QFT - Lecture 22

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0.1 Vertex Correction

Fig1

Problems:

- Ultraviolet divergence: repaired by renormalization
- Infrared divergence: Cancelled by divergence for bremsstrahlung

Fig2

1 Bremsstrahlung

$$iM = -ie\bar{u}(p')M_0(p', p-k)\frac{i(\not{p} - \not{k} + m)}{(p-k)^2 - m^2}\gamma^\mu \varepsilon^*_\mu(k)u(p) \tag{1}$$

$M_0(p', p-k) \approx M_0(p', p)$, plus same simplification as for Compton, and ignore $\not{k}$

$$= \text{Fig3} \cdot e\left[\frac{p'^*}{p'k} - \frac{p\varepsilon^*}{pk}\right] \tag{2}$$

$$\mathrm{d}\sigma = \frac{1}{2E_A 2E_B|v_A - v_B|}\prod_f \frac{\mathrm{d}^3 p_f}{(2\pi)^3}\frac{1}{2E_f} \cdot |M(p_A p_B \to \{p_f\}|^2 (2\pi)^4 \delta\left(p_A + p_B - \sum_f p_F\right) \tag{3}$$

$$\text{for us: } \mathrm{d}\sigma(p \to p' + \gamma) = \mathrm{d}\sigma(p \to p')\int \frac{\mathrm{d}^3 k}{(2\pi)^3}\frac{1}{2E_k}\sum_\lambda \left|\frac{p'\varepsilon^\lambda}{p'k} - \frac{p\varepsilon^\lambda}{pk}\right|^2, \quad E_k = |k| \tag{4}$$

From the $1/k^3$ factor and the k^2 factor of the integral we get a divergence.

Setting $E_k = \sqrt{\mu^2 + |k|^2}$ with μ photon mass tending to 0

$$= \cdots = \mathrm{d}\sigma(p - p')\frac{\alpha}{\pi}\log\left(\frac{-q^2}{\mu^2}\right)\log\left(\frac{-q^2}{m^2}\right) \tag{5}$$

$-q$ large, $q^2 = (E' - E)^2 - (q' - q)^2, \quad q = p' - p$

logarithmic divergence as the (artificial) photon mass goes to zero.

2 Vertex Correction

Fig4

We consider $\gamma^\mu \to \Gamma^\mu$

$$= iM = ie^2 \bar{u}(p')\Gamma^\mu(p, p')u(p)\frac{1}{q^2}\bar{u}(k')\gamma_\mu u(k) \tag{6}$$

2.1 Lorentz Invariance:

$\bar{u}(p')\Gamma^\mu u(p)$ is a four vector.

$$\Gamma^\mu = A\gamma^\mu B(p'^\mu + p^\mu) + C(p'^\mu - p^\mu) \tag{7}$$

$$A, B, C \text{ can be dependent on } e,\ m,\ p^2,\ p'^2,\ q^2 = (p' - p)^2 = -2pp'$$

$$pq = p(p' - p) = pp' - m^2,\ p'q = m^2 - pp'$$

$$\not{p} : \bar{u}(p')\not{p}u(p) = \bar{u}(p')mu(p)$$

$$\rightarrow A, B, C \text{ can only depend on } e, m, q^2.$$

2.1.1 Word's Identity

$$0 = \bar{u}(p')(\Gamma^\mu q_\mu)u(p) \tag{8}$$

$$= A\bar{u}(p')\gamma^\mu q_\mu u(p) + B\bar{u}(p')(p'^\mu + p^\mu)q_\mu u(p) + C\bar{u}(p')(p'^\mu - p^\mu)q_\mu u(p) \tag{9}$$

$$= \underbrace{A\bar{u}(p')(\not{p}' - \not{p})u(p)}_{=0} + \underbrace{B\bar{u}(p')(p'^\mu + p^\mu)(p'_\mu - p_\mu)u(p)}_{=} 0 + C\cdots \tag{10}$$

$$\rightarrow C = 0 \tag{11}$$

$$\Rightarrow \Gamma^\mu(p, p') = A(q^2)\gamma^\mu + B(q^2)(p'^\mu + p^\mu) \tag{12}$$

$$= \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2),\ \text{Gordon's Identity} \tag{13}$$

$$\tag{14}$$

F_1 and F_2 are called form factors and are important. On Wednesday they will be computed to the lowest (non trivial) order. In the lowest Order $F_1 = 1,\ F_2 = 0$.

2.1.2 Example:

Fig5