University of Salzburg — Lecturer: Roland Kwitt

Machine Learning (911.236)

Exercise sheet F (June 19, 2018)

Exercise 1. 10 P.

Let $w \in \mathbb{R}^d$ and $q \in (1, 2)$. Consider the l_q-norm

$$\|w\|_q = \left(\sum_{i=1}^{d} |w_i|^q \right)^{1/q}$$

Show that if

$$q = \frac{\log(d)}{\log(d) - 1}$$

then the function

$$R(w) = \frac{1}{2(q-1)} \|w\|_q^2$$

is (1/3)-strongly convex with respect to the l_1-norm over $\mathbb{R}^d$.

Exercise 2. 5 P.

In the lecture, we had the following lemma.

Lemma 1 (Strong-convexity properties)**.** *Let $w, u \in \mathbb{R}^d$ and $f : \mathbb{R} \to \mathbb{R}$.*

1. *The function $f(w) = \lambda \|w\|^2$ is 2λ-strongly convex.*
2. *If f is λ-strongly convex and g is convex, then $f + g$ is λ-strongly convex*
3. *If f is λ-strongly convex and u is a minimizer of f, then for any w,*

$$f(w) - f(u) \geq \frac{\lambda}{2} \|w - u\|^2$$

Give an example of a norm for which property (1) of Lemma 1 does not hold.

Total #points: 15 P.