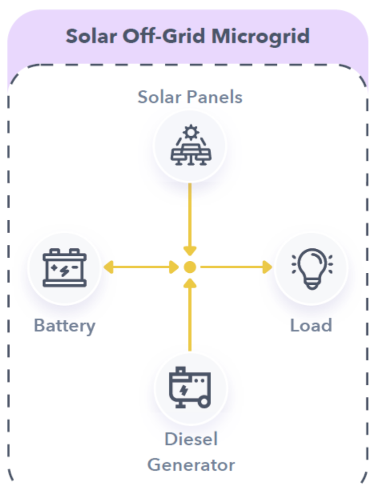
Liège University
Master 1 - Civil Engineer
Year 2021-2022

MATH0461-2 : Introduction to numerical optimization

Project : Solar off-grid microgrid



Auteurs :
Delcour Florian (S181063)
Waltregny-Dengis Karl (S176359)

Professeur :
Louveaux Quentin
Teaching assistant :
Bolland Adrien

Liège
November 2021

Abstract

In the current environmental context, use of green energies and optimization is strongly important to reduce our footprint on the planet. Solar panels combined with battery is a way to achieve this. However a trade-off has to be optimized between these which have high investment costs but low operating costs and using a genset (fossil fuels) which results in low investment costs but high operating costs.

Our task in this project is to identify the capacity components of the microgrid that minimize the sum of the installation costs and the operating costs over the lifetime of the system.

1 Q1

Our LP formulation for the problem is the following :

$$\min_{C^G, C^{PV}, E^{Bmax}} \underbrace{\pi^G \cdot C^G + \pi^{PV} \cdot C^{PV} + \pi^B \cdot E^{Bmax}}_{\text{installation costs}} + \underbrace{\pi^D \cdot \sum_{t=1}^{N} P_t^D}_{\text{operating costs}} \tag{1}$$

with π^G, π^{PV}, π^B are the cost per unit of installed capacities of PV, battery and genset, π^D the cost of the diesel and N is the lifetime of the system in hour. C^G, C^{PV} and E^{Bmax} are the capacities of the generator, the PV-panels and of the battery.

$$
\begin{aligned}
&s.t. \\
&P_t^{PV} + n_C^G \cdot P_t^D + P_t^{charge} = P_t^{load} + P_t^{discharge} && \forall t \in [1:N] \\
&n_C^G \cdot P_t^D \leq C^G && \forall t \in [1:N] \\
&P_t^{PV} \leq n_{PV} \cdot irra_t \cdot C_{PV} && \forall t \in [1:N] \\
&E_t^B = n^B \cdot E_{t-1}^B + n^{B+} \cdot P_{t-1}^{charge} \cdot \Delta t - \frac{P_{t-1}^{discharge}}{n^{B-}} \cdot \Delta t && \forall t \in [2:N] \\
&E_{t=1}^B = \frac{\sum_{i=1}^{n_{days}} E_{t=[(24 \cdot i)+1]}^B}{n_{days}} \\
&E_{t=N}^B = \frac{\sum_{i=1}^{n_{days}} E_{t=[24 \cdot i]}^B}{n_{days}} \\
&E_t^B \leq E^{Bmax} && \forall t \in [1:N]
\end{aligned}
$$

The first constraint is the equality constraint which says that everything that is produced at time t must be consume at time t. We take the centre of the micro-grid as the reference point. P_t^{charge} and $P_t^{discharge}$ are respectively the power flow charging the battery at time t and the power flow discharging the battery at time t. Because $n^{B+} < n^{B-}$, P^{charge} and $P^{discharge}$ cannot be different from 0 at the same time t. Indeed, it would mean that the battery is charging and discharging itself at the same time and so imply a loss of energy. It is a good way to test if our model is correct because if correctly implemented, the optimizer won't accept this loss of energy.

The second constraint assures the fact that the power deliver by the generator at each time t is bounded by its capacity.

