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QFT - Lecture 15

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Contents

1 Correlation functions

$$\langle\Omega| T\phi(x)\phi(y) |\Omega\rangle = \langle 0| T\phi(x)\phi(y)e^{-i\int \mathrm{d}t H_I(t)} |0\rangle \tag{1}$$

with $\phi(x)$, $\phi(y)$ being in the Heisenberg picture on the left and in the Interaction picture on the right.

$$H_I(t) = \int \frac{\lambda}{4!}\phi^4 \mathrm{d}^3x$$

with ϕ^4 in the interaction picture.

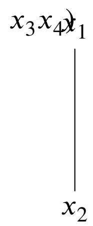
2 Wick's Theorem

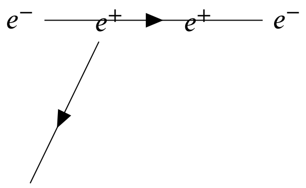
$$T\phi_1 \cdots \phi_n = N(\phi_1 \cdots \phi_n + \text{all possible contractions})$$

For example:

$$\langle 0| T\phi_1\phi_2\phi_3\phi_4 |0\rangle = \phi_1\phi_2\phi_3\phi_4 + \phi_1\phi_2\phi_3\phi_4 + \phi_1\phi_2\phi_3\phi_4 \tag{2}$$

Which can be represented as the Feynman Diagrams





$Eq1$:
lowest order: Num: $\langle 0| T\phi(x)\phi(y) |0\rangle = D_F(x-y) = \text{Fig2}$
first order: Num: $\langle 0| T\phi(x)\phi(y) |0\rangle (-i\lambda/4!) \int \mathrm{d}^4z\phi(z)^4 \langle 0|$
We now have 6 fields and must fiind all contractions. We can contract $\phi(x)$ and $\phi(y)$ with $\phi(z)$, which leads to $4 \cdot 3 = 12$ contractions. We can also contract $\phi(x)$ with $\phi(y)$ This leads to 3 contractions.
Which leads to

$$\frac{-i\lambda}{4!}\int \mathrm{d}^4z 3D_F(x-y)D_F(z-z)D_F(z-z) + 12D_F(x-z)D_F(y-z)D_F(z-z) \tag{4}$$

With the Feynman Propagator

$$D_F(x-y) = \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{i}{p^2+m^2+i\epsilon}e^{-ip(x-y)} \tag{5}$$

We can see that this integral becomes infinite for $x = y$, and integrating over all space again afterwards will definitely lead to infinity. We will ignore this for now, the divergences will disappear later.

For the 3 contractions we have the following Feynman Diagrams: Fig3

For the 12 contractions we have the following Feynman Diagrams: Fig4

Now considering a more complicated situation, with the third order:

$$\langle 0| T\phi(x)\phi(y)\frac{1}{3!}\left(\frac{-i\lambda}{4!}\right)^3 \int \mathrm{d}^4 z\phi^4 \int \mathrm{d}^4 w\phi^4 \int \mathrm{d}^4 u\phi(u) \tag{6}$$

We will consider one particular contraction, that being With the Feynmann Diagram Fig5

How many contractions give the same expression? We get 3! from interchanging verteces, $4 \cdot 3$ from the placement of contractions in the z vertex, we then get $4 \cdot 3 \cdot 2$ placements of the w vertex, and $4 \cdot 3$ from the u vertex. We have overcounted by a factor of 2, as we can interchange the contraction between the w and u fields, so we need to divide by 2.

we ignore $\frac{1}{3!}3!$ and we ignore $\frac{1}{4!} \cdot 4 \cdot 3$, $\frac{1}{4!}4!$. We then divide by the so called symmetry factor s, in our case $s = 2 \cdot 2 \cdot 2$. The first 2 comes from the line starting and ending at the z vertex, the 2nd from the line starting and ending at the u vertex, and the last because we can interchange the wu lines.

Another possibility of getting a symmetry factor is if two vertices are equivalent. Possible symmetry factors are as following

- line starting and ending at the same vertex
- equivalent lines
- equivalent vertices

These Symmetry factors are the reason why we should divide by the symmetry factor instead of leaving $\frac{1}{3!}$ and $\frac{1}{4!}$ in the third order term.

2.0.1 Summary:

$$\langle \Omega | T\phi(x)\phi(y) |\Omega\rangle = \frac{\text{Num}}{\text{Den}} \tag{7}$$

$$\text{Num: } \sum \text{all possible diagrams with two external points} \tag{8}$$

Feynman rules: Figs7

Fig8

Momentum Space Feynman Rules

Momentum Space Feynman Rules, correlation functions to Fourier space.

$$\langle\Omega|\, T\phi(x)\phi(y)\, |\Omega\rangle \mapsto \int \mathrm{d}^4x e^{ipx} \int \mathrm{d}^4y e^{ipy} \langle\Omega|\, T\phi(x)\phi(y)\, |\Omega\rangle \tag{9}$$

Omit exponent factors of external points, and the associated $\int \frac{\mathrm{d}^4p}{(2\pi)^4}$ Integrals.

$$f(x) \mapsto \hat{f}(p), \quad \hat{f}(p) \mapsto f(x), \quad f(x) = \int \frac{\mathrm{d}^4p}{(2\pi)^4} \hat{f}(p) e^{-ipx} \tag{10}$$

That way we can get $\hat{f}(p)$ by itself.

3 Exponentiation and cancellation

Consider Fig9

Then we have a product of two delta functions, which is nonsense

Let $V_i \in \{$The set of all different diagrams disconnected from external points$\}$.

$$A = (\text{Value of connected piece}) \cdot \prod_i \frac{1}{n_i!} V_i^{n_i}, \quad n_i = \text{number of } V_i.$$

$$\text{Num=} \sum_{\text{all connected}} (\text{value connected}) \cdot \underbrace{\prod_i \sum_{n_i} \frac{1}{n_1!} V_i^{n_i}}_{\exp(\sum_i V_i)}$$

$$\text{Den=} \exp(\sum_i V_i)$$

and therefore what we would like to have is

$$\frac{\text{Num}}{\text{Den}} = \sum \text{all connected}$$