Causal Reasoning by Alexander Bochman

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2 Logical Formal Theory of Causal Reasoning
3 Closing remarks

History: Overview of Bochman's Work on Causality

Relevant for my presentation:

- [1] *A Logical Theory of Causality*. 2021
- **[2] "Causal reasoning from almost first principles". 2024**

Early work:

- [3] "A logic for causal reasoning". 2003
- [4] "A causal approach to nonmonotonic reasoning". 2004

Exploring various aspects:

- [5] "Actual Causality in a Logical Setting". 2018
- [6] "Default Logic as a Species of Causal Reasoning". 2023
- [7] "An Inferential Theory of Causal Reasoning". 2023

History: Overview of Bochman's Work on Causality
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- historical context
- 2021: book - fundamental explanations, detailed introduction
- no references before about 2000
- explored various aspects of causal reasoning over years (summarised in book)
- most recent work 2024 with a concise and fundamental approach
- no co-authors

Relevance of causality

- "The interpretation of probability as propensity leads people to base their judgments of likelihood primarily on causal considerations, and to ignore information that does not have causal significance."[8]
- "The possibility to learn causal relationships from raw data has been on philosophers' dream list since the time of Hume (1711-1776)" [9]
- " causal intent , inference, implications, and recommendations — is common " (in the observational health literature) [10]

... but how to formalize causality in formal logic ?

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[9] Pearl. *Causality: Models, Reasoning, and Inference*. 2000

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- (impression of) causality is relevant in daily lives & science

1. Kahneman: humans tend to interpret their observations in causal terms / cause and effect
2. Pearl: causal inference from observations $\neq$ experiments
   - Structural Causal Model uses structural equation modeling
   - Pearl "The Book of Why: The New Science of Cause and Effect" (2018), 'do-calculus, counterfactuals'
   - **Causal inference** process of determining independent, actual effect of a particular phenomenon that is a component of a larger system.
   - **Causal reasoning** is the process of identifying causality: the relationship between a cause and its effect.
3. show, that hints about causation (and derived recommendations) are common in literature; "causation" is not mentioned directly

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Bochman's approach

- Causation is multifaceted: no formalization of concepts
- Approach: "investigate important variants of causal reasoning"[2]
- Definition of systems of reasoning:
  - Language consists of set of (causal) inference rules defined on a set of propositions
  - Semantics : valuations of propositions (with causal rules)
  - Causal Inference : formal derivations preserving rational semantics
- Distinction: rational semantics vs. causal theory
- Inherently nonmonotonic

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- concepts: causation, proposition, acceptance
- phil. terminology: rationality, normativity, reasons, explanations
- principles and constructions: inherently normative character
- normative: expressing value judgments / not stating facts
- flexible exploration

- Semantics based on causal principle of acceptance for propositions
- Causal theory describes "why" certain propositions should be accepted through causal rules.
- Rational semantics only show which propositions should be accepted but don't explain the original "why."
- **Distinction**: cannot reconstruct the full causal origins of accepted propositions from their semantics alone

Causal Calculus

- ... general logical formalism of causal reasoning [5]
- 2 layers: nonmonotonic semantics + logics of causal rules [4]

Causal Language

- Built on top of classical logic
- Language: set of causal rules
- Underlying language L: set of proposition

Basics: classical propositional language [1, p. 79]

- $\wedge, \vee, \neg, \rightarrow, t, f$: classical connectives & constants
- Th ($\vdash$): Syntactic *provability*, based on formal proof rules.
- $\vDash$: Semantic *entailment*, based on truth in all models.
- p,g,r finite sets of propositions; A, B, C classical propositions

Causal Calculus
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- 2021: two chapters about causal calculus:
  - "classic" Geffner 92, McCain and Turner 97, Lifschitz 97, Pearl
  - The causal calculus can be seen as a most natural and immediate generalization of classical logic that allows for causal reasoning.
- Knowledge can be stored as cause - effect relationships

Causal rule

- Causal binary relation / causal rule: $a \Rightarrow B$: *a causes B*
- Causal theory Δ: arbitrary set of causal rules

Principles of Acceptance

- Causal Acceptance Principle: B is *accepted iff* $a \Rightarrow B$, where all A in a are accepted
- **Preservation** Principle: If all propositions in a are accepted, and a causes B, then B should be accepted.
- Principle of **Sufficient Reason**: Any proposition should have a cause for its acceptance.

