QFT - Lecture 4

28.8.2024

Contents

1 Microstates and Macrostates

- what is the most likely outcome of tossing 3 coins?
- Microstates: state of all coins
  - heads: $s_i = 1$, tails $s_i = 0$.
  - all microstates are equally likely
- Macrostate: collective property
  - sum of states: $n = \sum_i s_i = \{0, 1, 2, 3\}$
- wWhich is the most likely macrostate?
  - $8 = 2^3$ possible microstates
  - Probabilities:
    * n = 0, P = 1/8
    * n = 1, P = 3/8...

Some Two-State models:

- Paramagnets
- two-sided box
- random walk

System: N spins, particles, steps, coins

- Isolated
- Independent (no interaction between spins/particles, no correlation between successive steps)
- Distinguishable (the order matters)
- Equal probability of states $s_i = \pm 1$

Macroscopic explanation of diffusion:
Net transport of energy of particles until thermodynamics equilibrium is reached

- $J = -D\nabla c$ Matter flux is proportional to gradient of concentration
- $Q = -\lambda \nabla T$ Heat flux is proportional to gradient of temperature
- what are the equilibrium conditions?

Microscopic explanation of diffuction

Net transport of energy or particles

  - through random thermal motion and particle collisions
  - until the most likely states are reached

- At any $T > 0K$, particles are in thermal motion
- Collisions between particles is a zigzag

Microscopic models of diffusion

1. Random walk
2. Algorithmic
3. Molecular Dynamics

- Reversible laws of motion
- Irreversible development: arrow of time
- Measure average property: distribution in box (left/right)

2 Einstein Crystal

There are N independent, distinguishable quantum harmonic oscillators.
They have discrete energy states $E_n = (n + \frac{1}{2})\hbar\omega$
The microstates of this crystal are $\{n_i\}; \{n_1, n_2, n_3, \cdots, n_N\}$
Macrostate $U_N = \sum_{i \in \{n_i\}} i = \sum_i n_i \hbar\omega + \frac{1}{2}\hbar\omega$

$$q = \frac{U_N - \frac{1}{2}\hbar\omega}{\hbar\omega} = \sum_i n_i \tag{1}$$

The macrostates are defined by (N, q).
Example: 3 oscillators.

$$q = 2 \tag{2}$$

we have 6 microstates for this macrostate. We will map this problem onto a two state model. We say that our oscillators are 3 boxes, where the walls are 1s and the balls are 0s. Then for example $\{0, 0, 2\} = 1100$. This fully describes our system with a 4-bit number. Then we calculate

$$N' = 4 = N - 1 + q = \text{ walls + balls} \tag{3}$$

$$n = q \tag{4}$$

$$\Rightarrow \Omega(N, q) = \frac{(N - 1 + q)!}{q!(N - 1)!} \tag{5}$$