# **Chapter 2 Renewable Energy Sources—Modeling and Forecasting**

#### 2.1 Introduction

Forecasts are essential to the integration of renewable power generation in electricity markets operations, since markets ought to be cleared in advance, while market participants shall then make decisions even before that. This is true for all types of electricity markets, that is, from real-time to futures markets, via the more classical day-ahead (forward) ones. For the reference case of conventional generators, power production forecasts are straightforward since, except for unit failures, one actually controls future electricity generation. In such a case, forecasts directly consist of potential schedules, which then translate to supply offers in the market. When it comes to renewable power generation, one is mostly left with Nature deciding on the future schedule of the power plants: wind power is only there when the wind blows and solar energy when the sun shines. Only hydro power is more dispatchable as the water originating from rainfall and snow melt can be stored in gigantic reservoirs. The nonstorability of other types of renewable energy sources, at least in a technologically and economically efficient manner today, magnifies this need for appropriate forecasts of renewable power generation. Here emphasis will be mainly placed on wind energy, which has so far been the leading form of renewable energy. The ideas and concepts presented could be extended to the case of, e.g., solar and wave energy since, from a mathematical point of view, the modeling and forecasting problems share a high level of similarity. Solar energy is becoming increasingly popular and present in a number of countries like Spain and Germany, among others. Wave energy is finally envisaged to become a natural complement to wind energy in the offshore energy mix, based on a number of demonstration projects today in the UK and Portugal, for instance.

It is sometimes argued that forecasts are there mainly to comfort decision-makers—here, the market and network operators, as well as power producers, and potentially end-consumers—while they are not really used or at least not used in an optimal manner in daily operations. However, employing the appropriate forecasts in a well-defined decision-making problem can tremendously improve the decisions to be made, while allowing controlling the risk brought in by unforeseen events. Indeed, a crucial starting point of this chapter is that forecasts are always wrong to

a certain extent. This should be accounted for in the various operational problems considered.

All aspects of renewable power forecasting cannot be covered within a single chapter of this book, nor can the necessary theoretical background on, e.g., stochastic processes, modeling, and estimation. Forecasting of renewable power generation relies on cross-disciplinary approaches taking roots in mathematics, statistics, meteorology, and power systems engineering. Most importantly, we aim at discussing here the various types of forecasts that exist for renewable power generation, being wind, solar or wave energy, and that are to be used as input to operational problems for electricity markets.

In Sect. 2.2, we introduce some of the necessary notation and definitions while placing ourselves in a stochastic process modeling framework. Necessary concepts related to stochastic processes are further developed in Appendix A. Subsequently, the various types of renewable energy forecasts that may be issued as input to decision-making problems are introduced in Sect. 2.3 based on examples, giving a pragmatic view of their characteristics. Emphasis is then placed in Sect. 2.4 on the quality of these forecasts, by covering their required and necessary properties, as well as some key scores and diagnostic tools for their evaluation. It is of utmost importance to fully appraise the quality of forecasts before to use them as input to decision-making and general operational problems. The way these forecasts may be generated from various sets of input data is then discussed in Sect. 2.5. Further readings are suggested at the end of this chapter.

#### 2.2 Renewable Power Generation as a Stochastic Process

Even though referring to either renewable energy or power modeling and forecasting, focus is always placed on the power variable. This is because it is actually power which is measured at renewable energy generation plants. It is then straightforward to obtain energy values for given periods of time if necessary, by integrating power observations over these time periods.

Owing to the combination of a large number of complex physical processes, also mixed with additional uncertainties in our understanding of these processes, there may always be a part of randomness in our knowledge of energy generation from renewable energy sources. For instance, for a wind farm, even if having a perfect picture of the theoretical power curve of each and every turbine (as provided by the turbine manufacturer), it is close to impossible to know for sure what the power curve of the wind farm composed by all these turbines may be. This uncertainty originates from shadowing effects among the set of turbines, turbulence effects, dust and insects on the blades, etc.

Accepting the fact that there are uncertainties in the process of renewable energy generation, it is hence considered as a stochastic process. Necessary basics related to the definition of stochastic processes are introduced in Appendix A. Consequently, power generation from renewable energy sources, such as wind and solar, will be referred to as *stochastic power generation* in the subsequent chapters.

**Definition 2.1 (The Renewable Energy Generation Stochastic Process).** In the most general case,

$$\{Y_{r,s,t}, r = r_1, \dots, r_m, s = s_1, \dots, s_n, t = 1, \dots, T\},$$
 (2.1)

is a multivariate stochastic process in space and in time, observed at a set of n locations,  $s = s_1, s_2, \ldots, s_n$ , and for successive time points  $t = 1, \ldots, T$ , describing power generation from a number m of different renewable energy sources,  $r = r_1, r_2, \ldots, r_m$ . The corresponding realizations of that stochastic process are denoted by

$$\{y_{r,s,t}, r = r_1, \dots, r_m, s = s_1, \dots, s_n, t = 1, \dots, T\}.$$
 (2.2)

This stochastic process may be univariate (m=1, in the above definition) if considering one type of renewable energy only, or multivariate (m>1) if jointly considering several forms of renewable energy generation, as for the example of wind and wave energy generation offshore. In the former case, the notation for the stochastic process may be simplified to  $\{Y_{(s,t)}\}$ . Similarly, while both the time and space dimensions may be jointly considered, it is often the case that (i) focus is on the spatial dimension only, e.g., as input to a power flow calculation, or (ii) focus is on the time dimension only, e.g., if dealing with renewable energy generation for a given location in an optimal storage operation problem. Notation would then simplify even more, by using the relevant subscript only, that is,  $\{Y_s\}$  and  $\{Y_t\}$  for the space and time cases, respectively.

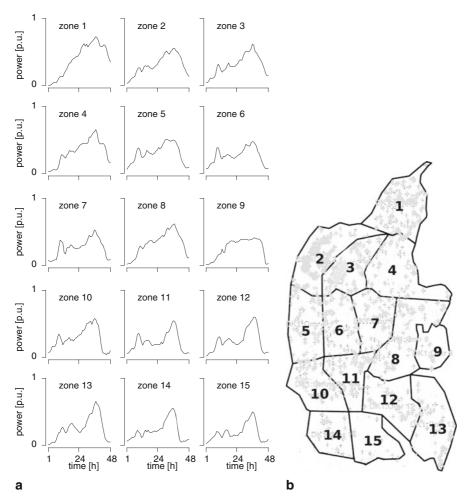
This stochastic process can be normalized for simplification, hence taking values between 0 and 1 at any time, any location and for all types of renewable energy,

$$Y_{r,s,t} \in [0,1], \quad \forall i, s, t.$$
 (2.3)

The above then also necessarily applies to all realizations  $y_{r,s,t}$ . The normalization is done individually by the nominal capacity of that type of renewable energy at this location and at this point in time. While it is fairly obvious that nominal capacity depends upon the renewable energy plant and therefore its location, one should not forget that nominal capacity can vary in time, e.g., due to maintenance planning and decommissioning/recommissioning of renewable energy assets.

The concepts introduced in the above are illustrated in the following example describing a univariate case, with wind power generation only, though observed at a number of locations, and for a long period of time.

Example 2.1 (Wind Power Generation for 15 Control Zones in Western Denmark) A dataset with wind power generation over the control area (split into 15 control zones) of Western Denmark, operated by Energinet.dk for a total nominal capacity of 2.515 GW, will be used as a basis for illustration in this chapter. This control area is commonly referred to as DK-1. Wind power generation over these 15 control zones can be considered as a univariate stochastic process  $\{Y_{s,t}\}$  in space and in time, in practice observed at 15 locations  $s = s_1, s_2, \ldots, s_{15}$  only. Figure 2.1 depicts an episode with two days in mid-February 2006 of wind power observations at these 15 control zones, with a hourly temporal resolution. These wind power observations for every zone are normalized by the relevant nominal capacity values. The generation



**Fig. 2.1** Episode from mid-February 2006 with two days of wind power observations at the 15 control zones forming the control area DK-1 of Energinet.dk. These measurements have an hourly temporal resolution and are normalized by the respective nominal capacities at every control zone. **a** Normalized power observations. **b** 15 control zones

patterns for neighboring zones have similar characteristics, while there is also a clear temporal dependence. These are important aspects when it comes to the modeling and forecasting of such a stochastic process.

# 2.3 The Various Types of Renewable Power Forecasts

Predictions of renewable energy generation can be obtained and presented in a number of different manners. The choice for the type of forecasts and their presentation somewhat depends upon the process characteristics of interest to the decision-maker,

and also upon the type of operational problem. For instance, a wind farm operator aiming to plan maintenance over the coming week may only be interested in simple deterministic-type of forecasts for wind and power generation at the level of this wind farm, and not in detailed space—time scenarios over the whole country.

The various types of renewable energy forecasts and their presentation are introduced below, starting from the most common point forecasts and building up towards the more advanced products that are probabilistic forecasts and scenarios. We finally mention some of the more exotic forecasts that are currently being issued with focus on predefined events.

# 2.3.1 Common Features of Renewable Power Forecasts

Forecasting is about foreseeing the future state of the process of interest, in this case, renewable energy generation, at a given location s or for a set of n locations  $s = s_1, s_2, \ldots, s_n$ , potentially with different forms of renewable energy sources at every location. Even though several locations and renewable energy forms may be considered, it is the temporal dimension that is of importance here. In contrast to spatial forecasts, we do not aim in this chapter at predicting the dynamics of the stochastic process at new locations. We do not attempt at issuing forecasts for new types of renewable energy sources either. The set of locations s and the energy mix are both fixed. Let us then place ourselves at time t and look at a future point in time t + k. For ease of notation, we only use time indices in the following when referring to values for the stochastic process. One should not forget that these may also be for several locations and types of renewable energy sources.

Emphasis is placed in the following on model-based approaches to forecasting. There exists a number of other approaches, e.g., based on expert judgments. For the example case of forecasting the electric demand (commonly referred to as load), it is often said that such expert judgments are very difficult to outperform by any model-based approach. For renewable energy forecasting, however, model-based approaches are to be preferred, since it would be much more difficult for experts to sharply foresee weather developments and their impact on corresponding renewable energy generation. Note that a difference should be made between a model, which comprises a mathematical representation of the processes considered, and a forecasting method, which is, instead, the process of issuing a prediction, based or not on a model.

**Definition 2.2.** A (model-based) forecast  $\hat{\cdot}_{t+k|t}$  of renewable power generation is an estimate of some of the characteristics of the stochastic process  $Y_{t+k}$  (where Y is for all locations and types of renewable energy sources) given a chosen model g, an estimated set of parameters  $\hat{\Theta}_t$  and the information set  $\Omega_t$  gathering all data and knowledge about the processes of interest up to time t. That information set is commonly employed to identify a model g and the set of parameters  $\Theta_t$ .

In the above definition, k is the *lead time*, though sometimes also referred to as *forecast horizon*. The 'hat' symbol expresses that  $\hat{l}_{t+k|t}$  is an estimate only: it reflects

the presence of uncertainty both in our knowledge of the process and inherent to the process itself. The notation 't + k|t' is based on the conditional symbol '|' in probability theory. The forecast for time t + k is conditional on our knowledge of the stochastic process up to time t, including the data used as input to the forecasting process, as well as the models identified and parameters estimated.

Whatever the type of forecast, forecasting is to be seen as a form of extrapolation. A model is built and fitted to a set of data, then applied for prediction purposes on totally new data. This conditionality of forecasts makes that they should implicitly be formulated as: "given the information set and assuming that the identified dynamics continue in the future, we can predict that . . . ". A forecaster somewhat makes the crucial assumption that the future will be like the past.

