Supplementary Material to "TI-FPCA: Effective and Interpretable Dimensionality Reduction with Transform-Invariant Functional Principal Component Analysis"

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1 Additional Experimental Results

Table 1. NormMSE of various models for oscillating data and varying number of components.

num. comp.	1	2	3	4	5	6	7	8	9	10
Weight length 512										
TI-FPCA	**0.630**	**0.462**	**0.399**	**0.344**	**0.301**	**0.265**	**0.238**	**0.217**	**0.200**	**0.185**
Autoencoder	0.884	0.715	0.571	0.486	0.454	0.439	0.430	0.423	0.417	0.412
FAE	0.904	0.750	0.623	0.517	0.433	0.428	0.426	0.427	0.426	0.424
FPCA	0.920	0.842	0.768	0.696	0.625	0.553	0.499	0.446	0.424	0.405
PCA	0.920	0.842	0.768	0.696	0.625	0.553	0.498	0.445	0.424	0.404
VAE	0.882	0.709	0.572	0.484	0.458	0.440	0.431	0.423	0.417	0.413
Weight length 256										
TI-FPCA	0.927	0.851	0.781	0.708	0.633	0.559	0.502	0.449	0.426	**0.404**
Autoencoder	**0.875**	0.710	0.572	**0.485**	0.452	0.444	0.432	**0.425**	**0.417**	0.412
FAE	0.897	0.740	0.623	0.513	**0.433**	**0.428**	**0.428**	0.428	0.425	0.426
FPCA	0.926	0.855	0.778	0.706	0.633	0.560	0.505	0.449	0.426	0.405
PCA	0.926	0.855	0.779	0.706	0.633	0.560	0.504	0.449	0.426	**0.404**
VAE	0.880	**0.699**	**0.560**	0.491	0.457	0.440	0.433	**0.425**	0.419	0.414

Table 2. NormMSEs and accuracies of various models for spiking data and varying number of components in the classification setting.

num. comp.	1	2	3	4	5	6	7	8	9	10
NormMSEs – (i) $T = 128, w_\ell = 256$										
TI-FPCA	**0.081**	**0.055**	**0.048**	**0.045**	**0.042**	**0.039**	**0.037**	**0.034**	**0.032**	**0.030**
Autoencoder	0.494	0.123	0.069	0.056	0.054	0.052	0.052	0.051	0.051	0.051
FAE	0.478	0.086	0.074	0.063	0.059	0.060	0.061	0.057	0.057	0.057
FPCA	0.866	0.764	0.668	0.581	0.507	0.432	0.367	0.309	0.254	0.210
PCA	0.866	0.764	0.668	0.581	0.507	0.432	0.367	0.309	0.254	0.210
VAE	0.491	0.104	0.072	0.057	0.054	0.052	0.052	0.051	0.051	0.051
NormMSEs – (ii) $T = 256 = w_\ell = 256$										
TI-FPCA	0.898	0.797	0.697	0.603	0.514	0.433	0.369	0.309	0.259	0.214
Autoencoder	0.517	0.125	0.079	**0.057**	0.055	**0.053**	0.054	0.053	0.053	**0.052**
FAE	0.487	**0.093**	**0.065**	**0.057**	**0.054**	0.054	0.054	0.053	0.053	0.053
FPCA	0.859	0.758	0.662	0.580	0.500	0.428	0.367	0.306	0.259	0.213
PCA	0.860	0.757	0.662	0.580	0.500	0.428	0.367	0.306	0.259	0.213
VAE	**0.462**	0.134	0.072	**0.057**	**0.054**	**0.053**	**0.052**	**0.052**	**0.052**	**0.052**
NormMSEs – (iii) $T = 256, w_\ell = 128$										
TI-FPCA	**0.454**	0.185	0.095	0.078	0.068	0.060	0.056	**0.054**	**0.051**	**0.050**
Autoencoder	0.502	0.134	0.070	**0.058**	**0.056**	0.056	**0.055**	0.055	0.055	0.055
FAE	0.488	**0.100**	**0.069**	0.060	0.057	0.057	0.057	0.056	0.057	0.055
FPCA	0.857	0.760	0.673	0.590	0.515	0.446	0.384	0.319	0.265	0.219
PCA	0.857	0.759	0.673	0.590	0.515	0.446	0.384	0.319	0.265	0.219
VAE	0.533	0.143	0.077	0.059	**0.056**	**0.055**	**0.055**	0.055	0.055	0.055
Accuracies – (i) $T = 128, w_\ell = 256$										
TI-FPCA	**99.7**	**99.8**	**99.8**	99.8	**99.8**	99.7	**99.8**	**99.8**	99.7	99.7
Autoencoder	93.5	99.0	99.7	**99.9**	99.8	**99.8**	99.7	**99.8**	**99.8**	**99.8**
FAE	95.0	99.5	99.6	99.6	**99.8**	**99.8**	99.5	99.7	99.7	**99.8**
FPCA	87.1	93.5	95.9	97.5	98.2	97.5	98.2	98.7	99.1	99.4
PCA	92.0	98.0	98.1	99.4	99.1	99.6	99.6	99.6	99.7	**99.8**
VAE	92.5	99.0	98.8	99.4	99.1	99.5	98.7	99.5	99.5	98.9
Accuracies – (ii) $T = w_\ell = 256$										
TI-FPCA	82.0	90.6	95.3	97.6	98.8	99.4	99.5	**99.9**	**99.9**	**100.0**
Autoencoder	94.5	98.2	**100.0**	**100.0**	**99.8**	**99.8**	**99.8**	99.8	99.6	99.8
FAE	94.5	**98.6**	99.8	99.7	99.5	99.4	99.6	99.4	99.5	99.5
FPCA	86.1	93.2	94.7	95.9	96.1	97.1	97.5	98.4	98.6	98.8
PCA	91.9	98.1	99.0	99.2	99.1	99.4	99.5	99.8	99.6	99.7
VAE	**95.5**	98.4	99.1	98.9	98.6	99.4	97.7	98.8	99.0	99.6
Accuracies – (iii) $T = 256, w_\ell = 256$										
TI-FPCA	83.9	97.1	**100.0**	**100.0**	99.8	99.8	**100.0**	99.9	99.8	99.8
Autoencoder	92.4	**99.0**	99.8	99.9	**100.0**	**100.0**	99.9	**100.0**	**100.0**	99.8
FAE	**94.0**	98.7	**100.0**	99.8	99.7	99.6	99.8	99.9	99.8	99.8
FPCA	87.8	94.3	95.3	96.6	97.2	97.6	98.1	98.7	99.2	99.6
PCA	93.5	98.0	98.9	99.7	99.3	99.6	99.6	99.6	99.8	**99.9**
VAE	90.5	98.3	99.6	99.0	99.4	99.3	99.3	99.6	98.7	99.6

Table 3. NormMSEs and accuracies of various models for spiking data and varying number of components in the outlier detection setting.

num. comp.	1	2	3	4	5	6	7	8	9	10
NormMSEs – (i) $T = 128, w_\ell = 256$										
TI-FPCA	**0.058**	**0.031**	**0.027**	**0.025**	**0.023**	**0.022**	**0.021**	**0.019**	**0.018**	**0.017**
Autoencoder	0.503	0.091	0.059	0.036	0.033	0.031	0.030	0.031	0.030	0.029
FAE	0.446	0.072	0.055	0.047	0.043	0.041	0.042	0.040	0.038	0.038
FPCA	0.805	0.708	0.618	0.526	0.440	0.371	0.305	0.253	0.208	0.172
PCA	0.805	0.708	0.618	0.526	0.440	0.371	0.305	0.253	0.208	0.171
VAE	0.503	0.113	0.049	0.037	0.034	0.031	0.031	0.030	0.029	0.029
NormMSEs – (ii) $T = w_\ell = 256$										
TI-FPCA	0.900	0.804	0.705	0.604	0.515	0.435	0.361	0.296	0.241	0.196
Autoencoder	0.529	0.119	0.051	**0.034**	**0.031**	**0.030**	**0.029**	**0.029**	**0.029**	0.029
FAE	**0.427**	**0.083**	0.049	0.038	0.035	0.033	0.033	0.032	0.032	0.032
FPCA	0.817	0.723	0.622	0.531	0.454	0.379	0.314	0.260	0.214	0.175
PCA	0.817	0.723	0.622	0.531	0.454	0.378	0.314	0.260	0.214	0.175
VAE	0.521	0.103	**0.046**	0.035	**0.031**	**0.030**	**0.029**	**0.029**	**0.029**	**0.028**
NormMSEs – (iii) $T = 256, w_\ell = 128$										
TI-FPCA	0.443	0.183	0.071	0.054	0.042	0.036	0.033	0.030	**0.028**	**0.027**
Autoencoder	0.479	0.100	0.059	0.035	**0.030**	0.030	**0.029**	**0.029**	0.029	0.029
FAE	**0.441**	**0.079**	0.054	0.037	0.035	0.033	0.031	0.031	0.031	0.031
FPCA	0.808	0.714	0.622	0.531	0.453	0.376	0.312	0.256	0.211	0.172
PCA	0.808	0.714	0.622	0.531	0.453	0.376	0.312	0.256	0.211	0.172
VAE	0.490	0.114	**0.049**	**0.034**	0.031	**0.029**	**0.029**	**0.029**	**0.028**	0.028
Accuracies – (i) $T = 128, w_\ell = 256$										
TI-FPCA	**100.0**	**99.9**	**99.8**	**99.9**	**99.8**	99.7	99.7	99.6	99.8	**99.8**
Autoencoder	92.2	99.1	**99.8**	99.8	**99.8**	**99.8**	100.0	99.9	99.8	**99.8**
FAE	91.6	97.9	99.6	99.6	99.6	**99.8**	99.6	**99.9**	99.9	99.8
FPCA	85.8	92.3	94.3	96.4	96.9	97.0	96.5	97.5	97.7	97.3
PCA	90.0	96.9	97.9	98.9	99.0	99.0	99.5	99.5	99.7	99.7
VAE	92.8	98.9	99.3	99.2	99.4	99.6	99.4	98.3	98.7	99.6
Accuracies – (ii) $T = w_\ell = 256$										
TI-FPCA	80.6	88.4	94.0	97.4	98.1	98.9	**99.6**	99.4	**99.6**	99.6
Autoencoder	89.0	97.9	**99.8**	**99.8**	**99.8**	99.6	**99.6**	**99.8**	**99.6**	**99.9**
FAE	91.6	97.0	99.2	99.4	99.4	99.2	99.0	99.3	99.5	99.6
FPCA	84.5	93.1	95.2	95.3	96.6	96.2	97.3	97.7	97.5	97.5
PCA	91.1	97.4	98.5	99.1	99.2	99.3	99.5	99.3	99.5	99.5
VAE	**94.1**	**98.9**	99.0	99.6	99.2	99.2	**99.6**	99.0	99.2	99.4
Accuracies – (iii) $T = 256, w_\ell = 128$										
TI-FPCA	84.2	95.8	**99.8**	99.9	**99.9**	**99.9**	99.8	**99.8**	**99.8**	99.8
Autoencoder	91.0	98.4	**99.8**	100.0	**99.9**	99.8	100.0	**99.8**	**99.8**	99.6
FAE	94.8	97.5	99.7	99.5	99.8	99.6	99.4	**99.8**	**99.8**	**99.9**
FPCA	89.1	91.8	95.8	97.0	97.4	97.9	98.5	98.7	98.7	98.7
PCA	93.6	96.7	98.0	99.2	99.3	99.5	99.7	99.6	**99.8**	99.6
VAE	**95.9**	**98.6**	99.6	99.7	99.8	99.8	99.1	99.7	94.1	99.6