Causal Theories and their Semantics
Causal rule
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Principles of Acceptance
Causal Acceptance Principle: B is *accepted iff* $a \Rightarrow B$, where all A in a are accepted
Preservation Principle: If all propositions in a are accepted, and a causes B, then B should be accepted.
Principle of **Sufficient Reason**: Any proposition should have a cause for its acceptance.

- causal theory Δ: arbitrary set of causal rules with constraints on *acceptance* of propositions
- Causal Acceptance Principle: A proposition capital B is accepted with respect to a causal theory delta, iff a set of propositions (lowercase) a causes B; where all propositions in a are accepted
- Preservation Principle: inference rules "transmit" acceptance
- Leibniz' Principle of Sufficient Reason: normative, propositions require reasons for their acceptance

Valuation on propositions for describing semantics

- function $v \in \{0,1\}^L$
- $v(A) = 1$: proposition A is accepted ('taken-true')
- $v(A) = 0$: non-acceptance is not rejection of A
- rejection: $v(\neg A) = 1$
- $\Delta(u) = \{B \mid a \Rightarrow B \in \Delta,\ a \subseteq u\}$

- Causal model: fixed point of (accepted) propositions $v = \Delta(v)$
- Least model $\Delta()$: smallest model
  - $u_0 = \emptyset, \quad u_1 = \Delta(u_0), \quad u_2 = \Delta(u_1), \ldots$
- Rational Semantics: set of all causal models of a theory

- For any set u of propositions and a causal theory Δ: $\Delta(u)$ the set of all propositions B that are directly caused by u in Δ
- $\Delta(u)$ monotonic operator; $\subseteq$ = subset
- causal model: determines that a proposition is accepted in a model if and only if it has a cause in this model.
- Fixed Point: applying the operator Δ to u doesn't change the set: $\Delta(u) = u$
- any causal theory has at least one causal model
- Causal Models: defined as a fixed point of the operator $\Delta(u)$.
- least model: iteratively applied, starting with empty set $\varnothing$
- Rational Semantics: full set of causal models (not just the least model) that satisfy Δ. These models provide a range of valid interpretations of the theory.

Causal theory

$$\text{Rained} \Rightarrow \text{Grasswet}$$

$$\text{Sprinkler} \Rightarrow \text{Grasswet}$$

$$\text{Rained} \Rightarrow \text{Streetwet}$$

Example
Causal theory
Rained ⇒ Grasswet
Sprinkler ⇒ Grasswet
Rained ⇒ Streetwet

- if *Rained* is accepted (with respect to such a causal theory)
- both *Grasswet* and *Streetwet* should be accepted
- any acceptable set of propositions that contains *Grasswet* should either contain *Rained* or *Sprinkler* as its causes
- defications from causes to their effects and from effects to their possible causes are essential parts of causal reasoning

Formal derivations "*metainferences*" among causal rules that always preserve the rational semantics:

- **Monotonicity:** If $a \Rightarrow A$ and $a \subseteq b$, then $b \Rightarrow A$.
- **Cut:** If $a \Rightarrow A$ and $a, A \Rightarrow B$, then $a \Rightarrow B$.

Reflexivity and Causal assumptions

- $A \Rightarrow A$ does **not** hold by default
- $A \Rightarrow A$ is a self-evident proposition that does not require further justification for its acceptance

- $\Rightarrow_\Delta$: all causal rules that are derivable from Δ; $a \Rightarrow_\Delta B$
- Any causal theory Δ is semantically equivalent to $\Rightarrow_\Delta$

Causal Inference
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- "If a rule a causes A holds and a is extended to a larger set b, the rule still holds."
- "If a causes A and the combined set of a,A casuse B, then a casues B can be inferred by combining these rules."
- Reflexivity postulate: first postulate of Tarski consequence
- Reflexivity would make propositions self-justified (self-evident)
- one of the key differences between causal inference and deductive consequence
- 'omission' creates the possibility of causal reasoning
- a causally implies B under the causal theory Δ
- The definition of $\Rightarrow_\Delta$ is not about a single, specific causal rule, but rather about a generalized statement capturing all causal relationships derivable from the causal theory Δ

