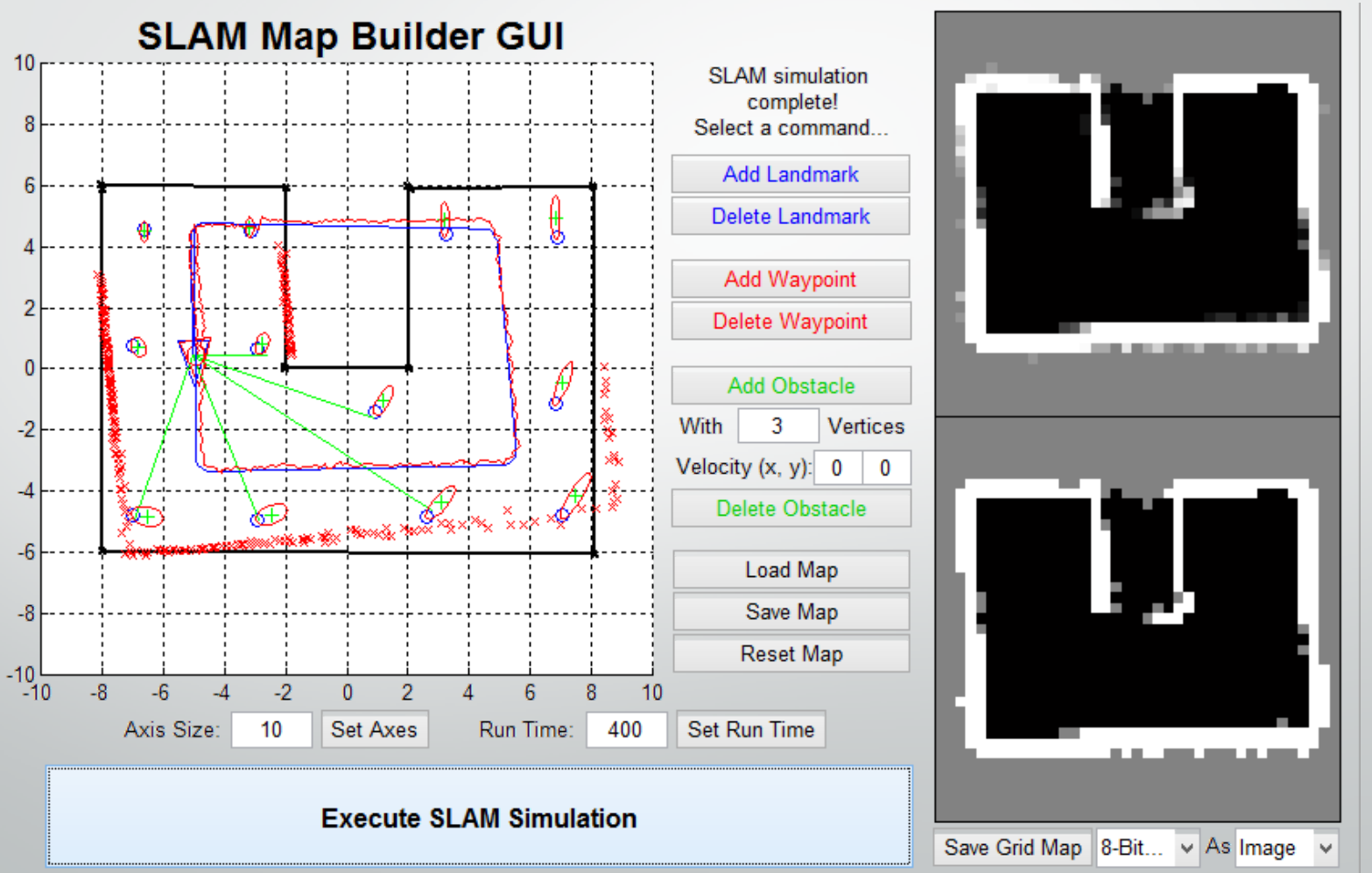
EKF toolbox

CV Toolbox



SLAM with EKF

EKF SLAM with correspondences



Bayesian Approaches
KF family & Others

Kalman Filters

- Kalman Filter
- Extended KF
- Unscendent KF

Others

- GraphSLAM
- FastSLAM
- Sparse Extended Information Filter
- Particle Filter
- Grid localization
- Monte Carlo Localization
- SLAM 3D

Thank you