Table 4. NormMSEs of various models for handwriting data and varying number of components.

num. comp.	1	2	3	4	5	6	7	8	9	10
Window length $T = 700$										
TI-FPCA	**0.19678**	**0.07352**	**0.06074**	**0.05701**	**0.05557**	**0.05453**	**0.05353**	**0.05274**	**0.05209**	0.05140
Autoencoder	0.33396	0.27798	0.22390	0.17056	0.19546	0.13699	0.13331	0.13047	0.13089	0.11474
FAE	0.33177	0.25203	0.20505	0.18767	0.13284	0.14730	0.13745	0.13098	0.12464	0.13046
FPCA	0.58668	0.46507	0.29278	0.19991	0.12351	0.09727	0.08003	0.06252	0.05897	0.05035
PCA	0.55888	0.42417	0.28315	0.18654	0.11854	0.09520	0.07898	0.06152	0.05816	**0.04860**
VAE	0.44787	0.29972	0.24056	0.20145	0.18606	0.16859	0.16071	0.17557	0.17893	0.17488
Window length $T = 1400$										
TI-FPCA	0.00591	0.00590	0.00589	0.00589	0.00587	0.00587	0.00586	0.00585	0.00584	0.00584
Autoencoder	0.00684	0.00638	0.00653	0.00614	0.00647	0.00653	0.00576	0.00605	0.00635	0.00564
FAE	0.00653	0.00619	0.00605	0.00602	0.00563	0.00539	0.00525	0.00570	0.00513	0.00523
FPCA	**0.00585**	**0.00480**	**0.00405**	**0.00373**	**0.00338**	0.00324	0.00306	0.00294	0.00289	0.00273
PCA	0.00586	0.00484	0.00412	0.00374	0.00339	**0.00318**	**0.00302**	**0.00291**	**0.00283**	**0.00271**
VAE	0.00671	0.00665	0.00668	0.00668	0.00667	0.00667	0.00700	0.00721	0.00768	0.00690

Table 5. NormMSEs and accuracies of various models for ErrP data and varying number of components in the classification setting.

num. comp.	1	2	3	4	5	6	7	8	9	10
NormMSEs – (i) sample length: 0.5 s, weight length: 1 s										
TI-FPCA	**0.622**	**0.429**	**0.365**	**0.318**	**0.282**	**0.255**	**0.233**	**0.217**	**0.203**	**0.191**
Autoencoder	0.838	0.708	0.681	0.655	0.637	0.628	0.621	0.604	0.590	0.567
FAE	0.934	1.169	1.173	0.969	1.003	0.857	0.894	0.858	0.886	0.839
FPCA	0.840	0.727	0.660	0.583	0.523	0.466	0.437	0.415	0.397	0.348
PCA	0.839	0.726	0.658	0.581	0.526	0.470	0.437	0.414	0.396	0.347
VAE	0.850	0.781	0.747	0.717	0.683	0.650	0.627	0.631	0.592	0.582
NormMSEs – (ii) sample and weight length: 0.5 s										
TI-FPCA	**0.844**	0.726	**0.650**	**0.572**	0.556	0.503	0.455	0.415	**0.396**	**0.348**
Autoencoder	0.848	**0.711**	0.688	0.665	0.640	0.620	0.608	0.594	0.574	0.568
FAE	0.891	1.057	0.907	1.215	1.080	0.766	0.824	0.840	0.843	0.787
FPCA	0.854	0.727	0.652	0.582	**0.553**	**0.502**	**0.437**	0.416	0.398	**0.348**
PCA	0.850	0.726	**0.650**	0.583	0.561	**0.502**	**0.437**	**0.414**	0.397	**0.348**
VAE	0.874	0.775	0.738	0.708	0.691	0.664	0.634	0.627	0.645	0.594
NormMSEs – (iii) sample length: 1 s, weight length: 0.5 s										
TI-FPCA	**0.784**	**0.679**	**0.609**	**0.562**	**0.515**	0.479	0.455	0.430	0.404	0.384
Autoencoder	0.861	0.724	0.708	0.678	0.667	0.651	0.646	0.632	0.626	0.619
FAE	0.927	1.027	1.134	1.098	1.094	0.925	1.005	0.848	0.877	0.939
FPCA	0.863	0.753	0.666	0.572	0.529	**0.472**	0.423	**0.403**	**0.385**	**0.368**
PCA	0.862	0.752	0.664	0.583	0.551	**0.472**	**0.422**	0.406	**0.385**	0.372
VAE	0.890	0.795	0.770	0.711	0.725	0.671	0.696	0.672	0.627	0.647
Accuracies – (i) sample length: 0.5 s, weight length: 1 s										
TI-FPCA	**97.0**	83.4	**97.**0	**97.0**	**97.0**	81.3	88.9	**97.0**	**97.0**	95.6
Autoencoder	**97.0**	96.5	96.8	96.4	95.9	90.3	92.6	95.1	84.7	93.3
FAE	**97.0**	**97.0**	94.6	92.8	95.1	95.5	94.9	96.3	91.4	93.2
FPCA	**97.0**	**97.0**	94.7	88.5	89.6	92.1	92.9	94.4	92.2	91.5
PCA	**97.0**	**97.0**	91.8	89.0	87.7	87.0	92.2	88.0	92.8	85.3
VAE	**97.0**	**97.0**	96.5	96.8	96.7	**96.5**	**97.0**	96.4	96.9	**96.9**
Accuracies – (ii) sample and weight length: 0.5 s										
TI-FPCA	**97.2**	**97.2**	90.6	94.5	88.8	93.0	**97.1**	**97.2**	92.0	84.3
Autoencoder	**97.2**	**97.2**	96.9	94.5	90.8	90.7	87.8	94.0	94.6	92.6
FAE	**97.2**	96.6	94.9	93.3	94.9	93.4	93.3	92.1	90.2	85.5
FPCA	**97.2**	**97.2**	92.3	92.6	91.2	88.5	94.0	96.7	91.6	91.5
PCA	**97.2**	96.8	93.7	91.7	93.0	86.0	85.7	85.9	84.7	88.5
VAE	**97.2**	97.0	**97.1**	**96.8**	**96.9**	**97.0**	97.0	97.1	**97.1**	**96.5**
Accuracies – (iii) sample length: 1 s, weight length: 0.5 s										
TI-FPCA	**93.7**	93.6	**93.7**	**93.7**	**93.6**	91.7	**93.7**	**93.3**	89.5	**93.7**
Autoencoder	**93.7**	93.5	84.1	88.9	89.7	87.4	88.9	88.9	91.8	89.4
FAE	**93.7**	**93.7**	93.1	85.8	91.3	80.6	92.3	88.8	91.6	80.7
FPCA	**93.7**	**93.7**	91.1	91.0	89.0	86.6	90.1	88.4	91.3	92.3
PCA	**93.7**	92.6	86.5	81.7	79.8	82.8	84.0	86.3	80.9	87.1
VAE	93.3	89.5	92.7	85.9	93.2	**92.7**	92.8	93.1	**93.1**	93.6

Table 6. NormMSEs and accuracies of various models for ErrP data and varying number of components in the outlier detection setting.

