

# Radiography

## Computed Tomography (CT)<sup>1 2</sup>

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<sup>1</sup>G. Dougherty. *Digital Image Processing for Medical Applications*. Cambridge University Press, 2009.

<sup>2</sup>D. Eppstein. *An Introduction to the Mathematics of Medical Imaging*. SIAM, 2008.



# Computed Tomography

## Introduction

Conventional X-Ray imaging produces *planar* images, i.e., projections of 3D objects onto 2D planes.

**Tomographic imaging**, e.g., computed tomography (CT), was developed to produce *transverse* images.

### Working principle

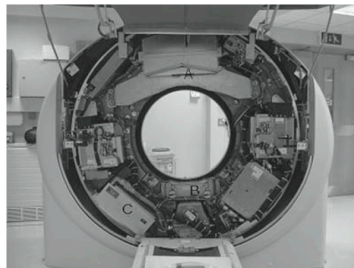
Scan a slice of tissue (with a narrow fan-shaped beam) from multiple angles → We obtain 1D projections of the object.

## Tomography

In tomographic imaging, we “reconstruct” the transverse image from the 1D projections of the object.

# Computed Tomography

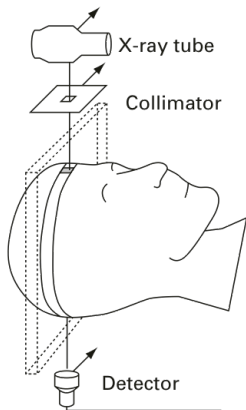
## Introduction



- Typically only one tube
- More filtering than projection radiography (previous lecture)
- Better approximation to monoenergetic source

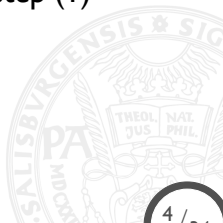
# Computed Tomography

## Working principle of a 1st generation CT scanner



### Iterate:

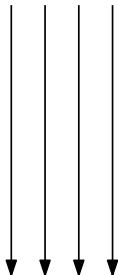
1. Sweep beam linearly across the patient's head
2. Turn off tube & rotate by small angle ( $\approx 1$  degree), then goto step (1)



# Computed Tomography

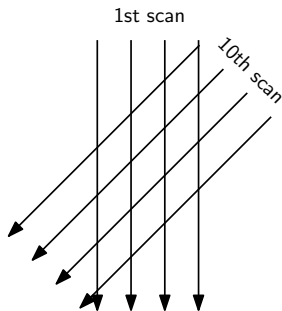
## Working principle of a 1st generation CT scanner

1st scan



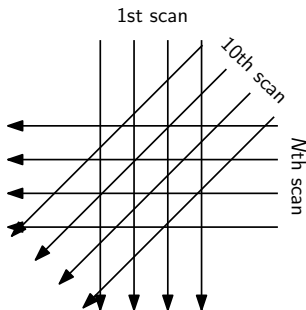
# Computed Tomography

## Working principle of a 1st generation CT scanner



# Computed Tomography

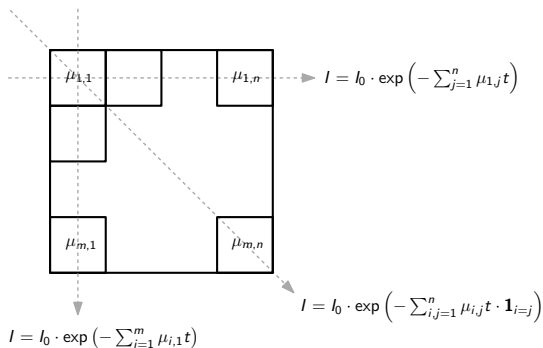
## Working principle of a 1st generation CT scanner



**Sampling interval & angle** determine pixel size; collimator width determines slice thickness.

# Computed Tomography

## Working principle



Remember  $I = I_0 \cdot e^{-\mu t}$ . Hence, the ray-sum along a path is

$$-\log\left(\frac{I}{I_0}\right) = \sum \mu_{ij} t \quad \text{with } i, j \text{ appropriately.}$$



# Computed Tomography

## Working principle

We have seen that the measured X-ray intensity depends on the sum of attenuation coefficients.

