Causal Reasoning by Alexander Bochman

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History: Overview of Bochman's Work on Causality

Relevant for my presentation:

- [1] *A Logical Theory of Causality*. 2021
- **[2] "Causal reasoning from almost first principles". 2024**

Early work:

- [3] "A logic for causal reasoning". 2003
- [4] "A causal approach to nonmonotonic reasoning". 2004

Exploring various aspects:

- [5] "Actual Causality in a Logical Setting". 2018
- [6] "Default Logic as a Species of Causal Reasoning". 2023
- [7] "An Inferential Theory of Causal Reasoning". 2023

- "The interpretation of probability as propensity leads people to base their judgments of likelihood primarily on causal considerations, and to ignore information that does not have causal significance."[8]
- "The possibility to learn causal relationships from raw data has been on philosophers' dream list since the time of Hume (1711-1776)" [9]
- " causal intent, inference, implications, and recommendations — is common " (in the observational health literature) [10]

... but how to formalize causality in formal logic?

[8] Kahneman. *Thinking, Fast and Slow*. 2013

[9] Pearl. *Causality: Models, Reasoning, and Inference*. 2000

[10] Haber et al. "Causal and Associational Language in Observational Health Research: A Systematic Evaluation". 2022

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Bochman's approach

- Causation is multifaceted: no formalization of concepts
- Approach: "investigate important variants of causal reasoning"[2]
- Definition of systems of reasoning:
  - Language consists of set of (causal) inference rules defined on a set of propositions
  - Semantics: valuations of propositions (with causal rules)
  - Causal Inference: formal derivations preserving rational semantics
- Distinction: rational semantics vs. causal theory
- Inherently nonmonotonic

- … general logical formalism of causal reasoning [5]
- 2 layers: nonmonotonic semantics + logics of causal rules [4]

Causal Language

- Built on top of classical logic
- Language: set of causal rules
- Underlying language L: set of proposition

Basics: classical propositional language [1, p. 79]

- $\wedge, \vee, \neg, \rightarrow, t, f$: classical connectives & constants
- Th ($\vdash$): Syntactic *provability*, based on formal proof rules.
- $\vDash$: Semantic *entailment*, based on truth in all models.
- *p,g,r* finite sets of propositions; *A, B, C* classical propositions

Causal rule

- Causal binary relation / causal rule: $a \Rightarrow B$: *a causes B*
- Causal theory Δ: arbitrary set of causal rules

Principles of Acceptance

- Causal Acceptance Principle: B is *accepted iff* $a \Rightarrow B$, where all A in a are accepted
- **Preservation** Principle: If all propositions in a are accepted, and a causes B, then B should be accepted.
- Principle of **Sufficient Reason**: Any proposition should have a cause for its acceptance.

Valuation on propositions for describing semantics

- function $v \in \{0, 1\}^L$
- $v(A) = 1$: proposition A is accepted ('taken-true')
- $v(A) = 0$: non-acceptance is not rejection of A
- rejection: $v(\neg A) = 1$
- $\Delta(u) = \{B \mid a \Rightarrow B \in \Delta,\ a \subseteq u\}$

- Causal model: fixed point of (accepted) propositions $v = \Delta(v)$
- Least model $\Delta()$: smallest model
  - $u_0 = \emptyset, \quad u_1 = \Delta(u_0), \quad u_2 = \Delta(u_1), \ldots$
- Rational Semantics: set of all causal models of a theory

Example

Causal theory

$$\text{Rained} \Rightarrow \text{Grasswet}$$

$$\text{Sprinkler} \Rightarrow \text{Grasswet}$$

$$\text{Rained} \Rightarrow \text{Streetwet}$$

Formal derivations "*metainferences*" among causal rules that always preserve the rational semantics:

- **Monotonicity:** If $a \Rightarrow A$ and $a \subseteq b$, then $b \Rightarrow A$.
- **Cut:** If $a \Rightarrow A$ and $a, A \Rightarrow B$, then $a \Rightarrow B$.

Reflexivity and Causal assumptions

- $A \Rightarrow A$ does **not** hold by default
- $A \Rightarrow A$ is a self-evident proposition that does not require further justification for its acceptance

- $\Rightarrow_\Delta$: all causal rules that are derivable from Δ; $a \Rightarrow_\Delta B$
- Any causal theory Δ is semantically equivalent to $\Rightarrow_\Delta$

