QFT - Lecture 24

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Contents

1 Vertex correction

Fig1.

$$\bar{u}(p')\delta\Gamma^\mu u(p) = 2ie^2 \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \int_0^1 \mathrm{d}x\mathrm{d}y\mathrm{d}z\delta(x+y+z-1)\frac{2}{D^3} \tag{1}$$

$$\cdot\bar{u}(p')\left[\gamma^\mu \underbrace{\left(-\frac{l^2}{2} + (1-x)(1-y)q^2 + (1-4z+z^2)m^2\right)}_{\text{leads to } F_1(q^2)-1} + \frac{i\sigma^{\mu\nu}}{2m}(2m^2 z(1-z))\right]u(p) \tag{2}$$

for $F_1(q^2)$ there is a UV divergence:

$$\int \frac{\mathrm{d}^4 l}{D^3} l^2 \to \int \mathrm{d}l l^3 \frac{l^2}{l^6} \tag{3}$$

a logarithmic divergence.

Confession: We have not included the factor $(\sqrt{Z})^{n+m}$ which we obtained when we simplified the LSZ-formula with amputation. In the lowest Order $Z \approx 1$. So until now it was okay not to include it. Our results so far had been to order $O(\alpha)$. It has also been fine for $F_2(q^2)$, as we had found that it was of order $O(\alpha)$.

For $F_1(q^2)$, we have the form $F_1(q^2) = 1 + O(\alpha)$ therefore we must include the factor Z.

We have:

$$\Gamma^\mu(p, p') = \text{fig2, but we need} \tag{4}$$

$$\sqrt{Z}^2\Gamma^\mu = Z\Gamma^\mu = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}}{2m} F_2(q^2) \tag{5}$$

Where we have redefined F_1 and F_2.

$$Z\Gamma^\mu = (1+\delta Z)(\gamma^\mu + \delta\Gamma^\mu) = \gamma^\mu + \delta\Gamma^\mu + \gamma^\mu\delta Z \tag{6}$$

$$\Rightarrow F_1(q^2) = 1 + \delta F_1(q^2) + \delta Z \tag{7}$$

where $\delta F_1(q^2) = F_1(q^2) - 1$ before the Z correction. It turns out that $\delta Z = -\delta F_1(0)$ from Peskin and Schröder 7.1

$$= 1 + \delta F_1(q^2) - \delta F_1(0) \tag{8}$$

2 Infrared divergence

Photon propagator

$$\frac{-ig_{\mu\nu}}{q^2 + i\epsilon} \mapsto \frac{-ig_{\mu\nu}}{q^2 - \mu^2 + i\epsilon} \tag{9}$$

Introduce a photon mass as regularization parameter. If we do that then

$$F_1(q^2) = 1 - \frac{\alpha}{2\pi}\log\left(-\frac{q^2}{m^2}\right)\log\left(-\frac{q^2}{m^2}\right),\ \text{large } -q^2 \tag{10}$$

We are interested in

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(p \to p') = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 \left[1 - \frac{\alpha}{\pi}\log\left(-\frac{q^2}{m^2}\right)\log\left(-\frac{q^2}{\mu^2}\right)\right] \tag{11}$$

We also had the Bremsstrahlung which was

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(p \to p' + \gamma) = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 \left[+\frac{\alpha}{\pi}\log\left(-\frac{q^2}{m^2}\right) + O(\alpha^2)\right] \tag{12}$$

It is not possible to distinguish $(p \to p')$ and $(p \to p' + \gamma)$, therefore we must add the cross sections together, with with the IR divergence disappears

3 The electron self-energy

in $\phi^4 - theory$:

$$\int \mathrm{d}^4 x e^{ipx} \langle\Omega| T\phi(x)\phi(0) |\Omega\rangle = \sum_\lambda \frac{iZ_\lambda}{p^2 - m_\lambda^2 + i\epsilon} = \text{sum of all diagrams two legs} \tag{13}$$