num. comp.	1	2	3	4	5	6	7	8	9	10
NormMSEs – (i) sample length: 0.5 s, weight length: 1 s										
TI-FPCA	**0.651**	**0.462**	**0.386**	**0.339**	**0.304**	**0.275**	**0.254**	**0.236**	**0.221**	**0.207**
Autoencoder	0.877	0.762	0.728	0.693	0.675	0.664	0.629	0.616	0.606	0.583
FAE	0.960	0.974	0.923	0.884	0.922	0.924	0.864	0.803	0.867	0.728
FPCA	0.878	0.771	0.697	0.611	0.576	0.541	0.508	0.458	0.409	0.381
PCA	0.877	0.771	0.696	0.613	0.575	0.542	0.513	0.465	0.419	0.382
VAE	0.904	0.840	0.809	0.749	0.770	0.700	0.733	0.665	0.640	0.623
NormMSEs – (ii) sample and weight length: 0.5 s										
TI-FPCA	0.877	0.767	0.703	0.637	0.593	0.549	0.517	0.490	0.463	0.422
Autoencoder	**0.869**	**0.765**	0.737	0.704	0.681	0.657	0.644	0.634	0.602	0.580
FAE	0.943	0.877	0.967	0.887	0.837	0.787	0.835	0.807	0.825	0.761
FPCA	0.884	0.770	**0.693**	0.609	0.579	**0.538**	**0.501**	**0.464**	**0.433**	**0.389**
PCA	0.884	0.769	**0.693**	**0.607**	**0.577**	0.545	0.507	0.468	0.439	0.396
VAE	0.908	0.798	0.849	0.764	0.734	0.737	0.721	0.662	0.648	0.635
NormMSEs – (iii) sample length: 1 s, weight length: 0.5 s										
TI-FPCA	**0.813**	**0.710**	**0.649**	**0.606**	**0.564**	**0.530**	0.502	0.474	0.450	0.429
Autoencoder	0.879	0.774	0.743	0.728	0.718	0.710	0.693	0.679	0.673	0.662
FAE	0.941	1.045	1.451	1.109	0.823	0.932	0.980	0.922	0.955	0.957
FPCA	0.880	0.778	0.707	0.636	0.595	0.539	**0.476**	**0.443**	**0.420**	0.405
PCA	0.879	0.778	0.706	0.632	0.612	0.540	0.486	0.446	0.422	**0.403**
VAE	0.930	0.860	0.830	0.827	0.786	NaN	0.759	0.738	0.737	0.706
Accuracies – (i) sample length: 0.5 s, weight length: 1 s										
TI-FPCA	**97.1**	**97.1**	**97.1**	**97.1**	**97.1**	96.1	**97.1**	89.6	**97.1**	**97.1**
Autoencoder	**97.1**	96.7	94.6	88.3	96.3	93.8	95.0	94.2	94.3	95.0
FAE	**97.1**	**97.1**	94.7	91.1	95.2	93.0	95.4	93.1	94.0	95.9
FPCA	**97.1**	**97.1**	95.0	89.4	91.8	95.1	96.4	93.3	88.5	94.9
PCA	96.3	96.9	90.1	87.5	93.1	86.7	87.8	87.4	87.0	85.3
VAE	97.0	96.7	96.8	96.5	96.5	**96.7**	96.7	**96.4**	96.4	96.5
Accuracies – (ii) sample and weight length: 0.5 s										
TI-FPCA	**96.9**	**96.9**	93.6	**96.9**	84.6	95.0	95.8	94.7	92.8	95.5
Autoencoder	96.8	96.7	95.4	95.8	90.2	95.8	93.7	95.5	93.5	90.1
FAE	**96.9**	95.9	94.7	95.4	85.8	90.8	92.3	92.5	**96.4**	93.6
FPCA	**96.9**	**96.9**	92.9	88.6	90.6	92.0	86.6	88.5	94.4	94.5
PCA	**96.9**	96.8	85.4	87.9	89.0	90.5	88.0	87.7	88.7	88.4
VAE	96.8	96.6	**96.5**	96.6	**96.4**	**96.4**	**96.4**	**96.2**	95.7	**96.5**
Accuracies – (iii) sample length: 1 s, weight length: 0.5 s										
TI-FPCA	**94.3**	**94.3**	**94.3**	92.8	**93.9**	**92.7**	**94.2**	**94.3**	90.6	**94.2**
Autoencoder	**94.3**	93.0	91.3	88.3	88.5	85.4	89.8	82.2	88.1	87.7
FAE	**94.3**	**94.3**	93.9	**93.8**	91.9	89.5	91.7	91.1	92.3	86.6
FPCA	**94.3**	**94.3**	90.0	90.8	91.0	92.1	92.4	90.5	**92.6**	92.2
PCA	94.1	93.4	82.5	84.3	86.2	84.6	85.1	79.8	85.8	86.3
VAE	94.2	93.5	85.7	92.6	93.6	92.6	92.7	93.5	**92.6**	93.9

Table 7. NormMSEs and accuracies of various models for MNIST data and varying number of components in the classification setting. "–" indicates that the gradients exploded for all repetitions.

num. comp.	1	2	3	4	5	6	7	8	9	10
NormMSEs – (i) samples of size 16×16										
TI-FPCA	**0.447**	**0.336**	**0.274**	**0.232**	**0.200**	**0.176**	**0.157**	**0.141**	**0.128**	**0.117**
Autoencoder	0.560	0.518	0.463	0.411	0.361	0.315	0.282	0.256	0.229	0.208
FAE	0.562	0.503	0.462	0.424	0.397	0.371	0.358	0.347	0.338	0.339
FPCA	0.597	0.551	0.514	0.477	0.444	0.418	0.394	0.371	0.351	0.332
PCA	0.597	0.551	0.514	0.477	0.444	0.418	0.394	0.371	0.351	0.332
VAE	0.683	0.949	1.068	–	–	–	–	0.986	–	–
NormMSEs – (ii) samples of size 28×28										
TI-FPCA	0.557	0.507	0.463	0.426	0.395	0.370	0.348	0.331	0.314	0.301
Autoencoder	0.485	**0.426**	**0.355**	**0.302**	**0.261**	**0.230**	**0.205**	**0.185**	**0.169**	**0.155**
FAE	**0.482**	0.431	0.395	0.368	0.352	0.338	0.334	0.326	0.317	0.322
FPCA	0.533	0.489	0.453	0.420	0.391	0.366	0.346	0.329	0.312	0.299
PCA	0.533	0.489	0.453	0.420	0.391	0.365	0.346	0.329	0.312	0.299
VAE	0.581	0.581	0.621	0.618	0.454	0.806	0.448	–	–	
NormMSEs – (iii) samples of size 56×56										
TI-FPCA	**0.524**	**0.423**	**0.351**	**0.308**	**0.276**	**0.250**	**0.229**	**0.211**	**0.196**	**0.183**
Autoencoder	0.703	0.590	0.524	0.487	0.444	0.393	0.362	0.326	0.301	0.275
FAE	0.702	0.595	0.551	0.520	0.498	0.482	0.468	0.458	0.446	0.436
FPCA	0.831	0.772	0.726	0.688	0.665	0.643	0.626	0.613	0.600	0.590
PCA	0.831	0.772	0.726	0.688	0.665	0.643	0.626	0.613	0.600	0.590
VAE	0.847	0.756	0.765	0.648	0.726	0.737	0.757	0.659	3.187	0.762
NormMSEs – (iv) randomly rotated samples										
TI-FPCA	**0.460**	**0.376**	**0.318**	**0.281**	**0.253**	**0.232**	**0.214**	**0.199**	**0.185**	**0.174**
Autoencoder	0.542	0.486	0.429	0.388	0.344	0.305	0.275	0.248	0.225	0.201
FAE	0.528	0.482	0.451	0.423	0.401	0.383	0.369	0.357	0.344	0.337
FPCA	0.560	0.518	0.476	0.448	0.427	0.406	0.386	0.366	0.347	0.332
PCA	0.560	0.518	0.476	0.448	0.427	0.406	0.386	0.366	0.347	0.332
VAE	0.566	0.544	0.641	0.575	0.410	7.330	0.425	–	–	0.998
Accuracies – (i) samples of size 16×16										
TI-FPCA	**19.5**	**27.4**	**30.8**	**32.2**	**32.7**	**32.7**	31.9	33.4	35.0	34.6
Autoencoder	16.6	21.6	20.7	21.6	24.1	28.2	26.4	24.8	23.6	24.2
FAE	15.5	19.3	20.8	23.7	24.6	22.6	22.8	20.6	23.2	23.1
FPCA	15.7	20.3	22.1	25.3	27.9	30.8	**34.6**	**34.5**	**35.5**	**36.5**
PCA	14.1	15.5	16.9	17.1	18.9	20.6	22.1	21.6	24.1	23.4
VAE	13.9	10.9	10.9	9.8	9.8	9.8	9.8	9.8	9.8	9.8
Accuracies – (ii) samples of size 28×28										
TI-FPCA	19.8	29.6	36.8	39.0	43.5	47.5	50.1	51.6	53.7	53.8
Autoencoder	40.6	**55.1**	**68.7**	**74.2**	**83.3**	**87.0**	**88.1**	**88.9**	**89.5**	**89.7**
FAE	**44.1**	53.0	63.4	71.2	76.7	80.6	80.5	80.7	82.8	81.1
FPCA	31.4	46.1	50.9	63.2	72.8	79.1	82.6	84.4	84.7	85.7
PCA	29.2	41.7	44.1	54.9	64.5	70.4	74.7	76.2	77.1	79.5
VAE	26.3	23.9	21.2	19.6	14.5	14.0	16.2	9.8	9.8	9.8
Accuracies – (iii) samples of size 56×56										
TI-FPCA	**20.1**	**35.6**	**39.3**	**43.5**	**46.3**	**47.8**	**47.7**	**46.8**	**50.0**	**48.5**
Autoencoder	12.4	13.0	24.4	18.7	18.1	18.8	20.7	21.2	19.7	21.8
FAE	11.1	11.8	18.3	20.5	19.4	20.5	23.4	27.2	26.5	30.9
FPCA	15.2	18.7	19.4	20.4	22.2	23.0	25.0	25.6	26.6	27.0
PCA	13.0	14.1	14.9	15.7	14.6	15.8	16.6	17.6	17.9	18.3
VAE	14.6	14.3	11.3	13.6	12.8	11.6	10.3	12.6	11.6	11.2
Accuracies – (iv) randomly rotated samples										
TI-FPCA	19.4	32.2	**36.9**	39.0	42.6	43.4	44.1	45.9	46.0	44.6
Autoencoder	23.9	**34.8**	36.7	38.3	39.5	42.5	42.6	44.1	45.7	45.9
FAE	24.2	26.8	30.9	36.3	41.1	40.1	40.9	38.8	41.7	38.2
FPCA	**28.2**	30.8	35.7	**44.7**	**47.1**	**48.1**	**53.4**	**56.0**	**57.4**	**57.5**
PCA	26.8	27.2	27.4	34.4	34.6	36.2	45.4	43.4	45.1	44.7
VAE	17.1	15.9	15.8	13.0	12.9	10.5	12.2	9.8	9.8	9.8

Table 8. NormMSEs and accuracies of various models for MNIST data and varying number of components in the outlier detection setting. "–" indicates that the gradients exploded for all repetitions.

