University of Salzburg | Lecturer: Roland Kwitt

Imaging Beyond Consumer Cameras – Proseminar (911.422)

Exercise sheet D (May 14, 2018) | Prepare for presentation on **May 22, 2018**

3D Sensors

Exercise 1. 10 P.

Lets consider the noise in measurement for ToF cameras, i.e., the photon-shot noise of the received signal. Lets say each of the samples $\hat{A}_i$ that we get is a measure of photon count, integrated over a short period of time. We can model this as a Poisson distribution.

Typically, $\hat{A}_i$ is in the range of tens or thousands. In that case, we can approximate the Poisson distribution by a Gaussian with parameters A and B, i.e.,

$$\hat{A}_i \sim \mathcal{N}(A, B) \ .$$

Your task is to derive expressions for the parameters A and B.

Possible solution strategy: In general, the probability mass function (pmf) of the Poisson distribution is given by

$$\mathrm{Pois}(x, \lambda) = \frac{\lambda^x e^{-\lambda}}{x!}, \lambda > 0, x \in \mathbb{N} \cup \{0\}$$

This captures the probability of x events in an interval where λ denotes the average number of these events.

We will assume, in our case, that $x \to \infty$ and $\lambda \to \infty$. In that setting, we can assume $P(x, \lambda)$ to be continuous. To solve this exercise, you will need the Stirling approximation, i.e.,

$$x! \approx x^x e^{-x} \sqrt{2\pi x}$$

First, take the log of $\mathrm{Pois}(x, \lambda)$, then use the Stirling approximation appropriately. Second, you can assume that the Gaussian fits best around the mean, i.e., at $x = \lambda$. Set $x = \lambda + \epsilon$ with $\epsilon/\lambda << 1$. If you use this expression, and after some algebraic manipulation, you should get terms of the form $\log(1 + \epsilon/\lambda)$ which you can expand using a MacLaurin series

$$\log\left(1 + \frac{\epsilon}{\lambda}\right) = \frac{\epsilon}{\lambda} - \frac{\epsilon^2}{2\lambda^2} + \cdots$$

The trick then is that you can ignore some terms (why?) and eventually rephrase the expressions so that you get the probability density of a Gaussian. From there, you can read-off the parameters A and B.

Exercise 2. 4 P.

Let

$$\hat{X} = \frac{\hat{A}_0 - \hat{A}_2}{2} \quad \text{and} \quad \hat{Y} = \frac{\hat{A}_1 - \hat{A}_3}{2}$$

From our previous exercise we should know by what distribution we can approximate $\hat{A}_i$.

Find the distribution of $\hat{X}$ and $\hat{Y}$ – No need for derivation. You can obviously also derive it, which is not that hard and will get you 2 bonus points.

Total #points: 14 P.