# 3D Sensors


(Source: https://www.google.com/selfdrivingcar/)

Slide credit to Radu Horaud, http://perception.inrialpes.fr

# 3D Sensors

## Overview

Basic principle: Measure depth based

1. on illuminating the scene with a controlled light source, *and*
2. then measuring the backscattered light.

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Basic principle: Measure depth based

1. on illuminating the scene with a controlled light source, *and*
2. then measuring the backscattered light.

Two classes of sensors:

1. *Time of Flight (ToF) sensors*: Measure *depth* by estimating the time delay from light emission to light detection.
2. *Projected-light sensors*: Combine the projection of a light pattern with a standard 2D camera and measure depth via triangulation.

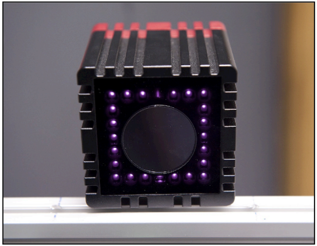
# 3D Sensors

## Examples – Types of ToF sensors

1. Point-wise ToF sensors: mounted on a two-dimensional pan-tilt scanning mechanism, also referred to as Light Detection and Ranging (LIDAR).
2. Matricial ToF sensors: estimate depth in a "single shot" using a matrix of ToF sensors (in practice, they use CMOS or CCD image sensors coupled with a lens system).

# 3D Sensors

## Examples – Matricial ToF sensors

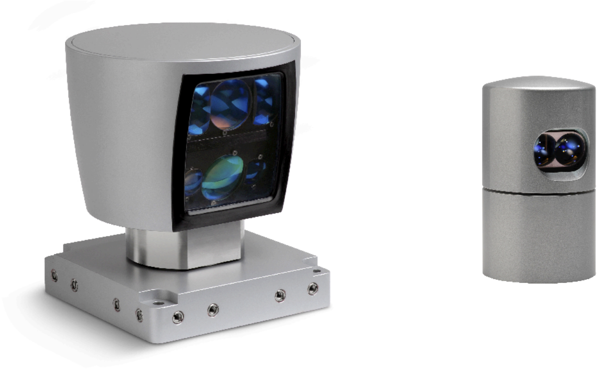



SR4000 (Swiss Ranger)

(Source: http://www.hizook.com, http://www.mesa-imaging.ch)

# 3D Sensors

## Examples – Point-wise ToF sensors



Velodyne HDL-64E & HDL-32E

(Source: http://velodynelidar.com/lidar/lidar.aspx)

# 3D Sensors

## Examples – Point-wise ToF sensors (Velodyne usage)

BAIDU self-driving car, NAVYA driverless shuttle
(Source: http://velodynelidar.com)

# 3D Sensors

## Examples – 3D Flash LIDAR cameras (direct ToF)


TigerEye 3D
(Source: http://www.advancedscientificconcepts.com/products/tigereye.html

# 3D Sensors

## Examples – Projected light sensors


Microsoft Kinect

(Source: https://de.wikipedia.org/wiki/Kinect)

# 3D Sensors

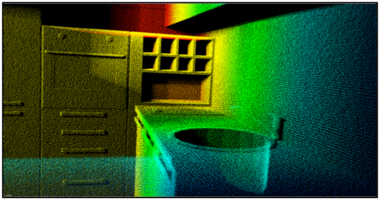
## Examples – Projected light sensors


Asus Xtion Pro Live

(Source: http://vr-zone.com)

# Time-of-Flight (ToF)

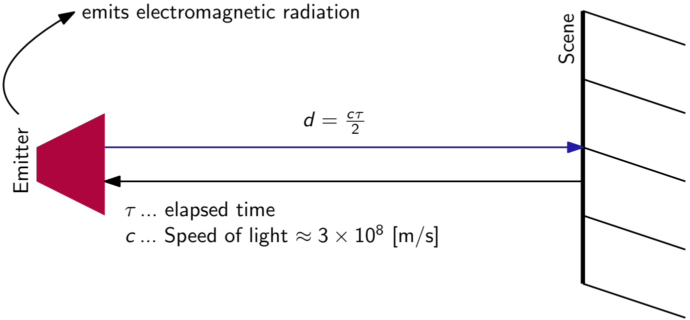
## Principles



Simulated ToF image, using "Blensor" http://blensor.org

# 3D Sensors

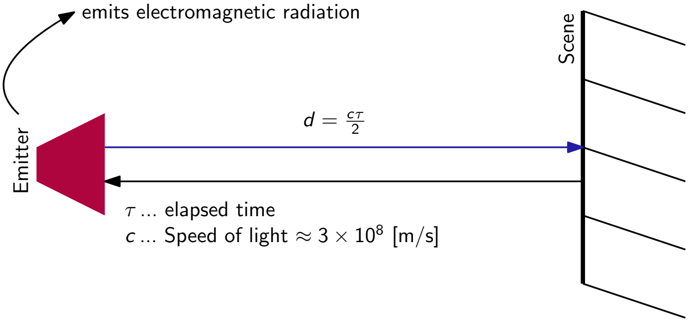
## ToF principles




What is the challenge?