num. comp.	1	2	3	4	5	6	7	8	9	10
NormMSEs – (i) samples of size 16×16										
TI-FPCA	**0.340**	**0.267**	**0.220**	**0.189**	**0.166**	**0.148**	**0.133**	**0.121**	**0.111**	**0.102**
Autoencoder	0.470	0.438	0.355	0.311	0.288	0.260	0.234	0.218	0.210	0.191
FAE	0.475	0.427	0.392	0.370	0.356	0.349	0.338	0.330	0.328	0.326
FPCA	0.506	0.461	0.429	0.406	0.384	0.364	0.347	0.332	0.318	0.304
PCA	0.506	0.461	0.429	0.406	0.383	0.364	0.347	0.331	0.317	0.304
VAE	0.473	0.457	0.368	–	–	–	–	–	–	–
NormMSEs – (ii) samples of size 28×28										
TI-FPCA	0.412	0.358	0.329	0.302	0.285	0.269	0.258	0.246	0.234	0.225
Autoencoder	**0.370**	**0.333**	**0.294**	**0.253**	**0.227**	**0.203**	**0.183**	**0.164**	**0.144**	**0.131**
FAE	0.394	0.349	0.321	0.295	0.297	0.285	0.277	0.274	0.276	0.263
FPCA	0.387	0.352	0.325	0.299	0.283	0.267	0.256	0.244	0.233	0.224
PCA	0.387	0.352	0.325	0.299	0.283	0.267	0.256	0.244	0.233	0.223
VAE	0.610	0.508	0.351	0.254	0.233	0.403	–	–	–	–
NormMSEs – (iii) samples of size 56×56										
TI-FPCA	**0.390**	**0.319**	**0.270**	**0.238**	**0.214**	**0.195**	**0.179**	**0.166**	**0.154**	**0.144**
Autoencoder	0.637	0.482	0.415	0.358	0.330	0.306	0.287	0.261	0.243	0.229
FAE	0.615	0.496	0.461	0.438	0.428	0.421	0.414	0.411	0.407	0.403
FPCA	0.800	0.736	0.675	0.630	0.601	0.577	0.554	0.539	0.524	0.514
PCA	0.800	0.736	0.675	0.630	0.601	0.577	0.554	0.539	0.524	0.514
VAE	0.777	0.657	0.600	0.526	0.648	0.661	0.513	0.454	0.385	0.439
NormMSEs – (iv) randomly rotated samples										
TI-FPCA	**0.325**	**0.277**	**0.242**	**0.219**	**0.200**	**0.184**	**0.171**	**0.160**	**0.150**	**0.141**
Autoencoder	0.440	0.383	0.324	0.290	0.254	0.231	0.211	0.190	0.175	0.150
FAE	0.426	0.386	0.354	0.334	0.325	0.318	0.310	0.309	0.301	0.294
FPCA	0.472	0.416	0.386	0.367	0.349	0.333	0.317	0.302	0.286	0.275
PCA	0.472	0.416	0.386	0.367	0.349	0.333	0.317	0.302	0.286	0.275
VAE	0.468	0.510	0.392	0.305	0.301	0.358	0.377	–	0.482	1.413
Accuracies – (i) samples of size 16×16										
TI-FPCA	90.2	**91.0**	**91.1**	**91.4**	**91.9**	**92.0**	**92.1**	**92.1**	**92.2**	**92.1**
Autoencoder	**90.3**	89.8	88.9	88.8	89.7	89.3	90.4	89.7	90.8	89.9
FAE	90.2	89.4	89.8	88.7	89.8	89.8	89.9	89.4	89.5	89.4
FPCA	90.2	90.2	90.1	90.1	90.1	90.1	90.2	90.2	90.2	90.3
PCA	89.8	88.3	88.4	87.6	89.3	89.2	89.6	87.9	89.0	89.5
VAE	**90.3**	90.2	90.3	90.3	90.3	90.3	90.3	90.3	90.3	90.3
Accuracies – (ii) samples of size 28×28										
TI-FPCA	89.9	90.7	90.5	90.5	90.6	90.6	90.8	91.0	91.0	91.4
Autoencoder	90.3	**91.3**	91.7	91.5	92.7	93.7	93.3	94.4	94.2	94.7
FAE	90.3	90.0	90.8	91.3	90.6	90.8	92.3	93.6	92.6	94.9
FPCA	**90.8**	90.8	**93.2**	**93.5**	**94.5**	**94.4**	**94.7**	**95.0**	**95.1**	**95.8**
PCA	89.9	88.8	91.7	92.4	92.7	93.7	94.0	94.0	94.4	94.5
VAE	89.3	90.7	90.7	90.5	90.6	91.1	90.3	90.3	90.3	90.3
Accuracies – (iii) samples of size 56×56										
TI-FPCA	**90.4**	**90.7**	**91.1**	**91.7**	**91.8**	**92.8**	**93.3**	**93.8**	**94.1**	**94.3**
Autoencoder	90.3	90.3	89.7	89.6	89.4	89.8	90.1	89.3	89.8	90.0
FAE	90.0	88.7	89.1	89.8	89.7	90.1	90.0	89.0	90.2	90.1
FPCA	90.3	90.3	90.2	90.1	90.2	90.2	90.2	90.2	90.1	90.1
PCA	89.1	89.7	87.2	87.9	87.6	89.6	89.7	89.1	89.7	89.6
VAE	89.0	86.1	88.9	89.5	89.9	88.3	90.2	89.6	90.3	90.1
Accuracies – (iv) randomly rotated samples										
TI-FPCA	90.1	**91.0**	**91.3**	**92.2**	**93.0**	**93.4**	**93.8**	**94.1**	**94.2**	**94.4**
Autoencoder	**90.3**	89.8	89.0	89.4	88.4	90.1	90.3	90.2	90.6	87.6
FAE	90.2	89.9	89.6	90.0	89.8	89.9	90.0	89.8	90.3	90.5
FPCA	90.2	90.0	90.0	90.5	91.1	91.2	91.4	91.5	91.5	91.6
PCA	89.7	88.4	87.3	89.0	90.1	89.5	89.4	90.8	91.0	91.2
VAE	90.2	90.2	89.8	90.3	90.2	90.3	90.2	90.3	90.3	90.3

2   Running Times of Model Trainings

Table 9. Mean running times of various models for oscillating data and varying number of components (standard deviation in brackets).

num. comp.	1	2	3	4	5	6	7	8	9	10
Weight length 512										
TI-FPCA	7.79 (±1.08)	11.96 (±1.55)	16.58 (±2.43)	20.63 (±2.41)	25.50 (±3.43)	29.62 (±4.07)	34.30 (±4.99)	38.87 (±5.38)	43.13 (±6.24)	48.06 (±7.19)
Autoencoder	16.15 (±2.93)	9.94 (±1.40)	9.23 (±0.71)	9.33 (±0.77)	9.34 (±0.73)	9.95 (±1.77)	10.20 (±1.88)	9.32 (±0.71)	9.51 (±0.75)	9.78 (±0.75)
FAE	18.85 (±2.48)	15.85 (±1.30)	14.66 (±0.49)	15.26 (±1.22)	14.64 (±0.48)	14.93 (±0.71)	15.08 (±1.26)	15.35 (±1.10)	14.88 (±0.93)	14.71 (±0.71)
FPCA	0.28 (±0.18)	0.29 (±0.18)	0.29 (±0.17)	0.35 (±0.25)	0.36 (±0.26)	0.36 (±0.15)	0.36 (±0.22)	0.33 (±0.14)	0.46 (±0.27)	0.37 (±0.16)
PCA	0.05 (±0.02)	0.04 (±0.02)	0.06 (±0.05)	0.05 (±0.02)	0.05 (±0.03)	0.05 (±0.04)	0.06 (±0.05)	0.06 (±0.02)	0.05 (±0.03)	0.06 (±0.03)
VAE	10.70 (±1.85)	9.70 (±0.66)	9.85 (±0.73)	9.81 (±0.71)	10.83 (±1.36)	10.15 (±0.92)	10.16 (±1.82)	10.18 (±1.00)	9.93 (±0.74)	9.91 (±0.70)
Weight length 256										
TI-FPCA	5.11 (±0.56)	5.73 (±2.22)	6.03 (±2.29)	6.20 (±1.81)	6.62 (±2.18)	7.04 (±2.66)	7.68 (±2.77)	7.64 (±2.67)	7.98 (±3.25)	8.03 (±2.70)
Autoencoder	13.54 (±0.37)	10.98 (±1.25)	9.03 (±0.48)	8.61 (±0.11)	8.73 (±0.11)	8.67 (±0.11)	8.70 (±0.13)	8.65 (±0.13)	8.76 (±0.12)	9.11 (±0.12)
FAE	16.50 (±0.23)	14.51 (±0.80)	14.41 (±0.40)	14.22 (±0.25)	14.23 (±0.24)	14.32 (±0.67)	14.10 (±0.40)	14.35 (±0.57)	14.14 (±0.49)	14.33 (±0.78)
FPCA	0.35 (±0.10)	0.28 (±0.14)	0.36 (±0.19)	0.34 (±0.15)	0.41 (±0.11)	0.36 (±0.16)	0.38 (±0.12)	0.38 (±0.08)	0.41 (±0.10)	0.39 (±0.09)
PCA	0.05 (±0.03)	0.05 (±0.02)	0.06 (±0.03)	0.05 (±0.02)	0.05 (±0.02)	0.05 (±0.02)	0.05 (±0.02)	0.06 (±0.02)	0.06 (±0.02)	0.08 (±0.04)
VAE	9.21 (±0.10)	9.07 (±0.15)	9.11 (±0.13)	9.03 (±0.13)	9.07 (±0.13)	9.18 (±0.17)	9.13 (±0.12)	9.57 (±0.21)	9.33 (±0.14)	9.27 (±0.13)

Table 10. Mean running times of various models for spiking data (standard deviation in brackets).

