Introduction to Numerical Optimization – Project

Solar off-grid microgrid

A solar off-grid microgrid is a small-scale electrical grid composed of photovoltaic (PV) panels converting solar energy into electricity, a battery storing electricity, and a dispatchable diesel generator (genset), which can be switched on and off to supply an electrical load. A sketch of the system configuration considered is presented in Figure 1. We are interested in optimally sizing the microgrid, that is, identifying the capacity of components that minimize the total cost of the system over its lifetime. The total cost of the microgrid is the sum of the overnight investment costs incurred when installing the various components and the operating costs incurred over the lifetime of the system, namely the fuel costs of the genset. An agent designing the system faces a trade-off between building a large installation, which typically leads to high investment costs but low (variable) operating costs, and deploying a small installation, which often results in low investment costs but high (variable) operating costs.

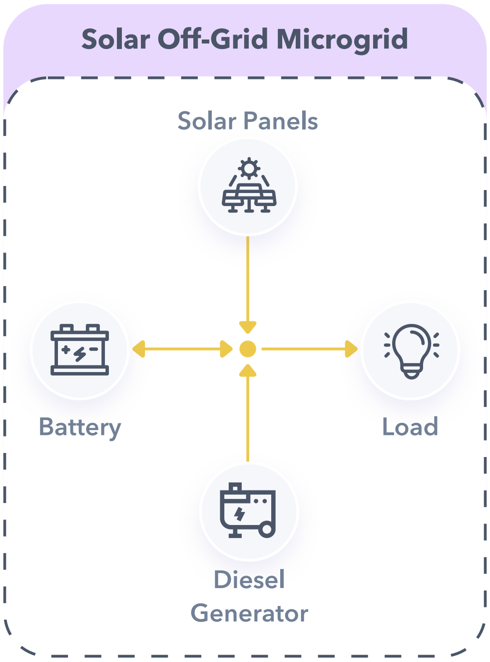

Figure 1: Microgrid configuration

Before formally introducing this benchmark problem, let us mention that we will use the notation $[\cdot]$ to indicate the physical unit of the symbol preceding it. In this section, $[W]$ denotes instantaneous power production in watts, $[W_p]$ denotes nameplate (manufacturer) power capacity, $[Wh]$ denotes energy in watt-hours and $[Wh_p]$ denotes nameplate (manufacturer) energy capacity. In addition, $[h]$ denotes the unit of time (hours) and [\$] denotes the currency considered in this problem. Finally, in the following, the time period $[t, t+1[$ between any two successive time steps t and $t+1$ corresponds to one hour (i.e., $\Delta t = 1$ hour) during which the consumption and the production levels are assumed constant.

In the system, the solar panels and the diesel generator provide electricity to supply the consumption of a load $P_t^C[W]$. On one hand, the solar panels harness solar radiation for electricity production. Formally, the quantity of power the panels produce $P_t^{PV}[W]$ at time t is proportional to the irradiance $i_t[Wh/Wh_p]$ at time t and to the installed (power) capacity of PV panels $C^{PV}[W_p]$ such that:

$$P_t^{PV} = \eta^{PV} i_t C^{PV} \ , \tag{1}$$

where $\eta^{PV}[\%]$ is the efficiency of the panels. In addition, the power production can be curtailed by covering the panel. On the other hand, the diesel generator can transform at any time t the energy contained in diesel $P_t^D[W]$ into electricity $P_t^G[W]$ with an efficiency $\eta^G[\%]$. The quantity of electricity a generator can produce is bounded by the (power) capacity of the device $C^G[W]$.

In order to make the most out of the solar panels installed, it is often necessary to store the energy produced when the sun shines in order to use it later. To do so, we can use batteries. Formally, a battery is an electrical device that can store energy up to a quantity $E^B[Wh_p]$, called (energy) capacity of the battery. Power can be charged in the battery during the time interval Δt with an efficiency $\eta^{B+}[\%]$ or discharged with an efficiency $\eta^{B-}[\%]$. In the case where energy is charged, the battery consumes energy that is produced. When the battery is discharged, it provides energy to the load. Finally, during the time interval Δt, the battery discharges itself by quantity $(1-\eta^B)$ times the energy that was stored in the battery, where $\eta^B[\%]$ is the efficiency of the battery.

Each component of the microgrid is associated with an investment cost that is proportional to the installed capacity. In addition, the electricity that is produced by the diesel generator comes with an operation cost proportional to the energy contained in the fuel. The objective is to find the capacity of PV panels, of the diesel generator and of the battery to install in order to minimize the sum of the investment and operation cost while ensuring that the power produced is effectively consumed in the microgrid. To this end, you will formulate the problem of controlling the microgrid during several years as a function of the three capacities you want to optimize. The different numerical values for the efficiencies are provided in Table 1 as well as the cost per unit of installed capacities and the cost of the diesel. In your model, you will consider that the hourly consumption and hourly irradiance of one typical day is repeated. The numerical values (in megawatt-hour) are provided in `data.jl`.

Symbol	Description	Value
η^{PV}	PV panels efficiency	18%
η^{B+}	Battery charge efficiency	85%
η^{B-}	Battery discharge efficiency	90%
η^{B}	Battery self-discharge efficiency	85%
η^{G}_{C}	Generator efficiency	90%
π^{PV}	Cost per installed capacity PV	$614\$/MW_p$
π^{B}	Cost per installed capacity of battery	$440\$/MWh_p$
π^{G}	Cost per installed capacity diesel generator	$1245\$/MW_p$
π^{D}	Cost per diesel power unit consumed	$23\$/MW_p$

Table 1: Parameters of the problem

Model

1. Formulate the problem mathematically as a linear optimization model.
   Hint: add a constraint on the state of charge to ensure that the initial state has a feasible physical value.

2. Solve the problem simulating one year and five years of system operation. Plot the production and consumption of the different elements of the microgrid as well as the state of charge of the battery during a typical week. Discuss the results.

3. Solve the previous problem using the simplex algorithm and using a barrier method. Report the time to solve the problem as a function of the number of years over which the microgrid is operated. Discuss the results.

4. Reduce the number of variables in the problem by writing a model which avoids the use of the state of charge of the battery as an explicit variable. Is the solving performance any better? Why?
   Hint: the state of charge at each time period may be replaced by a function of the power charged and discharged at previous time steps.

Sensitivity analysis

5. Assume the π^G parameter may be changed by a value $\Delta\pi^G$. Give the interval outside of which the optimal basis would change. In this interval, give the new optimal solution for any value of $\Delta\pi^G$. Interpret the results.

6. Provide an interpretation of the dual variables associated with equality constraints in the linear optimization model.

7. Assume the difference in production level of the generator between two successive time-steps is now limited to 10% of its maximum capacity. Give the new model formulation. How are the solution and the optimal basis of the previous model affected by the new constraint? How could they be reused to calculate the new optimal solution?

Robust and structural model

8. Give a robust formulation of the optimization problem assuming that the solar irradiance is subject to $\pm 7.5\%$ uncertainty. Discuss the uncertainty set in the formulation.

9. With the robust model, determine the minimum dimensioning of the PV panels to avoid having to install a diesel generator.

Deliverables

Students will work on this project in pairs or individually. For organizational reasons, each group is expected to send an email to `adrien.bolland@uliege.be` indicating the names of group members.

Each group will present its methods and findings on December 9. The exact format of the presentation will be given in due course. Each group is also expected to turn in a short report describing the problem formulations used and discussing findings by December 2.