### X-Ray CT Image Reconstruction

Solve for

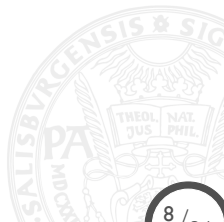
- the individual attenuation coefficients of each voxel
- and
- assign a value (depending on those coefficients) to each pixel in the 2D-array that describes the transverse image.

# Computed Tomography

## Working principle

Several **correction factors** needs to be implemented:

- X-Ray beam is (only approximately) mono-energetic and attenuation depends on energy.
- Effective X-Ray energy increases as it passes through patient – This effect is commonly known as “beam hardening”.

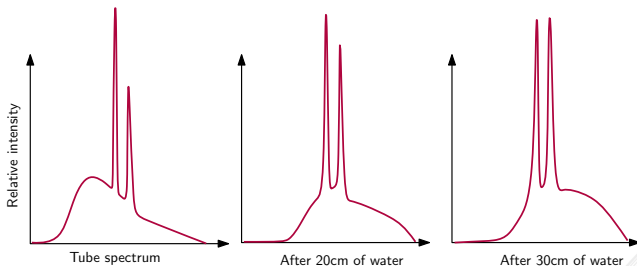


# Computed Tomography

## Working principle – Beam hardening

### Beam hardening

The mean energy of an X-Ray beam increases as it passes through an object / patient.

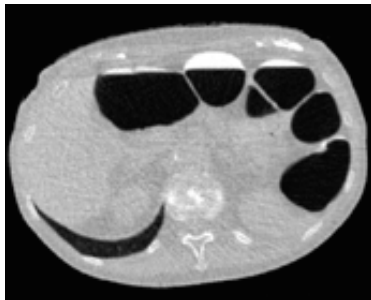
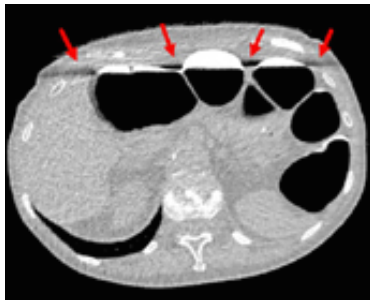


Lower energy photons are attenuated more easily, higher energy photons are attenuated less easily → dark streaks & cupping.

# Computed Tomography

## Working principle – Beam hardening

Beam hardening → streaking effects:

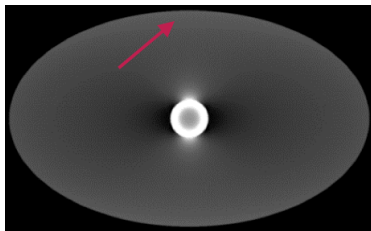
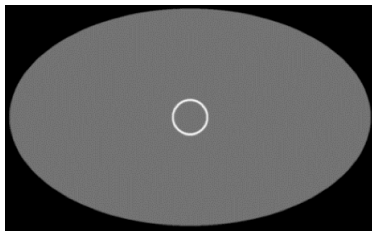


*Left: Streaking effect; Right: Virtual CT image*  
(Courtesy of Janne Nappi, PhD.)

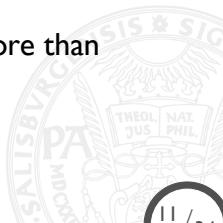
# Computed Tomography

## Working principle – Beam hardening

Beam hardening → “cupping” effects (simulated):



Beams passing through the center are “hardened” more than beams passing through edges.