num. comp.	1	2	3	4	5	6	7	8	9	10
Classification – setting (i) – $T = 128$, $w_\ell = 256$										
TI-FPCA	7.21 (±0.08)	9.58 (±0.14)	11.26 (±0.17)	13.08 (±0.17)	15.29 (±0.18)	17.16 (±0.26)	21.56 (±0.48)	23.09 (±0.70)	25.62 (±1.08)	31.76 (±1.45)
Autoencoder	12.44 (±0.38)	12.24 (±0.13)	12.42 (±0.10)	12.32 (±0.10)	12.25 (±0.06)	12.14 (±0.05)	12.19 (±0.07)	12.23 (±0.03)	12.53 (±0.06)	12.33 (±0.08)
FAE	15.73 (±0.12)	15.93 (±0.07)	16.70 (±0.09)	16.48 (±0.14)	16.34 (±0.10)	16.23 (±0.17)	16.32 (±0.37)	16.26 (±0.08)	16.16 (±0.08)	16.29 (±0.29)
FPCA	0.05 (±0.00)	0.06 (±0.00)	0.05 (±0.00)	0.05 (±0.00)	0.05 (±0.00)	0.06 (±0.00)	0.06 (±0.01)	0.06 (±0.00)	0.06 (±0.00)	0.06 (±0.00)
PCA	0.01 (±0.00)	0.02 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.02 (±0.00)
VAE	12.59 (±0.42)	12.58 (±0.20)	12.57 (±0.13)	12.46 (±0.07)	12.41 (±0.08)	12.70 (±0.14)	12.63 (±0.09)	12.48 (±0.06)	12.50 (±0.07)	12.72 (±0.29)
Classification – setting (ii) – $T = 256$, $w_\ell = 256$										
TI-FPCA	4.76 (±0.64)	4.94 (±0.66)	5.06 (±0.73)	5.14 (±0.72)	5.26 (±0.75)	5.46 (±0.82)	5.60 (±0.83)	5.76 (±0.88)	5.94 (±0.99)	6.07 (±1.01)
Autoencoder	16.48 (±4.16)	16.75 (±4.15)	17.53 (±4.39)	16.90 (±3.68)	16.84 (±3.26)	16.78 (±3.52)	16.65 (±3.19)	15.95 (±1.90)	15.88 (±1.10)	16.07 (±1.80)
FAE	23.74 (±3.89)	25.28 (±4.77)	25.81 (±4.41)	25.59 (±4.14)	25.58 (±4.61)	25.62 (±4.46)	25.34 (±4.41)	25.83 (±4.37)	25.71 (±4.71)	25.75 (±4.67)
FPCA	0.15 (±0.04)	0.16 (±0.05)	0.14 (±0.04)	0.15 (±0.04)	0.18 (±0.09)	0.14 (±0.04)	0.16 (±0.04)	0.16 (±0.04)	0.17 (±0.07)	0.31 (±0.46)
PCA	0.02 (±0.01)	0.02 (±0.01)	0.02 (±0.02)	0.02 (±0.01)	0.03 (±0.02)	0.02 (±0.01)	0.02 (±0.01)	0.03 (±0.02)	0.02 (±0.02)	0.03 (±0.01)
VAE	16.07 (±1.51)	15.96 (±1.30)	16.04 (±1.19)	16.20 (±1.56)	16.22 (±1.68)	16.35 (±1.47)	16.40 (±2.04)	16.99 (±2.68)	16.82 (±2.69)	16.68 (±3.05)
Classification – setting (iii) – $T = 256$, $w_\ell = 128$										
TI-FPCA	9.68 (±6.05)	13.89 (±8.91)	19.79 (±11.88)	18.91 (±5.47)	20.17 (±2.97)	23.86 (±3.61)	26.85 (±4.61)	30.27 (±5.08)	35.85 (±7.93)	36.80 (±5.48)
Autoencoder	14.32 (±0.59)	14.65 (±0.75)	15.74 (±2.11)	15.09 (±0.78)	14.91 (±0.38)	15.03 (±0.32)	15.16 (±0.50)	15.03 (±0.50)	15.72 (±1.21)	17.21 (±5.60)
FAE	22.72 (±3.48)	23.66 (±3.61)	29.89 (±16.60)	30.16 (±18.14)	30.18 (±17.93)	29.85 (±17.61)	29.19 (±17.38)	29.93 (±17.94)	29.95 (±16.41)	33.09 (±17.96)
FPCA	0.14 (±0.06)	0.14 (±0.05)	0.13 (±0.05)	0.13 (±0.04)	0.14 (±0.05)	0.13 (±0.06)	0.16 (±0.08)	0.15 (±0.06)	0.17 (±0.10)	0.19 (±0.12)
PCA	0.02 (±0.01)	0.02 (±0.01)	0.02 (±0.01)	0.02 (±0.01)	0.02 (±0.01)	0.02 (±0.02)	0.02 (±0.01)	0.02 (±0.02)	0.02 (±0.01)	0.03 (±0.01)
VAE	15.51 (±0.93)	15.51 (±1.07)	15.25 (±0.57)	15.91 (±1.87)	15.35 (±0.70)	16.03 (±2.14)	16.60 (±3.11)	16.15 (±2.61)	16.15 (±2.59)	16.49 (±3.25)
Outlier detection – setting (i) – $T = 128$, $w_\ell = 256$										
TI-FPCA	5.12 (±0.08)	6.75 (±0.13)	7.94 (±0.38)	9.10 (±0.24)	10.65 (±0.19)	11.95 (±0.16)	14.64 (±0.33)	15.67 (±0.46)	17.61 (±0.98)	20.66 (±0.56)
Autoencoder	7.84 (±0.10)	7.96 (±0.26)	8.01 (±0.09)	7.82 (±0.10)	7.84 (±0.08)	7.81 (±0.05)	7.84 (±0.11)	7.91 (±0.12)	8.20 (±0.09)	8.01 (±0.08)
FAE	10.13 (±0.11)	10.38 (±0.31)	10.71 (±0.14)	10.52 (±0.18)	10.43 (±0.19)	10.40 (±0.14)	10.42 (±0.16)	10.44 (±0.20)	10.41 (±0.23)	10.44 (±0.23)
FPCA	0.09 (±0.03)	0.09 (±0.03)	0.09 (±0.03)	0.09 (±0.03)	0.09 (±0.03)	0.09 (±0.03)	0.09 (±0.03)	0.09 (±0.03)	0.09 (±0.03)	0.09 (±0.02)
PCA	0.05 (±0.03)	0.05 (±0.03)	0.05 (±0.03)	0.05 (±0.03)	0.05 (±0.03)	0.05 (±0.03)	0.05 (±0.03)	0.05 (±0.03)	0.05 (±0.03)	0.05 (±0.02)
VAE	8.12 (±0.08)	8.10 (±0.07)	8.13 (±0.10)	8.24 (±0.30)	8.15 (±0.09)	8.20 (±0.10)	8.18 (±0.09)	8.15 (±0.06)	8.18 (±0.09)	8.17 (±0.11)
Outlier detection – setting (ii) – $T = w_\ell = 256$										
TI-FPCA	3.40 (±0.19)	3.51 (±0.11)	3.60 (±0.09)	3.66 (±0.07)	3.73 (±0.03)	3.82 (±0.04)	3.95 (±0.07)	4.06 (±0.10)	4.17 (±0.05)	4.26 (±0.05)
Autoencoder	8.93 (±0.17)	8.94 (±0.17)	9.01 (±0.12)	9.01 (±0.16)	9.04 (±0.15)	9.25 (±0.33)	9.26 (±0.15)	9.26 (±0.20)	9.48 (±0.16)	9.43 (±0.32)
FAE	12.78 (±0.20)	13.39 (±0.41)	13.65 (±0.32)	13.49 (±0.24)	13.47 (±0.24)	13.47 (±0.22)	13.55 (±0.27)	13.42 (±0.23)	13.51 (±0.25)	13.48 (±0.21)
FPCA	0.24 (±0.12)	0.25 (±0.21)	0.21 (±0.13)	0.21 (±0.09)	0.20 (±0.09)	0.20 (±0.08)	0.20 (±0.09)	0.19 (±0.09)	0.20 (±0.09)	0.20 (±0.08)
PCA	0.08 (±0.07)	0.08 (±0.07)	0.09 (±0.08)	0.10 (±0.08)	0.10 (±0.08)	0.10 (±0.08)	0.10 (±0.08)	0.09 (±0.08)	0.10 (±0.08)	0.10 (±0.08)
VAE	9.45 (±0.13)	9.40 (±0.15)	9.40 (±0.15)	9.42 (±0.14)	9.40 (±0.14)	9.46 (±0.12)	9.47 (±0.15)	9.58 (±0.27)	9.46 (±0.14)	9.48 (±0.15)
Outlier detection – setting (iii) – $T = 256$, $w_\ell = 128$										
TI-FPCA	4.79 (±0.05)	6.75 (±0.13)	8.62 (±0.14)	10.53 (±0.16)	12.61 (±0.48)	14.78 (±0.38)	16.35 (±0.26)	18.19 (±0.35)	20.52 (±0.32)	22.26 (±0.61)
Autoencoder	9.05 (±0.25)	8.99 (±0.18)	9.17 (±0.16)	8.97 (±0.17)	9.03 (±0.16)	8.99 (±0.16)	8.99 (±0.17)	9.02 (±0.21)	9.31 (±0.15)	9.10 (±0.15)
FAE	12.42 (±0.27)	12.91 (±0.29)	13.27 (±0.29)	13.20 (±0.24)	13.03 (±0.25)	13.10 (±0.22)	13.07 (±0.20)	12.94 (±0.26)	13.10 (±0.49)	13.14 (±0.25)
FPCA	0.17 (±0.08)	0.16 (±0.08)	0.16 (±0.08)	0.16 (±0.08)	0.16 (±0.08)	0.17 (±0.08)	0.16 (±0.08)	0.17 (±0.09)	0.16 (±0.07)	0.16 (±0.07)
PCA	0.05 (±0.06)	0.06 (±0.06)	0.06 (±0.07)	0.07 (±0.08)	0.06 (±0.07)	0.06 (±0.08)	0.06 (±0.08)	0.06 (±0.07)	0.06 (±0.07)	0.07 (±0.07)
VAE	9.15 (±0.16)	9.12 (±0.17)	9.16 (±0.15)	9.21 (±0.14)	9.20 (±0.12)	9.22 (±0.13)	9.20 (±0.17)	9.33 (±0.25)	9.26 (±0.13)	9.21 (±0.17)

Table 11. Mean running times of various models for handwriting data (standard deviation in brackets).