- $\mathcal{C}(u)$ is the set of propositions (A) that are caused by a set u: $\mathcal{C}(u) = \{A \mid u \Rightarrow A\}$
- Similar to the derivability operator $Th(u)$
- *Monotonicity*: adding more propositions to the set u, the set of consequences $\mathcal{C}(u)$ only grows or stays the same – it cannot shrink.
  - If $u \subseteq v$, then $\mathcal{C}(u) \subseteq \mathcal{C}(v)$
- *Deductive Closure*: $\mathcal{C}(u)$ always results in a *deductively closed set*: all logical consequences of the elements in $\mathcal{C}(u)$ are also included in $\mathcal{C}(u)$
  - $\mathcal{C}(u) = Th(\mathcal{C}(u))$
  - e.g.: if the rain (u) causes the street to be wet $(\mathcal{C}(u))$, it also causes the logical consequences $Th(\mathcal{C}(u))$ (e.g., a slippery street)
- Transitivity: if $A \Rightarrow B$ and $B \Rightarrow C$, then $A \Rightarrow C$

Non-inclusivity of $\mathcal{C}$

u is not necessarily a subset of $\mathcal{C}(u)$: $u \not\subseteq \mathcal{C}(u)$

- set of propositions that are caused by u (i.e., $\mathcal{C}(u)$) do not always contain u itself
- just because a proposition is in u it must not appear in the set of consequences caused by u
- Non-inclusivity related to (non-) reflexivity
- Contrast to derivability operator Th: original set is always included in the set of derivable propositions

Example: $\mathcal{C}$(rain)

a specific raining event (u) has a causal consequences ($\mathcal{C}(u)$, e.g. a wet street), but does not necessarily cause itself (but perhaps other raining events u')

Causal Operator $\mathcal{C}$ III

Non-idempotence of $\mathcal{C}$

- $\mathcal{C}(\mathcal{C}(u)) \neq \mathcal{C}(u)$
- a set of propositions that are caused by u ($\mathcal{C}(u)$) might cause another set of propositions $\mathcal{C}(\mathcal{C}(u))$
- causation propagates across several steps (cascading effects)

Example: $u = \{\text{rain}\}$

- 1. Step: Direct causal effects of u: $\mathcal{C}(u) = \{\text{wet street}\}$
  - causes logical consequences $Th(\mathcal{C}(u))$ directly: slippery street
- 2. Step: Effects of $\mathcal{C}(u) = \{\text{slippery street}\}$:
  - $\mathcal{C}(\mathcal{C}(u)) = \{\text{people slip on the street}, \text{traffic slows down}\}$
- 3. Step: Effects of $\mathcal{C}(\mathcal{C}(u))$:
  - $\mathcal{C}(\mathcal{C}(\mathcal{C}(u))) = \{\text{people get hurt}, \text{cloth get wet}\}$

- A **causal theory** is a set of causal rules (conditionals) that define the causal behavior within a certain framework.
- $\mathcal{C}_\Delta$: For any causal theory Δ, there is a least production relation, that includes all causal rules derivable from Δ.

If theory Δ states:

$$\{A \Rightarrow B, B \Rightarrow C\},$$

then for a premise set $u = \{A\}$, one can systematically derive

$$\mathcal{C}_\Delta(u) = \{B, C\},$$

representing both intermediate and direct effects triggered by A.

Causal Inference ($\Rightarrow$) vs. Deductive Consequence ($\vdash_\Rightarrow$)

- $\Rightarrow$: reasoning based on *cause-and-effect* relationships
- $\vdash_\Rightarrow$: propositional theory about the consequence relation (rules + results).
- $\Rightarrow$ has the same propositional theories as $\vdash_\Rightarrow$
- **But**: causal reasoning retains extra causal information beyond what is encoded in the propositional theories.

Example

- *Rained* $\Rightarrow$ *Grasswet*, *Sprinkler* $\Rightarrow$ *Grasswet* lead to *Grasswet*
- **But** the specific causal mechanisms (rain vs. sprinkler) are part of the causal model, not captured in the resulting propositional theory.

Equivalence

- Logically equivalent Δ are semantically equivalent
- Reverse does not hold!

- Two theories:

$$\Delta = \{A \Rightarrow B\}, \Phi = \{A \Rightarrow C\}$$

- different, determine same rational semantics $\Delta() = \Phi() = \emptyset$
- add causal rule $A \Rightarrow A$ to both
- $\Delta() = \{A, B\}, \quad \Phi() = \{A, C\}$

Axioms vs. Assumptions

- **Axiom**: $\emptyset \Rightarrow A$
- **Causal Assumption**: $A \Rightarrow A$
- (Relationship to abductive reasoning [1])

Original Example: Rained, Sprinkler, Streetwet

- by itself, this causal theory has a single empty causal model $\emptyset$
- add assumptions: $Rained \Rightarrow Rained \quad Sprinkler \Rightarrow Sprinkler$
- Rational semantics of this causal theory: three causal models
- $\{Rained, Grasswet, Streetwet\}$, $\{Sprinkler, Grasswet\}$, $\{Rained, Sprinkler, Grasswet, Streetwet\}$