The third one tells that the PV power production at each time t is bounded by the PV capacity multiplied by the irradiance at time t and the PV efficiency. It's not an equality constraint because the PV panels can be covered to curtailed the PV power production. Indeed, if the battery is already full and the load has already enough power, we must cover the panels to avoid power overload in the microgrid.

The next constraint expresses the state of the battery at time t so the quantity of energy that is stored in the battery at time t. The time step is $\Delta t = 1$ hour in our case so it simplifies the constraint. This state is defined as a function of the previous state of the battery, the charging power and discharging power both assumed constant during Δt.

However this constraint cannot be used at time step 1 because it depends on previous states. So it has been decided that the initial state of the battery will be the mean of all the values corresponding to the same hour as the initial state hour, for each day. This is the fifth constraint where $n_{days} = 365 \cdot n_{years}$. By doing this, the initial state of the battery has a stationary value and has not been arbitrarily decided by the optimizer. The same principle is applied to the last state (6th constraint).

The last constraint assures that the quantity of energy inside the battery at each time t is always smaller or equal to its full capacity.

2 Q2

It can be observed on Figure 1 and 3 that the state of charge of the battery depending on the time is a periodic function. Here the time duration is one week. There is 7 maximum for both graphs as they are 7 days in a week. Because of that, the period can be supposed to be equal to 24 hours. This result is expected because the data was formed by repeating $n_{years} * 365$ measurements collecting over one day.

In term of amplitude, the stored energy inside the battery is much larger for 5 years than for 1 years. It can be explain by the fact that when the lifetime of the micro grid increases, the operation costs become much larger than the investment costs. In fact, the consumption of fuel for the generator increases too much. It is now more cost-effective to prefer a large capacity of pv-panels in parallel with a great capacity of battery. A larger capacity for the pv-panels and the battery is chosen when the lifetime raises. If the capacities are larger, the power produced is also larger. It can be seen on Figure 4, the pv-panels produce much more than for one year.

We can add that it is not a good idea to increase only the capacity of the battery or the size of the PV panels because, if one of them increases, the other must also increase. If the capacity of the panels is too high, they will be curtailed during a given time. On the other hand, if the capacity of the battery is too high, the power produced by the panels will not be sufficient to charge it entirely.

On Figure 2 and 4, the power charge of the battery follows the pv-panels production. The P_t^{charge} curve is below the P_t^{PV} curve because the pv-panels have to feed the load at the same time (due to equality constraint on consumption and production).

As we can see on Table 1 and on the graphs, for 1 year of lifetime, a generator is still necessary because PV power production is not sufficient both to supply the load and to provide enough energy to the battery in order to feed the load during the night when there is no sun. For five years of lifetime, a bigger capacity of the battery joined to larger installation of pv-panels ensure the needs of the load for the entire day, the generator is not required anymore.

On Figure 1 and 2 there is a difference of amplitude for the state of the battery and the power of charge. This difference can be explained because the units are different. For the pv-panel, it is instantaneous power so the unity is the megawatt(MW) and for the charge in the battery it is megawatt-hour(MWh). The quantity of energy stored in the battery at time t is the integral of the instantaneous power during one hour (charge and discharge), plus the energy that was previously inside (minus a given part).