num. comp.	1	2	3	4	5	6	7	8	9	10
Window length $T = 700$										
TI-FPCA	46.78 (±0.40)	60.54 (±1.06)	75.44 (±1.59)	93.45 (±1.62)	118.14 (±26.93)	124.67 (±2.64)	142.63 (±4.40)	161.23 (±4.89)	175.22 (±5.71)	193.37 (±6.11)
Autoencoder	25.55 (±0.49)	24.70 (±0.58)	24.38 (±0.34)	24.23 (±0.17)	24.46 (±0.37)	24.40 (±0.38)	24.39 (±0.66)	24.31 (±0.41)	24.45 (±0.43)	24.18 (±0.17)
FAE	29.07 (±0.28)	30.07 (±0.21)	30.36 (±0.50)	30.45 (±0.53)	30.34 (±0.37)	30.90 (±0.35)	32.33 (±4.70)	33.53 (±8.11)	34.22 (±10.24)	30.79 (±0.78)
FPCA	0.19 (±0.04)	0.19 (±0.02)	0.17 (±0.02)	0.18 (±0.02)	0.18 (±0.02)	0.17 (±0.01)	0.18 (±0.01)	0.18 (±0.02)	0.18 (±0.02)	0.18 (±0.02)
PCA	0.00 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)
VAE	24.49 (±0.49)	24.81 (±0.68)	24.81 (±0.62)	24.58 (±0.31)	24.47 (±0.44)	24.52 (±0.71)	24.69 (±0.60)	24.61 (±0.42)	24.47 (±0.30)	24.47 (±0.12)
Window length $T = 1400$										
TI-FPCA	18.21 (±0.18)	18.58 (±0.15)	18.86 (±0.16)	18.84 (±0.13)	18.99 (±0.32)	19.11 (±0.23)	19.30 (±0.24)	19.36 (±0.21)	19.55 (±0.24)	19.84 (±0.45)
Autoencoder	28.34 (±0.33)	27.65 (±0.23)	27.59 (±0.57)	27.64 (±0.34)	27.60 (±0.21)	27.36 (±0.36)	27.34 (±0.11)	27.19 (±0.09)	27.31 (±0.55)	27.47 (±0.17)
FAE	39.34 (±1.05)	40.90 (±0.58)	40.96 (±0.55)	41.86 (±1.01)	41.02 (±0.49)	41.16 (±0.56)	40.77 (±0.43)	40.91 (±0.25)	42.28 (±0.28)	41.47 (±0.56)
FPCA	0.63 (±0.03)	0.63 (±0.03)	0.64 (±0.02)	0.65 (±0.02)	0.67 (±0.03)	0.74 (±0.15)	0.71 (±0.14)	0.73 (±0.09)	0.70 (±0.04)	0.70 (±0.03)
PCA	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)	0.01 (±0.00)
VAE	27.27 (±0.20)	27.23 (±0.18)	28.28 (±1.32)	27.52 (±0.57)	29.20 (±5.77)	34.96 (±21.95)	35.02 (±22.63)	35.31 (±22.68)	28.76 (±2.17)	27.85 (±0.60)

Table 12. Mean running times of various models for ErrP data (standard deviation in brackets).

| num. comp.	1	2	3	4	5	6	7	8	9	10
Classification – setting (i) – sample length: 0.5 s, weight length: 1 s										
TI-FPCA	7.75 (±0.13)	11.33 (±0.23)	15.38 (±0.28)	19.66 (±0.23)	24.66 (±1.46)	34.53 (±1.38)	37.20 (±0.93)	39.89 (±2.00)	45.67 (±2.51)	55.51 (±2.04)
Autoencoder	7.24 (±0.05)	7.38 (±0.06)	7.67 (±0.10)	7.52 (±0.11)	7.67 (±0.20)	7.83 (±0.14)	8.02 (±0.29)	7.90 (±0.29)	8.08 (±0.35)	7.93 (±0.13)
FAE	10.02 (±0.16)	10.73 (±0.14)	10.61 (±0.12)	10.58 (±0.12)	10.57 (±0.14)	10.54 (±0.11)	10.55 (±0.14)	10.56 (±0.24)	10.52 (±0.15)	11.01 (±0.12)
FPCA	0.38 (±0.09)	0.31 (±0.04)	0.34 (±0.02)	0.28 (±0.02)	0.34 (±0.02)	0.33 (±0.04)	0.34 (±0.03)	0.31 (±0.04)	0.34 (±0.03)	0.35 (±0.04)
PCA	0.05 (±0.00)	0.05 (±0.00)	0.06 (±0.00)	0.06 (±0.00)	0.06 (±0.00)	0.06 (±0.00)	0.06 (±0.00)	0.06 (±0.00)	0.06 (±0.00)	0.08 (±0.00)
VAE	8.09 (±0.40)	8.00 (±0.21)	8.09 (±0.22)	8.00 (±0.20)	8.28 (±0.28)	8.08 (±0.24)	7.99 (±0.08)	8.12 (±0.40)	8.05 (±0.15)	8.07 (±0.11)
Classification – setting (ii) – sample and weight length: 0.5 s										
TI-FPCA	1.33 (±0.00)	1.43 (±0.01)	1.52 (±0.01)	1.62 (±0.01)	1.72 (±0.02)	1.87 (±0.11)	2.05 (±0.14)	2.10 (±0.15)	2.11 (±0.03)	2.54 (±0.15)
Autoencoder	7.31 (±0.08)	7.40 (±0.12)	7.67 (±0.11)	7.53 (±0.10)	7.54 (±0.08)	7.56 (±0.10)	7.84 (±0.12)	7.73 (±0.04)	7.81 (±0.08)	7.82 (±0.07)
FAE	9.93 (±0.04)	10.66 (±0.15)	10.47 (±0.04)	10.41 (±0.06)	10.52 (±0.07)	10.50 (±0.06)	10.51 (±0.06)	10.55 (±0.20)	10.51 (±0.09)	11.05 (±0.28)
FPCA	0.34 (±0.01)	0.33 (±0.03)	0.32 (±0.03)	0.33 (±0.03)	0.33 (±0.03)	0.33 (±0.03)	0.37 (±0.09)	0.32 (±0.03)	0.35 (±0.03)	0.35 (±0.03)
PCA	0.05 (±0.00)	0.05 (±0.00)	0.06 (±0.00)	0.06 (±0.00)	0.06 (±0.00)	0.06 (±0.00)	0.06 (±0.00)	0.07 (±0.00)	0.07 (±0.00)	0.08 (±0.00)
VAE	7.89 (±0.09)	7.88 (±0.04)	7.98 (±0.18)	7.90 (±0.05)	8.18 (±0.09)	8.00 (±0.03)	7.96 (±0.06)	7.96 (±0.08)	7.98 (±0.12)	8.01 (±0.11)
Classification – setting (iii) – sample length: 1 s, weight length: 0.5 s										
TI-FPCA	5.94 (±0.06)	8.57 (±0.09)	11.29 (±0.19)	13.96 (±0.25)	16.43 (±0.43)	18.86 (±0.04)	21.39 (±0.15)	23.92 (±0.10)	26.52 (±0.14)	29.11 (±0.22)
Autoencoder	9.90 (±0.07)	10.21 (±0.20)	10.69 (±0.20)	10.69 (±0.06)	10.69 (±0.08)	10.74 (±0.11)	10.89 (±0.21)	10.90 (±0.06)	10.97 (±0.08)	10.88 (±0.10)
FAE	14.52 (±0.21)	15.86 (±0.24)	15.65 (±0.20)	15.53 (±0.21)	15.52 (±0.22)	15.53 (±0.26)	15.68 (±0.25)	15.66 (±0.40)	16.09 (±0.23)	15.87 (±0.18)
FPCA	0.69 (±0.02)	0.64 (±0.02)	0.64 (±0.02)	0.65 (±0.03)	0.64 (±0.01)	0.65 (±0.02)	0.65 (±0.01)	0.65 (±0.01)	0.67 (±0.03)	0.68 (±0.02)
PCA	0.09 (±0.00)	0.09 (±0.00)	0.09 (±0.00)	0.09 (±0.00)	0.09 (±0.00)	0.10 (±0.00)	0.10 (±0.00)	0.10 (±0.00)	0.10 (±0.00)	0.12 (±0.00)
VAE	11.02 (±0.20)	11.06 (±0.13)	11.03 (±0.12)	10.95 (±0.13)	11.19 (±0.15)	11.14 (±0.20)	11.01 (±0.12)	11.02 (±0.08)	11.13 (±0.09)	11.15 (±0.10)
Outlier detection – setting (i) – sample length: 0.5 s, weight length: 1 s										
TI-FPCA	1.72 (±1.18)	2.79 (±2.01)	3.88 (±2.72)	5.77 (±5.51)	6.47 (±5.04)	8.49 (±5.58)	9.02 (±6.70)	11.18 (±9.93)	12.21 (±10.13)	14.07 (±10.96)
Autoencoder	2.06 (±1.21)	2.06 (±1.23)	2.05 (±1.22)	2.06 (±1.22)	2.05 (±1.25)	2.07 (±1.29)	2.08 (±1.27)	2.18 (±1.48)	2.08 (±1.33)	2.07 (±1.31)
FAE	2.73 (±1.59)	3.00 (±1.60)	2.80 (±1.71)	2.77 (±1.69)	2.79 (±1.75)	2.77 (±1.68)	2.84 (±1.82)	2.78 (±1.73)	2.82 (±1.78)	3.21 (±1.98)
FPCA	0.18 (±0.05)	0.16 (±0.05)	0.16 (±0.06)	0.16 (±0.06)	0.24 (±0.05)	0.19 (±0.05)	0.22 (±0.10)	0.17 (±0.06)	0.17 (±0.07)	0.17 (±0.07)
PCA	0.02 (±0.01)	0.02 (±0.01)	0.02 (±0.01)	0.02 (±0.01)	0.02 (±0.01)	0.02 (±0.01)	0.02 (±0.01)	0.02 (±0.01)	0.03 (±0.01)	0.03 (±0.02)
VAE	2.16 (±1.31)	2.15 (±1.38)	2.12 (±1.38)	2.12 (±1.26)	2.32 (±1.18)	2.12 (±1.26)	2.11 (±1.27)	2.12 (±1.28)	2.18 (±1.41)	2.14 (±1.29)
Outlier detection – setting (ii) sample and weight length: 0.5 s										
TI-FPCA	0.57 (±0.18)	0.63 (±0.18)	0.70 (±0.20)	0.78 (±0.23)	0.84 (±0.22)	0.91 (±0.23)	0.98 (±0.24)	1.04 (±0.25)	1.12 (±0.25)	1.51 (±0.13)
Autoencoder	2.04 (±1.17)	2.02 (±1.18)	2.07 (±1.28)	2.05 (±1.22)	2.07 (±1.31)	2.09 (±1.38)	2.07 (±1.32)	2.06 (±1.32)	2.07 (±1.31)	2.08 (±1.35)
FAE	2.80 (±1.71)	3.06 (±1.75)	2.84 (±1.83)	2.84 (±1.86)	2.84 (±1.86)	2.86 (±1.87)	2.86 (±1.91)	2.85 (±1.89)	2.87 (±1.89)	3.12 (±1.77)
FPCA	0.18 (±0.04)	0.17 (±0.04)	0.27 (±0.15)	0.17 (±0.04)	0.18 (±0.04)	0.19 (±0.02)	0.18 (±0.03)	0.19 (±0.01)	0.19 (±0.01)	0.20 (±0.02)
PCA	0.02 (±0.01)	0.02 (±0.01)	0.03 (±0.02)	0.02 (±0.01)	0.02 (±0.01)	0.03 (±0.03)	0.02 (±0.01)	0.02 (±0.01)	0.02 (±0.01)	0.03 (±0.01)
VAE	2.16 (±1.32)	2.16 (±1.41)	2.14 (±1.35)	2.12 (±1.33)	2.31 (±1.27)	2.14 (±1.34)	2.17 (±1.40)	2.14 (±1.32)	2.19 (±1.42)	2.16 (±1.36)
Outlier detection – setting (iii) sample length: 1 s, weight length: 0.5 s										
TI-FPCA	1.50 (±1.15)	2.22 (±1.49)	2.99 (±1.95)	3.74 (±2.38)	4.82 (±2.69)	5.26 (±3.22)	6.03 (±3.79)	6.71 (±4.10)	7.58 (±4.67)	8.31 (±5.17)
Autoencoder	2.60 (±1.92)	2.66 (±2.08)	2.68 (±2.08)	2.67 (±2.09)	2.67 (±2.11)	2.69 (±2.18)	2.64 (±2.04)	2.63 (±2.01)	2.66 (±2.03)	2.64 (±2.00)
FAE	3.99 (±2.45)	4.35 (±2.60)	4.12 (±2.73)	4.10 (±2.74)	4.12 (±2.69)	4.15 (±2.78)	4.13 (±2.78)	4.13 (±2.76)	4.15 (±2.72)	4.15 (±2.73)
FPCA	0.34 (±0.10)	0.33 (±0.05)	0.33 (±0.07)	0.33 (±0.09)	0.34 (±0.06)	0.36 (±0.09)	0.33 (±0.09)	0.34 (±0.07)	0.35 (±0.07)	0.35 (±0.08)
PCA	0.03 (±0.01)	0.03 (±0.02)	0.03 (±0.02)	0.03 (±0.02)	0.03 (±0.02)	0.03 (±0.02)	0.03 (±0.02)	0.03 (±0.02)	0.04 (±0.02)	0.04 (±0.02)
VAE	2.71 (±1.98)	2.71 (±2.06)	2.70 (±2.05)	2.71 (±2.03)	2.91 (±1.93)	2.74 (±2.02)	2.73 (±2.01)	2.72 (±2.05)	2.77 (±2.10)	2.85 (±2.29)

