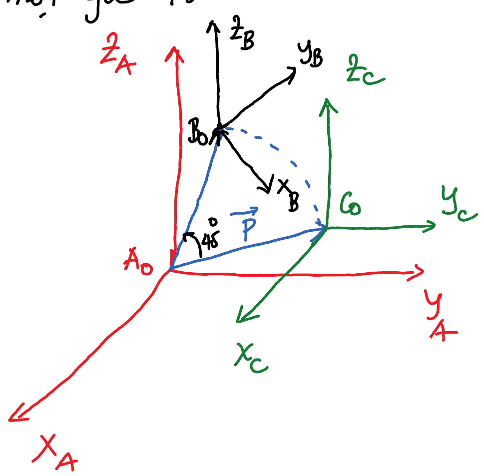
Đề 1

1, ${}^{A}P = [2, 8, 8]^T \quad {}^{B}P = [12, 20, -8]^T$

Hệ quy chiếu {B} ban đầu trùng {A} → Tịnh tiến hệ quy chiếu {B} theo vectơ $[P_x, P_y, P_z]^T$

→ Quay hệ quy chiếu {B} vừa thu được quanh trục z_A một góc 45°

$$\Rightarrow \{A\} \xrightarrow[{[P_x\ P_y\ P_z]}]{\text{Tịnh tiến}} \{C\} \xrightarrow[z_A,\ 45^\circ]{\text{Quay}} \{B\}$$



$${}^{A}_{C}T = \text{Trans}(P_x, P_y, P_z) = \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}^{A}X_C \quad {}^{A}Y_C \quad {}^{A}Z_C \quad {}^{A}P_{C_0}$

Chú ý: Phép quay là quay quanh trục $\underline{z_A}$

$${}^{A}P_{B_0} = \text{Rot}(z, 45)^\circ\, {}^{A}P_{C_0}$$

Ảnh của các vectơ ${}^{A}X_C$, ${}^{A}Y_C$ và ${}^{A}Z_C$ sau phép quay quanh trục z_A một góc 45° lần lượt là

$$\left.\begin{aligned} {}^{A}X_B &= \text{Rot}(z, 45^\circ)\, {}^{A}X_C \\ {}^{A}Y_B &= \text{Rot}(z, 45^\circ)\, {}^{A}Y_C \\ {}^{A}Z_B &= \text{Rot}(z, 45^\circ)\, {}^{A}Z_C \end{aligned}\right\} \quad {}^{A}_{B}T = \begin{bmatrix} {}^{A}X_B & {}^{A}Y_B & {}^{A}Z_B & {}^{A}P_{B_0} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow {}^{A}_{B}T = \text{Rot}(z, 45^\circ) \left[\begin{array}{ccc|c} {}^{A}X_C & {}^{A}Y_C & {}^{A}Z_C & {}^{A}P_{C_0} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\Rightarrow {}^{A}_{B}T = \text{Rot}(z, 45^\circ)\, {}^{A}_{C}T = \text{Rot}(z, 45^\circ).\text{Trans}(P_x\ P_y\ P_z)$$

<u>Chú ý</u>:

1, Nếu $\{A\} \xrightarrow[X_A,\ \alpha]{\text{Quay}} \{C\} \xrightarrow[{}^{C}P = [P_x\ P_y\ P_z]]{\text{Tịnh tiến}} \{D\} \xrightarrow[z_D,\ \beta]{\text{Quay}} \{B\}$

tịnh tiến theo vectơ trong hệ tọa độ {C}

Quay theo hệ trục z_D

$${}^{A}_{B}T = \text{Rot}(X_A, \alpha)\ \text{Trans}(P_x, P_y, P_z)\ \text{Rot}(z_D, \beta)$$

Thứ tự ngược ⟶

2, Nếu $\{A\} \xrightarrow[X_A,\ \alpha]{\text{Quay}} \{C\} \xrightarrow[{}^{A}P = [P_x\ P_y\ P_z]]{\text{Tịnh tiến}} \{D\} \xrightarrow[z_A,\ \beta]{\text{Quay}} \{B\}$

$${}^{A}_{B}T = \text{Rot}(z_A, \beta)\ \text{Trans}(P_x, P_y, P_z).\ \text{Rot}(X_A, \alpha)$$

Thứ tự xuôi ⟶

$${}^{A}_{B}T = \begin{bmatrix} \frac{\sqrt2}{2} & -\frac{\sqrt2}{2} & 0 & 0 \\ \frac{\sqrt2}{2} & \frac{\sqrt2}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt2}{2} & -\frac{\sqrt2}{2} & 0 & \frac{\sqrt2}{2}(P_x - P_y) \\ \frac{\sqrt2}{2} & \frac{\sqrt2}{2} & 0 & \frac{\sqrt2}{2}(P_x + P_y) \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Để tìm P_x, P_y, P_z ta có

$${}^{A}P = {}^{A}_{B}T \cdot {}^{B}P$$

$$\begin{bmatrix} 2 \\ 8 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 & \frac{\sqrt{2}}{2}(P_x - P_y) \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 & \sqrt{2}/2\,(P_x + P_y) \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ 20 \\ -8 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} 2 &= 6\sqrt{2} - 10\sqrt{2} + \frac{\sqrt{2}}{2}(P_x - P_y) \\ 8 &= 6\sqrt{2} + 10\sqrt{2} + \frac{\sqrt{2}}{2}(P_x + P_y) \\ 8 &= P_z - 8 \end{aligned} \Rightarrow \begin{cases} P_x - P_y = 2\sqrt{2} + 8 \\ P_x + P_y = 8\sqrt{2} - 32 \\ P_z = 16 \end{cases}$$

$$\begin{cases} P_x = 5\sqrt{2} - 12 \\ P_y = 3\sqrt{2} - 20 \\ P_z = 16 \end{cases}$$

Như vậy

$${}^{A}_{B}T = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 & 2 + 4\sqrt{2} \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 & 8 - 16\sqrt{2} \\ 0 & 0 & 1 & 16 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b)

$${}^{A}Q = {}^{A}_{B}T\, {}^{B}Q$$

$${}^{A}Q = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 & 2 + 4\sqrt{2} \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 & 8 - 16\sqrt{2} \\ 0 & 0 & 1 & 16 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 + 3\sqrt{2} \\ 8 - 15\sqrt{2} \\ 16 \\ 1 \end{bmatrix}$$