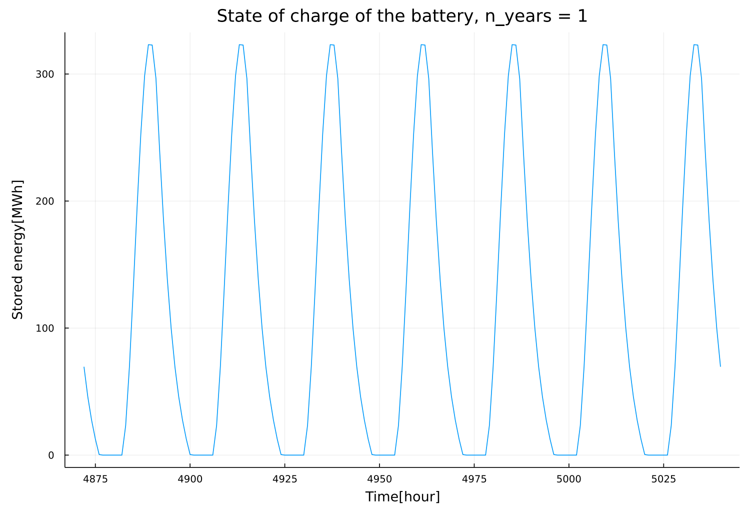


Figure 1 – State of charge of the battery (1 year)

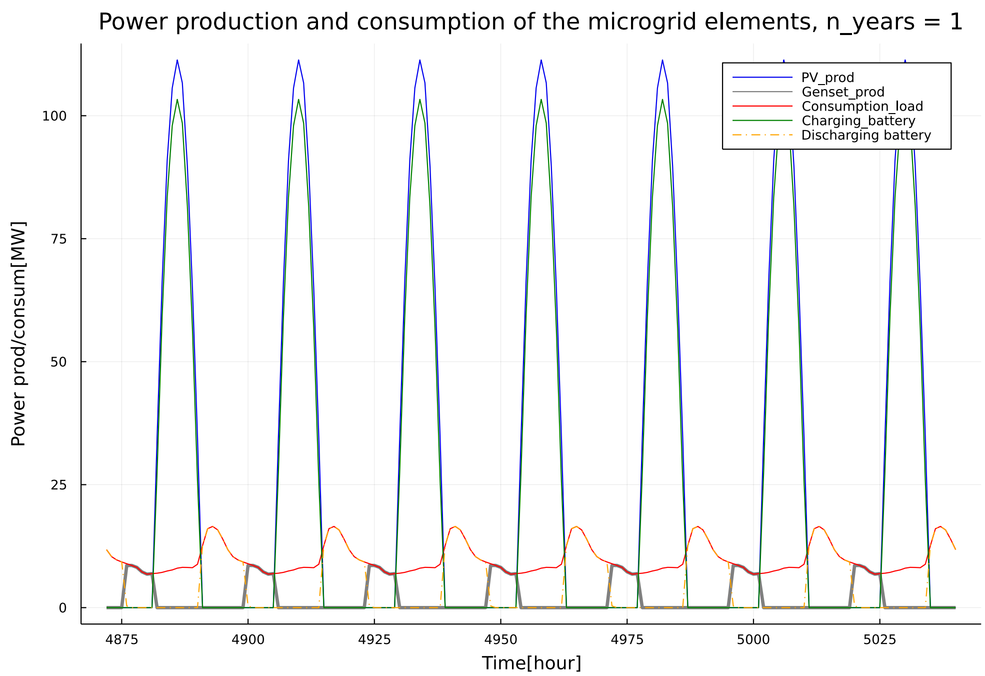


Figure 2 – Production and consumption (1 years)

In a traditional context, the pv-panels supply the load and charge the battery which will supply the load in the future with this energy. However let imagine that the cost for pv-panel and generator capacities are very high compare to the battery capacity and diesel costs. If the load need a lot of power at time t, it can append that the generator charges the battery in advance. If we assume that at time t, the panels and the generator (limited capacity) cannot produce a sufficient amount of power, it would be the role of the generator to feed the battery in advance (eg : low pv-panel capacity, low irradiance, big capacity of the