# 3D Sensors

## ToF principles




### What is the challenge?

It takes $\approx 3.3$ [ps] to cover a 1 [mm] path. For such a resolution, we would need a clock, capable of measuring 3.3 [ps] time steps!

# 3D Sensors

## ToF principles

Working principle: measure the absolute time that a light pulse needs to travel from a target object to a detector.

Pulsed modulation: measure the ToF directly

3D Surface

Source: `http://graphics.stanford.edu`

Emitter

Timer

Detector

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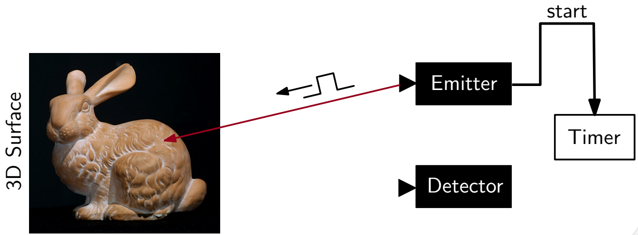


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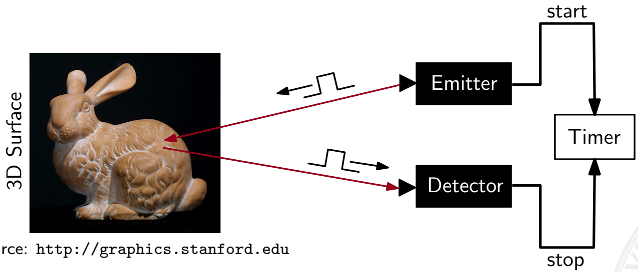


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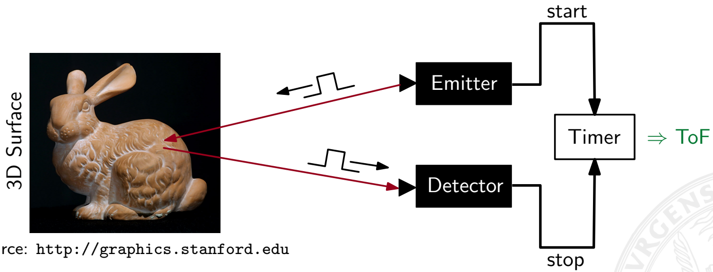


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# 3D Sensors

## ToF principles (Pulsed-modulation)

Advantages:

- High energy light pulses

  $\Rightarrow$ less influence of background illumination
- Illumination and observation directions are collinear

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Disadvantages:

- Arrival time must be measured very precisely
- Needs very short light pulses with fast rise/fall times
- High optical power
- Typically, these ToF sensors use lasers or laser diodes

# 3D Sensors

## ToF principles (CW-modulation)

Today, we focus on continuous wave (CW) modulation:

- uses continuous light waves
- detected wave after reflection has *shifted phase*
- *phase shift is proportional to distance* from reflected surface

3D Surface

e.g., 20 Mhz

Emitter

Detector

# 3D Sensors

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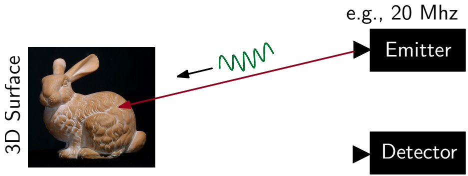

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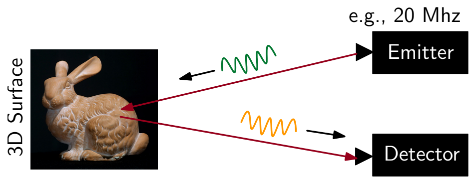

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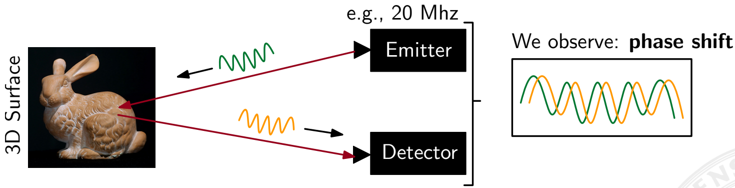

