Lưu Phương Thảo - RAI K62
213330790.

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CHƯƠNG 2. BIỂU DIỄN KHÔNG GIAN VÀ CÁC PHÉP BIẾN ĐỔI

Bài 2.1

a) $\{A\} \xrightarrow{\text{Quay } X} \{C\} \xrightarrow{\text{Quay } Z_C} \{D\} \xrightarrow{\text{tịnh tiến } Y} \{B\}$

$$ {}^{A}_{C}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{C}_{D}T = \begin{bmatrix} \cos\beta & -\sin\beta & 0 & 0 \\ \sin\beta & \cos\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

$$ {}^{A}_{B}T = {}^{A}_{C}T \cdot {}^{C}_{D}T \cdot {}^{D}_{B}T = \begin{bmatrix} \cos\beta & -\sin\beta & 0 & -3\sin\beta \\ \cos\alpha.\sin\beta & \cos\alpha.\cos\beta & -\sin\alpha & 3\cos\alpha.\cos\beta \\ \sin\alpha.\sin\beta & \sin\alpha.\cos\beta & \cos\beta & 3\sin\alpha.\cos\beta \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

b) Xét P trong {B} biết ${}^{B}P_P = [0 \quad -3 \quad 0 \quad 1]^T$ và $\alpha = 45°$, $\beta = 30°$.

$$ {}^{A}_{B}T = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 0 \\ \sqrt{2}/4 & \sqrt{6}/4 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/4 & \sqrt{6}/4 & -\sqrt{2}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

$$ {}^{A}P = {}^{A}_{B}T \cdot {}^{B}P = \cancel{\begin{bmatrix} 3/2 \\ -3\sqrt{6}/4 \\ -3 \end{bmatrix}} \begin{bmatrix} \frac{3}{2} & -\frac{3\sqrt{6}}{4} & -\frac{3\sqrt{6}}{4} & 1 \end{bmatrix}^T $$

c) Xác định Q trong {B} biết ${}^{A}P_Q = [2 \quad 2 \quad 2 \quad 1]^T$, $\alpha = 45°$, $\beta = 30°$.

$$ {}^{B}P_Q = {}^{B}_{A}T \cdot {}^{A}P_Q = {}^{A}_{B}T^{-1} \cdot {}^{A}P_Q = \begin{bmatrix} \sqrt{3}/2 & \sqrt{2}/4 & \sqrt{2}/4 & 0 \\ -1/2 & \sqrt{6}/4 & \sqrt{6}/4 & 0 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 2 \\ 1 \end{bmatrix} $$

$$ = \begin{bmatrix} \sqrt{2} + \sqrt{3} \\ \sqrt{6} - 1 \\ 0 \\ 1 \end{bmatrix} $$

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Bài 2.2 $\theta = 45°$, $\phi = 30°$

- ${}^{A}P$ quay quanh Z_A một góc θ:

$$ {}^{A}P' = C_Z(\theta) \cdot {}^{A}P = \begin{bmatrix} c\theta & -s\theta & 0 & 0 \\ s\theta & c\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sqrt{2}/2 \\ 3\sqrt{2}/2 \\ 3 \\ 1 \end{bmatrix}. $$

- ${}^{A}P$ quay quanh X_A một góc ϕ

$$ {}^{A}P'' = C_X(\phi) \cdot {}^{A}P' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\phi & -s\phi & 0 \\ 0 & s\phi & c\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -\sqrt{2}/2 \\ 3\sqrt{2}/2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sqrt{2}/2 \\ (3\sqrt{6}-6)/4 \\ (3\sqrt{2}+6\sqrt{3})/4 \\ 1 \end{bmatrix} $$

Bài 2.3

- Điểm P biểu diễn khi tịnh tiến theo vector dẫn ${}^{A}Q$ là:

$$ {}^{A}P' = \begin{bmatrix} p_x + q_x \\ p_y + q_y \\ p_z + q_z \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ -5 \\ 1 \end{bmatrix} $$

- Quay quanh X_A 1 góc $\phi = 90°$: ${}^{A}P'' = C_X(\phi) \cdot {}^{A}P'$.

$$ \rightarrow {}^{A}P'' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\phi & -s\phi & 0 \\ 0 & s\phi & c\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 4 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 4 \\ 1 \end{bmatrix}. $$