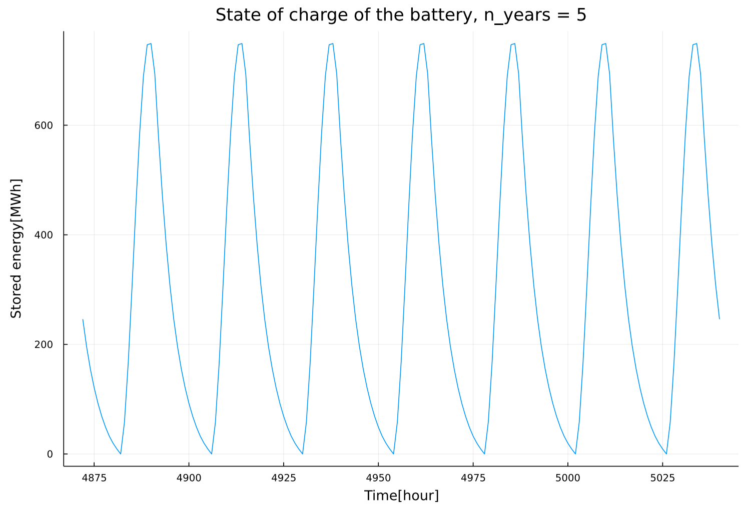


FIGURE 3 – State of charge of the battery (5 years)

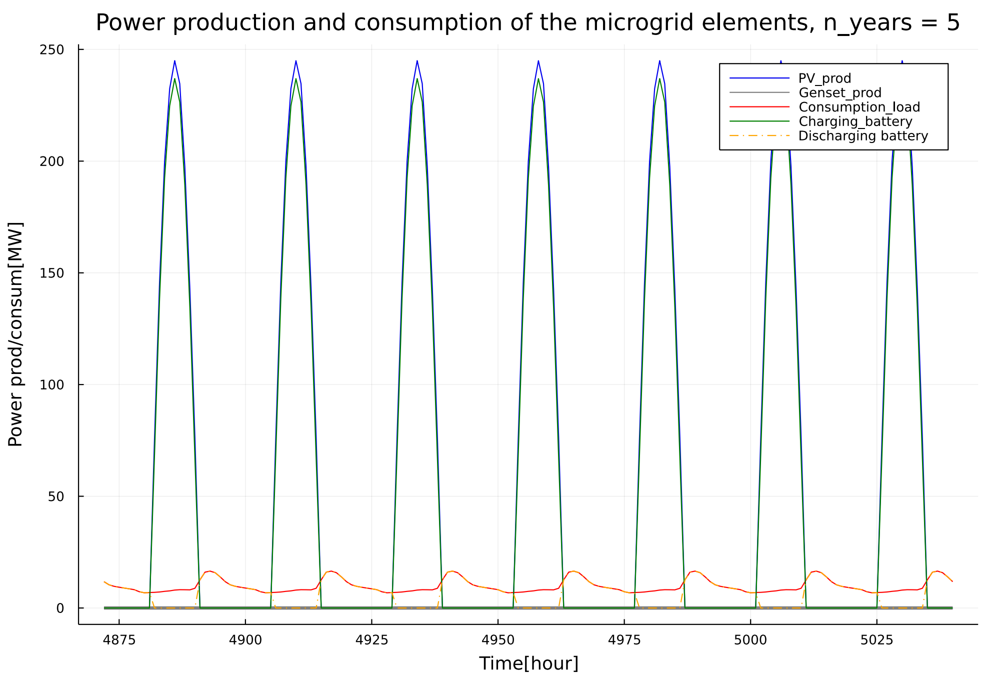


FIGURE 4 – Production and consumption (5 years)

battery, low price on diesel, big load, low generator capacity).

On Table 1, we can see that the optimal objective solution is about 1.04 million dollars for 1 year lifetime and 1.33 million dollars for 5 years lifetime. We noticed that the total cost is not linear with the number of years lifetime meaning that the installation costs are amortized over the lifetime. This explanation can be verified numerically on Table 1 where the total cost is the optimal solution.