Forecasts are issued as series of consecutive values  $\hat{i}_{t+k|t}$ ,  $k=1,2,\ldots,K$ , that is, for regularly spaced lead times up to the *forecast length K*. That regular spacing is called the *temporal resolution* of the forecasts. This will be illustrated when introducing the various types of renewable energy forecasts below. For instance, when one talks of 48-hour ahead forecasts with hourly resolution, this means that forecasts actually consist in forecast series gathering predicted power values for each of the following 48 h. Similarly, if predictions were to be issued on a regular spatial grid, one would talk of their spatial resolution. Here forecasts are for specific locations, not uniformly distributed on a grid, and therefore, the concept of spatial resolution does not make much sense.

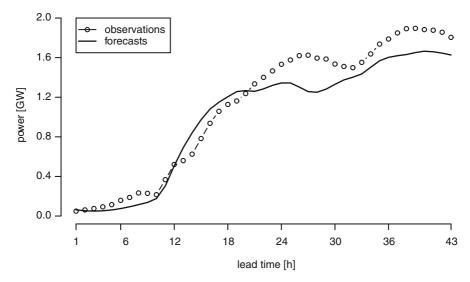
#### 2.3.2 Point Forecasts

When the renewable energy forecast issued at time t for t+k is single-valued, it is referred to as a point prediction and denoted by  $\hat{y}_{t+k|t}$ . The fact this forecast is single-valued makes that point forecasts issued in a deterministic or stochastic process framework look similar. However, they are not in essence. In a deterministic framework, the forecaster is somewhat sure that the prediction ought to realize—there is no uncertainty involved. In a stochastic process framework, instead,  $\hat{y}_{t+k|t}$  is an estimate only, hence acknowledging the presence of uncertainty.

**Definition 2.3.** A point forecast  $\hat{y}_{t+k|t}$  corresponds to the conditional expectation of  $Y_{t+k}$  given g,  $\hat{\Theta}$ , and the information set  $\Omega_t$ ,

$$\hat{y}_{t+k|t} = \mathbb{E}[Y_{t+k}|g, \Omega_t, \hat{\Theta}]. \tag{2.4}$$

In everyday words, the conditional expectation is the mean of all that may happen given our state of knowledge up to time t. Providing decision-makers with a forecast in the form of a conditional expectation translates to acknowledging the presence of uncertainty, even though it is not quantified and communicated.



**Fig. 2.2** Point forecasts of wind power generation issued on the 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark (for a nominal capacity of 2.515 GW on that day)

Example 2.2 (Point Forecasts of Wind Power Generation) Let us consider the example of point forecasts issued on 4th April 2007 at 00:00 UTC<sup>1</sup> for the whole onshore capacity of Western Denmark (2.515 GW at the time, see Example 2.1, depicted in Fig. 2.2, along with the corresponding observations obtained a posteriori. This forecast series has a hourly temporal resolution up to 43 h ahead.

It informs that the expected power generation on 5th April 2007 at 00:00 UTC should be 1.32 GW. There what the forecaster really says is that the predicted mean of all potential power production values is 1.32 GW. He or she is not telling about what could really happen, however. The actual power generated 24 h after the forecast is issued could range anywhere between 0 and 2.5 GW, and that would make a big difference! This will all depend upon the forecaster's skill and the inherent forecast uncertainty. In this case, the forecast error made a posteriori appears fairly small, since the observed power generation at that time was of 1.466 GW (still a 146 MW difference).

#### 2.3.3 Probabilistic Forecasts

This shortcoming of point predictions not giving the full picture about what *could* happen is of crucial importance when it comes to operational problems, where the costs potentially induced by the whole potential range of realizations that are likely

<sup>&</sup>lt;sup>1</sup> UTC actually stands for *Coordinated Universal Time*, which is a time standard by which we regulate time and clocks.

to occur is to be accounted for. This has therefore motivated the substantial research effort invested in the development of probabilistic forecasting methodologies for energy applications, with a strong emphasis on their optimal integration in operations research problems.

In contrast to point predictions, probabilistic forecasts aim at providing decision-makers with the full information about potential future outcomes. Let us use the same notation as before while dropping out the subscripts for location and type of renewable energy source. Recall that  $y_t$  is the power production measured at time t and corresponds to a realization of the random variable  $Y_t$ . Then let  $f_t$  and  $F_t$  be the *probability density function* (abbreviated pdf) and related *cumulative distribution function* (abbreviated cdf) of  $Y_t$ , respectively.

**Definition 2.4.** A probabilistic forecast issued at time t for time t + k consists in a prediction of the pdf (or equivalently, the cdf) of  $Y_{t+k}$ , or of some summary features.

Deterministic forecasts may be reinterpreted in a probabilistic framework as probability masses of 1 placed on these values predicted for the future state of the process—there is no uncertainty. The various types of probabilistic forecasts are detailed below, from quantile to density forecasts, and through prediction intervals.

#### 2.3.3.1 Quantile Forecasts

Let us now introduce the concept of quantile forecast based on the definition of the quantile of a cumulative distribution function as given in Def. A.6 of Appendix A.

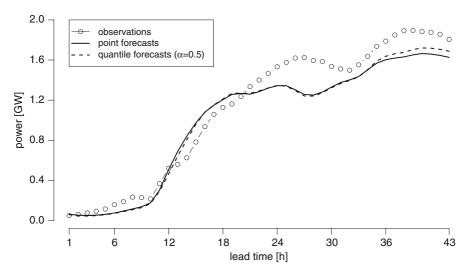
**Definition 2.5.** A quantile forecast  $\hat{q}_{t+k|t}^{(\alpha)}$  with *nominal level*  $\alpha$  is an estimate, issued at time t for lead time t+k, of the quantile  $q_{t+k}^{(\alpha)}$  for the random variable  $Y_{t+k}$ , given a model g, its estimated parameters  $\hat{\Theta}_t$  and the information set  $\Omega_t$ , i.e.,

$$P[Y_{t+k} \le \hat{q}_{t+k|t}^{(\alpha)} \mid g, \Omega_t, \hat{\Theta}] = \alpha. \tag{2.5}$$

By issuing a quantile forecast  $\hat{q}_{t+k|t}^{(\alpha)}$ , the forecaster tells at time t that there is a probability  $\alpha$  that renewable energy generation will be less than  $\hat{q}_{t+k|t}^{(\alpha)}$  at time t+k.

Quantile forecasts are of interest for a number of operational problems, since for a variety of loss functions (quantifying the cost of making a suboptimal decision, to be further introduced and discussed in Sect. 2.5.3), optimal decisions always relate to quantile forecasts with given nominal levels [2]. This is, for instance, the case for the design of optimal offering strategies by wind power producers, where optimal bids are quantile forecasts whose nominal level is a simple function of day-ahead and balancing market prices (see Chap. 7). Furthermore, quantile forecasts also define prediction intervals and, more generally, nonparametric probabilistic forecasts, as will be explained more extensively in the following. The concept of quantile forecasts is further illustrated below by building on the previous examples with wind power generation in Western Denmark.

Example 2.3 (Quantile Forecasts of Wind Power Generation) Fig. 2.3 depicts an example episode with quantile forecasts with a nominal level  $\alpha = 0.5$  (i.e., the



**Fig. 2.3** Quantile forecasts of wind power generation with a nominal level of 0.5 (i.e., the median) issued on 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark (for a nominal capacity of 2.515 GW on that day). Point forecasts and the corresponding observations are also shown

median), for the same period than in Fig. 2.2 and the same set-up as introduced in Example 2.1. For each lead time, these forecasts tell that wind power generation has a 50 % probability of being below (and, therefore, also above) the value they indicate. Their interpretation is hence quite different from that of the point forecasts considered before, since point forecasts, as conditional expectations, are not associated to any form of probability level. Note that, if forecast uncertainty is perfectly symmetric around point predictions, then  $\hat{q}_{t+k|t}^{(0.5)} = \hat{y}_{t+k|t}$ .

In the present case, if looking more closely at the 42-hour ahead lead time, while the previously discussed point forecasts tell that the expected power generation is 1.646 GW, the quantile forecast informs there is a 50% probability that power generation will be below (or above) 1.706 GW.

#### 2.3.3.2 Prediction Intervals

Quantile forecasts give a probabilistic information about future renewable power generation, in the form of a threshold level associated with a probability. Even though they may be of direct use for a number of operational problems, they cannot provide forecast users with a feeling about the level of forecast uncertainty for the coming period. For that purpose, *prediction intervals* certainly are the most relevant type of forecasts. Furthermore, prediction intervals are frequently used to make decisions under uncertainty using robust optimization (see, e.g., Chaps. 8 and 9).

**Definition 2.6.** A *prediction interval*  $\hat{I}_{t+k|t}^{(\beta)}$ , issued at time t for time t+k, defines a range of potential values for  $Y_{t+k}$ , for a certain level of probability  $(1-\beta)$ ,  $\beta \in [0,1]$ , its *nominal coverage rate*,

$$P[Y_{t+k} \in \hat{I}_{t+k|t}^{(\beta)} | g, \Omega_t, \hat{\Theta}] = 1 - \beta.$$
 (2.6)

It is equivalently referred to as an interval forecast.

Such an interval  $\hat{I}_{t+k|t}^{(\beta)}$  must be defined by its lower and upper bounds,

$$\hat{I}_{t+k|t}^{(\beta)} = [\hat{q}_{t+k|t}^{(\alpha)}, \ \hat{q}_{t+k|t}^{(\overline{\alpha})}], \tag{2.7}$$

where these bounds are quantile forecasts whose nominal levels  $\alpha$  and  $\overline{\alpha}$  verify that

$$\overline{\alpha} - \alpha = 1 - \beta. \tag{2.8}$$

This general definition makes that a prediction interval is not uniquely defined by its nominal coverage rate. It is thus also necessary to decide on the way it should be centered on the probability density function. Commonly, it is chosen to center it (in probability) on the median, so that there is the same probability that an uncovered realization  $y_{t+k}$  lies below or above that interval. This translates to

$$\alpha = 1 - \overline{\alpha} = \beta/2. \tag{2.9}$$

With this type of centering, the resulting intervals are called *central prediction intervals*. For example, central prediction intervals with a nominal coverage rate of 90 % (i.e.,  $(1 - \beta) = 0.9$ ) are defined by quantile forecasts with nominal levels of 5 and 95 %. Other types of intervals exist, e.g., shortest-length intervals and highest-density regions among others [5], depending upon the way they are chosen to summarize information from the full probabilistic distribution. An illustration is given below, for the case of wind power generation in Western Denmark.

Example 2.4 (Central Prediction Intervals of Wind Power Generation) Central prediction intervals of wind power generation with a nominal coverage rate of 90% (i.e.,  $(1 - \beta) = 0.9$ ), issued for the whole onshore capacity of Western Denmark and for the same period than in Figs. 2.2 and 2.3, are depicted in Fig. 2.4. They give a range of possibilities of power generation for every lead time, for a certain probability level, and therefore tell about how confident one may be about the point forecasts originally provided—the tighter they are, the higher the confidence is. The advantage is that they give a very visual information on the expected range of future events.

In the present case, these intervals, for instance, inform that there is a 90% probability that, 24 h in the future, wind power generation will be between 0.897 GW and 1.65 GW. There is only a 5% probability that wind power generation will actually be less than 0.897 GW, and similarly, only a 5% probability of being more than 1.65 GW.

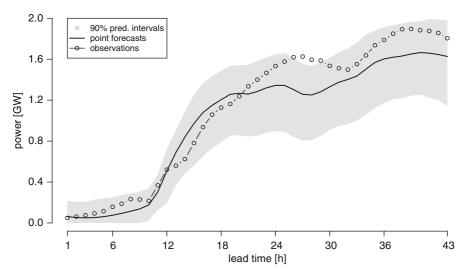


Fig. 2.4 Central prediction intervals of wind power generation with a nominal coverage rate of 90% issued on 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark (for a nominal capacity of 2.515 GW on that day). Point forecasts and the corresponding observations are also shown

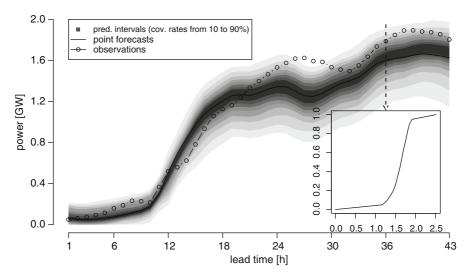
### 2.3.3.3 Density Forecasts

All the various types of predictions presented in the above, i.e., point, quantile, and interval forecasts, are in fact only partly describing the whole information about future renewable energy generation at every lead time. This whole information would be given by *density forecasts* for each point of time in the future.

