Exercice 1

In the course, we have shown 2 versions of the parallel computation of the maximum of n values.
Q1) Explain why the proposed CRCW PRAM algorithm that is using n^2 processors can indeed compare all needed values together in just one single phase. Remember that to find the maximum of a set of n values, each of them must at some point be compared to all the others.

Indeed, to extract the maximum of n values, each value, let us say, x, must be checked against all the n-1 other; in order to eliminate that value x, if one of the other n-1 values , eg y, is greater than this x. . Eg, (1,3,5,7); 1 gets eliminated if compared to 3,5,then 7. Still, we just have eliminated 1, and we do not know yet the maximum. The same has been done for 3, it gets compared to 1,5,7, so gets eliminated. Same for 5. The only value that, once compared to all the others has not been eliminated is indeed the maximum (here 7). The proposed CRCW algo implements this approach. The total number of comparisons done is (n^2-n).

Why in the sequential algorithm that you can easily write down, the time complexity ends up being in O(n) (and not O(n^2)).

In the sequential algorithm below, an optimisation of the brute force above approach is incorporated:

Max=-infinity;

For (int i=1, i<=n,i++) if T[i]>Max, Max=T[i]

=> at the end, Max contains the maximum, and the number of comparisons that have been done is simply (n-1). This may look strange that time is not O(n^2). This is thanks to the fact that after each comparison, the result of that test is kept in Max. So, at each loop turn, whenever Max is updated, it automatically eliminates the need to compare the previous Max values with any remaining (n-1) other values with which it may not have been compared yet. So the complexity of solving the problem in sequential is O(n). Obviously, we would aim for a PRAM algorithm whose total work (parallel time * number of procs) is also O(n). This is not the case for the version 1, as the work is O(n^2), so this is not optimal.

Q2) Explain why the proposed CRCW PRAM algorithm has to allow CR ?

Concurrent read operations are needed, because if eg, i=3, and j=4, one proc will have to read T[3] and T[4]. But in parallel, eg for i=4, and j=6, another proc will read also T[4] (and T[6]). So, at worst, a given value, T[k] is simultaneously read by n-1 processes.

Q3) Explain why the proposed CRCW PRAM algorithm has to allow Arbitrary CW ?

Some of these n-1 processes read T[k], 1<=k<n comparing T[k] with another value, T[l] 2<l<=n in parallel, can find that T[k]<T[l], so, need to write FALSE in m[k]. As all that need to write something, in parallel, must write the same value (FALSE), an ARBITRARY PRAM is sufficient. And we can see that the CW mode is needed.