	1 year	5 years
$C^{PV}[MW]$	743.93	1636.61
C^{G}[MW]	8.58	0
$E^{Bmax}[MWh]$	323.13	749.14
Optimal total cost [$]	1 041 153.77	1 334 498.86
Installation costs[$]	609 757.11	1 334 498.86
Operating costs [$]	431 396.66	0

TABLE 1 – Optimal solution and capacities of the microgrid components

3 Q3

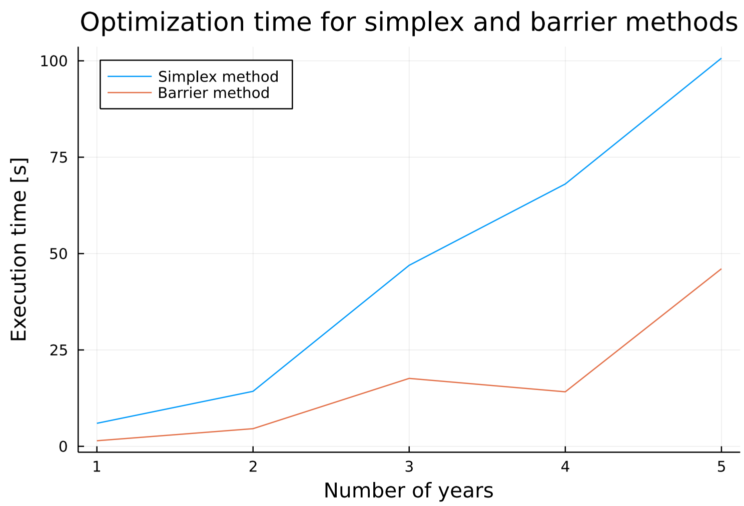


FIGURE 5 – execution time depending on the number of years

For this section, we keep the previous LP formulation of the problem and we only change the method used to solve it. The function `set_optimizer_attribute` available in `JuMP` allows us to access the parameter `Method` of the Gurobi optimizer. To compute the optimization time, we access to the Gurobi model attribute `Runtime`. Then, the problem is solved for 1, 2, 3, 4 and 5 years. It can be observed on Figure 5 that the simplex method increases with the number of years like an exponential. Whereas the execution time of the barrier method increases much less with the number of years. To be more precise, the model is repeated for each year 5 times. The mean is taken and plotted. Thanks to that, we are more robust in term of time's variation between two launch. The disadvantage is that the code is slower because the operation is repeated.

The barrier method also called interior-point method reaches a best solution by traversing the interior of the feasible region whereas the simplex method evolves from vertices to vertices following the edges of a polyhedron to find the optimal solution. At worst, the simplex algorithm visits all the vertices and so the complexity is exponential whereas the barrier method has a polynomial complexity. In our case, the barrier method outperforms the simplex method because we have a large model where the many simplex pivot operations are expensive in complexity compare to barrier method.

4 Q4

To understand how we reduce the number of variables from our previous formulation, let's wright what are the equations for the state of charge of the battery at time step t, $t-1$, $t-2$, ... :

$$\begin{aligned}
E_t^B &= E_{t-1}^B \cdot n^B + n^{B+} \cdot P_{t-1}^{charge} - P_{t-1}^{discharge}/n^{B-} \\
E_{t-1}^B &= E_{t-2}^B \cdot n^B + n^{B+} \cdot P_{t-2}^{charge} - P_{t-2}^{discharge}/n^{B-} \\
E_{t-2}^B &= E_{t-3}^B \cdot n^B + n^{B+} \cdot P_{t-3}^{charge} - P_{t-3}^{discharge}/n^{B-} \\
&\vdots \\
E_{t=2}^B &= E_{t=1}^B \cdot n^B + n^{B+} \cdot P_{t=1}^{charge} - P_{t=1}^{discharge}/n^{B-}
\end{aligned}$$

where we replaced $\Delta t = 1$ hour and we considered $E_{t=1}^B = 0$ because we did not succeed to rewrite this constrainst from the first model. Then after that let's replace the value of $t-1$ in the equation at time step t by its expression depending on $t-2$. Repeat the operation until reaching time step $t = 2$.