**Definition 2.7.** (A *density forecast*  $\hat{f}_{t+k|t}$  (or  $\hat{F}_{t+k|t}$  if focusing on cdfs) issued at time t for time t+k, is a complete description of the pdf (or cdf) of  $Y_{t+k}$  conditional on a given model g, estimated parameters  $\hat{\Theta}_t$  and the information set  $\Omega_t$ .

For a wide range of decision-making problems related to renewable energy management and to its integration into electricity markets, density forecasts in the form of predictive densities  $\hat{f}_{t+k|t}$  or predictive cdfs  $\hat{F}_{t+k|t}$  are a necessary input to related operational problems. Representative examples include the design of optimal offering strategies, to be dealt with in Chap. 7, or the optimal quantification of reserve requirements, accounting for all uncertainties involved [11].

Example 2.5 (Density Forecasts of Wind Power Generation) Figure 2.5 displays density forecasts of wind power generation issued for the whole onshore capacity of Western Denmark and for the same period than in Figs. 2.2, 2.3, and 2.4. Density forecasts are visualized as a river-of-blood fan chart based on central prediction intervals with nominal coverage rates  $(1 - \beta) \in \{0.1, 0.2, \dots, 0.9\}$ . Even though they may not be easy to interpret visually, they permit to fully characterize wind power generation for every individual lead time: for instance, Fig. 2.5 shows, for the case



**Fig. 2.5** Predictive densities of wind power generation issued on 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark (for a nominal capacity of 2.515 GW on that day). Point forecasts and the corresponding observations are also shown. The predicted cumulative distribution function for the 36-hour lead time is depicted as an example piece of information that can be extracted from such forecasts

of the 36-hour ahead lead time, the type of cumulative distribution function that can be extracted from the set of density forecasts, hence describing all possibilities for wind power generation at that lead time. Point forecasts and the corresponding measurements are also shown for comparison.

Remember that, for this example, the point forecasts told that the expected power generation for 5th April 2007 at 00:00 UTC was of 1.32 GW. Here the probabilistic forecasts inform that there is a  $5\,\%$  probability that power production will be less than 896 MW, a  $25\,\%$  probability of power being less than 1.181 GW, and a  $25\,\%$  probability of being more than 1.477 GW.

Whatever the type of probabilistic forecasts considered, they may be obtained based on parametric or nonparametric approaches, which are discussed in a further section below

#### 2.3.4 Scenarios

Probabilistic forecasts give substantial information about the characteristics of the stochastic process of interest, i.e., renewable power generation, for the coming future. However, they only concentrate on detailing marginal densities for each lead time, location and renewable energy type, independently. For instance, if considering several locations and types of renewable energy, say, wind and wave energy,

the interdependence between locations and renewable energy sources would not be described. Maybe, more importantly, the temporal dependence structure of potential forecast errors is disregarded. For instance, if forecasts errors are strongly correlated in time, it means that a large forecast error at time t+k is most likely followed at time t+k+1 by another large error. In contrast, if that dependence is weak, forecast errors for future lead times may be seen as completely random.

Such information about dependence in time, space, and among types of renewable energy sources, may be crucial as input to a number of operational problems arising from an integrated management of their energy output. One may think of

- (i) the optimal operation of a virtual power plant composed by a wind farm and storage,
- (ii) optimal offshore maintenance planning that necessitates the joint forecasting of the power output of wind and wave energy devices at some offshore location,
- (iii) stochastic unit commitment accounting for wind power capacities spread over a control zone,
- (iv) market-clearing methods aiming to optimally accommodate the output of renewable energy sources.

Scenarios are first introduced here in their most simple form as time trajectories. They will be presented in a more generic manner in the modeling section below, by also looking at other dimensions as well. Emphasis is therefore placed on a single type of renewable energy source for a single location (or for a capacity aggregation). As preliminary, it is required to introduce the multivariate random variable

$$Z_t = \{Y_{t+k}, \ k = 1, \dots, K\}.$$
 (2.10)

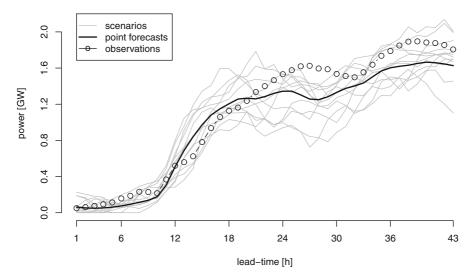
which gathers the random variables characterizing the stochastic power generation process for the K following lead times, hence covering their marginal densities as well as their interdependence structure. Denote by  $F_{Z_t}$  its multivariate cdf.

**Definition 2.8.** Scenarios issued at time t and for a set of K successive lead times, i.e., with  $k \in \{1, 2, ..., K\}$ , are samples of  $\hat{F}_{Z_t}$ , namely the predicted cdf of  $Z_t$ . They consist in a set of J time trajectories

$$\hat{z}_{t}^{(j)} = [\hat{y}_{t+1|t}^{(j)}, \hat{y}_{t+2|t}^{(j)}, \dots, \hat{y}_{t+K|t}^{(j)}]^{\top} \quad j = 1, \dots, J.$$
 (2.11)

The corresponding observation is  $z_t = [y_{t+1}, \dots, y_{t+K}]^{\top}$ .

Indeed, even in the most simple case, where  $Z_t$  would be multivariate Gaussian, communicating a forecast distribution for  $Z_t$  is complex since it would consist in a set of conditional expectations for the successive lead times, associated with a conditional covariance matrix summarizing the second-order characteristics of  $Z_t$ . Overall, it may not even be possible to fully characterize the K-dimensional distribution of  $Z_t$ . This is why one, instead, issues time trajectories such as that in Eq. (2.11). If used as input to operational problems, they should be seen as equally likely samples of the predictive distribution of  $Z_t$ . The resulting time trajectories comprise scenarios like those commonly used as input to operational problems in



**Fig. 2.6** Scenarios (12 time trajectories) of wind power generation issued on 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark (for a nominal capacity of 2.515 GW on that day). Point forecasts and the corresponding observations are also shown

a stochastic programming framework (see, e.g., Chap. 5). An illustration is given below based on the real-world example of Western Denmark.

Example 2.6 (Scenarios of Wind Power Generation) For the same period and conditions as in Fig. 2.5, scenarios of short-term wind power generation in the form of time trajectories are depicted in Fig. 2.6. These 12 scenarios jointly inform on the marginal densities for each lead time (even though having more scenarios would clearly be beneficial), while they also tell about the temporal correlation of the generation process, since they represent plausible alternative paths into the future. The original point forecasts, as well as the corresponding observations obtained a posteriori, are also shown for comparison.

The scenarios reflect some of the properties of predictive densities shown in Fig. 2.5, for instance, their uncertainty increasing when getting further in the future. For every lead time, however, they only give a discrete representation of these densities, here with 12 equally likely values of power generation.

The optimal set of scenarios, i.e., time, space-time, and/or multivariate trajectories, readily depends upon the type of operational problem to be solved, as well as the dependencies that are known to be of primary importance. Raising the complexity and dimension of these scenarios clearly has a cost—the simpler they are, the better.

#### 2.3.5 Event-Based Predictions

Since the mid-2000s, a number of renewable power forecast users have expressed the need for more targeted types of predictions, based on specific events that have the

most impact on electricity markets and power systems operations. This has therefore led to coining the term of *event-based* predictions, where the events may be the so-called ramps (large gradients of power production over a short time period), intense power variability (periods with power fluctuations of high magnitude), etc.

Event-based forecasting has its root in meteorology and climate sciences, where events are defined by setting a threshold on the value of a continuous variable, e.g., in our case, "renewable power generation being greater than 50 % of the portfolio's nominal capacity". The particularity of an event is that observations take values in {0, 1} only, depending upon the event realizing or not. Related probability forecasts take values in [0, 1], hence informing about the probability of that particular event realizing. These forecasts are evaluated for each observation time, potentially as a function of the lead time.

In a generic manner, write

$$g: z \to g(z; \theta),$$
 (2.12)

a functional allowing to define an event based on a time trajectory z and a parameter set  $\theta$ . A number of functionals could easily be defined for events relevant to renewable power forecasts users. The functional g may be generalized to be applied to the multivariate random variable  $Z_t$  of Eq. (2.10), with the probability of observing the event based on the random variable  $Z_t$  as output.

Example 2.7 (Ramp Events over 6-hour Time Windows) Let us place ourselves at time t. Given a window of size h centered on lead time k, one may introduce the functional g as

$$g(z_t; k) = 1 \left\{ \left( \max_{i \in \mathcal{S}} y_{t+i} - \min_{i \in \mathcal{S}} y_{t+i} \right) \ge 0.5 \right\},$$

$$\mathcal{S} = \{ k - 3, k - 2, \dots, k + 2, k + 3 \}, \tag{2.13}$$

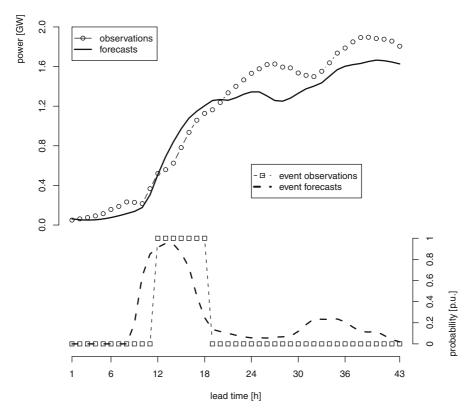
where  $y_{t+i}$  is the  $i^{th}$  component of  $z_t$ . In the above, 1{.} is an indicator operator, being equal to 1 if the condition expressed within brackets realizes, and to 0 otherwise. This functional defines a ramp event over 6-hour time windows centered on the lead time k, with the maximum absolute variation in power generation being (or not) greater than 50 % of nominal capacity.

As an extension, it is now possible to introduce the concept of event-based prediction.

**Definition 2.9.** An event-based forecast issued at time t for time t + k consists in a forecast  $\hat{g}(Z_t; \theta)$  of the probability  $g(Z_t; \theta)$  of observing the event defined by g.

Event-based forecasts can be directly issued based on statistical models such as generalized linear models (GLMs) [10]. Alternatively, they may be a filtered version of the complex scenarios presented before. In that case, the event-based forecasts  $\hat{g}(Z_t;\theta)$  is obtained by applying the functional g to the predicted set of time trajectories,

$$\hat{g}(Z_t; \theta) = \frac{1}{J} \sum_{i=1}^{J} g(\hat{z}_t^{(j)}; \theta). \tag{2.14}$$



**Fig. 2.7** Probability forecasts of ramp events issued on 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark (for a nominal capacity of 2.515 GW on that day), along with the original point forecasts of wind power generation. Ramps are defined as a change of more than 500 MW in power generation within a 6-hour time window. Ramp forecasts are filtered from 1000 scenarios of short-term wind generation. Related observations, for both power generation and ramp events, are also shown

i.e., as the share of time trajectories predicting this event.

Event-based forecasts obtained from trajectories may be appealing in practice owing to the difficulty to process complex temporal, spatio-temporal, and multivariate dependencies with naked eyes. Working based on specific events of interest readily simplifies the analysis.

