QFT - Lecture 25

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Contents

1 Renormalization of electric charge

Consider corrections to photon propagator

Vacuum polatization diagram: Fig1

$$= (-ie)^2(-1)\int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{tr}\left[\gamma^\mu \frac{i(\not{k}+m)}{k^2-m^2+i\epsilon}\gamma^\nu \frac{i(\not{k}+\not{q}+m)}{(k+q)^2-m^2+i\epsilon}\right] \tag{1}$$

$$= i\Pi_2^{\mu\nu}(q) \tag{2}$$

where we have (-1) and the trace because of the fermion loop.

More generally: Fig 2

$\Pi^{\mu\nu}(q)$ is a tensor. It is comprised of the tensors that are included in the calculation. Then

$$\Pi^{\mu\nu}(q) = Cq^\mu q^\nu + Dg^{\mu\nu}, \text{ by Lorentz invariance} \tag{3}$$

We use Ward's Identity:

$$q_\mu \Pi^{\mu\nu} = 0 \tag{4}$$

Which we apply to eq3

$$\Rightarrow Cq^2q^\nu + Dq\nu = 0 \tag{5}$$

$$\Rightarrow (Cq^2 + D)q^\nu = 0 \tag{6}$$

$$\Rightarrow Cq^2 + D = 0 \tag{7}$$

$$\Rightarrow D = -Cq^2 \tag{8}$$

Which we substitute into the previous equation

$$\Pi^{\mu\nu}(q) = C(q^\mu q^\nu - q^2 g^{\mu\nu} \tag{9}$$

and we redefine $-C = \Pi(q^2)$.

$$= (q^2 g^{\mu\nu} - q^\mu q^\nu)\Pi(q^2) \tag{10}$$

2 Sum of all correction to the poton propagator

denoted by Fig 3

$$= \text{Fig 4} \tag{11}$$

Consider a low q^2 process ($q^2 << m^2$):

Fig5

What if q^2 is not small?

$$e_0 \to \frac{e_0}{\sqrt{1-\Pi(q^2)}} = e\frac{\sqrt{1-\Pi(0)}}{\sqrt{1-\Pi(q^2)}} \tag{12}$$

The apparent charge is dependent on q^2.

That means there is an Effective potential from an electron that is

$$V(r) = -\frac{\alpha}{r}\left(1 + \frac{\alpha}{4\sqrt{\pi}}\frac{e^{-2mr}}{(mr)^{3/2}} + \cdots\right) \tag{13}$$

Plotting this, with $V(r) = \frac{-\alpha_{eff}(r)}{r}$: Fig6

When the elctrons are closer together, they have a stronger attraction. Fig7