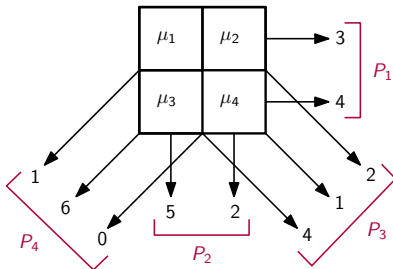
# Computed Tomography

## Projection & Backprojection

We observe that

$$-\log \left( \frac{I}{I_0} \right)$$

is the line integral of linear attenuation coefficients at an effective energy. Next is an example of 4 voxel and their projections ( $P_i$ ):



# Computed Tomography

## Projection & Backprojection

The question is “How do we obtain attenuation coefficients for each voxel”?

Various algorithms exist, e.g.,

- Backprojection
- Filtered backprojection
- Direct Fourier reconstruction

Backprojection (based on example from previous slide)

$$\begin{array}{c} P_1 \\ \rightarrow \end{array} \begin{array}{|c|c|} \hline 3 & 3 \\ \hline 4 & 4 \\ \hline \end{array} \begin{array}{c} P_2 \\ \rightarrow \end{array} \begin{array}{|c|c|} \hline 8 & 5 \\ \hline 9 & 6 \\ \hline \end{array} \begin{array}{c} P_3 \\ \rightarrow \end{array} \begin{array}{|c|c|} \hline 9 & 7 \\ \hline 13 & 7 \\ \hline \end{array} \begin{array}{c} P_4 \\ \rightarrow \end{array} \begin{array}{|c|c|} \hline 10 & 13 \\ \hline 19 & 7 \\ \hline \end{array}$$

No effort is taken to *distribute* the values over the voxel  $\rightarrow$  suboptimal!

# Computed Tomography

## Projection & Backprojection

**Problem:** We get *high values*!

**Countermeasures:**

1. Remove the sum in any of the projections (here: 7).

$$\begin{array}{c|c} 10 & 13 \\ \hline 19 & 7 \end{array} \rightarrow \begin{array}{c|c} 3 & 6 \\ \hline 12 & 0 \end{array}$$

2. Normalize by the highest common factor:

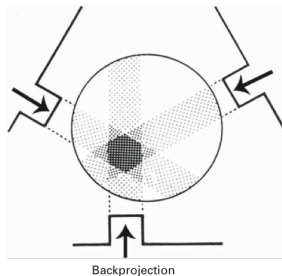
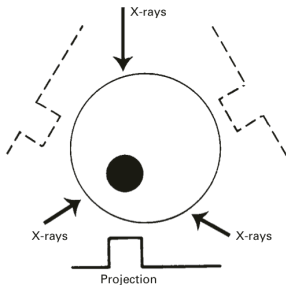
$$\begin{array}{c|c} 3 & 6 \\ \hline 12 & 0 \end{array} \rightarrow \begin{array}{c|c} 1 & 2 \\ \hline 4 & 0 \end{array}$$

In our case, this gives the attenuation values that actually led to the projection results!



# Computed Tomography

## Working principle – Problems of “simple” backprojection



### Star artifacts

The more images we have from different angles, the less prominent the object appears.

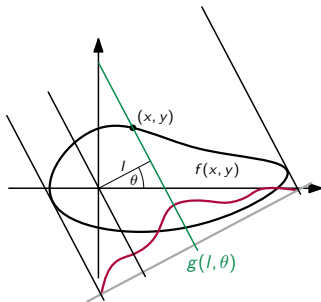
Why? Projections at an angle to the image grid intersect incomplete pixel.

# Computed Tomography

## Projection & backprojection principles

The projection line through the point  $(x, y)$  can be specified as

$$x \cos(\theta) + y \sin(\theta) = l \quad (\text{"Hessesche Normalform"})$$



$$g(l, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos(\theta) + y \sin(\theta) - l) dx dy$$

# Computed Tomography

## Projection & Backprojection

What is  $g(\cdot, \cdot)$ ?