Table 13. Mean running times of various models for MNIST data and varying number of components in the classification setting (standard deviation in brackets).

num. comp.	1	2	3	4	5	6	7	8	9	10
(i) samples of size 16 × 16										
TI-FPCA	31.73 (±0.79)	49.35 (±1.77)	60.63 (±2.53)	74.49 (±2.87)	89.84 (±3.31)	132.92 (±8.41)	144.46 (±10.59)	166.55 (±15.01)	244.62 (±9.69)	237.56 (±15.26)
Autoencoder	60.65 (±0.85)	61.72 (±0.28)	61.71 (±0.43)	62.82 (±0.68)	62.30 (±0.70)	61.95 (±0.50)	61.68 (±0.30)	61.79 (±0.38)	62.02 (±0.67)	62.89 (±1.07)
FAE	67.20 (±0.38)	67.26 (±0.76)	67.51 (±0.54)	68.89 (±1.17)	67.86 (±0.53)	67.90 (±0.65)	67.33 (±0.78)	67.35 (±0.54)	67.41 (±0.88)	68.26 (±0.94)
FPCA	0.58 (±0.01)	0.59 (±0.01)	0.59 (±0.01)	0.60 (±0.01)	0.61 (±0.01)	0.62 (±0.01)	0.62 (±0.01)	0.63 (±0.01)	0.69 (±0.14)	0.70 (±0.02)
PCA	0.23 (±0.22)	0.16 (±0.00)	0.17 (±0.00)	0.18 (±0.01)	0.19 (±0.01)	0.19 (±0.01)	0.19 (±0.01)	0.20 (±0.01)	0.21 (±0.01)	0.25 (±0.01)
VAE	62.42 (±0.45)	62.39 (±0.33)	62.11 (±0.30)	62.25 (±0.37)	62.20 (±0.33)	63.68 (±0.58)	62.92 (±0.65)	62.53 (±0.96)	62.65 (±0.51)	62.98 (±0.39)
(ii) samples of size 28 × 28										
TI-FPCA	11.81 (±0.17)	12.51 (±0.15)	13.50 (±0.29)	13.24 (±0.13)	13.93 (±0.23)	14.36 (±0.25)	15.07 (±0.43)	15.69 (±0.17)	16.16 (±0.27)	16.67 (±0.19)
Autoencoder	83.75 (±0.81)	84.02 (±0.66)	84.85 (±0.75)	86.71 (±0.81)	90.61 (±14.22)	85.10 (±0.84)	85.18 (±0.75)	84.69 (±0.68)	85.11 (±1.18)	85.84 (±1.00)
FAE	83.33 (±1.67)	83.39 (±2.34)	86.41 (±2.08)	86.14 (±2.31)	84.52 (±1.39)	85.28 (±2.23)	84.53 (±1.52)	83.77 (±1.71)	83.78 (±1.61)	83.36 (±1.75)
FPCA	2.15 (±0.01)	2.14 (±0.02)	2.19 (±0.03)	2.22 (±0.10)	2.25 (±0.12)	2.19 (±0.01)	2.23 (±0.04)	2.23 (±0.02)	2.39 (±0.37)	2.40 (±0.05)
PCA	0.36 (±0.00)	0.38 (±0.01)	0.39 (±0.00)	0.40 (±0.00)	0.41 (±0.00)	0.42 (±0.00)	0.42 (±0.01)	0.43 (±0.00)	0.44 (±0.00)	0.53 (±0.01)
VAE	85.56 (±0.68)	85.21 (±0.85)	85.08 (±0.86)	84.96 (±0.82)	85.27 (±0.79)	87.18 (±0.54)	86.58 (±0.68)	85.91 (±0.69)	85.96 (±0.71)	86.13 (±0.62)
(iii) samples of size 56 × 56										
TI-FPCA	218.03 (±21.90)	228.42 (±9.93)	278.45 (±8.34)	329.19 (±16.02)	379.76 (±22.39)	428.18 (±18.60)	482.44 (±23.82)	550.70 (±55.78)	593.93 (±43.96)	755.55 (±286.26)
Autoencoder	200.30 (±27.23)	204.23 (±26.80)	204.25 (±26.33)	209.59 (±27.44)	206.88 (±26.71)	203.42 (±26.45)	203.21 (±23.95)	202.49 (±26.51)	203.69 (±25.40)	207.78 (±26.03)
FAE	151.83 (±48.44)	134.83 (±21.13)	141.14 (±15.72)	137.23 (±19.04)	138.07 (±20.55)	136.73 (±21.57)	136.71 (±22.30)	137.09 (±20.96)	137.43 (±19.07)	137.55 (±19.29)
FPCA	24.57 (±3.09)	25.18 (±2.99)	25.51 (±3.12)	33.06 (±25.44)	24.77 (±2.83)	25.71 (±3.26)	25.06 (±2.63)	25.64 (±3.07)	25.34 (±3.00)	26.00 (±3.10)
PCA	1.52 (±0.18)	1.62 (±0.24)	1.65 (±0.20)	1.70 (±0.25)	1.80 (±0.26)	1.70 (±0.22)	1.72 (±0.23)	1.72 (±0.23)	1.77 (±0.24)	2.22 (±0.34)
VAE	206.53 (±28.34)	205.34 (±26.56)	204.43 (±28.36)	204.46 (±28.01)	206.33 (±27.56)	212.79 (±28.40)	207.03 (±25.23)	207.45 (±27.90)	205.29 (±26.91)	205.39 (±27.26)
(iv) randomly rotated samples										
TI-FPCA	55.65 (±3.84)	72.18 (±4.71)	108.55 (±10.05)	129.46 (±18.42)	197.18 (±26.39)	301.70 (±43.66)	336.16 (±49.98)	365.56 (±70.56)	618.93 (±61.27)	755.12 (±71.04)
Autoencoder	123.01 (±5.84)	133.53 (±17.76)	134.39 (±13.41)	134.44 (±12.56)	127.09 (±6.55)	132.91 (±16.52)	132.22 (±21.79)	129.72 (±15.73)	134.75 (±16.37)	138.24 (±17.68)
FAE	121.19 (±9.59)	124.90 (±7.88)	123.97 (±10.47)	126.12 (±9.45)	122.12 (±6.64)	123.29 (±4.66)	121.74 (±2.86)	121.91 (±6.00)	121.71 (±6.92)	123.27 (±4.55)
FPCA	4.65 (±1.37)	14.15 (±28.47)	15.24 (±31.59)	4.09 (±0.27)	4.01 (±0.60)	4.37 (±0.54)	4.02 (±0.67)	5.70 (±3.40)	4.28 (±0.33)	4.71 (±0.34)
PCA	0.57 (±0.07)	0.58 (±0.07)	0.63 (±0.07)	0.61 (±0.06)	0.60 (±0.08)	0.61 (±0.08)	0.64 (±0.08)	1.43 (±2.07)	0.70 (±0.13)	1.89 (±2.50)
VAE	140.56 (±18.32)	128.95 (±8.44)	135.32 (±16.61)	134.17 (±14.31)	129.17 (±5.92)	129.09 (±10.81)	125.98 (±14.08)	124.46 (±10.39)	128.24 (±8.85)	128.89 (±6.70)