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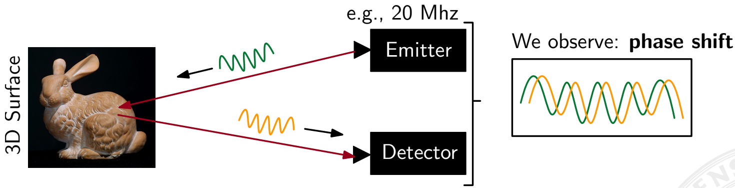


*A way to recover the phase shift is by cross-correlation between the emitted and received signal!*

# 3D Sensors

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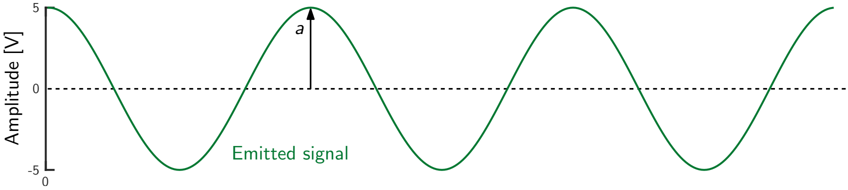

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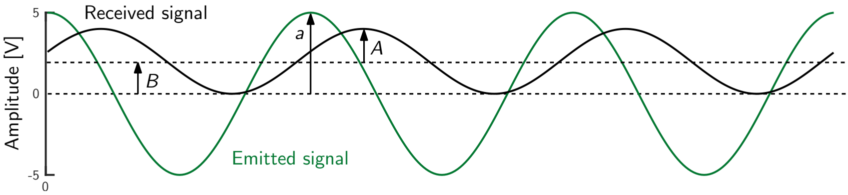

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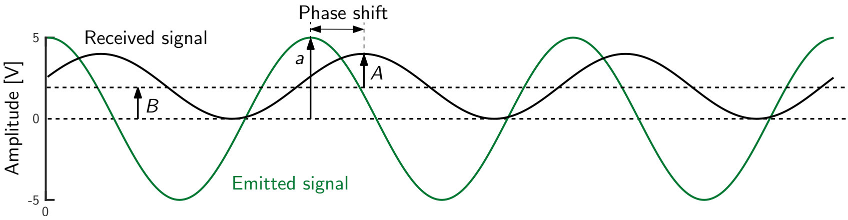


In this illustration, $A = 2$, $a = 5$ and bias $B = 2$.

# 3D Sensors

## ToF principles (CW-modulation)

Phase shift recovery through cross-correlation: We know that the relationship between distance $d$, light speed $c$ and ToF $\tau$ is

$$d = \frac{1}{2} c \tau$$

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Assuming a constant bias $B$ and a cosine signal with modulation frequency $f$ and amplitude $a$, the emitted $s(t)$ and received $r(t)$ signals are given as

$$s(t) = a\cos(2\pi ft) \quad \text{and} \quad r(t) = A\cos(2\pi f(t-\tau)) + B$$

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$B$ is an offset, due to ambient illumination!

# 3D Sensors

## ToF principles (CW-modulation)

*Using cross-correlation between the emitted and received signal, we can recover the phase.*

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Let's look at the following integral first:

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The solution to this integral is (the correlation function)

$$C(x) = \frac{aA}{2} \cos(\underbrace{2\pi f \tau}_{\phi} + 2\pi f x) + B$$

# 3D Sensors

## ToF principles (4-bucket algorithm)

We can now evaluate $C(x)$ at four selected phases (aka 4-bucket algorithm), i.e.,

$$
\begin{aligned}
2\pi f x_0 &= 0^\circ \\
2\pi f x_1 &= 90^\circ \\
2\pi f x_2 &= 180^\circ \\
2\pi f x_3 &= 270^\circ
\end{aligned}
$$

to recover the three unknowns $A$, $B$ and $\tau$.

# 3D Sensors

## ToF principles

Using standard trigonometric relationships, we obtain

$$\phi = 2\pi f \tau = \tan^{-1}\left(\frac{C(x_3) - C(x_1)}{C(x_0) - C(x_2)}\right)$$

$$A = \frac{1}{2a}\sqrt{(C(x_3) - C(x_1))^2 + (C(x_0) - C(x_2))^2}$$

$$B = 0.25 \sum_i C(x_i)$$

# 3D Sensors

## ToF principles (phase wrapping)

Can you think of any problems related to that approach?