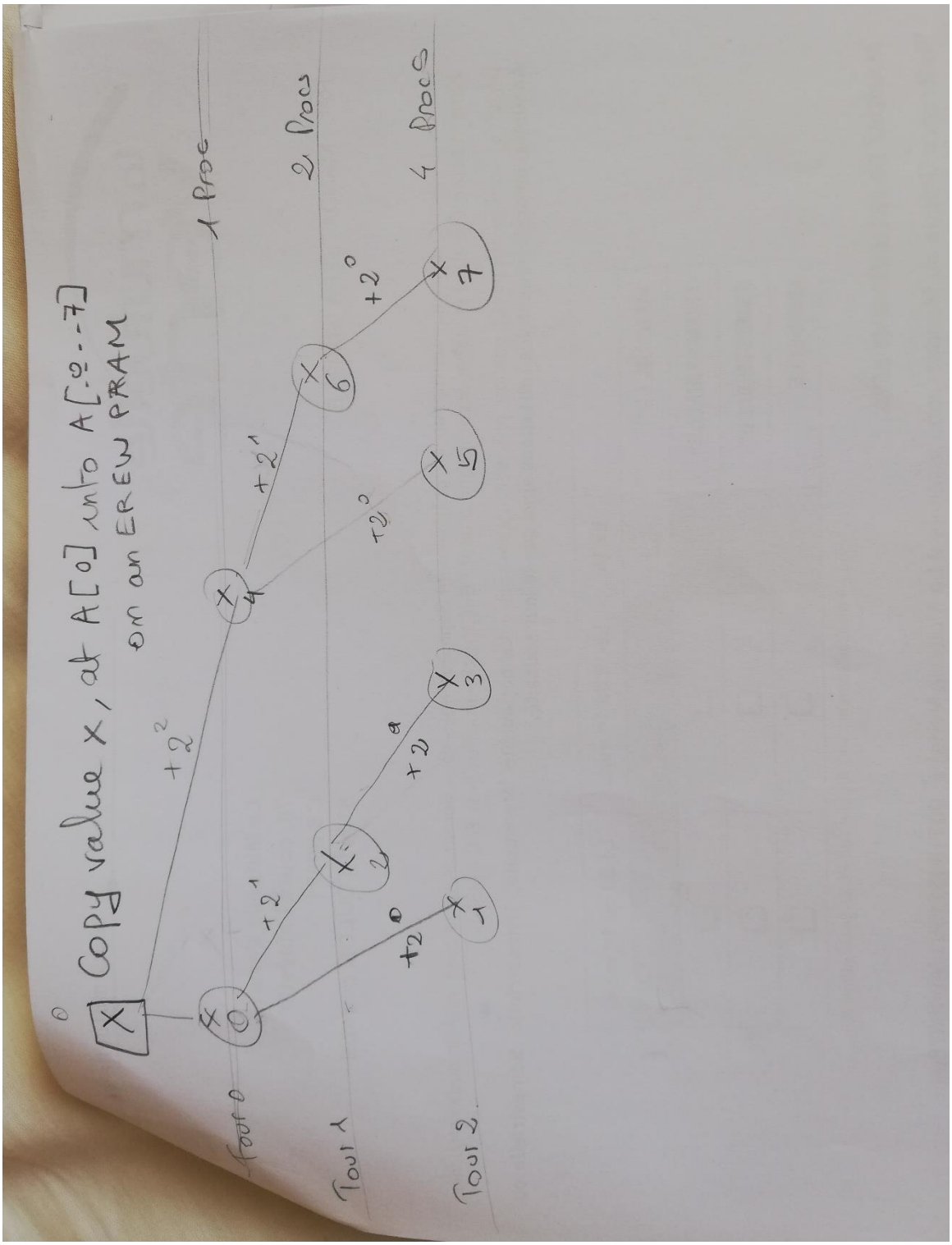
Q4) Sketch how to simulate a C.R. of this algorithm, on an E.R PRAM.

For the C.R simulation, write down the complete algorithm, assuming the value to be copied to the n processors, is stored in an array of size n, at index 0. Assume that n=2^m, so, you can iterate in O(m) parallel steps. Highlight why the simulation has only O(log n) parallel time complexity, given the number of data is n.