Example 2.8 (Ramp Probability Forecasts of Wind Power Generation) To finalize the series of examples based on wind power forecasts issued on 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark (for a nominal capacity of 2.515 GW on that day), Fig. 2.7 shows ramp probability forecasts along with the original point forecasts for that period. The ramp forecasts are obtained from 1000 scenarios such as those shown in Fig. 2.6, where a ramp event is defined as a change of 500 MW (or more) in power generation within a time window of 6 h.

The forecasts tell that the probability of observing a ramp event increases steadily for lead times between 8 and 13 h ahead, while it decreases for further lead times. The predicted probability is then small for lead times greater than 28 h ahead, though it is more than 0, indicating that such an event could occur.

Even though these event-based forecasts may be appealing from a visual point of view, and also owing to their simplicity in comparison with scenarios, it is clear that their value as input to operational problems may be more limited. Scenarios of renewable energy generation, informing on the full characteristics of the stochastic processes involved, are a must-have when it comes to operational problems related to electricity markets and power systems operations. For that reason, event-based forecasts will not be extensively dealt with in the remainder of this chapter.

# 2.4 Aspects of Forecast Verification

Predictions ought to be evaluated prior to be used as input to operational problems. It would be particularly difficult to analyze the results obtained for the operational problem at hand if not knowing whether the input forecasts were good or not. After briefly covering general aspects of forecast verification below, focus is given to some of the key concepts, criteria, and diagnostic tools to be used for evaluation of point and probabilistic forecasts, as well as scenarios. An extensive overview of forecast verification concepts, although applied to meteorological forecasting, is available in [6].

#### 2.4.1 General Framework

For predictions in any form, one must differentiate between their quality and their value. Forecast *quality* corresponds to the ability of the forecasts to genuinely inform of future events by mimicking the characteristics of the processes involved. Forecast *value* relates, instead, to the benefits from using forecasts in a decision-making process.

Consequently, forecast quality is independent of the operational problem at hand, while this is not the case of forecast value. In the remainder of the book, we will mainly seek to exploit the value of forecasts, insofar as these forecasts will be considered as input to operational problems. In contrast, here, our aim is to provide the reader with the basics necessary to appraise the quality of predictions.

Visually inspecting the forecasts so as to get a feel for their quality is most certainly advisable. The most relevant manner to assess forecast quality, nonetheless, is to carry out quantitative assessments based on a number of criteria, potentially complemented with a qualitative appraisal using related diagnostic tools. Such quantitative and qualitative assessments must be performed on an independent evaluation period, i.e., a period with data that have not been used in any way for model identification and

estimation purposes. Over this period, the forecasts should also be issued as if in real operational conditions. In the following, it will be considered that predictions are verified on an evaluation period of length T with T sufficiently large. Typically, one wants to have several months of forecasts and corresponding measurements available for verification, so as to draw meaningful conclusions. Employing evaluation sets of limited lengths would bring significant uncertainty in the quality assessment.

Importantly for an objective and critical appraisal of evaluation results, benchmark approaches should be considered. A benchmark method is characterized by its apparent simplicity while already allowing to provide competitive forecasts. The most relevant ones are persistence and climatology. The former is based on the idea that the most recent observation is the best forecast for the coming future, while the latter considers that the best prediction is to be derived from knowledge of the long-term statistics.

**Definition 2.10.** Placing ourselves at time t and looking in the future at time t + k, the *persistence* forecast is given by the last available measurement,

$$\hat{\mathbf{y}}_{t+k|t} = \mathbf{y}_t, \tag{2.15}$$

while the climatology one is the mean of all available past measurements. If having m past observations, this yields

$$\hat{y}_{t+k|t} = \frac{1}{m} \sum_{i=0}^{m-1} y_{t-i}.$$
(2.16)

Illustrative examples of persistence and climatology-based forecasts are given below.

Example 2.9 (Persistence and Climatology Forecasts) The observed wind power generation at time t is equal to 1.25 GW, while the average of all past observations is 892 MW. A persistence forecast for any time t + k in the future is 1.25 GW, while the climatology one is 892 MW.

At time t+1, when having a new observed power value of 1.08 GW, the persistence forecast for any time t+1+k becomes 1.08 GW, while the climatology forecast is still 892 MW.

#### 2.4.2 Point Forecasts

Point forecasts of renewable power generation are for a continuous variable. Consequently, the base quantity for evaluating point forecasts is the *forecast error*.

**Definition 2.11.** The *forecast error*  $\varepsilon_{t+k|t}$  is the difference between observed and predicted values,

$$\varepsilon_{t+k|t} = y_{t+k} - \hat{y}_{t+k|t}. \tag{2.17}$$

The subscripts employed are the same as for the forecast it is linked to.

Note that the forecast error may be normalized so that verification results may be comparable for different renewable energy generation sites. If so, normalization is most commonly performed by the nominal capacity of the site of interest. Alternatively, some use the average power generation over the evaluation period. The former convention will be used here, while the terms "forecast error" and "normalized forecast error" will be used interchangeably.

Example 2.10 (Forecast Error and its Normalized Version) A forecast issued at time t indicates that the expected wind power generation at time t+1 is 1.25 GW. The observation, obtained a posteriori, is 1.08 GW. The corresponding forecast error is hence -170 MW. Given a nominal capacity of 2.515 GW, the normalized forecast error is -6.76% of that nominal capacity.

The first error criterion that may be computed is the *bias* of the forecasts, which corresponds to the systematic part of the forecast error. It may be corrected in a straightforward manner using simple statistical models.

**Definition 2.12.** The *bias* is the mean of all errors over the evaluation period of length T, considered indifferently. For lead time k, this writes

$$\operatorname{bias}(k) = \frac{1}{T} \sum_{t=1}^{T} \varepsilon_{t+k|t}.$$
 (2.18)

This summary measure does not tell much about the quality of point forecasts, only about a systematic error that should be corrected. For a better appraisal of the forecasts, it is advised to use scores defined based on particular *loss functions* (to be further introduced and discussed in Sect. 2.5.3). Loss functions assign a penalty to forecast errors, as a proxy of the cost of these errors for those making decisions based on such forecasts. Considering a quadratic loss function yields the definition of the *root mean square error* (*RMSE*).

**Definition 2.13.** The *RMSE* is defined as the square root of the sum of squared errors over an evaluation set of length T,

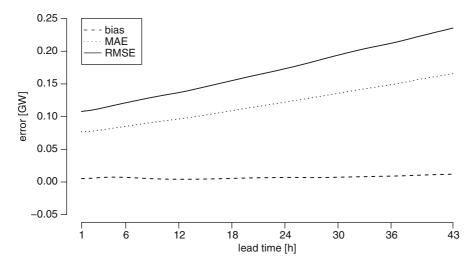
$$RMSE(k) = \sqrt{\frac{1}{T} \sum_{t=1}^{T} (\varepsilon_{t+k|t})^2}.$$
 (2.19)

Alternatively, the *mean absolute error* (MAE) criterion is based on a linear loss function.

**Definition 2.14.** The MAE is defined as the average of the absolute forecast errors over an evaluation set of length T,

$$MAE(k) = \frac{1}{T} \sum_{t=1}^{T} |\varepsilon_{t+k|t}|.$$
 (2.20)

All the above error criteria are independent of the length of the evaluation set. They are normalized by its length. They may also be normalized by the nominal capacity of



**Fig. 2.8** Evaluation of point forecasts of wind power generation for Western Denmark with bias, MAE and RMSE criteria, over a period of almost two years

the renewable energy site (or aggregation) of interest, and then referred to as Nbias, NRMSE, and NMAE.

Example 2.11 (Evaluation of Point Forecasts of Wind Power Generation) Point forecasts of wind power generation issued for the whole Western Denmark area, with lead times between 1 and 43 h ahead, are evaluated over a period of almost two years. Figure 2.8 shows the result of such an evaluation, as the evolution of bias, MAE, and RMSE as a function of the lead time. Recall that the nominal capacity of that area is 2.515 GW.

The forecasts are almost unbiased, while the forecast accuracy degrades almost linearly as the lead time increases. RMSE values are greater than MAE values owing to the nature of their underlying loss functions (squared errors for the RMSE, and errors in absolute value for the MAE).

For the example of the 24-hour ahead lead time, the MAE and RMSE of the forecasts are 122 MW and 173 MW. If considering their normalized versions instead, NMAE and NMRSE values are 4.86 % and 6.89 % of the nominal capacity, respectively.

#### 2.4.3 Probabilistic Forecasts

The quality of probabilistic forecasts of renewable energy is dominated by their probabilistic calibration and their overall skill. These aspects are treated one after the other in the following. They may be relevant for all types of probabilistic forecasts, i.e., quantile, interval and density forecasts. The quality of probabilistic forecasts is

here generally evaluated in a nonparametric framework: no assumption is made about the shape of predictive densities. Indeed, a particular predictive density  $\hat{f}_{t+k|t}$  issued at time t for time t+k is simply characterized by sets of quantile forecasts  $\hat{q}_{t+k|t}^{(\alpha_i)}$  with nominal levels  $\alpha_i$ ,  $i=1,\ldots,m$ .

#### 2.4.3.1 Probabilistic Calibration

The first requirement for probabilistic forecasts is for them to consistently inform about the probability of events. This directly leads to the concept of *probabilistic calibration*, also referred to as *reliability*. The general definition of probabilistic calibration is introduced for the most general density forecasts, while it may then be derived for quantile and interval forecasts in a straightforward manner.

**Definition 2.15.** Probabilistic forecasts for a given lead time k, defined by their predictive cdfs  $\hat{F}_{t+k|t}$ , are said to be probabilistically calibrated if

$$\hat{F}_{t+k|t}(Y_{t+k}) \sim U[0,1],$$
 (2.21)

that is, if the forecast probabilities are consistent with the observed ones.

That definition is difficult to use in practice, since only one realization of  $Y_{t+k}$  is observed. One, therefore, employs a frequentist approach for the assessment of probabilistic calibration, based on an evaluation set of sufficient length. There it is verified that the *probability integral transform* (PIT)  $\hat{F}_{t+k|t}(y_{t+k})$  of the measurements follows a uniform distribution over the unit interval. This may be performed in a nonparametric set-up by assessing the reliability of each of the defining quantile forecasts. For that purpose, the *indicator variable*  $\xi_{t,k}^{(\alpha)}$  is introduced as follows:

**Definition 2.16.** The indicator variable  $\xi_{t,k}^{(\alpha)}$ , for a given quantile forecast  $\hat{q}_{t+k|t}^{(\alpha)}$  and corresponding realization  $y_{t+k}$  is defined as

$$\xi_{t,k}^{(\alpha)} = 1\{y_{t+k} < \hat{q}_{t+k|t}^{(\alpha)}\} = \begin{cases} 1, & \text{if } y_{t+k} < \hat{q}_{t+k|t}^{(\alpha)}, \\ 0, & \text{otherwise} \end{cases}$$
(2.22)

i.e., as a binary variable indicating if the quantile forecasts actually cover, or not, the renewable power measurements.

Example 2.12 (Indicator Variable and Quantile Forecasts) Quantile forecasts of wind power generation are issued at time t and time t+1, with lead time k and for a nominal level of 0.8. For instance, the forecasts for time t+k and t+k+1 are 1.86 GW and 1.93 GW, respectively. The power measurements, obtained a posteriori, are 1.43 GW and 1.98 GW. As a consequence the indicator variable takes the value 1 for time t+k and 0 for time t+k+1.

These '1's and '0's are nicknamed *hits* and *misses*. It is by studying the binary time-series  $\{\xi_{t,k}^{(\alpha)}\}$  of indicator variable values that one can assess the reliability of quantile forecasts, for given nominal level  $\alpha$  and lead time k. Adapting the definition of probabilistic calibration, quantile forecasts  $\hat{q}_{t+k|t}^{(\alpha)}$  with nominal level  $\alpha$  are

probabilistically calibrated if

$$\xi_{t,k}^{(\alpha)} \sim B(\alpha),$$
 (2.23)

that is, if the corresponding indicator variable  $\xi_{t,k}^{(\alpha)}$  is distributed Bernoulli with chance of success  $\alpha$ .