- $\mathcal{C}(u)$ is the set of propositions (A) that are caused by a set u: $\mathcal{C}(u) = \{A \mid u \Rightarrow A\}$
- Similar to the derivability operator $Th(u)$
- *Monotonicity* : adding more propositions to the set u, the set of consequences $\mathcal{C}(u)$ only grows or stays the same – it cannot shrink.
  - If $u \subseteq v$, then $\mathcal{C}(u) \subseteq \mathcal{C}(v)$
- *Deductive Closure* : $\mathcal{C}(u)$ always results in a *deductively closed set*: all logical consequences of the elements in $\mathcal{C}(u)$ are also included in $\mathcal{C}(u)$
  - $\mathcal{C}(u) = Th(\mathcal{C}(u))$
  - e.g.: if the rain (u) causes the street to be wet $(\mathcal{C}(u))$, it also causes the logical consequences $Th(\mathcal{C}(u))$ (e.g., a slippery street)
- Transitivity : if $A \Rightarrow B$ and $B \Rightarrow C$, then $A \Rightarrow C$

Causal Operator $\mathcal{C}$ I
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Non-inclusivity of $\mathcal{C}$

u is not necessarily a subset of $\mathcal{C}(u)$: $u \not\subseteq \mathcal{C}(u)$

- set of propositions that are caused by u (i.e., $\mathcal{C}(u)$) do not always contain u itself
- just because a proposition is in u it must not appear in the set of consequences caused by u
- Non-inclusivity related to (non-) reflexivity
- Contrast to derivability operator Th: original set is always included in the set of derivable propositions

Example: $\mathcal{C}(\textbf{rain})$

a specific raining event (u) has a causal consequences ($\mathcal{C}(u)$, e.g. a wet street), but does not necessarily cause itself (but perhaps other raining events u')

Causal Operator $\mathcal{C}$ II

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Causal Operator $\mathcal{C}$ III

Non-idempotence of $\mathcal{C}$

- $\mathcal{C}(\mathcal{C}(u)) \neq \mathcal{C}(u)$
- a set of propositions that are caused by u ($\mathcal{C}(u)$) might cause another set of propositions $\mathcal{C}(\mathcal{C}(u))$
- causation propagates across several steps (cascading effects)

Example: $u = \{\text{rain}\}$

- 1. Step: Direct causal effects of u: $\mathcal{C}(u) = \{\text{wet street}\}$
  - causes logical consequences $Th(\mathcal{C}(u))$ directly: slippery street
- 2. Step: Effects of $\mathcal{C}(u) = \{\text{slippery street}\}$:
  - $\mathcal{C}(\mathcal{C}(u)) = \{\text{people slip on the street, traffic slows down}\}$
- 3. Step: Effects of $\mathcal{C}(\mathcal{C}(u))$:
  - $\mathcal{C}(\mathcal{C}(\mathcal{C}(u))) = \{\text{people get hurt, cloth get wet}\}$

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- A **causal theory** is a set of causal rules (conditionals) that define the causal behavior within a certain framework.
- $\mathcal{C}_\Delta$: For any causal theory Δ, there is a least production relation, that includes all causal rules derivable from Δ.

If theory Δ states:

$$\{A \Rightarrow B, B \Rightarrow C\},$$

then for a premise set $u = \{A\}$, one can systematically derive

$$\mathcal{C}_\Delta(u) = \{B, C\},$$

representing both intermediate and direct effects triggered by A.

Causal Theories and Expanded Notions

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representing both intermediate and direct effects triggered by A.

Causal Inference ($\Rightarrow$) vs. Deductive Consequence ($\vdash_\Rightarrow$)

- $\Rightarrow$: reasoning based on *cause-and-effect* relationships
- $\vdash_\Rightarrow$: propositional theory about the consequence relation (rules + results).
- $\Rightarrow$ has the same propositional theories as $\vdash_\Rightarrow$
- **But**: causal reasoning retains extra causal information beyond what is encoded in the propositional theories.

Example

- *Rained* $\Rightarrow$ *Grasswet*, *Sprinkler* $\Rightarrow$ *Grasswet* lead to *Grasswet*
- **But** the specific causal mechanisms (rain vs. sprinkler) are part of the causal model, not captured in the resulting propositional theory.

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- **But** the specific causal mechanisms (rain vs. sprinkler) are part of the causal model, not captured in the resulting propositional theory.

- **Causal Inference ($\Rightarrow$)**: Refers to reasoning based on *cause-and-effect* relationships defined by a causal theory.
- **Deductive logic**: the propositional theory tells everything about the consequence relation (rules + results).
- Causal inference relation $\Rightarrow$ has the same propositional theories as the corresponding consequence relation $\vdash_\Rightarrow$

Equivalence

- Logically equivalent Δ are semantically equivalent
- Reverse does not hold!