You end up with the following expression (note that here, we only replace for E_{t-3}^B to avoid a too long equation) :

$$\begin{aligned}
E_t^B = \Big[&(E_{t-3}^B \cdot n^B + n^{B+} \cdot P_{t-3}^{charge} - P_{t-3}^{discharge}/n^{B-}) \cdot (n^B)^2 \\
&+ n^{B+} \cdot P_{t-2}^{charge} - P_{t-2}^{discharge}/n^{B-} \Big] \cdot n^B \\
&+ n^{B+} \cdot P_{t-1}^{charge} - P_{t-1}^{discharge}/n^{B-}
\end{aligned}$$

which can be simplify to this one :

$$E_t^B = \sum_{i=1}^{t-1} \Big[n^{B+} \cdot P_i^{charge} - P_i^{discharge}/n^{B-}) \cdot (n^B)^{t-1-i} \Big] \tag{2}$$

This new equation must be verified at each time step t. However the goal was to totally suppress the variable E^B. This can be done by remembering that the last equation of our previous model was that E_t^B should be lower than E^{Bmax}. The final constraint is :

$$E^{Bmax} \geq \sum_{i=1}^{t-1} \Big[n^{B+} \cdot P_i^{charge} - P_i^{discharge}/n^{B-}) \cdot (n^B)^{t-1-i} \Big] \tag{3}$$

The performance are much lower with this new model compare to the first one. Indeed, it takes about 85 seconds for this new model whereas the previous achieve the optimization in 2 seconds. It can be explain by the fact that without the variable E^B, we loose a lot of time to compute sum that has been already computed. Indeed, the variable E_B was helpful to stored theses computations and to avoid to recompute previous computed sums.

5 Q5

In this question, it is asked to compute the sensitivity of the model through the parameter π_G. For this purpose, the function `lp_sensitivity_report` is used. This function sends back the variation of π_G that are acceptable without changing the optimal basis. It means that if the parameter goes below $\pi^G - lowerbound$ or above $\pi_G + upperbound$, the optimal basis would change. The values are given in the Table 2 :

π^G interval	1 year
lower bound[$/MW]	1238.49
upper bound[$/MW]	1245.46

TABLE 2 – Interval outside of which the optimal basis would change

So if $\pi^G \notin [\pi^G - 6.51; \pi^G + 0.46]$, the optimal basis has changed meaning that the optimal solution in the previous basis is not optimal in the new basis because the basic solution of the primal and dual problems are not both feasible in this new basis. The new solution is computed by changing the objective function. The new objective function is :

$$\min_{C^G, C^{PV}, E^{Bmax}} (\pi^G + \Delta\pi^G) \cdot C^G + \pi^{PV} \cdot C^{PV} + \pi^B \cdot E^{Bmax} + \pi^D \cdot \sum_{i=1}^{N} P_i^D \tag{4}$$

The execution time of this part is much longer than previously.

The optimal solution of the new function increases when $\Delta\pi^G$ is positive and decreases when $\Delta\pi^G$ is negative.

6 Q6

The primal solution in the optimal basis remains optimal while π^G stays inside the interval previously defined. However, the corresponding dual solution might change since a cost coefficient is perturbed in the objective function.

On the other hand, the interval associated with a constraint is the interval of the allowed perturbation of the corresponding right-hand side coefficient without changing the optimal basis. In this interval the current dual solution remains optimal, but the optimal primal solution might change.