Using this indicator variable, the *empirical level* of quantile forecasts can be defined, estimated, and eventually compared with their nominal one.

**Definition 2.17.** The *empirical level*  $\hat{a}_k^{(\alpha)}$ , for a nominal level  $\alpha$  and lead time k, is obtained by calculating the mean of the  $\{\xi_{t,k}^{(\alpha)}\}$  time-series over an evaluation set of length T,

$$\hat{a}_{k}^{(\alpha)} = \frac{1}{T} \sum_{t=1}^{T} \xi_{t,k}^{(\alpha)} = \frac{n_{k,1}^{(\alpha)}}{n_{k,0}^{(\alpha)} + n_{k,1}^{(\alpha)}},$$
(2.24)

where  $n_{k,1}^{(\alpha)}$  and  $n_{k,0}^{(\alpha)}$  correspond to the sum of hits and misses, respectively. More specifically, they are calculated as

$$n_{k,1}^{(\alpha)} = \#\{\xi_{t,k}^{(\alpha)} = 1\} = \sum_{t=1}^{T} \xi_{t,k}^{(\alpha)}, \tag{2.25}$$

$$n_{k,0}^{(\alpha)} = \#\{\xi_{t,k}^{(\alpha)} = 0\} = T - n_{k,1}^{(\alpha)}.$$
 (2.26)

The difference between nominal and empirical levels of quantile forecasts is to be seen as a probabilistic bias.

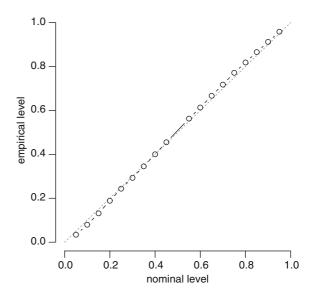
Example 2.13 (Nominal and Empirical Levels of Quantile Forecasts) Let us build on Example 2.12, by considering quantile forecasts issued for a lead time k and for a nominal level of 0.8, over an evaluation set with 200 forecasts and corresponding observations. After counting the number of times the observations were lower or greater than the quantile forecasts, the sum of hits and misses are  $n_{k,1}^{(0.8)} = 153$  and  $n_{k,0}^{(0.8)} = 47$ , yielding an empirical level of 0.765. It is slightly less than the nominal one.

Prediction intervals and nonparametric predictive densities are characterized by a set of quantile forecasts. Consequently, their reliability may be evaluated similarly by verifying the empirical level of their defining quantile forecasts. Such an approach can also be used for parametric probabilistic forecasts by extracting quantiles for given nominal levels. The overall deviation from perfect probabilistic calibration is quantified by the so-called *reliability* or *discrepancy* index.

**Definition 2.18.** For a given lead time k, the *reliability* (also called *discrepancy*) index  $\Delta(k)$  corresponds to the average absolute difference between nominal and empirical levels,

$$\Delta(k) = \frac{1}{m} \sum_{i=1}^{m} |\alpha_i - \hat{a}_k^{(\alpha_i)}|, \qquad (2.27)$$

Fig. 2.9 Reliability diagram for the evaluation of the reliability of probabilistic forecasts for the whole onshore capacity of Western Denmark over a period of almost 2 years. The lead time considered here is 12-hour ahead. The probabilistic forecasts are defined by a number of quantile predictions, with nominal levels between 0.05 and 0.95, and with increments of 0.05, except for the median



for interval or density forecasts defined by a set of quantile forecasts with nominal levels  $\alpha_i$ , i = 1, ..., m.

Instead of calculating discrepancy indices, probabilistic calibration may be appraised visually based on reliability diagrams plotting empirical vs. nominal levels of the quantiles defining density forecasts.

Example 2.14 (Reliability Diagrams and Discrepancy Index) Probabilistic forecasts of wind power generation issued for the whole Western Denmark area, with lead times between 1 and 43 h ahead, are evaluated over a period of almost two years. These probabilistic forecasts take the form of predictive densities defined by a number of quantile predictions, with nominal levels between 0.05 and 0.95, and with increments of 0.05, except for the median.

The calibration of these probabilistic forecasts is performed by evaluating that of quantile forecasts for all nominal levels. The resulting comparison of empirical and nominal levels is summarized in the reliability diagram such as that in Fig. 2.9, for the example of 12-hour ahead forecasts. Their correspondence appears to be very good, with a low discrepancy index value of 0.012. This discrepancy index value corresponds to the average difference between what was observed (line with circle markers) and the ideal case of perfect calibration (dotted line). Results for other lead times may be looked at similarly.

#### 2.4.3.2 Skill of Probabilistic Forecasts

As for the case of the bias for point forecasts, the assessment of probabilistic calibration only informs about a form of bias of probabilistic forecasts. The fact that they may be perfectly calibrated does not guarantee that the forecasts are really good,

for instance, they may not be able to discriminate among situations with various uncertainty levels, while these aspects are of crucial importance in decision-making.

The overall quality of probabilistic forecasts may be assessed based on skill scores for quantile, interval and density forecasts. The first skill score that may be used is the *negative quantile-based score* (*NQS*).

**Definition 2.19.** For predictive densities  $\hat{f}_{t+k|t}$  defined by sets of quantile forecasts  $\hat{q}_{t+k|t}^{(\alpha_i)}$  with nominal levels  $\alpha_i$ ,  $i=1,\ldots,m$ , and corresponding measurements  $y_{t+k}$ , the NQS is given by

$$NQS(k) = \frac{1}{T} \frac{1}{m} \sum_{t=1}^{T} \sum_{i=1}^{m} (\alpha_i - \xi_{t,k}^{(\alpha_i)}) (y_{t+k} - \hat{q}_{t+k|t}^{(\alpha_i)}),$$
 (2.28)

over an evaluation set of length T.

This score is negatively oriented and admits a minimum value of 0 for perfect probabilistic predictions. It may be readily used for quantile forecasts (then the sum over i in Eq. (2.28) disappears) and for prediction intervals (with m = 2 in Eq. (2.28)).

When focus is on predictive densities only, the most widely used score is the *continuous ranked probability score (CRPS)*.

**Definition 2.20.** The *CRPS* for predictive densities  $\hat{F}_{t+k|t}$  and corresponding measurement  $y_{t+k}$ , is calculated as

$$CRPS(k) = \frac{1}{T} \sum_{t=1}^{T} \int_{x} \left( \hat{F}_{t+k|t}(x) - H(x - y_{t+k}) \right)^{2} dx,$$
 (2.29)

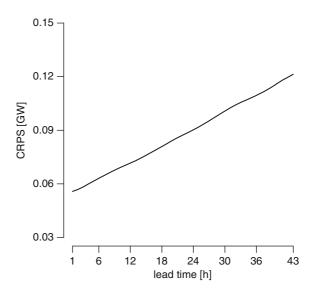
over an evaluation set of length T. H(x) is the Heaviside step function, taking the value 1 for  $x \ge y$  and 0 otherwise.

The CRPS evaluates the area between the predictive cdf and that of the observation (which would have been the perfect forecast). It is a proper skill score: it is minimal when the true distribution of events is used as predictive density. It is a negatively-oriented score (the lower the better) and has the same unit than the variable of interest, while taking a minimum of 0. Note that it can be directly compared to the MAE criterion (defined by Eq. (2.20)) used for point forecasts, since the CRPS is its generalization in a probabilistic forecasting framework.

The computation of the CRPS can directly rely on analytical formulas in the case of some parametric predictive densities, e.g., Gaussian (also for truncated and censored). In the nonparametric case, it is obtained using numerical integration through predictive cdfs as defined by the set of quantile forecasts. Asymptotically, as the number m of quantiles considered tends towards infinity, one exactly has CRPS = 2NQS.

Example 2.15 (Skill of Probabilistic Forecasts) For the same dataset with probabilistic forecasts of wind power generation as in Example 2.14, and for the same period as in Example 2.11, the skill of these forecasts is evaluated as a function of the lead time, with the CRPS criterion. The NQS criterion is not shown, though it

Fig. 2.10 Skill evaluation of probabilistic forecasts for the whole onshore capacity of Western Denmark over a period of almost 2 years, as given by the evolution of the CRPS score as a function of lead time. The probabilistic forecasts are defined by a number of quantile predictions, with nominal levels between 0.05 and 0.95, and with increments of 0.05, except for the median



could also be employed by summing quantile score values for all defining nominal levels. Remember that these probabilistic forecasts are given by a number of quantile forecasts, with nominal levels between 0.05 and 0.95, and with increments of 0.05, except for the median. Linear interpolation through these quantiles yields complete predictive densities and related cumulative distribution functions. The results of the forecast skill evaluation are displayed in Fig. 2.10.

The decrease in forecast skill is fairly linear, similar to the point forecast verification results shown and discussed in Example 2.11, from a CRPS value of 56 MW for the first lead time to a value of 121 MW for the 43-hour ahead lead time. These values are slightly less than the MAE values in Fig. 2.8, hence telling that these probabilistic forecasts have higher skill (and could be seen as more informative) than the previously studied point predictions.

#### 2.4.4 Scenarios

Scenarios ought to be seen as an extension of probabilistic forecasts in a multivariate framework, in the form of sample trajectories from multivariate probabilistic predictions. They should therefore be evaluated in a manner consistent with that described for probabilistic forecasts in the above, based on probabilistic calibration and skill.

One the one hand, probabilistic calibration of scenarios may be assessed based on the multivariate generalization of calibration for probabilistic forecasts (Def. 2.15). Since it may be too difficult to directly use that definition for evaluating the probabilistic calibration of scenarios, alternative proposals were made in the literature, relying on the concept of ranks. Owing to the complexity of these approaches to the reliability assessment of scenarios, these aspects are overlooked here. The interested reader is referred to [14] and references therein for a complete treatment.

On the other hand, the skill of scenarios is more straightforward to appraise. Recall that the skill scores for probabilistic forecasts presented in Sect. 2.4.3.2 are to be computed for each lead time, location and type of renewable energy sources. They therefore overlook the interdependence structure that is modeled by scenarios. In order to assess skill, a multivariate generalization is necessary for the case of scenarios. Today, the lead skill score for that purpose is the *energy score*, which is a direct generalization of the CRPS defined by Eq. (2.29).

**Definition 2.21.** For a given set of time trajectories  $\hat{z}_t^{(j)} j = 1, \dots, J$ , issued at time t, the *energy score* is given by

$$\operatorname{Es}_{t} = \frac{1}{J} \sum_{j=1}^{J} \|z_{t} - \hat{z}_{t}^{(j)}\|_{2} - \frac{1}{2J^{2}} \sum_{i=1}^{J} \sum_{j=1}^{J} \|\hat{z}_{t}^{(i)} - \hat{z}_{t}^{(j)}\|_{2}, \tag{2.30}$$

where  $\|.\|_2$  is the K-dimensional Euclidean norm (also called  $l^2$  norm). It is then calculated and averaged over an evaluation set of length T,

$$Es = \frac{1}{T} \sum_{t=1}^{T} Es_t.$$
 (2.31)

The energy score is a strictly proper skill score, in the sense that it is minimal only when the true distribution is used for generating trajectories. It is a negatively-oriented score (the lower the better), while having the same unit than the variable of interest.

Example 2.16 (Skill of Scenarios of Wind Power Generation) A set of 12 scenarios of wind power generation was shown in Fig. 2.6, as issued on 4th April 2007 at 00:00 UTC for the whole onshore capacity of Western Denmark, and for a nominal capacity of 2.515 GW. Applying the formula of Eq. (2.30) for assessing their quality based on the energy score yields a score value of 0.765 GW.

