Preload-dependent power laws in detachment dynamics of probabilistic fasteners

Before detachment from an elastic fabric substrate occurs, mushroom arrays do not in general follow a linear force-strain relationship characterized by the "per mushroom" modulus E. The non-linearity of the force response as the mushrooms load the fibers is apparent in the raw force-distance curves presented in Figure 1. For preloads that exceed 20 N, there in no single slope that characterises the curves in the stage following the preload, but before the detachment process.

The non-linearity of the loading regime represents a preliminary challenge to the validity of fibre bundle models, which attempt to capture the microscopic features of the feature-fabric interaction. In a basic form, such a model would feature a linear loading regime $F = E\lambda$, followed by a failure regime $\exp(-\frac{E\lambda}{\sigma_0})^m$, with the latter term representing the survival probability of a feature at strain λ. Since our curves disobey linearity at low λ, we need to, first, explain the non-linearity, and, second, present a modified model.

An intuitively appealing explanation is the notion of *activation* – in the initial loading, the textile fibers are not stretched, and as such do not carry load. In other words, the initial deformation is "for free". We attempted to model activation by dividing the force by an activity coefficient $\xi = (1 - \exp -\frac{\lambda}{\lambda_m})^{-1}$. By the strain λ we under the non-dimensionalised gap size after correcting for the zero-force "loading" point $\lambda = \frac{d-d_0}{d_{\max}}$ and by λ_m is meant $\frac{L_m}{d_{\max}}$. L_m must then be a constant independent from preload and feature density, varied over a conservative range from 114 to 441 features per 25·25 mm^2. The presumed role of L_m as a geometric parameter is required by the fact that fibers are assumed to attach to only one feature, as such being

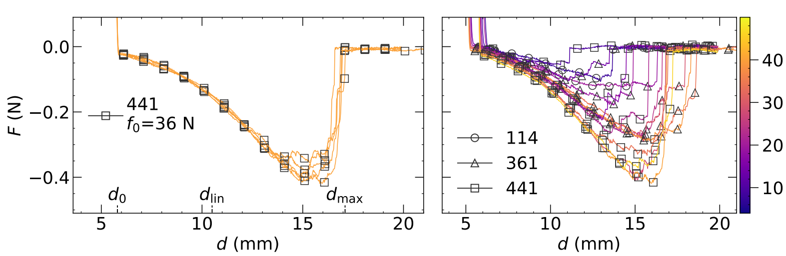

Figure 1: Raw force-distance curves of mushroom-patterned silicone rubber adhesive pads detaching from a textile substrate consisting of nylon fibers.

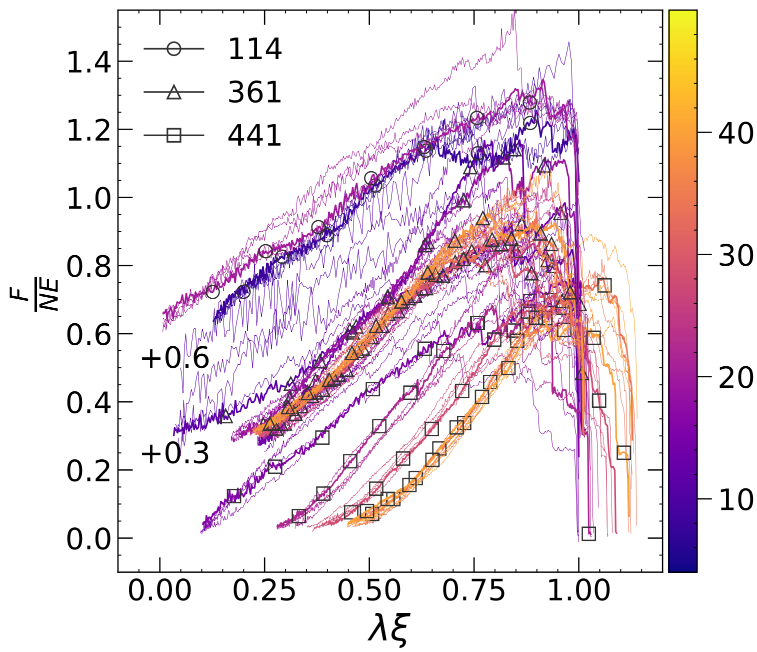

Figure 2: Force-distance curves taken from Figure 1 converted to force-activated strain curves. Strain is multiplied by the activation coefficient ξ. Force is normalized by E, which is the average slope obtained from linear regression of the unnormalized force-strain curves.

unable to perceive the mushroom densities.

We do not find evidence for the validity of activation in our data: from the fact that some force-distance curves *are* actually linear at low strain, it should be clear that we can not expect one L_m to take care of reducing all loading regimes to a linear load line. Thus, we attempted to collapse the data by varying L_m between 50 μm to 5 mm to achieve collapse. However, Figure 2, in which we multiply the x-axis by ξ, shows that even an activation parameter optimized *per curve* does not result in a model that describes the loading process.

Rather, the fiber stretching imposes a power law on the force-distance curve, which we show in Figure 3 as a straight line through on a plot of $\log \frac{f}{NA}$ against $\log \lambda$.