- if we fix  $l$  and  $\theta \Rightarrow$  line integral of  $f(x, y)$
- if we fix  $\theta \Rightarrow$  projection of  $f(x, y)$  at angle  $\theta$

In other words,  $g(\cdot, \cdot)$  is the **Radon transform** of  $f(x, y)$ .

## Image Generation

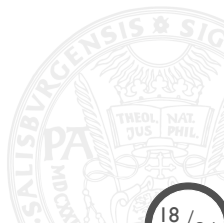
Projections are acquired for a selection of  $l$  and  $\theta$  values and then CT image is *reconstructed* from these projections.

# Computed Tomography

## Projection & Backprojection

### Backprojection (Variant I)

- $g$  is only measured at certain  $l$
- coarse sampling  $\rightarrow$  many points will not be assigned a value!
- Variant I is what we have seen so far!



# Computed Tomography

## Projection & Backprojection

### Backprojection (Variant 2) – Better Option!

For each angle  $\theta$ , go through all sampling points in an image, find the corresponding  $l$  and take the  $g(l, \theta)$  value.

$$b_{\theta}(x, y) = g(x \cos(\theta) + y \sin(\theta), \theta)$$

$$\begin{aligned} f_b(x, y) &= \frac{1}{\pi} \int_0^{\pi} b_{\theta}(x, y) d\theta = \frac{1}{\pi} \int_0^{\pi} g(x \cos(\theta) + y \sin(\theta), \theta) d\theta \\ &= \frac{1}{\pi} \int_0^{\pi} [g(l, \theta)]_{l=x \cos(\theta)+y \sin(\theta)} d\theta \end{aligned}$$

Note: since  $g(l, \theta)$  is only measured at certain  $l$ , we need to interpolate.

$f_b(x, y)$  is the **backprojection summation** image.

# Computed Tomography

## Radon Transform – An alternative view

A line in  $\mathbb{R}^2$  is the set of points satisfying

$$ax + by = c$$

where  $a^2 + b^2 \neq 0$ . Equally, we can write

$$\frac{ax}{\sqrt{a^2 + b^2}} + \frac{by}{\sqrt{a^2 + b^2}} = \frac{c}{\sqrt{a^2 + b^2}}.$$

Now,

$$\left( \frac{a}{\sqrt{a^2 + b^2}}, \frac{b}{\sqrt{a^2 + b^2}} \right) \in \mathbb{S}^1$$

where  $\mathbb{S}^1$  is the unit circle in  $\mathbb{R}^2$ .



# Computed Tomography

## Radon Transform – An alternative view

Consequently, we can parametrize a line by the unit vector  $\mathbf{w} \in \mathbb{S}^1$  and  $t \in \mathbb{R}$ . The line  $l_{\mathbf{w},t}$  is the set of points satisfying

$$\langle \mathbf{w}, (x, y) \rangle = t$$

We can also parametrize  $\mathbf{w}$  as

$$\mathbf{w}(\theta) = (\cos(\theta), \sin(\theta)), \theta \in [0, 2\pi]$$

and we have

$$\cos(\theta)x + \sin(\theta)y = t$$

Also, the vector **perpendicular** to  $\mathbf{w}$  is parametrized by

$$\hat{\mathbf{w}}(\theta) = (-\sin(\theta), \cos(\theta)), \theta \in [0, 2\pi].$$

# Computed Tomography

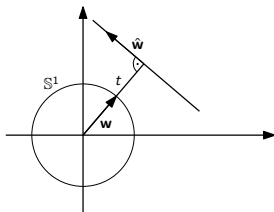
## Radon Transform – An alternative view

It holds that

$$\forall s \in \mathbb{R} : \langle \mathbf{w}, (t\mathbf{w} + s\hat{\mathbf{w}}) \rangle = t$$

which allows us to parametrize  $l_{\mathbf{w},t}$  as the set of points

$$l_{\mathbf{w},t} = \{t\mathbf{w} + s\hat{\mathbf{w}} | s \in (-\infty, \infty)\}$$



$\hat{\mathbf{w}}$  is chosen s.t.  $\det(\mathbf{w}\hat{\mathbf{w}}) = +1$ .