# 2.5 Model-Based Approaches to Generating Renewable Power Forecasts

Renewable power forecasting has its wealth of models and methods that have appeared in the scientific literature and in commercial software over the last couple of decades. The development of forecasting approaches intensified since the beginning of the new millennium with the boost in the deployment of renewable energy capacities. Most of these approaches were first introduced for wind energy, reflecting its leading role in this deployment. Since then, similar methodologies have appeared for, e.g., solar and wave energy.

As it is impossible (neither is it our aim) to describe all approaches to renewable energy forecasting, our objective is instead to cover some of the basic aspects of interest to readers aiming to use them as input to operational problems related to electricity markets. For that, we place ourselves in a probabilistic framework, where probabilistic forecasts comprise a basis product, while point forecasts and scenarios are to be derived from these probabilistic forecasts.

# 2.5.1 Overview of Forecasting Methodologies and Required Inputs

Power generation from all forms of renewable energy sources is directly influenced by some of the weather variables, e.g., wind (and air density to a lesser extent) for wind energy, solar irradiance, and temperature for solar energy, and finally, significant wave height and period for wave energy. In parallel, the power output of renewable energy devices is a function of the technology embedded, which impact the way the energy originally provided by the weather variables is transformed into electric energy. Consequently, for all forms of renewable energy sources, the core aspects of the prediction exercise include (i) the appraisal of external conditions that drive the energy conversion process, and (ii) the energy conversion process itself, that is, how the potential energy in external conditions is eventually converted to electrical power.

The mathematical modeling involved in these two core aspects may rely on physical and statistical concepts, optimally on a combination of both. The share of physical and statistical expertise to be blend in to get the best forecasts will depend on the lead times of interest, world location (linking to local climatology), and maybe, the level of expertise of those laying the mathematical models and forecasting method.

For the latter aspect, the effect of technology can be directly modeled based on physical and statistical approaches. For the former aspect however, one is bound to rely on weather forecast providers, since all the modeling and computing involved in predicting relevant weather variables may be too cumbersome for the intended application. Note, nevertheless, that some consider predicting these relevant weather variables themselves, most often based on statistical time-series models such as those encountered in the finance literature.

Two types of data are employed as input to the various existing forecasting methodologies. On the one hand, relevant observations at the site of interest (and potentially offsite) of power generation and of relevant meteorological variables permit to better appraise the local process dynamics. This set of observations is generically denoted  $\psi_t$ . On the other hand, forecasts of the relevant weather variables, as provided by any public or private weather forecasting office, permit to give an overall picture of how input weather variables may evolve in the coming hours to days. This set of input meteorological forecasts is denoted by  $\Gamma_t$ .

Example 2.17 In the case of wind power forecasting, the set  $\psi_t$  of observations available at time t may be such that

$$\psi_t = \{ y_t, y_{t-1}, \dots, u_t, u_{t-1}, \dots, \omega_t, \omega_{t-1}, \dots \},$$
 (2.32)

where y is for power measurements, while u and  $\omega$  are for observed wind speed and direction respectively. These variables will be different when looking at solar and wave energy prediction, e.g., solar irradiance and cloud cover for the former, and wave height and period for the latter. In parallel, the set  $\Gamma_t$  of meteorological forecasts available at a given time t may take the form of

$$\Gamma_t = \{\hat{u}_{t+1|t}, \hat{u}_{t+2|t}, \dots, \hat{u}_{t+K|t}, \hat{\omega}_{t+1|t}, \hat{\omega}_{t+2|t}, \dots, \hat{\omega}_{t+K|t}\},$$
(2.33)

i.e., with forecasts of wind speed and direction for the site or portfolio of interest, for all lead times up to the forecast length K. Again, these variables will be different for the solar and wave energy cases.

Finally, in a generic manner, any prediction of renewable energy generation issued at time t, being point, probabilistic, scenario or event-based forecast is a linear or nonlinear function of these sets  $\psi_t$  and  $\Gamma_t$  (or of some of their subsets).

# 2.5.2 Issuing Probabilistic Forecasts

Probabilistic forecasts of renewable power generation are the most general form of prediction, informing on the whole range of potential power outcomes for every lead time. They were illustrated in the form of predictive densities in Fig. 2.5, as well as in the simpler version of prediction intervals in Fig. 2.4. The way they may be generated based on parametric and nonparametric approaches is presented below.

#### 2.5.2.1 Parametric Approaches

The interest of parametric approaches is to rely on parametric assumptions, which can be seen as predefined shapes for predictive densities. These shapes are fully characterized by a few parameters only, hence easing subsequent estimation.

**Definition 2.22.** Following a parametric assumption, the distribution of the stochastic process at time t + k is such that

$$Y_{t+k} \sim F(y; \theta_{t+k}), \tag{2.34}$$

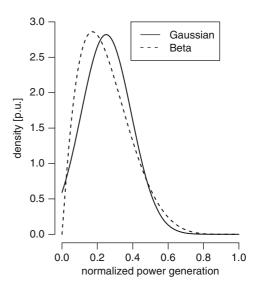
where F is the parametric distribution of choice, and where  $\theta_{t+k}$  is a set of parameters fully determining the distribution F. A probabilistic forecast issued at time t for  $Y_{t+k}$  is then obtained after predicting the value of the set of parameters  $\theta_{t+k}$ , i.e.,

$$Y_{t+k} \sim \hat{F}_{t+k|t},\tag{2.35}$$

where

$$\hat{F}_{t+k|t} = F(y; \hat{\theta}_{t+k|t}).$$
 (2.36)

Fig. 2.11 Probabilistic forecasts of normalized wind power generation based on Gaussian and Beta distributions, with the same mean and variance of 25 % and 2 % of the nominal capacity



The choice for F comes from an expert guess of the forecaster or after a thorough empirical analysis of the stochastic process based on available data. Furthermore, it is the set of parameters  $\theta_{t+k}$  that is to be modeled and then predicted based on the information available at a given time t.

Example 2.18 (Gaussian and Beta Predictive Densities with Same Mean and Variance) Consider forecasting wind power generation (normalized by nominal capacity) for a specific portfolio and for lead times of several hours ahead, based on a Gaussian assumption for predictive densities,

$$Y_{t+k} \sim \mathcal{N}(\hat{\mu}_{t+k}, \hat{\sigma}_{t+k}^2), \tag{2.37}$$

for instance, with a mean  $\hat{\mu}_{t+k} = 25\%$  of nominal capacity and a variance  $\hat{\sigma}_{t+k}^2 = 2\%$ . Alternatively, the forecast may be based on a Beta assumption,

$$Y_{t+k} \sim \text{Beta}(\hat{\alpha}_{t+k}, \hat{\beta}_{t+k}).$$
 (2.38)

As an example, for the same mean and variance, the parameters of the Beta distribution would be  $\hat{\alpha}_{t+k} = 2.09$  and  $\hat{\beta}_{t+k} = 6.28$ . Even if the mean and variance are the same, the resulting distributions look different, as illustrated in Fig. 2.11. This is because the skewness and kurtosis of Beta distributions are also a function of their mean and variance, while this is not the case for Gaussian distributions.

The choice for one assumption or the other will provide different types of information on the range of potential outcomes and their probability. In both cases, the set of parameters  $\theta_{t+k}$  to be modeled and predicted is of dimension two only. Formulating a Gaussian assumption may not be the best choice, since it would not reflect the double-bounded nature of wind power generation (between zero and the nominal capacity of the portfolio).

Despite the fact that a wide range of candidate distributions F exist for the probabilistic forecasting of renewable energy generation, the Gaussian one (and its truncated and censored versions) certainly is the most commonly employed in practice. Other possibilities which are not necessarily more complex should be envisaged depending upon the type of renewable energy, the forecast range, or upon the size and geographical spreading of the portfolio more generally. As an example, Beta distributions are a good alternative to Gaussian ones when it comes to wind power. If more specifically focusing on very-short term fluctuations of wind power generation, Generalized logit-Normal (GL-Normal) should be preferred. Likewise, when forecasting wave energy flux and wave power generation, log-Normal and GL-Normal distributions, respectively, are relevant candidate distributions. Note that no particular results were reported so far for the case of solar energy.

Any chosen distribution F can be fully characterized by the set of parameters  $\theta_{t+k}$ . Mathematical models are then used to describe the future evolution of this set of parameters, as a function of the information set discussed before. For the  $i^{\text{th}}$  element (out of l) of  $\theta_{t+k}$ , this writes

$$\theta_{i,t+k} = g_i(\psi_t, \Gamma_t) + \varepsilon_{i,t+k}, \quad i = 1, \dots, l,$$
(2.39)

with  $\varepsilon_{i,t+k}$  a centered noise term, and with  $\psi_t$  and  $\Gamma_t$  the set of observations and forecasts available at time t. The  $g_i$  models may be linear or not depending upon the forecaster's expertise and the result of an empirical analysis of the data available.

Example 2.19 The previous example is continued here. Note that if choosing a Beta assumption, given that  $\alpha_{t+k}$  and  $\beta_{t+k}$  may be directly linked to a mean  $\mu_{t+k}$  and a variance  $\sigma_{t+k}^2$  parameter, it may still be more natural to work with these latter two quantities. The mean  $\mu_{t+k}$  of predictive densities is commonly modeled as a nonlinear function of wind forecasts for that lead time,

$$\mu_{t+k} = g_{\mu}(\hat{u}_{t+k|t}, \hat{\omega}_{t+k|t}) + \varepsilon_{\mu,t+k},$$
(2.40)

with  $\varepsilon_{\mu,t+k}$  a centered noise term. This model reflects the power curve converting the available energy in the wind to electric power, for the wind portfolio of interest. In parallel, the variance  $\sigma_{t+k}^2$  could be modeled as a constant, in the most simple case,

$$\sigma_{t+k}^2 = c_k + \varepsilon_{\sigma,t+k},\tag{2.41}$$

with  $\varepsilon_{\sigma,t+k}$  also a centered noise term. The constant  $c_k$  is most certainly different for every lead time k. For both models, the parameters of  $g_{\mu}$  and the constant  $c_k$  would have to be estimated from data.

#### 2.5.2.2 Nonparametric Approaches

In contrast with the parametric approaches presented in the above, the nonparametric ones do not rely on any specific assumption regarding the shape of predictive densities. As a consequence, it is necessary to fully characterize the distribution F

instead of having to model and predict a limited number of parameters only. Since it is not possible to do so, F is commonly summarized by a set of quantiles with appropriately chosen nominal levels.

**Definition 2.23.** In a nonparametric setup, the distribution of the stochastic process at time t + k is such that

$$Y_{t+k} \sim F_{t+k},\tag{2.42}$$

where  $F_{t+k}$  is summarized by a set of Q quantiles

$$F_{t+k} = \{ q_{t+k}^{(\alpha_i)}, \ 0 \le \alpha_1 < \dots < \alpha_i < \dots < \alpha_O \le 1 \}, \tag{2.43}$$

that is, with chosen nominal levels spread over the unit interval. A probabilistic forecast issued at time t for  $Y_{t+k}$  is then obtained based on a set of quantile forecasts for these Q nominal levels,

$$Y_{t+k} \sim \hat{F}_{t+k|t},\tag{2.44}$$

where

$$\hat{F}_{t+k|t} = \{\hat{q}_{t+k|t}^{(\alpha_i)}, \ 0 \le \alpha_1 < \dots < \alpha_i < \dots < \alpha_Q \le 1\}. \tag{2.45}$$

A model is to be proposed for each of the Q quantiles permitting to define nonparametric distributions,

$$q_{t+k}^{(\alpha_i)} = g_i(\psi_t, \Gamma_t) + \varepsilon_{i,t+k}, \quad i = 1, \dots, Q,$$
(2.46)

with  $\varepsilon_{i,t+k}$  a centered noise term, and with  $\psi_t$  and  $\Gamma_t$  the set of observations and forecasts available at time t. Here again, these  $g_i$  models may be linear or not depending upon the forecaster's expertise and the result of an empirical analysis of the data available. The number of models and the corresponding potential computational burden may rapidly increase in comparison with the parametric approaches presented before as the number Q of defining quantiles gets large. Indeed, while most parametric distributions may be fully characterized with two or three parameters only, nonparametric distributions will require a minimum of Q=18-20 quantiles to provide a satisfactory description. This was, for instance, the case of the representation of probabilistic forecasts in Fig. 2.5, where 18 quantiles were used for describing distributions of wind power generation for every lead time, i.e., with  $\alpha_i \in \{0.05, 0.1, \ldots, 0.45, 0.55, \ldots, 0.9, 0.95\}$ .

