University of Salzburg | Lecturer: Roland Kwitt

Imaging Beyond Consumer Cameras – Proseminar (911.422)

Exercise sheet A (March 6, 2018) | Prepare for presentation on **March 13, 2018**

Preliminaries/X-Ray Imaging

Exercise 1. 2 P.

Use either Convert3D, ITKSnap or 3DSlicer to load the following MRI image:

download by clicking

Figure out the (1) image size (in voxel), (2) the physical voxel size, (3) image orientation, (4) image origin and (5) the range of the intensity values. For fun, flip the first axis (e.g., using Convert3D) and see what changes.

Exercise 2. 8 P.

Experimentally, the loss in intensity (from I_0 to I) of light traveling through a thin cuvette (of width d) filled with some solution is proportional to (1) the travelled path length l and proportional (2) to the concentration c of the solution in the cuvette. Derive the Beer-Lambert law from these two observations, i.e., the law that relates absorption $A = \log(I_0/I)$ to c, and d.

$$A = \log\left(\frac{I_0}{I}\right) = ?\ .$$

After you have derived the expression of A, how does this relate to the attenuation of X-Ray beams (cf. slides).

Exercise 3. 10 P.

Show that binomial selection at the detector under a Poisson source yields another Poisson source. Then, illustrate discuss effect of the initial number of emitted photons on the signal-to-noise ratio (SNR). Details are given below.

Assume that an X-Ray source transmits photons of a certain energy. These photons pass through an object and hit a specific pixel on the detector. The X-Ray source is modelled by a Poisson random variable N with mean N_0. The probability mass function (PMF) of a Poisson distribution is

$$P[N = n] = \frac{1}{n!}e^{-N_0}N_0^n$$

where n is the number of events (photons) that occur with average rate N_0.

We model the detector as a Bernoulli random variable M with probability p. This is a reasonable model, since emitted photons either pass unaffected or interact with some object (independent events). We know that

$$P[M = m|N = n] = \binom{n}{m} p^m (1-p)^{n-m}$$

Your task is to derive the distribution of $P[M = m]$ (Hint: use total probability).

For your second subtask, note that the SNR is defined as

$$\text{SNR} = \frac{E[N]}{\sigma_N}$$

where N is the standard deviation. *Now, what does this mean for our Poisson source?*

Bonus question. What happens if, at the detector, we only capture a fraction (say ν) of incident photons (modeled via another Bernoulli random variable Y)?

Total #points: 20 P.