# Computed Tomography

## Radon Transform – An alternative view

Formally, the **Radon transform** is the integral transform

$$\mathcal{R}f(t, \mathbf{w}) = \int_{-\infty}^{\infty} f(t\mathbf{w} + s\hat{\mathbf{w}}) ds$$

mapping functions defined in  $\mathbb{R}^2$  to functions defined on  $\mathbb{R} \times \mathbb{S}^1$ .

**Remark.** We assume that functions  $f$  are in the *natural domain*<sup>3</sup> of the Radon transform, i.e.,  $f$

1. is regular enough so that the restriction to  $l_{\mathbf{w},t}$  is integrable.
2. goes to 0 rapidly so that the improper integrals converge.

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<sup>3</sup>D. Eppstein. *An Introduction to the Mathematics of Medical Imaging*. SIAM, 2008.

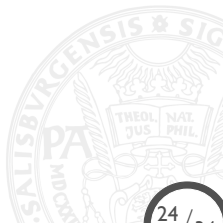
# Computed Tomography

## Radon Transform – An alternative view

**Backprojection** essentially *averages* the values of  $\mathcal{R}f$  over the lines that pass through a point, i.e.,

$$f_b(x, y) = \frac{1}{2\pi} \int_0^{2\pi} \mathcal{R}f(\langle(x, y), \mathbf{w}(\theta)\rangle, \theta) d\theta$$

We have seen the discretization of this idea before in our examples.



# Computed Tomography

## Interpretation of CT image values – Hounsfield units (HU)

Instead of using the attenuation coefficients directly as gray values, **Hounsfield units (HU)** are used:

$$HU = 1000 \cdot \frac{\mu - \mu_{H_2O}}{\mu_{H_2O}}$$

(relative to the attenuation of water; truncated to the nearest integer)

**Why do we want to use Hounsfield units?** Minimize dependence on the energy of X-Ray beam; produces unitless values!

# Computed Tomography

## Interpretation of CT image values – Hounsfield units (HU)

A brief overview of Hounsfield units:

Tissue type	HU
Bone	1000+
Hemorrhage	60-110
Liver	50-80
Muscle	44-69
Blood	42-58
Gray matter	32-44
White matter	24-36
Heart	24
Cerebrospinal fluid	0-22
Water	0
Fat	-20 to -100
Lung	-300
Air	-1000

(Note: Very dense bone has HU values of  $\approx 3000$ , so the range is  $\approx 4000 \rightarrow$  we will need 12 [bits])

# Computed Tomography

## Some sources of noise & artifacts

Main source of noise: **Quantum noise**<sup>4</sup>

(result of the statistical nature of X-ray emission)

Signal-to-noise Ratio (SNR)  $\propto \sqrt{N}$ , where  $N$  denotes the number of X-ray quanta / pixel.

→ better SNR if we increase  $N$ , but also more radiation!

### Countermeasures

- Larger voxels give better SNR (but reduced resolution)
- Smoothing during reconstruction (but resolution degrades)
- Better quantum efficiency of detectors

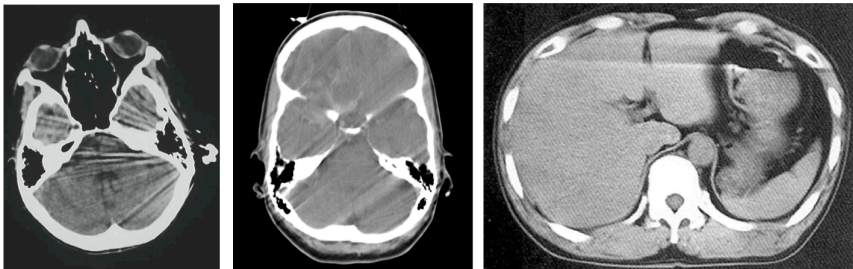
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<sup>4</sup>each photon is a “quantum”, i.e., a specific quantity of energy. Roughly speaking, due to the independent nature of photons, we get an uneven distribution of photons in an image area; this shows up as noise!

