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Projektarbeit 1 (T2_2000)

im Rahmen der Prüfung zum

Bachelor of Science (B.Sc.)

des Studienganges Informatik

an der Dualen Hochschule Baden-Württemberg Karlsruhe

von

Vorname Nachname

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Eidesstattliche Erklärung

Ich versichere hiermit, dass ich meine Projektarbeit 1 (T2_2000) mit dem Thema:

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gemäß § 5 der „Studien- und Prüfungsordnung DHBW Technik“ vom 29. September 2017 selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe. Die Arbeit wurde bisher keiner anderen Prüfungsbehörde vorgelegt und auch nicht veröffentlicht.

Ich versichere zudem, dass die eingereichte elektronische Fassung mit der gedruckten Fassung übereinstimmt.

Karlsruhe, den 6. Januar 2023

Nachname, Vorname

Sperrvermerk

Die nachfolgende Arbeit enthält vertrauliche Daten der:

SAP SE
Dietmar-Hopp-Allee 16
69190 Walldorf, Deutschland

Der Inhalt dieser Arbeit darf weder als Ganzes noch in Auszügen Personen außerhalb des Prüfungsprozesses und des Evaluationsverfahrens zugänglich gemacht werden, sofern keine anderslautende Genehmigung vom Dualen Partner vorliegt.

Abstract

- English -

This is the starting point of the Abstract. For the final bachelor thesis, there must be an abstract included in your document. So, start now writing it in German and English. The abstract is a short summary with around 200 to 250 words.

Try to include in this abstract the main question of your work, the methods you used or the main results of your work.

Abstract

- Deutsch -

Dies ist der Beginn des Abstracts. Für die finale Bachelorarbeit musst du ein Abstract in deinem Dokument mit einbauen. So, schreibe es am besten jetzt in Deutsch und Englisch. Das Abstract ist eine kurze Zusammenfassung mit ca. 200 bis 250 Wörtern.

Versuche in das Abstract folgende Punkte aufzunehmen: Fragestellung der Arbeit, methodische Vorgehensweise oder die Hauptergebnisse deiner Arbeit.

Inhaltsverzeichnis

List of abbreviations

DE	Differential Equation
DFT	Discrete Fourier Transform
FEM	Finite Element Method
FT	Fourier Transform
FFT	Fast Fourier Transform
IDFT	Inverse Discrete Fourier Transform
IFFT	Inverse Fast Fourier Transform
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation

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0.1 Heat Equation

The heat conduction within a medium can be described using the following partial differential equation (PDE):

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u \tag{0.1}$$

With u being a function of space and time and α being a positive constant. For this paper u will be defined in terms of one spacial dimension:

$$u := u(x,t) \tag{0.2}$$

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} \tag{0.3}$$

$$x \in \chi \subset \mathbb{R} \quad t \in \tau \subset \mathbb{R} \tag{0.4}$$

$$x_0 \le x \le x_n \quad t_0 \le t \le t_n \tag{0.5}$$

[1]
In order to not only model the conduction of heat within a medium but also a heating process, a new function $h : \chi \times \tau \to \mathbb{R}$ is introduced:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} + h(x,t) \tag{0.6}$$

For this paper it is assumed that initial and boundary values are known:

$$a, b \in \mathbb{R} \tag{0.7}$$

$$f : \chi \to \mathbb{R} \tag{0.8}$$

$$u(x_0,t) = a \quad u(x_n,t) = b \tag{0.9}$$

$$u(x,t_0) = f(x) \tag{0.10}$$

Applying the Fourier transform (FT) w.r.t x to 0.6 yields the inhomogeneous ordinary differential equation (ODE):

$$\hat{u} = \mathfrak{F}(u) \quad \hat{h} = \mathfrak{F}(h) \tag{0.11}$$

$$\frac{d}{dt}\hat{u} = -\alpha\omega^2 \hat{u} + \hat{h} \tag{0.12}$$

A solution to 0.12 is given by:

$$\hat{u} = \hat{u}_0 + \hat{u}_p \tag{0.13}$$

Where $\hat{u}_0$ is the homogeneous solution and $\hat{u}_p$ is the particular integral. In order to solve this ODE for the particular integral $\hat{h}$ has to be known. [2] The choice of h is, except to some restrictions, arbitrary. Therefore an approximate solution to 0.12 $\hat{u}_a$ is obtained by the forward euler scheme:

$$\frac{d}{dt}\hat{u} \approx \frac{\Delta\hat{u}}{\Delta t} \tag{0.14}$$

$$\hat{u}_{t+1} = \hat{u}_t + \Delta t(-\alpha\omega^2\hat{u} + \hat{h}) \tag{0.15}$$

$$\hat{u}_a = [\hat{u}_{t_0}, ..., \hat{u}_{t_n}] \tag{0.16}$$

In order to apply the euler scheme successfully an initial condition $\hat{u}_0$ has to be known. This initial condition is obtained by applying the discrete Fourier transform (DFT) to an initial temperature distribution along x:

$$\hat{u}_0 = \mathfrak{F}(f(x)) \tag{0.17}$$

[3]
The forward euler scheme is used here because it is fairly easy to implement. By applying the inverse discrete Fourier transform (IDFT) to $\hat{u}_a$ an approximate solution to 0.6 can be obtained. To decrease computing time, the fast Fourier transform (FFT) and inverse fast Fourier transform (IFFT) is used instead of the DFT and IDFT.

0.2 Finite Element Method

The finite element method (FEM) is a method to approximate solutions for differential equations (DE) within a certain domain Ω. Assume that a DE is given by:

$$m, n \in \mathbb{N} \quad \zeta \in \Omega \subset \mathbb{R} \quad m \geq 1 \tag{0.18}$$

$$\frac{\partial^m y}{\partial \zeta^m} - g(y) = r(\zeta, t) \tag{0.19}$$

It is assumed that g is a linear function that can also contain partial derivatives of y w.r.t. time, y takes the value 0 at the boundary Γ and $y(\zeta, 0) = f(\zeta)$. An approximate solution to y is given by μ, which is expressed as a sum of basis functions contained in the set ϕ:

$$\mu(\zeta, t) = \sum_{j=1}^{N} c_j(t)\phi_j(\zeta) \tag{0.20}$$

The residual is defined as:

$$\mathfrak{r} = \frac{\partial^m \mu}{\partial \zeta^m} - g(\mu) - r(\zeta, t) \tag{0.21}$$

Furthermore the residual is required to orthogonal to all basis functions:

$$\langle \mathfrak{r}, \phi_k \rangle = 0 \quad \forall \phi_k \in \phi \tag{0.22}$$

Since the functions in ϕ are known, it is only required to find the coefficients $c_j(t)$ in 0.20. To find those coefficients 0.25 needs to be expressed as follows:

$$\int_\Omega \frac{\partial^m \mu}{\partial \zeta^m} \phi_k \, d\zeta - \int_\Omega g(\mu)\phi_k \, d\zeta = \int_\Omega r(\zeta, t)\phi_k \, d\zeta \quad \forall \phi_k \in \phi \tag{0.23}$$

If μ is substituted with 0.20 the following is obtained:

$$\sum_{j=1}^{N} (\int_\Omega \frac{\partial^m \phi_j}{\partial \zeta^m} \phi_k \, d\zeta - g(\int_\Omega \phi_k \phi_j))c_j(t) = \int_\Omega r(\zeta, t)\phi_k \, d\zeta \quad \forall \phi_k \in \phi \tag{0.24}$$

It is also necessary to apply integration by parts to the first integral term taking into account that y at Γ is 0:

$$\int_\Omega \frac{\partial^m \phi_j}{\partial \zeta^m} \phi_k \, d\zeta = -\int_\Omega \frac{\partial^{m-1} \phi_j}{\partial \zeta} \frac{\partial \phi_k}{\partial \zeta} \, d\zeta \quad \forall \phi_k \in \phi \tag{0.25}$$

This formulation leads to a system of ODEs or a system of linear equations that can be solved either analytically or numerically.

This formulation of FEM can be applied to 0.6:

$$\Omega = \chi \quad \Gamma = \{x_0, x_n\} \tag{0.26}$$

$$y(\zeta, t) = u(x, t) \quad g(u) = \frac{1}{\alpha}\frac{\partial u}{\partial t} \tag{0.27}$$

$$r(\zeta, t) = -\frac{1}{\alpha}h(x, t) \tag{0.28}$$

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[1] Gustafsson, B. „Heat Conduction“. In: *Fundamentals of Scientific Computing.* Berlin, Heidelberg: Springer Berlin Heidelberg, 2011, S. 255–262. URL: https://doi.org/10.1007/978-3-642-19495-5_16.

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