Example 2.20 Similar to the parametric approach in the above, consider the issue of probabilistic forecasting of wind power generation for a given portfolio and lead time k. Models for the quantiles  $q_{t+k}^{(\alpha_i)}$  may actually be formulated in a manner similar to that for  $\mu_{t+k}$  in the parametric case, that is,

$$q_{t+k}^{(\alpha_i)} = g_{\alpha_i}(\hat{u}_{t+k|t}, \hat{\omega}_{t+k|t}) + \varepsilon_{i,t+k}, \quad i = 1, \dots, Q,$$
 (2.47)

with  $\varepsilon_{i,t+k}$  a centered noise term. Such models will also have some shape similar to that of the power curve for the wind portfolio, except that it will reflect given thresholds in terms of potential outcomes of the stochastic process.

# 2.5.3 Extracting Single-Valued Forecasts

Because probabilistic forecasts may be too difficult to handle and interpret by a number of forecast users, it is often the case that single-valued forecasts are to be extracted from probabilistic density forecasts. Note, however, that they could also be directly generated if aiming to skip the step of producing full density forecasts of renewable power generation. This is the case if directly predicting the conditional expectation of renewable power generation (as in Eq. (2.40)), or if modeling and predicting quantiles for a specific nominal level (as in Eq. (2.47)).

Indeed, two types of single-valued forecasts were introduced in Sect. 2.3: those corresponding to the conditional expectation of the stochastic process for every lead time, and quantile forecasts that inform on a probabilistic threshold (for a given nominal level) for the range of outcomes to be expected. These various definitions of single-valued forecasts are to be linked to the so-called *loss function* of forecast users, already mentioned a few times in previous sections, which will then permit to extract single-valued predictions from density forecasts. Formally, a loss function  $L(\hat{y}, y)$  gives the perceived loss of the forecast user if provided with the forecast  $\hat{y}$  while the value y then materializes.

A special relevant case of a loss function is the quadratic one,

$$L_2(\hat{y}, y) = (y - \hat{y})^2. \tag{2.48}$$

At time t a point forecast  $\hat{y}_{t+k|t}$  for time t+k is the value of the process such that it minimizes the expected loss for the forecast user for all potential realizations of the process, given our state of knowledge at that time. This translates to

$$\hat{\mathbf{y}}_{t+k|t} = \arg\min_{\hat{\mathbf{y}}} \mathbb{E}[L_2(\hat{\mathbf{y}}, \mathbf{y}) \mid \mathbf{y} \sim F_{t+k}, g, \hat{\Theta}_t]. \tag{2.49}$$

The value that is optimal in terms of this problem is that given in Eq. (2.4), that is, the conditional expectation of  $Y_{t+k}$ .

Quantile forecasts are related to another type of loss function  $L(\hat{y}, y)$  to be assumed for the forecast user. This loss function is the piecewise linear one, sometimes nicknamed as the "pinball" loss,

$$L_{\alpha}(\hat{y}, y) = \begin{cases} \alpha |\hat{y} - y|, & \text{if } y \leq \hat{y}, \\ (1 - \alpha)|\hat{y} - y|, & \text{otherwise.} \end{cases}$$
 (2.50)

Similar to the case of Eq. (2.49), quantile forecast can be defined as the solution of an optimization problem in a decision-theoretic framework,

$$\hat{q}_{t+k|t}^{(\alpha)} = \arg\min_{\hat{y}} \mathbb{E}[L_{\alpha}(\hat{y}, y) \mid y \sim F_{t+k}, g, \hat{\Theta}_t], \tag{2.51}$$

i.e., as the quantity minimizing the expected pinball loss for any potential realization of  $Y_{t+k}$  with cumulative distribution function  $F_{t+k}$ .

Example 2.21 (Extracting Single-Valued Forecasts from Predictive Densities) Let us build on Example 2.18, where predictive densities were issued based on Gaussian and

Beta parametric assumptions with the same mean and variance parameters ( $\mu_{t+k} = 0.25$  and  $\sigma_{t+k}^2 = 0.02$ ).

Consider here two types of loss functions, based on which single-valued forecasts should be extracted from these predictive densities. These loss functions are the quadratic one introduced in Eq. (2.48), and the piecewise linear one of Eq. (2.50), with  $\alpha = 0.5$ .

In the case of the quadratic loss function, the optimal single-valued prediction to be extracted is given by optimization problem (2.49). It yields  $\hat{y}_{t+k|t} = 0.25$  for both distributions (i.e., their expected value). Looking at the piecewise linear case instead, solving optimization problem (2.51) gives different single-valued forecasts for the two predictive densities:  $\hat{y}_{t+k|t} = 0.25$  for the Gaussian distribution and  $\hat{y}_{t+k|t} = 0.23$  for the Beta distribution. These single-valued forecasts correspond to their median. They are not the same owing to the different shape of these predictive densities, as can be seen from Fig. 2.11.

# 2.5.4 Issuing Scenarios

Scenarios of renewable power generation comprise the most important type of forecast input to operational problems when it comes to renewable energy in electricity markets. They ought to represent both

- (i) the *marginal predictive densities* of power generation for each lead time, location and renewable energy source individually, and
- (ii) the *interdependence structure* in power generation through time, space and type of renewable energy sources.

The central ideas about the generation of scenarios of renewable power generation are detailed below, with considerations related to building joint predictive densities from marginal ones, to the modeling of interdependence structures, and finally to the generation of scenarios themselves.

#### 2.5.4.1 From Marginal to Joint Predictive Densities

In the most general set-up, let us assume that at any given time t, one aims at predicting the full set of characteristics of the multivariate random variable

$$Z_t = \{Y_{r,s,t+k}, r = r_1, \dots, r_m, s = s_1, \dots, s_n, k = 1, \dots, K\}.$$
 (2.52)

This multivariate random variable jointly considers m types of renewable energy sources,  $r = r_1, \ldots, r_m$ , a set of locations  $s = s_1, \ldots, s_n$ , as well as the set of lead times  $k = 1, \ldots, K$ .

Based on the concepts presented before, predictive densities of renewable power generation are available for the random variables  $Y_{r,s,t+k}$ . That for the  $i^{th}$  type of renewable energy source  $r_i$ , the  $j^{th}$  location  $s_j$  and the  $k^{th}$  lead time is denoted  $\hat{F}_{i,j,t+k|t}$ .

The overall set of predictive densities is consequently written  $\{\hat{F}_{i,j,t+k|t}\}$ . These densities are referred to as *marginal* since being issued for each lead time, location, and energy source, individually, hence not informing about the interdependence structure of  $Z_t$ .

In order to overcome this lack of information about such interdependence, the set of marginal predictive densities may be augmented so as to become a joint predictive density.

**Definition 2.24 (Joint Predictive Density).** A joint predictive density  $\hat{F}_{Z_t}$  issued at time t for m types of renewable energy sources  $r = r_1, \ldots, r_m$ , a set of locations  $s = s_1, \ldots, s_n$  and a set of look-ahead times  $k = 1, \ldots, K$ , can be obtained by complementing the set of marginal predictive densities  $\{\hat{F}_{i,j,t+k|t}\}$  with a *copula* model,

$$\hat{F}_{Z_t} = \{ \{ \hat{F}_{i,j,t+k|t} \}, C(\delta r, \delta s, \delta k) \}, \tag{2.53}$$

where the copula model  $C(\delta r, \delta s, \delta k)$  gives the whole information about the interdependence among sources, locations and lead times.  $\delta r, \delta s$  and  $\delta k$  are here to denote variations in these variables.

An exhaustive introduction to copulas and copula-based modeling is available in [13]. The above definition follows from the probabilistic calibration property of probabilistic forecasts introduced in Sect. 2.4.3. Indeed, in the case where every marginal predictive density  $\hat{F}_{i,j,t+k|t}$  is calibrated,

$$U_{i,j,t+k|t} = \hat{F}_{i,j,t+k|t}^{-1}(Y_{i,j,t+k}), \quad U_{i,j,t+k|t} \sim U[0,1].$$
 (2.54)

One can then introduce at time t the multivariate random variable  $U_t$  with marginal distributions  $U_{i,j,t+k|t}$ ,  $\forall i, j, k$ . The so-called copula function permits to define the joint cumulative distribution of  $U_t$ . Example contributions of joint predictive densities based on a Gaussian copula and various different forms of covariance models are given in the following.

#### 2.5.4.2 Gaussian Copula and Alternative Covariance Models

Applying a Gaussian Copula

The simplest and most convenient copula to use is the Gaussian one. Its basis consists in transforming each uniform random variable  $U_{i,j,t+k|t}$  to standard Gaussian with

$$\Phi^{-1}\left(U_{i,j,t+k|t}\right) \sim \mathcal{N}(0,1),$$
 (2.55)

where  $\Phi^{-1}$  is the inverse of the cumulative distribution function for a standard Gaussian distribution.

Consequently, the copula model  $C(\delta l, \delta s, \delta k)$  permitting to summarize the whole interdependence structure reduces to a covariance structure. It can be similarly referred to as correlation structure, since the Gaussian variables involved have unit

variance. In addition, considering that this covariance structure is separable, the multidimensional (and potentially complex) covariance  $C(\delta r, \delta s, \delta k)$  reduces to

$$C(\delta r, \delta s, \delta k) = C_r(\delta r)C_s(\delta s)C_k(\delta k), \tag{2.56}$$

that is, as the direct product of three one-dimensional covariance functions.

Example 2.22 (The Wind-Wave Offshore Installation) One may look at a wind-wave offshore installation, for which multivariate scenarios are to be issued. For simplicity, only one lead time (say, 24 h ahead) is considered. Predictive densities of wind and wave power generation are produced for that lead time. Employing a Gaussian copula, the interdependence structure in Eq. (2.56), for wind and wave power, reduces to  $C_r(\delta_r)$ , with

$$C_r(\delta_r) = \rho(r_1, r_2), \tag{2.57}$$

which is a single correlation coefficient describing the interdependence between wind  $(r_1)$  and wave  $(r_2)$  power generation, at that location and for that lead time.

Example 2.23 (Spatio-Temporal Dependencies in Wind Power Generation) A more advanced case is that of spatio-temporal dependencies of wind power generation (following the setup of Sect. 2.2 and the test case of Western Denmark) for which multivariate scenarios may be required as input to problems for optimal power system operations. Predictive densities of wind power generation are available for all 15 control zones and for lead times between 1 and 43 h ahead. Based on Eq. (2.56), the interdependence structure is summarized as

$$C(\delta s, \delta k) = C_s(\delta s)C_k(\delta k), \tag{2.58}$$

for which the spatial and temporal dependence structures have to be described individually. This is dealt with in the following.

#### Covariance Modeling

Different strategies exist for the modeling and estimation of the covariance functions involved. In the case where the dimension is low, as for the offshore wind-wave energy setup of Example 2.22, it may be easier to summarize all information with a single correlation matrix  $\Sigma = {\rho_{ij}}$ , where each and every element  $\rho_{ij}$  may indifferently inform of dependence among locations, lead times and types of renewable energy sources.