Bài 2.5

a) {A} quay quanh Z_A một góc $\theta = 45°$; tịnh tiến theo X_A 4 đvị, theo Y_A 3 đvị $\longrightarrow$ {B}

$$ \rightarrow {}^{A}_{B}T = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 4 \\ \sin\theta & \cos\theta & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 & 4 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

b)
$$ {}^{B}_{A}T = {}^{A}_{B}T^{-1} = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 & -7\sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

Bài 2.6

$$ {}^{B}_{C}T = {}^{B}_{A}T \cdot {}^{A}_{U}T \cdot {}^{U}_{C}T = {}^{B}_{A}T \cdot {}^{U}_{A}T^{-1} \cdot {}^{C}_{U}T^{-1} = \begin{bmatrix} 0,5 & 0,6 & 0,69 & -1,61 \\ -0,75 & 0,7 & -0,35 & 11,3 \\ -0,43 & -0,52 & 1,4 & -1,97 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

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$\boxed{\text{Bài 2.4}}$

- {B} quay quanh Z_B 1 góc θ:

$$C_Z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Quay quanh X_B 1 góc ϕ.

$$C_X(\phi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 \\ 0 & \sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Tịnh tiến học thu đc dọc theo Z_B 5 đvị.

$$L_Z = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 1 \end{bmatrix}$$

- $${}^A_BT = C_Z(\theta) \cdot C_X(\phi) \cdot L_Z = \begin{bmatrix} -2{,}5\cos(\theta+\phi) + 2{,}5\cos(\theta-\phi) \\ -2{,}5\cos(\theta+\phi) - 2{,}5\cos(\theta-\phi) \\ 5\cos(\phi) \\ 1 \end{bmatrix}$$

Với $\theta = 135^\circ$, $\phi = 60^\circ$;

- $${}^AP = {}^A_BT \cdot {}^BP = \Bigg[$$

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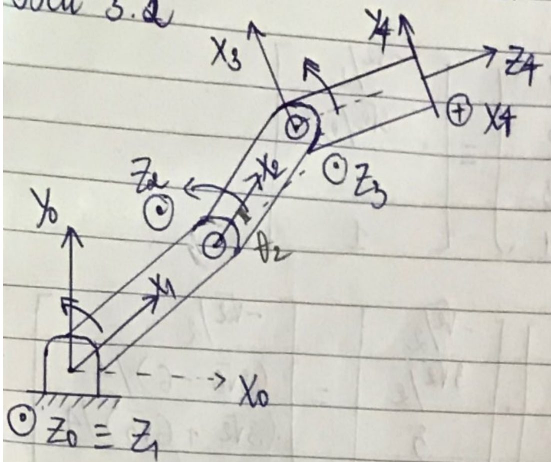
CHƯƠNG 3: ĐỘNG HỌC ROBOT

Bài 3.2



	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	l_1	0	θ_2
3	0	l_2	0	$\theta_3 + 90^\circ$
4	90°	0	l_3	270°

với: l_1: khoảng cách giữa Z_1 và Z_2
l_2: khoảng cách giữa Z_2 và Z_3.

$$ {}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ c0^\circ . s\theta_1 & c0^\circ . c\theta_1 & s0^\circ & -0.s0^\circ \\ s0^\circ . s\theta_1 & s0^\circ . c\theta_1 & c0^\circ & 0.c0^\circ \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^2_3T = \begin{bmatrix} -s\theta_3 & -c\theta_3 & 0 & l_2 \\ c\theta_3 & -s\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^3_4T = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -l_3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_4T = {}^0_1T . {}^1_2T . {}^2_3T . {}^3_4T = \begin{bmatrix} \cancel{\sin(\theta_1+\theta_2+\theta_3)} & \cancel{-s(\theta_1+\theta_2+\theta_3)} & \cancel{c(\theta_1+\theta_2+\theta_3)} \\ \cancel{-c(\theta_1+\theta_2+\theta_3)} & \cancel{c(\theta_1+\theta_2+\theta_3)} & \cancel{s(\theta_1+\theta_2+\theta_3)} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} $$

$$ = \begin{bmatrix} s(\theta_1+\theta_2+\theta_3) & -s(\theta_1+\theta_2+\theta_3) & c(\theta_1+\theta_2+\theta_3) & l_1.c\theta_1 + l_2.c(\theta_1+\theta_2) + l_3.c(\theta_1+\theta_2+\theta_3) \\ -c(\theta_1+\theta_2+\theta_3) & c(\theta_1+\theta_2+\theta_3) & s(\theta_1+\theta_2+\theta_3) & l_1.s\theta_1 + l_2.s(\theta_1+\theta_2) + l_3.s(\theta_1+\theta_2+\theta_3) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