7 Q7

For this question we create 2 additional constraints to express the fact that the production of the generator can only change by 10 %. Here are the two constraints added to the model of question 1 :

$$(P_i^D - P_{i-1}^D) \cdot n^G \leq C^G \cdot 0.1$$
$$(P_i^D - P_{i-1}^D) \cdot n_G \geq -C_G \cdot 0.1$$

The results from the optimization are available in Table 3 :

	1 year
$C^{PV}[MW]$	747.48
C^G[MW]	28.696
$E^{Bmax}[MWh]$	325.73
Optimal cost [$]	1 103 115.31

TABLE 3 – Optimization results for question 7

Compare to the results of Q2, the capacity of each part of the micro-grid increases and the optimal solution also. The change in capacity of the generator between two time step is now limited. Then the generator will sometimes not be able to produce the amount of power it has to. To compensate this, a part of the power of the generator is supply by the pv-panels so its capacity increases. Moreover, a part of the power produced by the generator is stocked inside the battery to prepare enough energy in advance, so the capacity of the battery raises also. Thirdly the capacity of the generator increases to be able to produce more energy at time t even if the change in production is limited.

For this question we can apply the same principle than for the previous one. In this case, the objective function is unchanged while the constraints are change. By adding those constraints, the primal solution may change while the dual is not affected. To calculate the new optimal solution, the dual problem can be used to "reoptimize" the model without starting from 0 and then to find the primal.

8 Q8

Assuming that the solar irradiance is subject to $\epsilon = \pm 7.5\%$ uncertainty, we must provide a pessimistic model also called robust model in order to be prepare for best ($\epsilon = +7.5\%$) and worst ($\epsilon = -7.5\%$) scenarios. Indeed, the irradiance is not a deterministic variable anymore and we must consider at each time t all the possible situations to cover the whole uncertainty range. The uncertainty set for the irradiance is defined at each time step t as :

$$|\epsilon| \leq 0.075$$

$$(1-\epsilon) \cdot \hat{irra}_t \leq irra_t \leq (1+\epsilon) \cdot \hat{irra}_t$$

where $\hat{irra}_t$ is the irradiance value at time t without uncertainty (same as the deterministic model).
The best case corresponds to $\epsilon = 0.075$ which is when the irradiance is at its maximum at each time step t. This case does not need to be handle because the PV-panels can be cover to avoid overload power in the microgrid.
The worst case is the opposite, so when the irradiance is at the lowest at each time step t meaning that the PV power production will be the lowest. We can thus write our new constraint that satisfies all the possible values in the uncertainty set :

$$P_t^{PV} \leq n^{PV} \cdot (1-\epsilon) \cdot \hat{irra}_t \cdot C^{PV}$$

The results are given in Table 4. As expected, we see that robust optimization is more pessimistic than the deterministic one. Indeed, the total cost is 36 865$ larger for the robust optimization because there is less PV capacity and so more use of the genset meaning more diesel consumption.

	Robust	Deterministic
$C^{PV}[MW]$	695.49	743.93
C^G[MW]	8.93	8.58
$E^{Bmax}[MWh]$	325.73	323.13
Optimal cost [$]	1 078 018.70	1 041 153.77

TABLE 4 – Comparison between robust and deterministic optimization results, over 1 year

9 Q9

To determine the minimum dimensioning of the PV panels to avoid having to install a diesel generator, using the previous robust model, we removed the variables C^G and P_t^D.
Results are in Table 5 and we found that the minimum PV panels capacity is 1769.3 MW. This result was expected because it almost corresponds to the results we obtained from the model of question 1 over 5 years, without generator aswell. As the power load is still the same, the power produced by the battery is the same meaning that the battery capacity does not change. However, to obtain the same results for the PV capacity, we must remember that we used a pessimistic model, thus to provide this quantity of energy to the battery, the PV panels capacity must increase by $\frac{1}{1-\epsilon} \cdot 1636.61 = 1769.3$ MW.

	Robust	Deterministic
$C^{PV}[MW]$	1769.3	1636.61
$E^{Bmax}[MWh]$	749.14	749.14
Optimal cost [$]	1 415 975.29	1 334 498.86

TABLE 5 – Comparison between robust and deterministic optimization results, without genset