Example 2.24 (Interdependence Among n-Locations) Let us focus on one single form of renewable energy generation, say, wind energy, for a given lead time (24 h ahead), at n locations  $s = s_1, s_2, \ldots, s_n$ . Predictive densities of wind power generation are produced for that lead time and these n locations. The whole spatial covariance structure  $C_s(\delta_s)$  can be summarized in the form of a correlation matrix  $\Sigma = {\rho_{ij}}$  of dimension  $n \times n$ , where

$$\rho_{ij} = \rho(s_i, s_j), \quad i, j \in \{1, 2, \dots, n\},$$
(2.59)

that is, by the value of the correlation coefficient for each pairwise association of sites.

When the dimension of the problem gets high, it may not be possible to handle the full description of the interdependence structure by evaluating all correlation values in  $\Sigma$ . It may then be beneficial to parametrize such a dependence with covariance functions.

Example 2.25 (Spatio-Temporal Dependencies in Wind Power Generation (Continued)) For the case of spatio-temporal dependencies of wind power generation (as a continuation of Example 2.23 with 15 control zones and 43 lead times, a correlation matrix allowing to summarize all interdependencies would have a dimension of  $645 \times 645$  (i.e., 15 times 43). It would translate to 416,025 correlation values to be calculated in order to fully characterize the interdependence structure. One may propose covariance functions instead, for both spatial and temporal dimensions. For instance, assuming an exponential decay in time and in space, thus considering a rapidly decreasing interdependence with increasing  $\delta s$  and  $\delta k$ , this yields

$$C_s(\delta s) = \exp(\delta s/\nu_s),$$
 (2.60)

and

$$C_k(\delta k) = \exp(\delta k/\nu_k),$$
 (2.61)

where  $v_s$  and  $v_k$  are range parameters in space and in time, controlling the speed of the exponential decay. The model hence rely on two parameters only, which is considerably less than the original 416,025 correlations that should have been calculated.

Following this parametric covariance modeling, a covariance matrix may then be derived, if necessary, for the subsequent generation of scenarios. This is performed by evaluating the correlation values in  $\Sigma$  for all associations of locations and lead times.

The parameters of the covariance models can be estimated in varied statistical frameworks, for instance, with exponential smoothing for the tracking of the correlation matrix  $\Sigma$ , or with weighted least squares or maximum likelihood techniques for the parameters of covariance functions (for more details, see e.g., [14]).

#### 2.5.4.3 Generation of Scenarios

Based on the above, one has at a given time t joint probabilistic forecasts  $\hat{F}_{Z_t}$  covering all potential locations, lead times and types of renewable energy sources. Following Eq. (2.53), these are formed by marginal predictive densities  $\{\hat{F}_{i,j,t+k|t}\}$  and by a copula model simplifying the covariance structure  $C(\delta r, \delta s, \delta k)$ .

For issuing a set of J scenarios at time t, one first needs to sample a number J of realizations  $z^{(j)}$  from a multivariate Gaussian random variable Z, with mean 0

and covariance matrix  $\Sigma$  as estimated directly or derived from parametric modeling based on covariance functions,

$$Z \sim \mathcal{N}(0, \Sigma).$$
 (2.62)

Using the cumulative distribution function  $\Phi$  for a standard Gaussian random variable, as well as the predictive cumulative distribution functions  $\{\hat{F}_{i,j,t+k|t}\}$  for every lead time, location and renewable energy source, these multivariate Gaussian realizations are transformed into trajectories of wind power generation having the appropriate marginal distributions,

$$\hat{z}_{t}^{(j)} = \hat{F}_{i,j,t+k|t}^{-1} \left( \Phi(z^{(j)}) \right), \quad j = 1, \dots, J.$$
 (2.63)

This is illustrated below for the example of the Western Denmark dataset, already used in some of the previous examples.

Example 2.26 (Space–Time Trajectories for Western Denmark) We consider again the dataset with wind power generation at the 15 control zones of the Energinet.dk control area in Western Denmark, for which an episode with two days of wind power generation was depicted in Fig. 2.1.

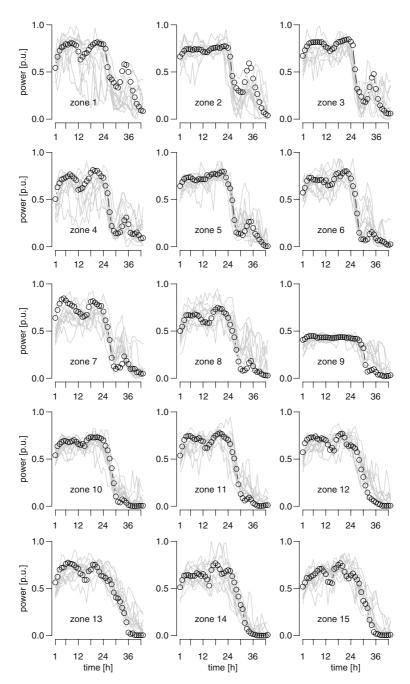
Based on input meteorological forecasts and on statistical methods, nonparametric probabilistic forecasts of wind power production are generated every 6 h, with a forecast length of 43 h. In parallel, a parametric space–time covariance structure is chosen, being separable and with exponential decay in time and in space. The range parameters for these exponential covariance models in space and in time are of 125 km and 7 h, respectively. For comparison, the maximum distance between the centroid of the control zones is of 250 km, between zones 1 and 15. By combining the nonparametric probabilistic forecasts and the covariance structure, scenarios of wind power generation can be issued.

As an illustration, Fig. 2.12 gathers a set of 12 spatio-temporal scenarios for this dataset, issued on 15th January 2007 at 00:00 UTC and normalized by the nominal capacity of every control zone.

# 2.6 Summary and Conclusions

This chapter presents the most common inputs to operational problems related to renewable energy sources in electricity markets. It therefore places particular emphasis on various forms of forecasts, since markets are cleared a fair amount of time before actual operation.

These forecasts are introduced in their various forms, while insisting on the necessity to appraise them in a probabilistic framework. Indeed, renewable energy forecasts will always have a non-negligible share of uncertainty. Such uncertainty is, at the very least, contingent on external conditions (most generally, the weather conditions) and to the time-varying state of the energy conversion systems. The type



**Fig. 2.12** Scenarios with 43-hour ahead wind power generation at the 15 control zones of Western Denmark issued on the 15th January 2007 at 00:00 UTC, normalized by nominal capacity. These are based on input nonparametric probabilistic forecasts and on a parametric separable space–time covariance structure with exponential decay. The decay parameters in space and in time are of 125 km and 7 h, respectively. Observations, obtained a posteriori, are also shown

of forecasts to be used as input to operational problems will depend upon the nature of the problem itself, and on the way the corresponding optimization problem is formulated. Not using the appropriate type of forecasts in operational problems may certainly lead to suboptimal decisions and policies, even though these may be more transparent and easier to understand.

Basics of forecast verification were covered, in order for the reader to acquire background knowledge on forecast quality. It is intuitively expected that higher-quality forecasts will yield better policies and decisions. Evaluating the quality of probabilistic forecasts is, however, a difficult task. A complete coverage would require some further reading, suggested below. Similarly, when aiming at building appropriate models for generating these forecasts, extensive literature exists. Only main principles were discussed in this chapter.

Nowadays, the renewable energy forecasting field is extremely active and dynamic. It is hence expected that new forecasting methods and products will be proposed and used in operational problems related to electricity markets in the coming years.

# 2.7 Further Reading

This chapter aimed at giving a compact overview of renewable power forecasting, laying down some of the main concepts and key characteristics of the various types of forecasts to be used as input to operational problems. For those interested in a more extensive coverage of renewable power forecasting, we refer to [7] for the case of wind energy, mainly focusing on a physical approach to the problem, hence overlooking some of the statistical aspects of relevance. In parallel, for the case of solar energy, a good overview can be obtained from [1] and [8]. Finally, some relevant approaches and discussion on forecasting for wave energy can be found in [17] and [18]. Advanced works on the generation of scenarios of renewable power production include those in [12] and [15].

Readers who are interested in learning more about the question of forecast quality, and about how to thoroughly evaluate prediction before to use them in operational problems, may find extensive information in [9] for point predictions, in [3] and [16] for probabilistic forecasts, and lastly, in [14] for scenarios of renewable power generation. More generally, [4] and [19] give a good overview on aspects of multivariate probabilistic forecast evaluation. Finally, a discussion on point forecasts and how to optimally extract them from probabilistic predictions is given in [2].

#### **Exercises**

**2.1.** Quantile forecasts with nominal level  $\alpha = 0.3$  are issued at time t for the following 6 h, for power generation at a solar power plant with a nominal capacity of 12 MW:

$$\hat{q}_{t+1|t}^{(0.3)} = 2MW, \ \hat{q}_{t+2|t}^{(0.3)} = 9MW, \ \hat{q}_{t+3|t}^{(0.3)} = 11MW, \ \hat{q}_{t+4|t}^{(0.3)} = 10MW.$$

What is the predicted probability that solar power generation will be above 9 MW at time t + 2?

**2.2.** A number of 1-hour ahead point forecasts are issued for power generation at this same solar power plant (in MW):

$$\{2, 3.5, 4.2, 5.6, 7.4, 5.6, 6.4, 5.3, 6.7, 8.6, 9.3, 4.7\}.$$
 (2.64)

The corresponding observations are obtained a posteriori:

$$\{1.8, 3.9, 4, 5.1, 7.2, 6.1, 6.7, 5.9, 6.6, 8.3, 10.5, 6.2\}.$$
 (2.65)

Evaluate these point forecasts with the common scores introduced in this chapter, i.e., bias, MAE, and RMSE, calculated both in MW and in their normalized version, i.e., as percentage of installed capacity.

**2.3.** A nonparametric predictive density for power generation at a wind farm of 35 MW is issued, at time t for time t + k, based on a set of quantile forecasts with increasing nominal levels, i.e.,

$$\hat{q}_{t+k|t}^{(0.05)} = 6$$
MW,  $\hat{q}_{t+k|t}^{(0.25)} = 9$ MW,  $\hat{q}_{t+k|t}^{(0.5)} = 11$ MW,  $\hat{q}_{t+k|t}^{(0.75)} = 14$ MW,  $\hat{q}_{t+k|t}^{(0.95)} = 17$ MW.

Based on these forecasts, define central prediction intervals with nominal coverage rates of  $50\,\%$  and  $90\,\%$  for that lead time.

**2.4.** A parametric predictive density for the normalized power generation at a wind farm is issued at time t + k in the form of a Beta distribution such that

$$Y_{t+k} \sim \text{Beta}(6.28, 2.09).$$

Deduce from that predictive density a number of point and interval forecasts:

- 1. the point forecast that would be extracted under a quadratic loss function,
- 2. the point forecast that would be extracted under a pinball loss function, with  $\alpha = 0.4$ ,
- 3. the central prediction interval with nominal coverage rate  $(1 \beta) = 0.8$ .

Finally, what is the (predicted) probability that the normalized power generation will be greater than 25 % of nominal capacity?

**2.5.** Two forecasters generate at time t different parametric predictive densities of normalized wind power generation for time t + k. The first one is

$$Y_{t+k} \sim \text{Beta}(2,5),$$

while the second one is

$$Y_{t+k} \sim \mathcal{N}(0.4, 0.01).$$

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Which of the two forecasters predict the highest probability that power generation will be greater than 50 % of nominal capacity at time t + k?

**2.6.** Parametric predictive densities for the normalized power generation at a wind farm are issued at time t for time t+1 and t+2, based on a Beta distribution assumption, with

$$Y_{t+1} \sim \text{Beta}(2,5), \ Y_{t+2} \sim \text{Beta}(5,4).$$

In addition, the interdependence structure between the two lead times is modeled with a Gaussian copula with a covariance matrix

$$\Sigma = \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix}.$$

Based on these predictive densities and this interdependence structure, issue 100 scenarios of wind power generation and evaluate them with the energy score, given that the normalized power observations for times t + 1 and t + 2 are  $y_{t+1} = 0.3$  and  $y_{t+2} = 0.38$ .

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