

# Exam Parallelism Fabricé

For  $i=1$  to  $N$  do if  $T[i]=1$  return  $i$

// there is a 1 in the array

$T = O(N)$

First one

Array  $B$ .

prelim

= For  $i=1$  to  $N$  in parallel do each

$B[i] = A[i]$

modif . [ For all  $i, j$  such that  $i < j$  in //  
if  $B[i]=1$  &  $B[j]=1$   
then  $B[j]=0$

Q) if  $B[i]$  and  $B[j]$  are = 1 too, this means that  $B[j] \neq A[j]$  is not the first one occurrence

So at the end of the for all loop, if  $A[k] = B[k] = 1$ , there is not any  $l < k$  such that  $A[l] = 1$ , so  $A[k]$  is the cell having the first 1.

And, in all other  $B[x]$  having  $A[x]=1$ , there is now a 0

Which PRAM do we need for this code?

for each  $i, j$ ,  $i < j$  do in parallel

// time =  $O(1)$  if  $(B[i]=1 \text{ \& } B[j]=1)$   
then  $B[j] = 0$

while a proc reads a cell, eg  $B[l]$  another may read also that same cell  $B[k]$  where the  $k$  may be = to the  $l$ .

so  $\in R$  is needed,  $O(n^2)$  procs but more precisely only  $\frac{(N)(N+1)}{2}$

• CW arbitrary is needed writing all 0

if a write to  $B[j]$  is triggered =  $\sum_{i=1}^N i = \frac{N \times (N+1)}{2}$

1	2	3	4	...	N
2	3	4	...	N	
3	4	...	N		
...					
(N)					

A =  
00010101  
1 2 3 4 5 6 7 8

Ex,  $i, j$   
(4, 5)  
(4, 8)

or (4, 8)  $i=8$   
(6, 8)

$B[8]=0$   
written in  
// by  $P_4$  and  $P_6$



Final op -

POSITION variable is a var in global shared mem.

for all  $i$  in  $\parallel$  do

if  $(B[i] = 1)$  POSITION =  $i$ , exit.

$T \parallel = O(1)$ , EREW needed because we know only one cell in B has value 1

Initial

:

No work optimal algorithm, because we have for first one:  $O(N^2)$  proc, time  $O(1)$ , but we know seq. work =  $O(N)$ .

### Reducing number of procs

var shared in global memory  
function  $r2$  - there is a one on a CRCW PRAM

for all  $i, (1..N)$  in  $\parallel$  do  
 $O(1) \parallel$  if  $(A[i] == 1)$  // there is a one  
time. there is a one = 1

CW PRAM arbitrary mode is needed because many procs may find at the same time a cell in A that contains 1. Still, they all want to write 1

1) Ex:  $(0, 0, 0, 0, 1, 1, 1, 0)$  with  $x = 2$

$C = [0 \ 0 \ 1 \ 1]$  then exec first one on C. CRCW  $O(1)$   
// time also, using  $O(N/x)^2$  procs.

2) Given the index in C showing 1, being  $k$  (here 3)  
it corresponds to  $\{A[(k-1)*x+1] \dots A[k*x]\}$   
a "1" stored within subarray

So, in  $\parallel$ , take  $x$  PRAM procs to work on this subarray, and apply final op on this subarray  
// time  $O(1)$  on an EREW

3) It takes  $O(1)$  time using  $O(N/x)^2$  proc at maximum

4) If  $x = \sqrt{N}$ ,  $O(N/x)^2 = O(\frac{N}{\sqrt{N}})^2 = O(\frac{\sqrt{N} \cdot \sqrt{N}}{\sqrt{N}})^2 = O(N)$  processors

of a CRCW PRAM. As  $\parallel$  time is  $O(1)$ , work is  $O(N)$  which is now of the same order as sequential work