Subsequently, we re-fit all curves in Figure 2 to an equation which combines a power law loading regime, followed by the aforementioned probabilistic decay factor:

$$FN^{-1} = A\lambda^{\alpha} \exp -(\frac{E\lambda}{\sigma_0}^m) \tag{1}$$

Good fits to the data were accomplished with least squares optimization. We

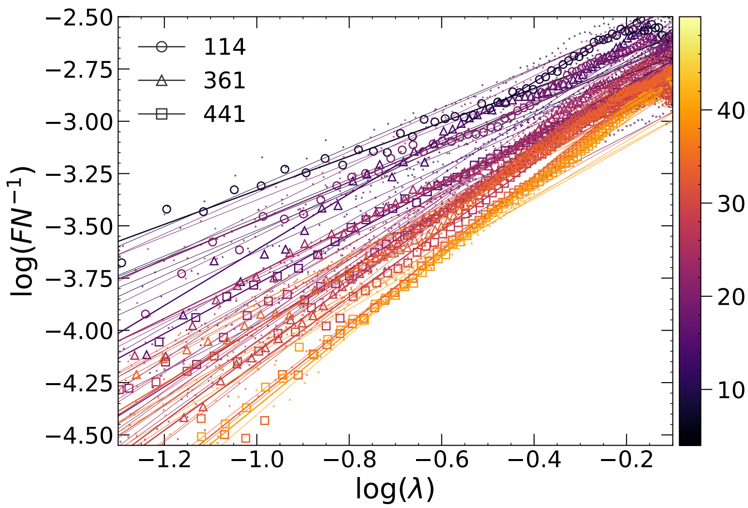

Figure 3: Log(force)-low(strain) curves with straight lines found with linear regression.

used the powers and pre-factors from Figure 3 to initialize the fits, and σ_0 and m were initialized with linear regression on one representative curve using a linearised version of Equation 1 with $\alpha = 1$, which was sufficient to find reasonable values for all curves. E was left fixed. Figure shows that fits describe the loading regime excellently.

The failure regime of arrays with small N $1 \cdot 10^2$ can not reasonably expected to follow $\exp -(\frac{E\lambda}{\sigma_0}^m)$, since the latter factor assumes a continues distribution of critical failure stresses, which requires $N \to \infty$. A result is a dramatic overestimation of m: we fit values as high as 20.

A surprising result of the present analysis is that power law α varies between 1 and $\frac{3}{2}$, as seen in Figure . We analyze the emergence of a power law with strength $\frac{3}{2}$ as strain hardening caused by the presence of entanglements between the fibers ("crosslinks"). Stretching of the fibers causes a concomitant tensing-up at the intersection points, resulting in a progressively stiff network of fibers as more strain is applied. The dependence of the power on preload can be seen as the presence of a critical active fiber density beneath which the formation of intersection points is unlikely. At low densities, we simply measure the (linear, at all low strains) elasticity of the nylon strands.

We note that strain hardening with a power law of strength $\frac{3}{2}$ is a common feature seen in crosslinked polymer networks of semi-flexible polymers.

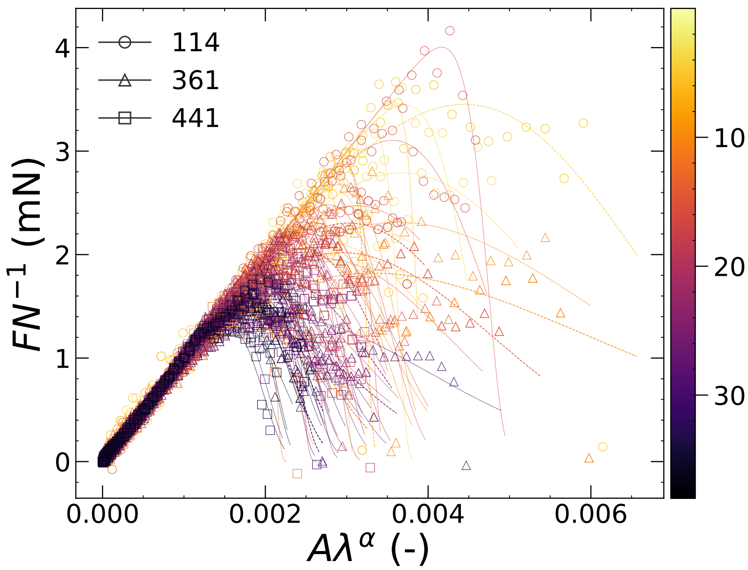

Figure 4: Fits of force-strain curves to Equation 1. The loading regime is successfully collapsed by inserting the power law λ^{α}.

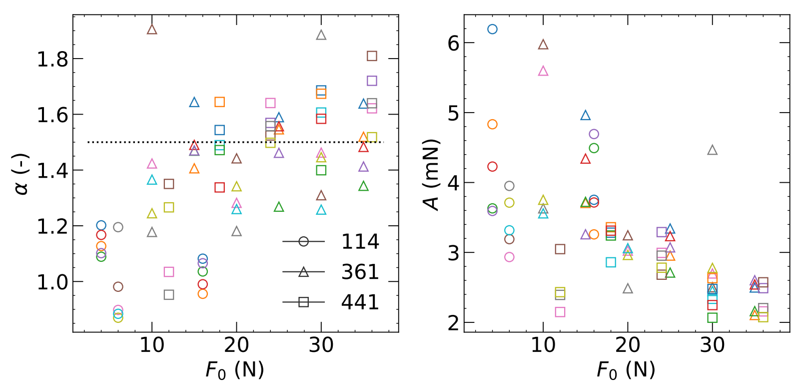

Figure 5: Fit parameters α and A as functions of preload F_0 as obtained from least squares optimization of force-strain data to Equation 1. The horizontal dashed line indicates the value $\frac{3}{2}$.