Assignment #3:
This assignment combines the unsteady blade loads with the elastic structural response of the wind turbine rotor. Consider a simple structural model for a wind turbine as sketched in Figure 1.

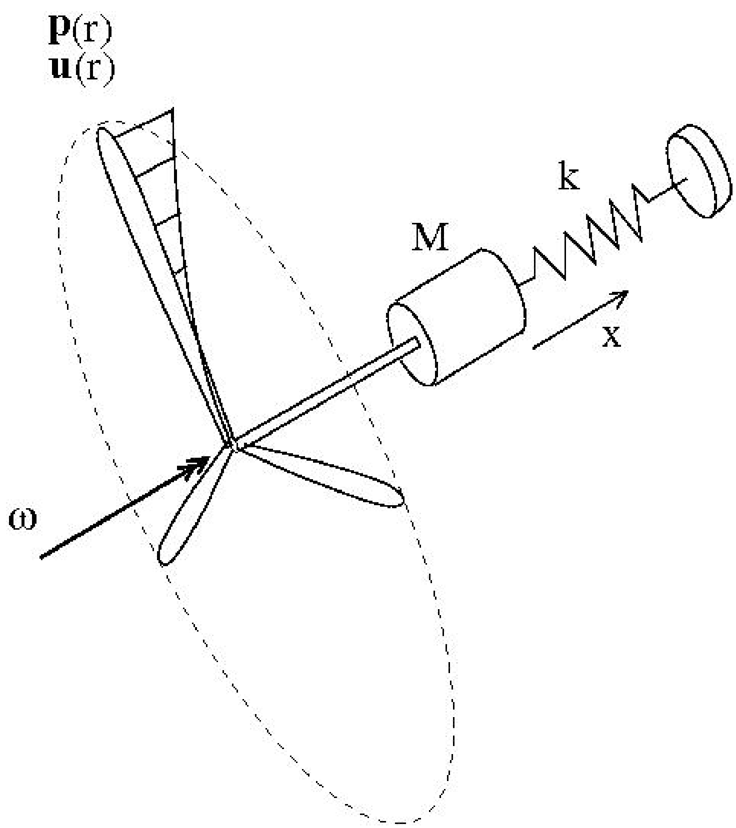

Figure 1: A simple structural model of a wind turbine rotor. M is the mass of the nacelle, drivetrain and generator.

The tower stiffness fore-aft is modelled as a simple spring with a spring stiffness, $k_1=1.7\cdot10^6$N/m. The blade deflections are modelled using a modal approach as a linear combination of the three lowest eigenmodes (1f = first flap, 1e first edge and 2f=second flap) as

$$\underline{u}_n(r) = \begin{pmatrix} u_y(r) \\ u_z(r) \end{pmatrix} = q_n^1(t)\begin{pmatrix} u_y^{1f} \\ u_z^{1f} \end{pmatrix} + q_n^2(t)\begin{pmatrix} u_y^{1e} \\ u_z^{1e} \end{pmatrix} + q_n^3(t)\begin{pmatrix} u_y^{2f} \\ u_z^{2f} \end{pmatrix} \quad \text{n denotes the blade number 1,2 or 3}$$

That is there is 3 DOFs per blade (q_n^1, q_n^2, q_n^3 n=1,2,3).

The DOFs are thus

$$\underline{GX} = \begin{pmatrix} x \\ q_1^1 \\ q_1^2 \\ q_1^3 \\ q_2^1 \\ q_2^2 \\ q_2^3 \\ q_3^1 \\ q_3^2 \\ q_3^3 \end{pmatrix}$$

Q#1: Derive the mass and stiffness matrices and the generalized force vector. Further, describe how to calculate the relative wind speeds at the rotor blades. Estimate the tower for-aft natural frequency.

Structural data:
Mass M=446000kg
Blade mass distribution:

r [m]	m [kg/m]
2.800	1189.51
11.00	1113.62
16.87	868.87
22.96	735.79
32.31	566.23
41.57	477.51
50.41	385.14
58.53	291.66
65.75	221.77
71.97	153.75
77.19	108.93
78.71	95.18
80.14	82.34
82.71	68.28
84.93	54.47
86.83	40.65
88.45	25.20
89.17	15.42

The mode shapes are uploaded to CampusNet assuming a pitch angle of -3.34^o, and their corresponding eigenfrequencies are ω^{1f}= 3.93 rad/s, ω^{1e} =6.10 rad/s and ω^{2f} =11.28 rad/s. Note and explain how the mode shapes change with the pitch angle.

Q#2: First assume that the tower is stiff and only consider one elastic blade (3-DOF) and solve the system in time domain for a turbulent inflow for a mean wind speed of 8 m/s. Show how to include inertia loads when computing the root bending moments and try and quantify their effect.

Please provide 1) blade tip deflection in flapwise and edgewise direction, 2) blade root flap and edgewise bending moment at r = 2.8m.

Q#3: Now assume all three blades and the tower to be elastic (10-DOF) and repeat Q#2 but also include the tower deflection.

Describe how the elastic velocities are included when computing the relative wind speed.
If your code becomes very unstable, you may stabilize it by including some small structural damping (see Eq. 12.39 in Aerodynamics of wind turbines book and using around 3% logarithmic decrement damping value).