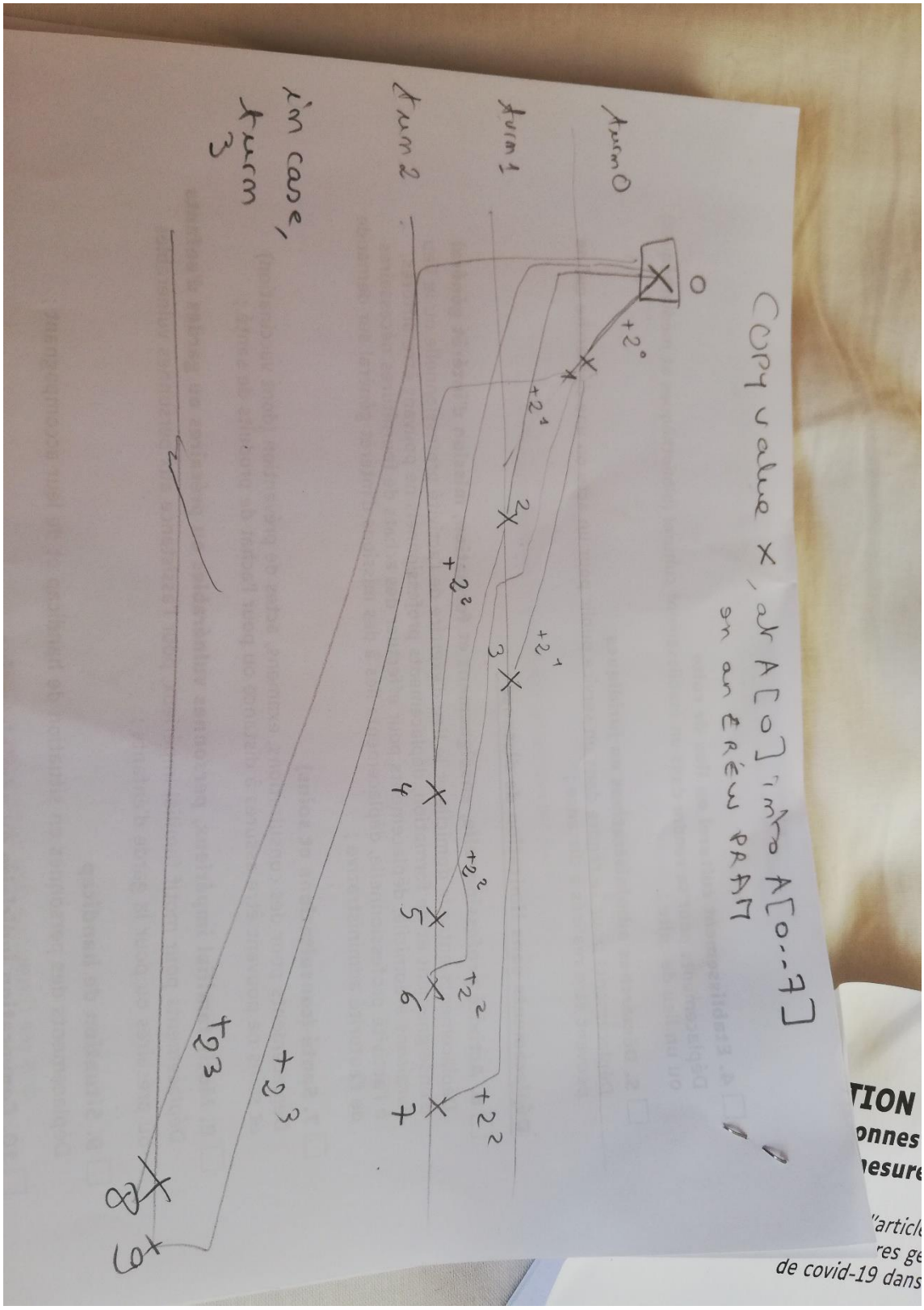
The simulation of a CR operation will be done in this specific context, but, consider that it is a general method to simulate a CR operation. (same will happen for the CW arbitrary operation).

To simulate a concurrent read operation of T[k] by O(n) procs : one proc only reads T[k] , it duplicates it in an auxiliary array A, at addresses eg A[1], A[2]. Then, two proc in parallel read A[1] and A[2]. Proc 1 duplicates A[1] in A[3] and A[4], while Proc 2 duplicates A[2] in A[5] and A[6], etc… Let us write that clearly: In order to do so, we will assume we start at A[0] and not A[1], and, more precisely, procs will not fill up A in order as you will see.

We may think of this sort of “pseudo tree” that depicts data movement, on which one can see which procs ids should be active at each step, etc…



Or, because writing the resulting algorithm of that data movement schema is not easy, we can think of another schema for data to be copied and by which proc, at each parallel step/turn.



Now, writing the algorithm, and “testing” that it makes the job is more easy

Q5) Sketch how to do the same for the C.W arbitrary mode, on an E.W. PRAM.

Basic idea: each proc j, 0<=j<n writes down the value (to be eventually written at address T[i]) in A[j]. If a process j has nothing to write at T[i], then A[j] remains empty. We may either “compact” A so all values to be written are at the left hand side of A, or we can live with an auxiliary array A, that holds some “empty” values. Let us in the sequel assume all j have to write down a value. So, no need to compact A. (still, one small exercise eg for the exam, could be to write down such an algorithm to compact A); and all A[j], 0<=j<n holds a value.