- Aim: integrating causal reasoning into a more comprehensive reasoning system
- **Classical entailment** (formal logical reasoning): integral part of the causal system
- Supraclassical: extends and includes classical entailment, to naturally incorporate classical logic
- "Causal reasoning is not a replacement or competitor of logical (deductive) reasoning, but its complement (or extension)" [1]
- *(Strengthening)* If $A \vDash B$ and $B \Rightarrow C$, then $A \Rightarrow C$;
- *(Weakening)* If $A \Rightarrow B$ and $B \vDash C$, then $A \Rightarrow C$;
- *(And)* If $A \Rightarrow B$ and $A \Rightarrow C$, then $A \Rightarrow (B \wedge C)$;
- *(Truth and Falsity)* $t \Rightarrow t$; $f \Rightarrow f$.

Further Properties of Causal Reasoning

- **Default causal theory**: pair $(\Delta, \mathcal{D})$, $\mathcal{D}$ a subset of causal assumptions
- **Nonmonotonicity**
  - Context-Sensitive Causal Reasoning
  - Causal acceptance is directional
- **Structural Equation Models**: representation in the causal calculus
- Counterfactual Equivalence: related to SEM
- Negative Causal Completion: negation as default $(\neg p \Rightarrow \neg p)$

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- Explainable AI [2]
- Causal attribution (actual causality) in legal theory [5]

Take away messages

Bochman's causal reasoning...

- Unified approach: integrate Pearl's approach to causation, classical entailment within causal reasoning
- Nonmonotonic reasoning
- Asymmetry in Semantics
- Roots in Historical Reasoning
- Connection to Inferentialism (rule-based vs. representational)

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References I

[1] Alexander Bochman. *A Logical Theory of Causality*. The MIT Press, Aug. 17, 2021. ISBN: 978-0-262-36620-5. DOI: 10.7551/mitpress/12387.001.0001.

[2] Alexander Bochman. "Causal reasoning from almost first principles". In: *Synthese* 203.1 (Jan. 4, 2024), p. 19. ISSN: 1573-0964. DOI: 10.1007/s11229-023-04442-6.

[3] Alexander Bochman. "A logic for causal reasoning". In: *Proceedings of the 18th international joint conference on Artificial intelligence*. IJCAI'03. San Francisco, CA, USA: Morgan Kaufmann Publishers Inc., Aug. 9, 2003, pp. 141–146.

[4] Alexander Bochman. "A causal approach to nonmonotonic reasoning". In: *Artificial Intelligence* 160.1 (Dec. 1, 2004), pp. 105–143. ISSN: 0004-3702. DOI: 10.1016/j.artint.2004.07.002.

[5] Alexander Bochman. "Actual Causality in a Logical Setting". In: *Proceedings of the Twenty-Seventh International Joint Conference on Artificial Intelligence*. Twenty-Seventh International Joint Conference on Artificial Intelligence {IJCAI-18}. Stockholm, Sweden: International Joint Conferences on Artificial Intelligence Organization, July 2018, pp. 1730–1736. ISBN: 978-0-9992411-2-7. DOI: 10.24963/ijcai.2018/239.

[6] Alexander Bochman. "Default Logic as a Species of Causal Reasoning". In: *Proceedings of the International Conference on Principles of Knowledge Representation and Reasoning* 19.1 (Aug. 1, 2023). Conference Name: Proceedings of the 20th International Conference on Principles of Knowledge Representation and Reasoning, pp. 117–126. ISSN: 2334-1033. DOI: 10.24963/kr.2023/12.

[7] Alexander Bochman. "An Inferential Theory of Causal Reasoning". In: *Logic, Rationality, and Interaction*. Ed. by Natasha Alechina, Andreas Herzig, and Fei Liang. Cham: Springer Nature Switzerland, 2023, pp. 1–16. ISBN: 978-3-031-45558-2. DOI: 10.1007/978-3-031-45558-2_1.

[8] Daniel Kahneman. *Thinking, Fast and Slow*. First Edition. New York: Farrar, Straus and Giroux, Apr. 2, 2013. 512 pp. ISBN: 978-0-374-53355-7.

[9] Judea Pearl. *Causality: Models, Reasoning, and Inference*. Google-Books-ID: wnGU_TsW3BQC. Cambridge University Press, Mar. 13, 2000. 412 pp. ISBN: 978-0-521-77362-1.

[10] Noah A. Haber et al. "Causal and Associational Language in Observational Health Research: A Systematic Evaluation". In: *American Journal of Epidemiology* 191.12 (Nov. 19, 2022), pp. 2084–2097. ISSN: 1476-6256. DOI: 10.1093/aje/kwac137.