Table 14. Mean running times of various models for MNIST data and varying number of components in the outlier detection setting (standard deviation in brackets).

num. comp.	1	2	3	4	5	6	7	8	9	10
(i) samples of size 16 × 16										
TI-FPCA	32.81 (±0.35)	45.94 (±0.61)	66.83 (±0.80)	73.87 (±1.09)	89.17 (±1.16)	106.76 (±6.84)	147.96 (±11.45)	155.02 (±14.01)	173.05 (±21.16)	229.10 (±22.92)
Autoencoder	62.57 (±1.12)	62.91 (±0.92)	62.89 (±0.75)	64.24 (±0.95)	63.10 (±0.46)	62.62 (±0.25)	62.62 (±0.27)	62.75 (±0.38)	62.84 (±0.49)	63.75 (±0.56)
FAE	68.77 (±0.49)	69.09 (±0.75)	69.06 (±0.44)	69.14 (±0.50)	70.37 (±0.53)	69.86 (±0.83)	69.63 (±0.73)	69.89 (±1.60)	69.53 (±0.78)	70.23 (±0.43)
FPCA	0.16 (±0.00)	0.16 (±0.01)	0.19 (±0.09)	0.21 (±0.15)	0.16 (±0.14)	0.21 (±0.14)	0.17 (±0.01)	0.17 (±0.01)	0.16 (±0.01)	0.22 (±0.12)
PCA	0.03 (±0.00)	0.03 (±0.00)	0.03 (±0.00)	0.03 (±0.00)	0.03 (±0.00)	0.03 (±0.01)	0.03 (±0.00)	0.03 (±0.00)	0.03 (±0.00)	0.04 (±0.00)
VAE	64.58 (±0.91)	64.26 (±0.74)	64.16 (±0.52)	64.17 (±0.49)	64.11 (±0.48)	64.10 (±0.49)	64.60 (±0.80)	64.43 (±0.55)	64.20 (±0.45)	64.13 (±0.49)
(ii) samples of size 28 × 28										
TI-FPCA	16.45 (±3.27)	17.17 (±3.56)	17.65 (±3.73)	18.71 (±3.96)	19.13 (±3.51)	19.62 (±3.54)	19.65 (±2.83)	19.85 (±2.73)	20.56 (±2.66)	20.97 (±2.68)
Autoencoder	89.18 (±8.14)	91.86 (±10.40)	89.00 (±9.17)	93.42 (±11.73)	90.30 (±9.48)	89.61 (±9.12)	90.03 (±9.83)	90.04 (±9.92)	90.16 (±9.88)	94.19 (±12.38)
FAE	85.66 (±9.72)	85.46 (±10.26)	85.62 (±9.99)	85.94 (±9.03)	89.46 (±10.64)	87.38 (±10.31)	88.51 (±9.75)	92.30 (±16.58)	93.49 (±20.43)	88.51 (±10.39)
FPCA	0.60 (±0.28)	0.62 (±0.28)	0.64 (±0.30)	0.59 (±0.24)	0.76 (±0.48)	0.70 (±0.45)	0.82 (±0.78)	0.72 (±0.36)	0.60 (±0.20)	0.58 (±0.22)
PCA	0.06 (±0.01)	0.05 (±0.00)	0.05 (±0.00)	0.06 (±0.01)	0.06 (±0.01)	0.06 (±0.00)	0.06 (±0.00)	0.07 (±0.01)	0.07 (±0.01)	0.08 (±0.01)
VAE	92.31 (±10.24)	90.96 (±9.32)	90.75 (±9.36)	90.79 (±9.53)	90.70 (±9.41)	90.86 (±9.65)	91.19 (±9.13)	90.75 (±9.00)	91.10 (±9.48)	91.63 (±9.95)
(iii) samples of size 56 × 56										
TI-FPCA	173.62 (±1.13)	300.60 (±1.86)	273.67 (±0.91)	327.75 (±3.94)	433.89 (±10.67)	471.29 (±8.56)	463.20 (±1.17)	505.72 (±2.56)	555.49 (±1.98)	596.01 (±4.38)
Autoencoder	166.55 (±2.69)	169.78 (±0.50)	169.50 (±0.74)	172.04 (±0.88)	170.58 (±0.85)	169.80 (±0.67)	169.72 (±0.83)	169.33 (±0.59)	169.94 (±0.93)	172.72 (±1.14)
FAE	111.34 (±1.11)	113.15 (±0.91)	113.77 (±1.45)	118.60 (±0.86)	116.36 (±1.13)	115.66 (±1.39)	116.78 (±0.95)	116.07 (±0.88)	115.71 (±1.77)	116.07 (±1.51)
FPCA	2.53 (±0.12)	2.71 (±0.68)	2.71 (±0.53)	2.52 (±0.03)	2.54 (±0.04)	2.54 (±0.07)	2.51 (±0.03)	2.88 (±1.08)	2.54 (±0.04)	2.57 (±0.03)
PCA	0.14 (±0.01)	0.13 (±0.00)	0.14 (±0.00)	0.14 (±0.00)	0.14 (±0.00)	0.14 (±0.01)	0.14 (±0.00)	0.15 (±0.01)	0.15 (±0.00)	0.18 (±0.00)
VAE	170.94 (±1.20)	171.72 (±1.38)	170.58 (±1.22)	170.18 (±1.38)	170.66 (±1.21)	171.13 (±1.37)	171.91 (±0.95)	171.94 (±1.05)	171.69 (±1.06)	172.37 (±1.57)
(iv) randomly rotated samples										
TI-FPCA	38.94 (±11.96)	54.54 (±15.61)	80.83 (±22.48)	102.24 (±21.49)	163.24 (±46.91)	216.14 (±74.89)	252.39 (±72.71)	324.30 (±112.47)	422.06 (±101.15)	535.58 (±138.15)
Autoencoder	85.04 (±0.31)	85.67 (±0.18)	87.93 (±0.47)	87.16 (±1.04)	85.78 (±0.35)	85.67 (±0.44)	85.94 (±0.39)	85.53 (±0.48)	87.63 (±0.56)	86.95 (±0.43)
FAE	82.90 (±1.48)	82.97 (±0.89)	82.80 (±1.94)	82.99 (±1.08)	83.60 (±1.32)	83.89 (±1.08)	84.23 (±1.07)	84.37 (±2.36)	83.65 (±1.54)	83.60 (±1.20)
FPCA	0.40 (±0.02)	0.40 (±0.01)	0.40 (±0.01)	0.41 (±0.02)	0.42 (±0.01)	0.41 (±0.01)	0.42 (±0.01)	0.43 (±0.01)	0.41 (±0.01)	0.41 (±0.02)
PCA	0.05 (±0.00)	0.05 (±0.00)	0.05 (±0.00)	0.05 (±0.00)	0.06 (±0.00)	0.06 (±0.01)	0.06 (±0.00)	0.06 (±0.00)	0.06 (±0.00)	0.07 (±0.00)
VAE	87.03 (±0.52)	87.29 (±0.65)	86.67 (±0.69)	86.92 (±0.88)	87.06 (±0.65)	87.73 (±0.82)	87.70 (±0.94)	86.82 (±0.55)	87.59 (±0.89)	87.36 (±0.58)

3   Projection of MNIST data onto TI-FPCs
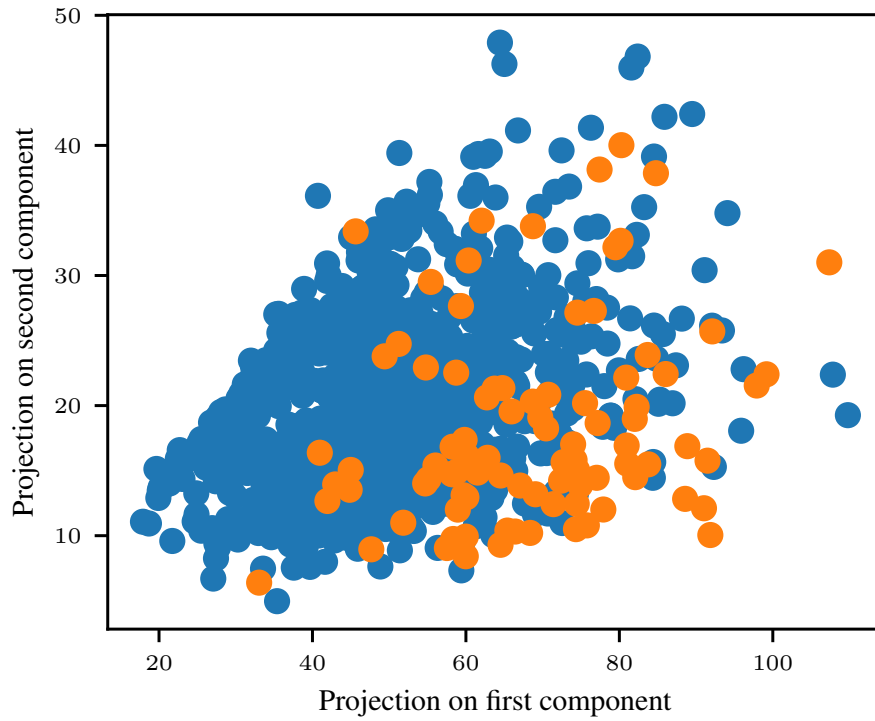
Fig. 1. Projections of 1024 samples from the MNIST data set onto the first two TI-FPCs in the outlier detection setting, i. e., fitted to images of the digit "8". Orange: Images of the digit "8". Blue: Images of other digits.