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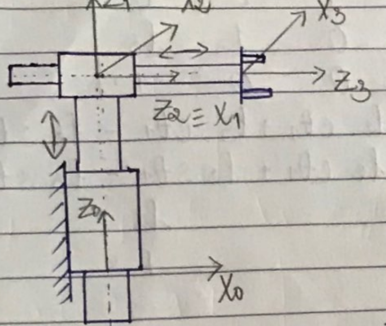
Bài 3.3



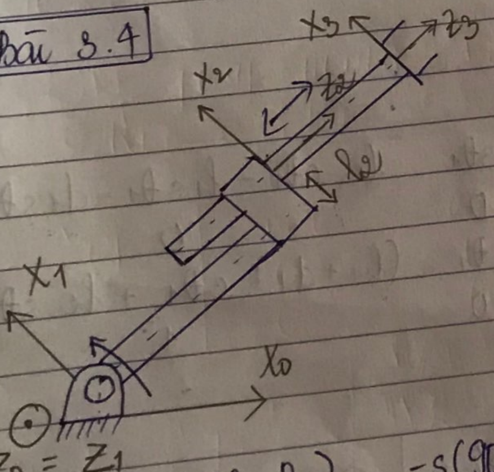
	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	l_1	0
2	90°	0	0	90°
3	0	0	$l_2 + d_2$	0

$$ {}^0_1T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^1_2T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2_3T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_2 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

$$ {}^0_3T = {}^0_1T \cdot {}^1_2T \cdot {}^2_3T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -l_2 - d_2 \\ 1 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Bài 3.4



	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	$90° + \theta_1$
2	90°	l_2	l_2	0
3	0	0	l_3	0.

$$ {}^0_1T = \begin{bmatrix} c(90° + \theta_1) & -s(90° + \theta_1) & 0 & 0 \\ s(90° + \theta_1) & c(90° + \theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -s\theta_1 & -c\theta_1 & 0 & 0 \\ c\theta_1 & -s\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

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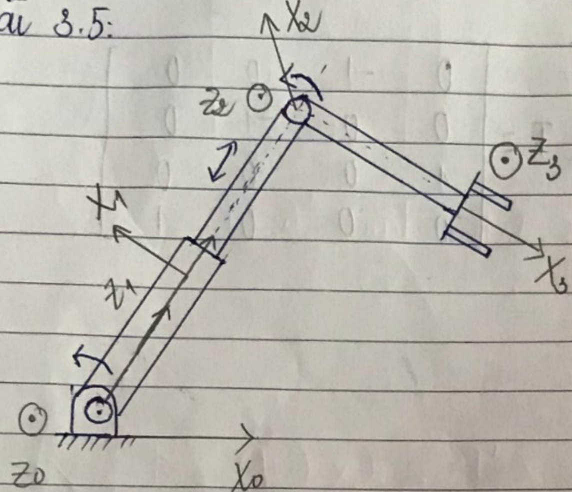
$$
{}^1_2T = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 0 & -1 & -l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^2_3T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_3T = {}^0_1T \cdot {}^1_2T \cdot {}^2_3T = \begin{bmatrix} -s\theta_1 & 0 & c\theta_1 & l_2 . c\theta_1 + l_3 . c\theta_1 - l_2 s\theta_1 \\ c\theta_1 & 0 & s\theta_1 & l_2 c\theta_1 + l_2 s\theta_1 + l_3 s\theta_1 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Bài 3.5:



