Homework Sheet 03

October 26, 2024

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1 Particle in External Electromagnetic Field

1.1

The canonical momentum is

$$P = \frac{\partial L}{\partial \dot{x}} = m\dot{x} + qA \tag{1}$$

which is related to the linear momentum p as

$$P = p + qA \tag{2}$$

1.2

We calculate the Hamiltonian

$$H = \dot{x} \cdot \frac{\partial L}{\partial \dot{x}} - L \tag{3}$$

$$= \dot{x}(m\dot{x} + qA) - \frac{1}{2}m\dot{x}^2 + q(V - \dot{x}A) \tag{4}$$

$$= \frac{1}{2}m\dot{x}^2 + qA\dot{x} + q(V - \dot{x}A) \tag{5}$$

$$= \frac{1}{2}m\dot{x}^2 + qV \tag{6}$$

$$= \frac{p^2}{2m} + qV \tag{7}$$

$$= \frac{1}{2m}(P - qA)^2 + qV \tag{8}$$

This is the total energy if V and A do not depend on t. As it wasn't specified I will assume that they don't depend on t, so the Hamiltonian is the total Energy.