- Two theories:

$$\Delta = \{A \Rightarrow B\}, \Phi = \{A \Rightarrow C\}$$

- different, determine same rational semantics $\Delta() = \Phi() = \emptyset$
- add causal rule $A \Rightarrow A$ to both
- $\Delta() = \{A, B\}, \quad \Phi() = \{A, C\}$

Equivalence
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- different, determine same rational semantics $\Delta() = \Phi() = \emptyset$
- add causal rule $A \Rightarrow A$ to both
- $\Delta() = \{A, B\}, \quad \Phi() = \{A, C\}$

- Two causal theories are logically equivalent, if each can be obtained from the other using the postulates of causal inference
- Reverse: rational semantics does not fully determine the content of the original causal theory.
- develops a notion of *strong semantic qeuivalency*
- Example: single model $\emptyset$, no props. accepted

Axioms vs. Assumptions

- **Axiom**: $\emptyset \Rightarrow A$
- **Causal Assumption**: $A \Rightarrow A$
- (Relationship to abductive reasoning [1])

Original Example: Rained, Sprinkler, Streetwet

- by itself, this causal theory has a single empty causal model $\emptyset$
- add assumptions: $\mathit{Rained} \Rightarrow \mathit{Rained}$ $\quad \mathit{Sprinkler} \Rightarrow \mathit{Sprinkler}$
- Rational semantics of this causal theory: three causal models
- $\{\mathit{Rained}, \mathit{Grasswet}, \mathit{Streetwet}\}$, $\{\mathit{Sprinkler}, \mathit{Grasswet}\}$, $\{\mathit{Rained}, \mathit{Sprinkler}, \mathit{Grasswet}, \mathit{Streetwet}\}$

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- Law of causality (Leibniz): any accepted proposition should have an accepted cause
- A proposition A will be called an **axiom** of a causal theory Δ if the rule $\emptyset \Rightarrow A$ belongs to Δ;
- axioms do not require justification, must be accepted in any model
- A proposition A will be called a **causal assumption** of a causal theory if the rule $A \Rightarrow A$ belongs to it.
- self-evident propositions, can be incorporated into a causal model when cosistent
- any axiom will also be an assumption, though not vice versa
- As a result, causal theories admit in general multiple causal models depending on the assumptions we actually accept
- abductive reasoning: Bochman 2007

Supraclassical Causal Reasoning

- Aim: integrating causal reasoning into a more comprehensive reasoning system
- **Classical entailment** (formal logical reasoning): integral part of the causal system
- Supraclassical: extends and includes classical entailment, to naturally incorporate classical logic
- "Causal reasoning is not a replacement or competitor of logical (deductive) reasoning, but its complement (or extension)" [1]
- *(Strengthening)* If $A \vDash B$ and $B \Rightarrow C$, then $A \Rightarrow C$;
- *(Weakening)* If $A \Rightarrow B$ and $B \vDash C$, then $A \Rightarrow C$;
- *(And)* If $A \Rightarrow B$ and $A \Rightarrow C$, then $A \Rightarrow (B \wedge C)$;
- *(Truth and Falsity)* $t \Rightarrow t$; $f \Rightarrow f$.

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- *(Truth and Falsity)* $t \Rightarrow t$; $f \Rightarrow f$.

- ancient principle ex nihilo nihil fit ('Nothing comes from nothing")
- ex falso quodlibet ("from falsehood, anything")

Further Properties of Causal Reasoning

- **Default causal theory**: pair $(\Delta, \mathcal{D})$, $\mathcal{D}$ a subset of causal assumptions
- **Nonmonotonicity**
  - Context-Sensitive Causal Reasoning
  - Causal acceptance is directional
- **Structural Equation Models**: representation in the causal calculus
- Counterfactual Equivalence: related to SEM
- Negative Causal Completion: negation as default $(\neg p \Rightarrow \neg p)$

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- Defaults in Causal Reasoning: special kind of assumptions, must accept unless there is reason not to (not rejected $\neg A$)
- nonmonotonic formalisms, adding auxiliary 'presumptions' to an inference rule such that only their refutation could lead to cancellation of the rule
- SEM: cause-and-effect recipes
- new truths into the causal framework can invalidate earlier conclusions about causation.
- When **negation** is viewed as default, negative propositions are exempted from the **need of causal explanation**; in other words, they do not need causes for their acceptance.

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- Explainable AI [2]
- Causal attribution (actual causality) in legal theory [5]

Applications of Causal Reasoning

- Explainable AI [2]
- Causal attribution (actual causality) in legal theory [5]

Bochman's causal reasoning...