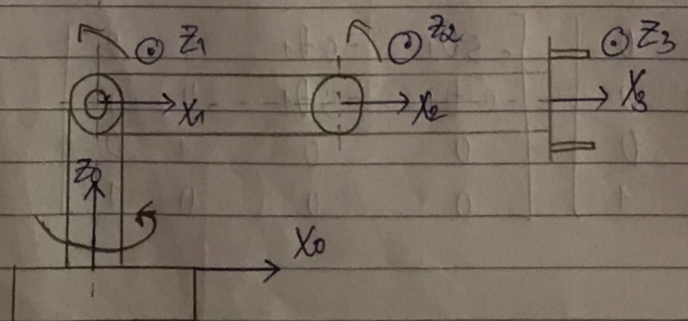
	α_{i-1}	a_{i-1}	d_i	θ_i
1	90°	0	0	θ_1
2	-90°	0	$l_1 + d_1$	0
3	0	l_2	0	θ_3

$$
{}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^1_2T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & l_1 + d_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & l_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_3T = {}^0_1T \cdot {}^1_2T \cdot {}^2_3T = \begin{bmatrix} c\theta_1 . c\theta_3 & -s\theta_1 . s\theta_3 & -s\theta_1 & l_2 . c\theta_1 - l_1 s\theta_1 - d_1 s\theta_1 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ s\theta_1 . c\theta_3 & s\theta_1 . s\theta_3 & c\theta_1 & (l_1 + d_1) c\theta_1 + l_2 s\theta_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Bài 3.6:



	α_{i-1}	a_{i-1}	d_i	θ_i
1	90	0	l_1	θ_1
2	0	l_2	0	θ_2
3	0	l_3	0	θ_3

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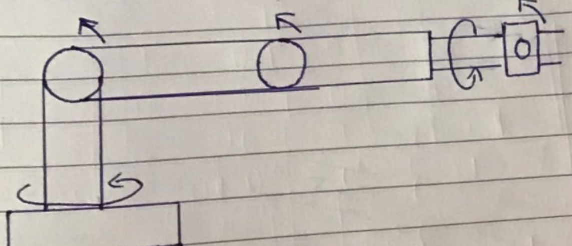
$$ {}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ 0 & 0 & -1 & -l_1 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_2 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

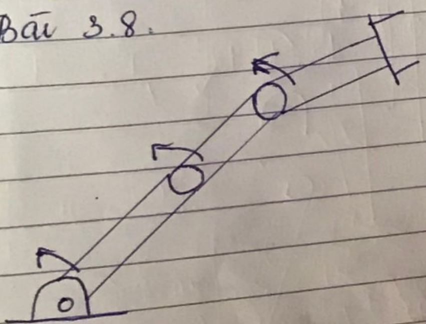
$$ {}^2_3T = \begin{bmatrix} c\theta_3 & s\theta_3 & 0 & l_3 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_3T = {}^0_1T \cdot {}^1_2T \cdot {}^2_3T = \begin{bmatrix} c(\theta_1+\theta_2+\theta_3) & -s(\theta_1+\theta_2-\theta_3) & 0 & l_2c\theta_1 + l_3c(\theta_1+\theta_2) \\ 0 & 0 & -1 & -l_1 \\ s(\theta_1+\theta_2+\theta_3) & c(\theta_1+\theta_2-\theta_3) & 0 & l_2s\theta_1 + l_3s(\theta_1+\theta_2) \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Bài 3.7:



Bài 3.8.



$$ {}^0_4T = \begin{bmatrix} s(\theta_1+\theta_2+\theta_3) & -s(\theta_1+\theta_2+\theta_3) & c(\theta_1+\theta_2+\theta_3) & l_1c\theta_1 + l_2c(\theta_1+\theta_2) + l_3c(\theta_1+\theta_2+\theta_3) \\ -c(\theta_1+\theta_2+\theta_3) & c(\theta_1+\theta_2+\theta_3) & s(\theta_1+\theta_2+\theta_3) & l_1s\theta_1 + l_2s(\theta_1+\theta_2) + l_3s(\theta_1+\theta_2+\theta_3) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Có $\begin{cases} P_x = l_1c\theta_1 + l_2c(\theta_1+\theta_2) + l_3c(\theta_1+\theta_2+\theta_3) \\ P_y = l_1s\theta_1 + l_2s(\theta_1+\theta_2) + l_3s(\theta_1+\theta_2+\theta_3) \\ P_z = 0 \end{cases}$

$$ P_x^2 + P_y^2 = l_1^2 + l_2^2 + l_3^2 + 2c\theta_2 l_1l_2 - 2c(\theta_2+\theta_3)l_1l_3 - 2c\theta_3 l_2l_3 $$

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