Cache-optimierte QR-Zerlegung

Bachelor Kolloquium

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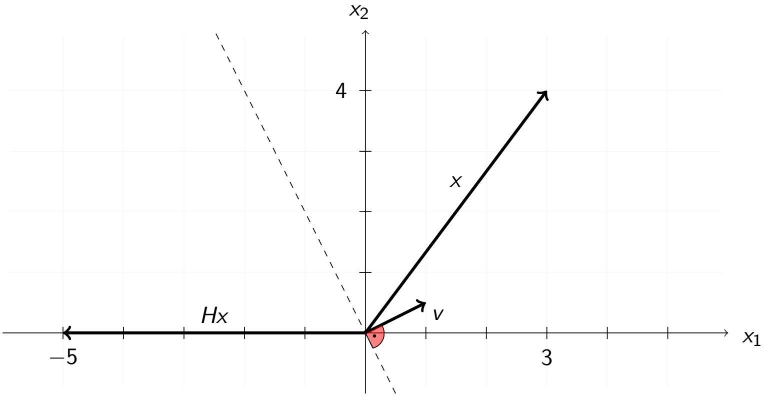
QR-Zerlegung

- $Ax = b$
- $A = QR$
- $QRx = b \quad \Longleftrightarrow Rx = Q^T b$

Householder-Transformation

$$H = I - 2\frac{vv^T}{v^T v}$$



Householder-Transformation

- Householder Vektor berechnen
  - Ansatz $Hx = \alpha e_1$
  - Normieren $v_1 = 1$
  - $\tau = \dfrac{2}{v^T v} \quad \Longrightarrow \quad H = I - 2\dfrac{vv^T}{v^T v} = I - \tau vv^T$
- Householder-Transformation anwenden

$$HA = (I - \tau vv^T)A = A - \tau(vv^T)A = A - \tau v(v^T A)$$

QR-Zerlegung mittels Householder

- $A = QR$

$$H_1 A = \begin{pmatrix} * & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{pmatrix}$$

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$$H_1 = \left(\hat{H}_1\right) \quad , \quad H_2 = \left(\begin{array}{c|c} I_1 & 0 \\ \hline 0 & \hat{H}_2 \end{array}\right) \quad , \quad H_i = \left(\begin{array}{c|c} I_{i-1} & 0 \\ \hline 0 & \hat{H}_i \end{array}\right)$$

QR-Zerlegung mittels Householder

- $A = QR$

$$H_2 H_1 A = \begin{pmatrix} * & * & * & * \\ 0 & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & * & * \end{pmatrix}$$

-

$$H_1 = \left(\hat{H}_1\right) \quad , \quad H_2 = \left(\begin{array}{c|c} I_1 & 0 \\ \hline 0 & \hat{H}_2 \end{array}\right) \quad , \quad H_i = \left(\begin{array}{c|c} I_{i-1} & 0 \\ \hline 0 & \hat{H}_i \end{array}\right)$$

QR-Zerlegung mittels Householder

- $A = QR$

$$H_3 H_2 H_1 A = \begin{pmatrix} * & * & * & * \\ 0 & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \end{pmatrix}$$

-

$$H_1 = \left(\hat{H}_1\right) \quad , \quad H_2 = \left(\begin{array}{c|c} I_1 & 0 \\ \hline 0 & \hat{H}_2 \end{array}\right) \quad , \quad H_i = \left(\begin{array}{c|c} I_{i-1} & 0 \\ \hline 0 & \hat{H}_i \end{array}\right)$$

QR-Zerlegung mittels Householder

- $A = QR$

$$H_3 H_2 H_1 A = \begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & r_{1,4} \\ v_2^{(1)} & r_{2,2} & r_{2,3} & r_{2,4} \\ v_3^{(1)} & v_3^{(2)} & r_{3,3} & r_{3,4} \\ v_4^{(1)} & v_4^{(2)} & v_4^{(3)} & r_{4,4} \end{pmatrix}$$

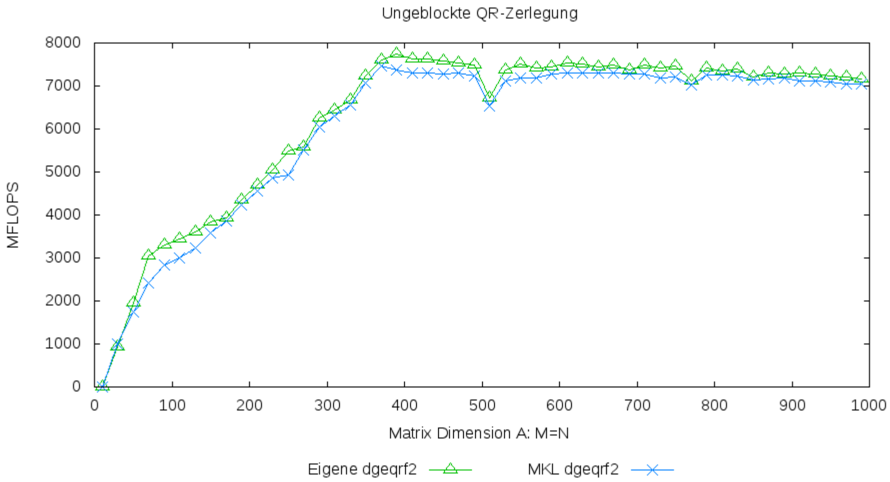
-

$$H_1 = \left(\hat{H}_1\right) \quad , \quad H_2 = \left(\begin{array}{c|c} I_1 & 0 \\ \hline 0 & \hat{H}_2 \end{array}\right) \quad , \quad H_i = \left(\begin{array}{c|c} I_{i-1} & 0 \\ \hline 0 & \hat{H}_i \end{array}\right)$$