- Unified approach: integrate Pearl's approach to causation, classical entailment within causal reasoning
- Nonmonotonic reasoning
- Asymmetry in Semantics
- Roots in Historical Reasoning
- Connection to Inferentialism (rule-based vs. representational)

Take away messages
Bochman's causal reasoning...
- Unified approach: integrate Pearl's approach to causation, classical entailment within causal reasoning
- Nonmonotonic reasoning
- Asymmetry in Semantics
- Roots in Historical Reasoning
- Connection to Inferentialism (rule-based vs. representational)

- It serves as a natural basis for integrating Pearl's approach to causation while also accommodating classical entailment within causal reasoning.
- A critical feature is the **asymmetry**: while the language determines its semantics, the reverse (semantics determining the language) is not true. This challenges conventional representational approaches.
- ties back to historical views on causation from Aristotle, Leibniz, and Hume
- It formalizes meaning through **rules of inference** (causal relationships). It provides a philosophical basis for understanding asymmetrical relationships in causal reasoning systems.
- **holistic approach** in his theory, where propositions make sense primarily in relation to the broader web of inferences rather than in isolation.

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Overview

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5 Spare Slides
Classical propositional language

6 Example slides

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History: Other relevant authors and papers

Some of the most influential authors:

- [11] Pearl. “Embracing causality in default reasoning”. 1988
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Overview
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Syntactic Provability ($\vdash$)

Definition: Th ($\vdash$) represents syntactic provability, meaning a proposition A can be formally derived using axioms and formal inference rules. Provability is about whether a proposition A can be formally derived from a set of axioms or premises Γ using a given set of syntactic proof rules within a formal system.

Key Idea:

- A proposition A is provable (written as $Th \vdash A$ if there is a formal proof sequence for A.
- Relies entirely on the structure and rules of a formal system (e.g., axioms, inference rules like Modus Ponens).
- Does not involve interpretations or models; it's purely rule-based reasoning.
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Semantic Entailment ($\models$)

Definition: $\models$ represents semantic entailment, meaning a proposition A is true in any model where the assumptions Γ hold. Entailment is about whether A is true in every possible model (interpretation) where a set of premises Γ is true.

Key Idea:

- Determines logical consequence based on truth in all models, not formal derivation.
- Like checking universal truths: something is true in all possible worlds where the premises hold.
- Depends on truth values assigned to propositions in all models of the logic. Assesses whether a logical proposition holds in a model-theoretic sense.

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Propositional Atoms (p, g, r)

Definition: Propositional atoms are the smallest, indivisible units of logic, representing atomic facts.
Example:

$$p = \text{"It is raining"}, \quad g = \text{"The grass is wet"}, \quad r = \text{"It is windy"}.$$

No further breakdown is possible for these logical units.

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Definition: Classical propositions are statements formed by combining propositional atoms with logical connectives $(\wedge, \vee, \neg, \rightarrow)$ or truth constants (t, f).
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$$A = p \wedge g, \quad B = p \vee \neg r, \quad C = \neg g \rightarrow r.$$

These statements are more complex and their truth values depend on logical interpretation.

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- Syntactic Provability ($\vdash$): Formal derivation based on rules.
- Semantic Entailment ($\vDash$): Truth across all models.
- Propositional Atoms: Indivisible units (p, g, r).
- Classical Propositions: Composite logical statements (A, B, C).

These concepts bridge syntactic and semantic aspects of logic, fundamental to understanding relationships in formal reasoning systems.

Summary

- Syntactic Provability ($\vdash$): Formal derivation based on rules.
- Semantic Entailment ($\vDash$): Truth across all models.
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These concepts bridge syntactic and semantic aspects of logic, fundamental to understanding relationships in formal reasoning systems.

- **Provability**: $Th \vdash Grasswet$
  Derive *Grasswet* syntactically using axioms like $Rained \to Grasswet$.
- **Entailment**: $Rained \vDash Grasswet$
  Grasswet is true in every model where *Rained* is true.
- **Propositional Atoms**: *Rained*, *Grasswet*, and *Streetwet* are indivisible facts used to construct more complex statements.
- **Classical Proposition**: $Rained \to (Streetwet \lor Grasswet)$
  A composite logical statement describing causal or correlative relationships.

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Examples

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We can highlight text. There are also a darker option, and a yellow option.

The same options are also available in math mode: $a + b = c$

2024-12-10

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└─Example slides
└─Highlighting Text

Highlighting Text

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Blocks

Block

This is a regular block.

- This is an item in a block.

Block

This is an alert block.

- This is an item in an alert block.

Block

This is an example block.

- This is an item in an example block.

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Causal Reasoning by Alexander Bochman
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