References III

[11] Judea Pearl. "Embracing causality in default reasoning". In: *Artificial Intelligence* 35.2 (June 1, 1988), pp. 259–271. ISSN: 0004-3702. DOI: 10.1016/0004-3702(88)90015-X.

[12] Vladimir Lifschitz. "On the logic of causal explanation". In: *Artificial Intelligence* 96.2 (Nov. 1, 1997), pp. 451–465. ISSN: 0004-3702. DOI: 10.1016/S0004-3702(97)00057-X.

[13] Hector Geffner. "Causal theories for nonmonotonic reasoning". In: *Proceedings of the eighth National conference on Artificial intelligence - Volume 1*. AAAI'90. Boston, Massachusetts: AAAI Press, July 29, 1990, pp. 524–530. ISBN: 978-0-262-51057-8.

[14] Hudson Turner. "A logic of universal causation". In: *Artificial Intelligence* 113.1 (Sept. 1, 1999), pp. 87–123. ISSN: 0004-3702. DOI: 10.1016/S0004-3702(99)00058-2.

[15] D. Dash. "Causal Logic Models". In: 2012.

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History: Other relevant authors and papers

Some of the most influential authors:

- [11] Pearl. "Embracing causality in default reasoning". 1988
- [9] Pearl. *Causality*. 2000
- [12] Lifschitz. "On the logic of causal explanation". 1997
- [13] Geffner. "Causal theories for nonmonotonic reasoning". 1990
- [14] Turner. "A logic of universal causation". 1999
- [15] Dash. "Causal Logic Models". 2012

... and many others

Overview

Syntactic Provability ($\vdash$)

Definition: Th ($\vdash$) represents syntactic provability, meaning a proposition A can be formally derived using axioms and formal inference rules. Provability is about whether a proposition A can be formally derived from a set of axioms or premises Γ using a given set of syntactic proof rules within a formal system.

Key Idea:

- A proposition A is provable (written as $Th \vdash A$ if there is a formal proof sequence for A.
- Relies entirely on the structure and rules of a formal system (e.g., axioms, inference rules like Modus Ponens).
- Does not involve interpretations or models; it's purely rule-based reasoning.
- Following strict rules to deduce a conclusion.

Semantic Entailment ($\vDash$)

Definition: $\vDash$ represents semantic entailment, meaning a proposition A is true in any model where the assumptions Γ hold. Entailment is about whether A is true in every possible model (interpretation) where a set of premises Γ is true.

Key Idea:

- Determines logical consequence based on truth in all models, not formal derivation.
- Like checking universal truths: something is true in all possible worlds where the premises hold.
- Depends on truth values assigned to propositions in all models of the logic. Assesses whether a logical proposition holds in a model-theoretic sense.

Propositional Atoms (p, g, r)

Definition: Propositional atoms are the smallest, indivisible units of logic, representing atomic facts.
Example:

$$p = \text{"It is raining"}, \quad g = \text{"The grass is wet"}, \quad r = \text{"It is windy"}.$$

No further breakdown is possible for these logical units.

Classical Propositions (A, B, C)

Definition: Classical propositions are statements formed by combining propositional atoms with logical connectives $(\wedge, \vee, \neg, \rightarrow)$ or truth constants (t, f).
Example:

$$A = p \wedge g, \quad B = p \vee \neg r, \quad C = \neg g \rightarrow r.$$

These statements are more complex and their truth values depend on logical interpretation.

Summary

- Syntactic Provability ($\vdash$): Formal derivation based on rules.
- Semantic Entailment ($\vDash$): Truth across all models.
- Propositional Atoms: Indivisible units (p, g, r).
- Classical Propositions: Composite logical statements (A, B, C).

These concepts bridge syntactic and semantic aspects of logic, fundamental to understanding relationships in formal reasoning systems.

- **Provability**: $Th \vdash Grasswet$
  Derive *Grasswet* syntactically using axioms like $Rained \rightarrow Grasswet$.
- **Entailment**: $Rained \vDash Grasswet$
  Grasswet is true in every model where *Rained* is true.
- **Propositional Atoms**: *Rained*, *Grasswet*, and *Streetwet* are indivisible facts used to construct more complex statements.
- **Classical Proposition**: $Rained \rightarrow (Streetwet \vee Grasswet)$
  A composite logical statement describing causal or correlative relationships.

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Highlighting Text

We can highlight text. There are also a darker option, and a yellow option.

The same options are also available in math mode: $a + b = c$

Blocks

Block

This is a regular block.

- This is an item in a block.

Block

This is an alert block.

- This is an item in an alert block.

Block

This is an example block.

- This is an item in an example block.