Benchmark

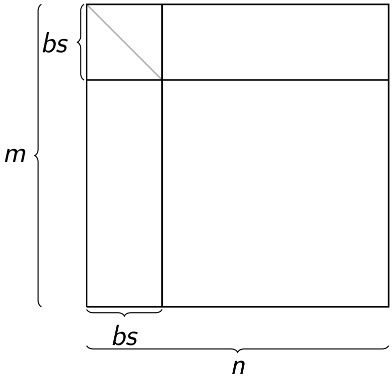
- Rechner
- Peak performance
- Flops
- Aufwand QR Householder

Ungeblockte QR



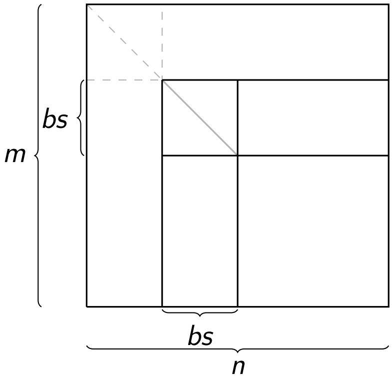
Geblockte QR-Zerlegung

- Matrix A



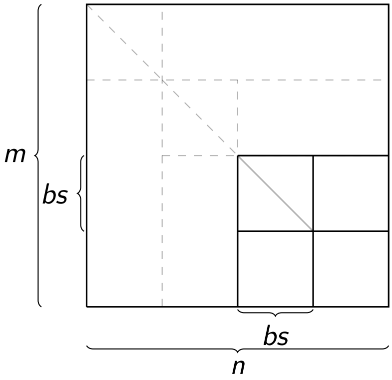
Geblockte QR-Zerlegung

- Matrix A



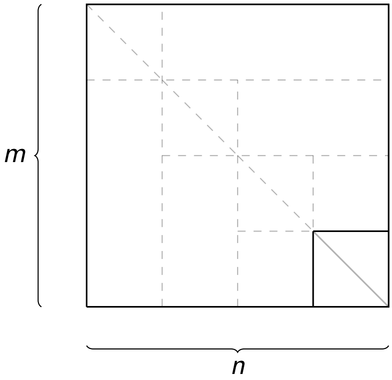
Geblockte QR-Zerlegung

- Matrix A



Geblockte QR-Zerlegung

- Matrix A



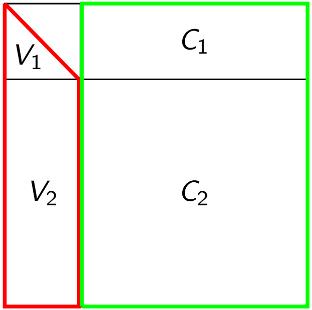
Mehrere Householder-Transformationen anwenden

- Ansatz

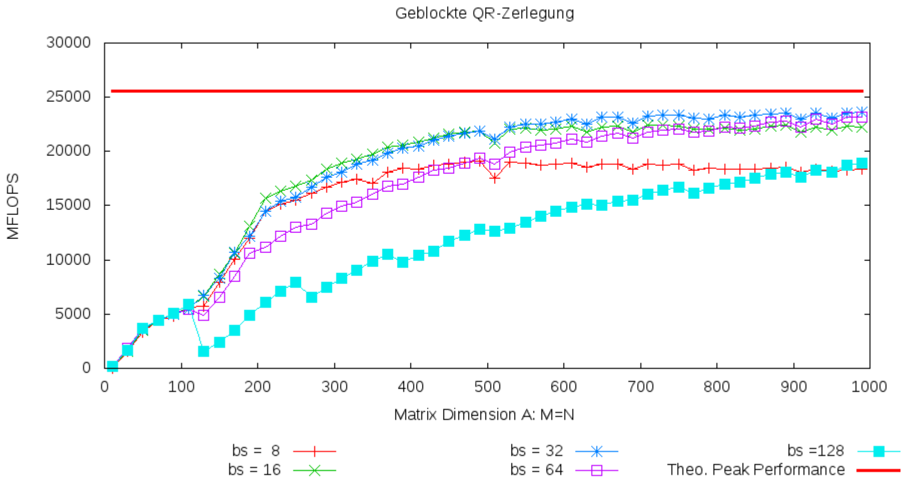
  $$\hat{H} = H_1 H_2 ... H_k = I - VTV^T \quad \text{mit} \quad H_i = I - \tau_i v_i v_i^T$$

- Householder-Transformationen anwenden

  $$C \leftarrow \hat{H} C = C - VTV^T C$$



Verschiedene Blockgrößen



Geblockte QR - Blocksizes