If we would simulate a CONSISTENCY CW PRAM, we would need to assume that all these values are equal. If not, after the compaction phase if needed, we would raise an error. (again, this can be a small exercise, eg for the exam, to effectively write down an algorithm to check the need to raise such an error).

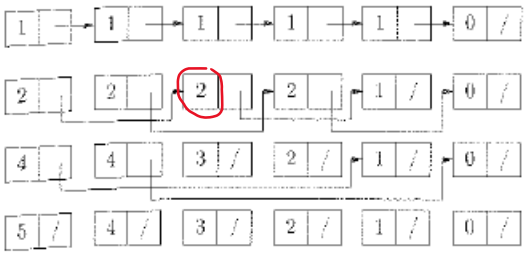
Here, in the context of computing the maximum on a CRCW PRAM, we only need an ARBITRARY CW PRAM (because, if several processes write a value, it happens that they write the same value, here equal to False!), meaning that we have simply to select one value among the candidate values. In that case, simply what we could do is to select just one cell in A, eg. A[0] to be written as an arbitrary chosen value in T[i]. Even, we would just need to check in advance that A[0] holds a value (so the possible need above to compact the values at the left hand side of A, or, to be able to select one non-empty cell in A).

Exercice 2

Q1) What does the following algorithm applied to a chained linked list of elements compute?

Start with this list once initialized 

It will have in d[i] for each i, the distance of the element in this list from the end of the list. See that



Q2) What is its parallel time complexity on a PRAM (considering you can use the most powerful PRAM you need) ? Hint : how many times is the condition of the while loop executed ?

By putting next[i]=next[next[i]], at each while loop next[i] points to an element that is twice farther from the end of the list than it was at the preceding step. So, in at most log n loops, each element will have next[i] = to NIL. So the number of iterations will be at most log n. Then, in each iteration, we must evaluate the number of instructions in the for each. It has 3 instructions (the test, and the two assignments). So total time complexity is O(log n) parallel time.

Q3) Which PRAM variant is needed at least, not to increase the parallel time complexity ? Hint: is it possible that 2 processes read data of the same list item at the same PRAM instruction ? (consider an instruction, eg, an addition with two operands as being one single instruction, ie., do not decompose even more one such operation, like an addition, into its corresponding assembly code)

We need an CR PRAM. Indeed, we can see that d[next[i]] requires to read the d variable of a given element, while, in parallel, another processor of the PRAM may read that same d variable of the same element . See the red circled d[i], of the 3rd element in the picture above. This value (2), is read by the processor in charge of the 1st element, and it is also read by the 3rd processor , in order to update the d variable of the 3rd element (to become 3).

We do not need CW however. So, only a CREW PRAM is needed.

Algorithme

Initialisation

  for each processor i in // **do**

    if $next[i] =$ NIL **then** $d[i] = 0$ **else** $d[i] = 1$

Boucle principale

  while ($\exists$ objet i t.q. $next[i] \neq$ NIL) **do**

    for each processor i in // **do**

      if $next[i] \neq$ NIL **then** $d[i] \leftarrow d[i] + d[next[i]]$; $next[i] \leftarrow next[next[i]]$

SpareQ) How to modify the algorithm so only an EREW PRAM is required. Hint: you can use local additional variables. Indeed, each PRAM processor can store information in its local memory

If next[i] <> NIL then { temp=d[next[i]]; d[i]=d[i]+temp;}

Exercice 3

In the algorithm provided for the parallel computation of the maximum (version 2), the course has shown a classical way to derive a work optimal PRAM algorithm.
Write down, using the pseudo PRAM language, the proposed work optimal algorithm.

For computing the maximum, in a way that is work optimal, we allocate to each of the PRAM processors logn data, instead of just one data. So, before running the parallel maximum PRAM algorithm on each of the n/log n data, each processor must first locally compute the maximum of the logn data. Then, the PRAM algorithm will compute the max of these n/logn maximum local values.

We will write this algorithm very precisely, by taking clearly in consideration the global memory addresses that each of the n/log n processors will handle