QFT - Lecture 5

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Contents

D(x-y) $= \langle 0| \varphi(x)\varphi(y) |0\rangle$ (1)

$$\varphi(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_p e^{-ipx} + a_p^\dagger e^{ipx} \right] \tag{2}$$

Interpretation of $\varphi(x) |0\rangle$?
Particle localized at x?

$$\varphi(x) |0\rangle = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{2E_p} e^{-ipx} |p\rangle , \quad |p\rangle = \sqrt{2E_p} a_p^\dagger |0\rangle \tag{3}$$

(Abb1.1)

$$D(x-y) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{1}{2E_p 2E_q} e^{iqy - ipx} \langle q| |p\rangle \tag{4}$$

The Delta function collapses one of the integrals, here q is chosen.

$$= \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{2E_p} e^{-ip(x-y)}, \quad -r = x - y. \tag{5}$$

$$= \frac{1}{2(2\pi)^2} \int_0^\infty \mathrm{d}p \frac{p^2}{\sqrt{p^2+m^2}} \underbrace{\int_0^\pi \underbrace{\mathrm{d}\theta \sin(\theta)}_{=-\mathrm{d}u} e^{ipr \overbrace{\cos(\theta)}^{u}}}_{II} \tag{6}$$

Choose r along the z axis, where it and the p vector span the angle θ, Abb1.2

$$(II) = -\int_{-1}^{1} \mathrm{d}u e^{ipru} = -\frac{e^{ipr} - e^{-ipr}}{ipr} \tag{7}$$

$$(\mathrm{tot}) = \frac{i}{2(2\pi)^2 r} \int_0^\infty \frac{\mathrm{d}p p^2}{\sqrt{p^2+m^2}} \frac{e^{ipr} - e^{-ipr}}{p} \tag{8}$$

$$= \frac{1}{2(2\pi)^2 r} \int_{-\infty}^{\infty} \frac{p}{\sqrt{p^2+m^2}} e^{ipr} \approx \frac{e^{-mr}}{\sqrt{r}} \tag{9}$$

Look up Fourier Transform, Get Bessel function, Look at asymptotic behavior.

That means, that $D(x-y)$ is non-zero everywhere. This makes sense, if the particle is not localized, meaning it has tails. Not a violation of Causality.

0.1 Causality in QFT

A measurement at x does not affect a measurement at y, if $x - y$ is spacelike.
We then calculate the commutator of $\varphi(x)$ and $\varphi(y)$. It should be 0.

$$[\varphi(x),\varphi(y)] = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\frac{1}{\sqrt{2E_p2E_q}}\left(\left[a_pe^{-ipx,}, a_q^\dagger e^{iqy}\right] + \left[a_p^\dagger e^{ipx}, a_q e^{-iqy}\right]\right) \tag{10}$$

$$= \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{2E_p}\left(e^{-ip(x-y)} - e^{ip(x-y)}\right) \tag{11}$$

$$= D(x-y) - D(y-x) \tag{12}$$

Assume, that x and y are spacelike.

Put $y = (0,0,0,0)$ and $x = (t,0,0,z)$. $z > t$ because x spacelike.

there exists a Lorentz Transformation such that $x \mapsto x'$, $t \mapsto t' = 0$.

This is satisfied for $\beta = -t/z$. Then $t' = \gamma(t + \beta z) = 0$.

$$= D(x-y) - \underbrace{D(y-x)}_{=D(x-y)}\ (3-vectors) \tag{13}$$

$$= 0 \tag{14}$$

1 Klein-Gordon Propagator.

Let $x^0 > y^0$.

$$[\varphi(x),\varphi(y)] = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{E_p}\left(\underbrace{e^{-ip(x-y)}}_{D(x-y)} - \underbrace{e^{ip(x-y)}}_{D(y-x)}\right) \tag{15}$$

We transform $\vec{p} \mapsto -\vec{p}$

$$= \int \frac{\mathrm{d}^3p}{(2\pi)^3}\left[\left.\frac{e^{-ip(x-y)}}{2E_p}\right|_{p^0=E_p} - \left.\frac{e^{-ip(x-y)}}{2E_p}\right|_{p^0=-E_p}\right] \tag{16}$$

$$\int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{-2\pi i}\int \mathrm{d}p^0\frac{e^{-ip(x-y)}}{p^2-m^2} \tag{17}$$

We have two poles, one at $p^0 = E_p$ and one at $p^0 = -E_p$. Abb2.1

$$= i\int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{e^{-ip(x-y)}}{p^2-m^2} \tag{18}$$

1.1 Retarded propagator

For any x^0 and y^0. The Retarded Operator:

$$D_R(x-y) = \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{i}{p^2-m^2}e^{-ip(x-y)}\begin{cases} D(x-y) - D(y-x) & ,\ x^0 > y^0 \\ 0 & ,\ x^0 < y^0 \end{cases} \tag{19}$$

$$= \Theta(x^0 - y^0)[D(x-y) - D(y-x)]\ \text{Heaviside function} \tag{20}$$

$iD_R(x-y)$ is a Green's function for the Klein Gordon equation.

$$(\partial^2+m^2)iD_R(x-y) = \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{(i)i}{p^2-m^2}\left((-ip_\mu)(-ip^\mu)+m^2\right)e^{ip(x-y)} \tag{21}$$

$$= \int \frac{\mathrm{d}^4p}{(2\pi)^4}e^{ip(x-y)} = \delta(x-y) \tag{22}$$

2 Feynman propagator

$$D_F(x-y = \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{i}{p^2-m^2}e^{-ip(x-y)} \tag{23}$$

Here, the integration path is different. We integrate below the first pole, and above the second.

$$= \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{i}{p^2-n^2+i\epsilon}e^{-ip(x-y)}, \text{ Abb3.1} \tag{24}$$

The Feynman Propagator is used a lot.
When $x^0 > y^0$: Loop has to be closed below, this gives $D(x-y)$.
When $x^0 < y^0$: Loop has to be closed above, this gives $D(y-x)$.

$$D_F(x-y) = \begin{cases} D(x-y) & , x^0 > y^0 \\ D(y-x) & , x^0 < y^0 \end{cases} = \langle 0|\, T\{\varphi(x)\varphi(y)\}\, |0\rangle \tag{25}$$

T is the time ordering symbol. It is defined as

$$T\varphi(x)\varphi(y) = \varphi(x)\varphi(y), \text{ when } x^0 > y^0 \tag{26}$$

$$T\varphi(x)\varphi(y) = \varphi(y)\varphi(x), \text{ when } y^0 < y^0 \tag{27}$$

Next Topic: Chapter 3 in Peskin and Schröder. Before that, discuss Group Theory.