Thus for $p^2 \nearrow\!\!\!\!\searrow m^2$ ($m =$ single part mass):

$$\sum \text{all diagrams two legs} = \text{fig 3} \tag{14}$$

$$= \frac{i}{p^2 - m_0^2 + i\epsilon} + \cdots \tag{15}$$

$$= \frac{iZ}{p^2 - m^2 + i\epsilon} \tag{16}$$

in QED:
Consider

$$\int \mathrm{d}^4 x e^{ipx} \langle\Omega| T\psi(x)\bar{\psi}(0) |\Omega\rangle = \tag{17}$$

$$\text{fig4} \tag{18}$$

$$= \underbrace{\frac{i(\not{p} + m_0)}{p^2 - m_0^2 + i\epsilon}}_{\text{free field propagator}} + \underbrace{\frac{i(\not{p} + m_0)}{p^2 - m_0^2}\left[-i\Sigma_2(\not{p})\right]\frac{i(\not{p} + m_0)}{p^2 - m_0^2 + i\epsilon}}_{\text{electron self-energy}} + \cdots \tag{19}$$

$$-i\Sigma_2(\not{p}) = (-ie)^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\gamma^\mu \frac{i(\not{k} + m_0)}{k^2 - m_0^2 + i\epsilon}\gamma_\mu \frac{-i}{(p-k)^2 + i\epsilon} \tag{20}$$

This is divergent. We introduce Feynman parameters. complete the square, shift the momentum to $l = k - xp$. We use Pauli-Villands cutoff Λ momentum integral. And Wick rotate.

Result:

$$\Sigma_2(\not{p}) = \frac{\alpha}{2\pi} \int_0^1 \mathrm{d}x (2m_0 - x\not{p}) \log\left(\frac{x\Lambda^2}{(1-x)m_0^2 - x(1-x)p^2} \right) \tag{21}$$

Define one-particle irreducible diagrams (1PI):

Any diagram that cannot be split in two by removing a single line. Fig5

Let $-i\Sigma - 2$ be the sum of all 1PI diagrams

$$-iSigma(\not{p}) = 1PI = fig6 \tag{22}$$

We proceed with all diagrams. We want to find the sum of all diagrams like we did in ϕ^4 theory.

$$= \frac{i(\not{p} + m_0)}{p^2 - m^2} + \frac{i(\not{p} + m_0)}{p^2 - m^2} \left[-i\Sigma(\not{p}) \right] \frac{i(\not{p} + m_0)}{p^2 - m^2} + \cdots \tag{23}$$

$$= \frac{i}{\not{p} - m_0} \left(1 + \frac{\Sigma(\not{p})}{\not{p} - m_0} \right), \quad \frac{i(\not{p} + m_0)}{p^2 - m_0^2} := \frac{i}{\not{p} - m_0} \tag{24}$$

$$= \frac{i}{\not{p} - m_0} \frac{i}{1 - \frac{\Sigma(\not{p})}{\not{p} - m_0}} = \frac{i}{\not{p} - m_0 - \Sigma(\not{p})} \tag{25}$$

The pole is shifted from $\not{p} = m_0$ to $\not{p} = m_0 + \Sigma(\not{p} = m) =: m$

Then, the sum of all diagrams:

$$= \frac{iZ}{\not{p} - m + i\epsilon} \tag{26}$$

close to the pole $\not{p} = m : \Sigma(\not{p}) = \Sigma(\not{p} = m) + \Sigma'(\not{p} = m)(\not{p} - m)$

$$\not{p} - m_0 - \Sigma(\not{p}) = \not{p} - m_0 - \Sigma(\not{p} = m) - \Sigma'(\not{p} = m)(\not{p} - m) + \cdots \tag{27}$$

$$= (\not{p} - m)\left(1 - \Sigma'(\not{p} = m) + \cdots\right) \tag{28}$$

$$\Rightarrow Z^{-1} = 1 - \Sigma'(\not{p} = m) + \cdots \tag{29}$$

$$\delta m = m - m_0 = \Sigma_2(\not{p} = m) = \Sigma_2(\not{p} = m_0) \tag{30}$$

$$= \frac{\alpha m_0}{2\pi} \int_0^1 \mathrm{d}x (2 - x) \log\left(\frac{x\Lambda^2}{(1-x)^2 m_0^2} \right) = \frac{3\alpha m_0}{2\pi} \left[\frac{1}{4} + \log\frac{\Lambda}{m_0} \right] \tag{31}$$

For reasonable values of Λ, $\frac{\delta m}{m_0}$ is small. Let m_0 be dependent on Λ such that m is not small.