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We can easily compute the depth $d$ from the recovered phase as

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This *ambiguity* is referred to as phase wrapping!

$$d = \left(\frac{\phi}{2\pi} + n\right) d_{\text{max}}, \quad \text{with} \quad n = 0, 1, 2, \ldots$$

Here, $n$ denotes the number of wrappings.

*Example*: for $f = 30$ [Mhz], the unambiguous range is 0-5 [m].

# 3D Sensors

## ToF principles (phase wrapping)

The CCD sensor plays several roles:

1. The incoming photons are converted to electron charges
2. Clocking
3. Signal processing (i.e., demodulation)

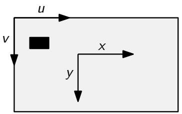
After the demodulation, the signal $C(\psi)$, i.e.,

$$C(\psi) = \frac{aA}{2}\cos(2\pi f\tau + \underbrace{2\pi fx}_{\psi}) + B$$

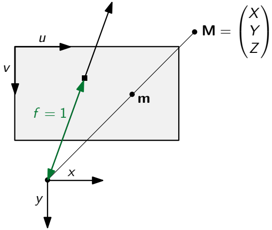
is integrated at four equally-spaced intervals within one modulation period.

# 3D Sensors

## ToF principles – From depth to Euclidean coordinates




$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \approx \mathbf{A} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$




The ToF camera measures the depth $d$ from the 3D point $\mathbf{M}$ to the optical center, hence

$$d = \|\mathbf{M}\|$$

# 3D Sensors

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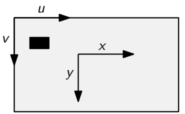


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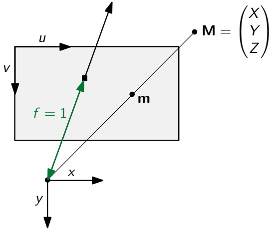


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We also know (from similar triangles) that

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = Z \begin{pmatrix} X/Z \\ Y/Z \\ 1 \end{pmatrix} = Z \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

# 3D Sensors

## From depth to Euclidean coordinates

Now, the point $\mathbf{m}$ has $(u, v)$-coordinates

$$\mathbf{p} = [u\ v\ 1]^\top$$

and $(x, y)$ coordinates

$$\mathbf{m} = [x\ y\ 1]^\top \quad \text{with} \quad \mathbf{m} = \mathbf{A}^{-1}\mathbf{p} \ .$$

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Next, we have $Z = \|\mathbf{M}\| \ / \ \|\mathbf{m}\|$ and consequently

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = Z \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \frac{\|\mathbf{M}\|}{\|\mathbf{m}\|} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \frac{d}{\|\mathbf{A}^{-1}\mathbf{p}\|}\mathbf{A}^{-1} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

# 3D Sensors

## Distortions

We observe distortions due to ...

- Phase warping (already covered)

  $\Rightarrow$ leads to ambiguities in distance measurements!

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- Photon-shot noise

  $\Rightarrow$ affects the precision of distance measurements
- Saturation & Motion blur

# 3D Sensors

## Distortions – More details on photon-shot noise

Interestingly, the noise which affects the distance measurement can be approximated by a Gaussian with variance

$$\sigma_d = \frac{c}{4\pi f\sqrt{2}}\frac{\sqrt{B}}{A}$$

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Interestingly, the noise which affects the distance measurement can be approximated by a Gaussian with variance

$$\sigma_d = \frac{c}{4\pi f\sqrt{2}}\frac{\sqrt{B}}{A}$$

We observe

- As $A$ gets larger, precision improves
- As $B$ gets larger, precision gets worse
- $B$ depends on background illumination and amplitude $A$
- An increase in $B$ due to increase in $A$ does not hurt (since $\sqrt{B}$)
- Increase $f \Rightarrow$ better precision, but loss in max. measurable $d$

# 3D Sensors

## Distortions – Other noise sources

Other noise sources are, e.g.:

- Thermal noise caused by the receiver signal amplifier
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A typical strategy to counteract noise effects is signal averaging over several periods.

# 3D Sensors

## Distortions – Signal averaging

Typical averaging intervals are between 1 [ms] and 100 [ms]. In case of $f = 30$ [Mhz], e.g., we get a *period* $T$ of

$$T = 33.3 \times 10^{-9} \approx 33 \text{ [ns]}$$

and consequently, for averaging over 1 [ms], we cover $3 \times 10^4$ periods, or, for 100 [ms], $3 \times 10^6$ periods.

The averaging interval length is called the integration time.