# Computed Tomography

Some sources of noise & artifacts

Artifact from **patient motion**:

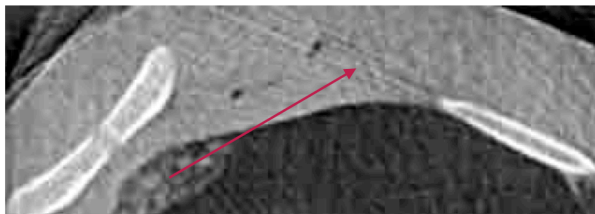


Characteristic streaks and “ghosting” (i.e., the image appears as if it is composed of superimposed images).

# Computed Tomography

Some sources of noise & artifacts

Artifact from **beam hardening**:



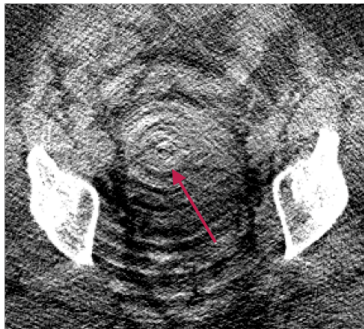
Cause: already discussed (see previous slides)!



# Computed Tomography

Some sources of noise & artifacts

Artifact from **calibration issues**:



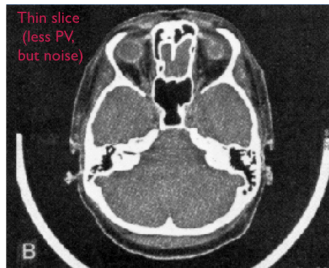
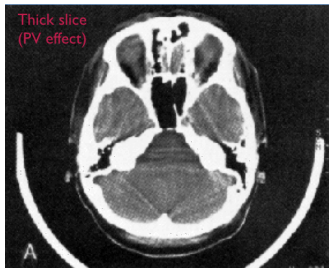
Cause: poorly calibrated or defective detector elements; shows up as bright/dark ring(s) around the center of rotation.



# Computed Tomography

Some sources of noise & artifacts

Artifact from **partial volume effects**:



Cause: Object only *partially* overlaps a slice. For example, a highly attenuating material might appear with the density of soft tissue!

# Computed Tomography

## Advances in scanner technology

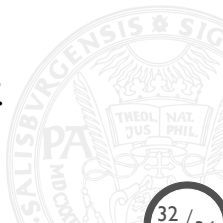
Early CT scanners acquired *a slice at a time*. Below is a sketch of the procedure (with stationary patient table):

1. Gantry spins  $360^\circ$  in one direction (one slice)
2. Gantry spins back (another slice)
3. Gantry stops and the patient table moves
4. Goto step (1)

Very time-consuming, hence long scan times!

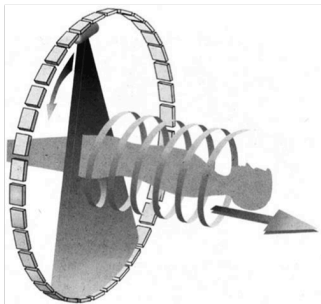
What is used today?

Continuously rotating gantries → spiral or helical CT.



# Computed Tomography

## Advances in scanner technology



The data is acquired while the patient is moved through the scanner. The trajectory of X-Ray the beam traces out a helix.

Remark: This requires changes in the backprojection algorithm!

# Computed Tomography

## Applications of CT

CT is primarily used to acquire images of

- chest,
- lungs,
- abdomen,
- bones.

Good modality to diagnose pulmonary (i.e., lung) disease. Why?  
Lungs are hard to diagnose using MRI or Ultrasound.

