FlowVB: Automatic determination of the number of mixture components in Flow Cytometry with Variational Bayes

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Motivation: GMMs for clustering

Starting point:

Gaussian Mixture Models (GMMS) are a popular method for clustering data

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- Aware of covariance-structure
- Mathematically convenient
- Can be fit (relatively) easily with the *Expectation-Maximization* (EM) Algorithm

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We are trying to solve these two problems.

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But:

Closed form solutions don't exist for all parameters of the distribution

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- Therefore optimizing over M is an degenerate (ill-posed) problem

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Task

Develop an algorithm to fit SMMs using Variational Bayes

Demo - Robustness to noise

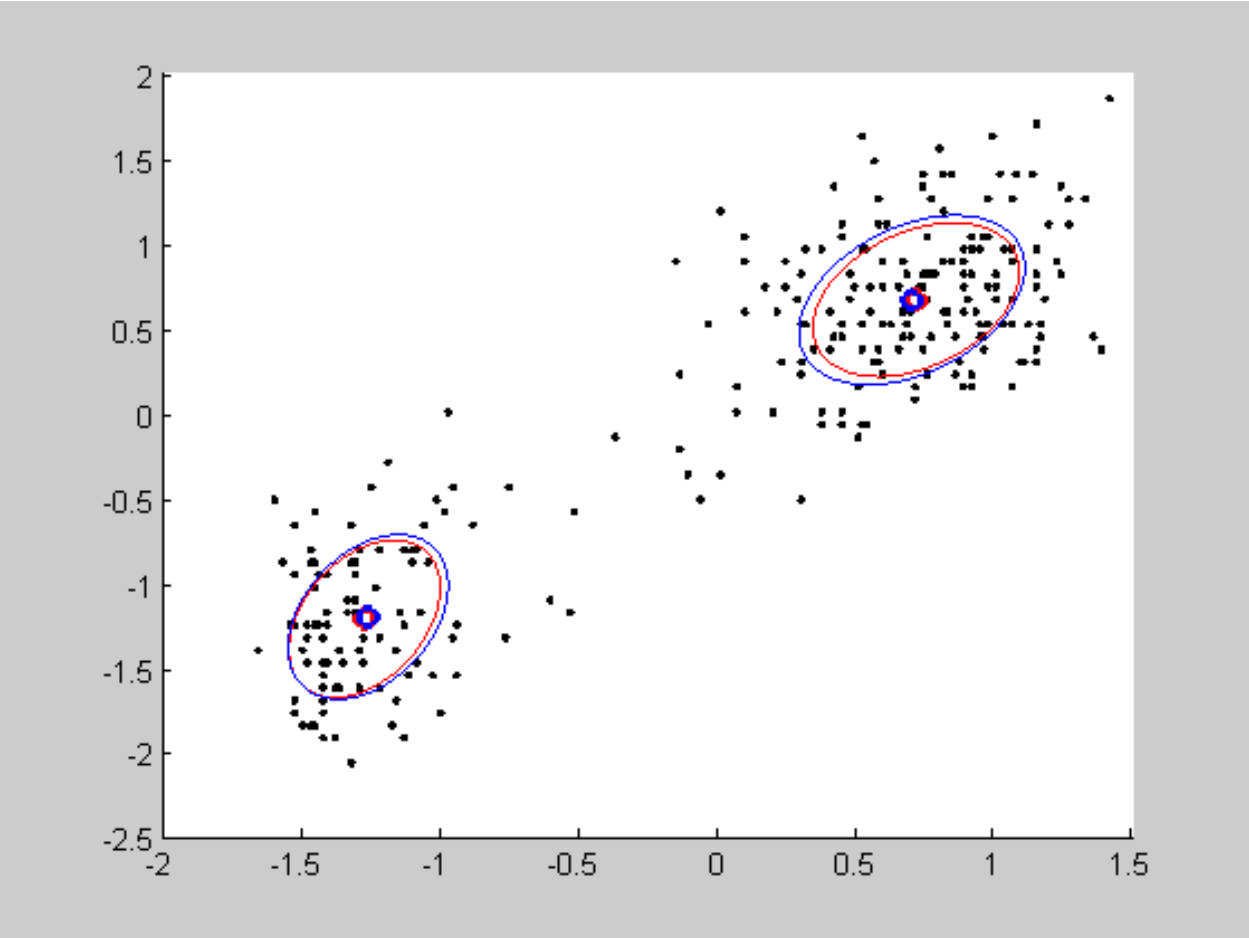


Figure: GMM (blue) vs. SMM(red) fit with VB without added noise

Demo - Robustness to noise

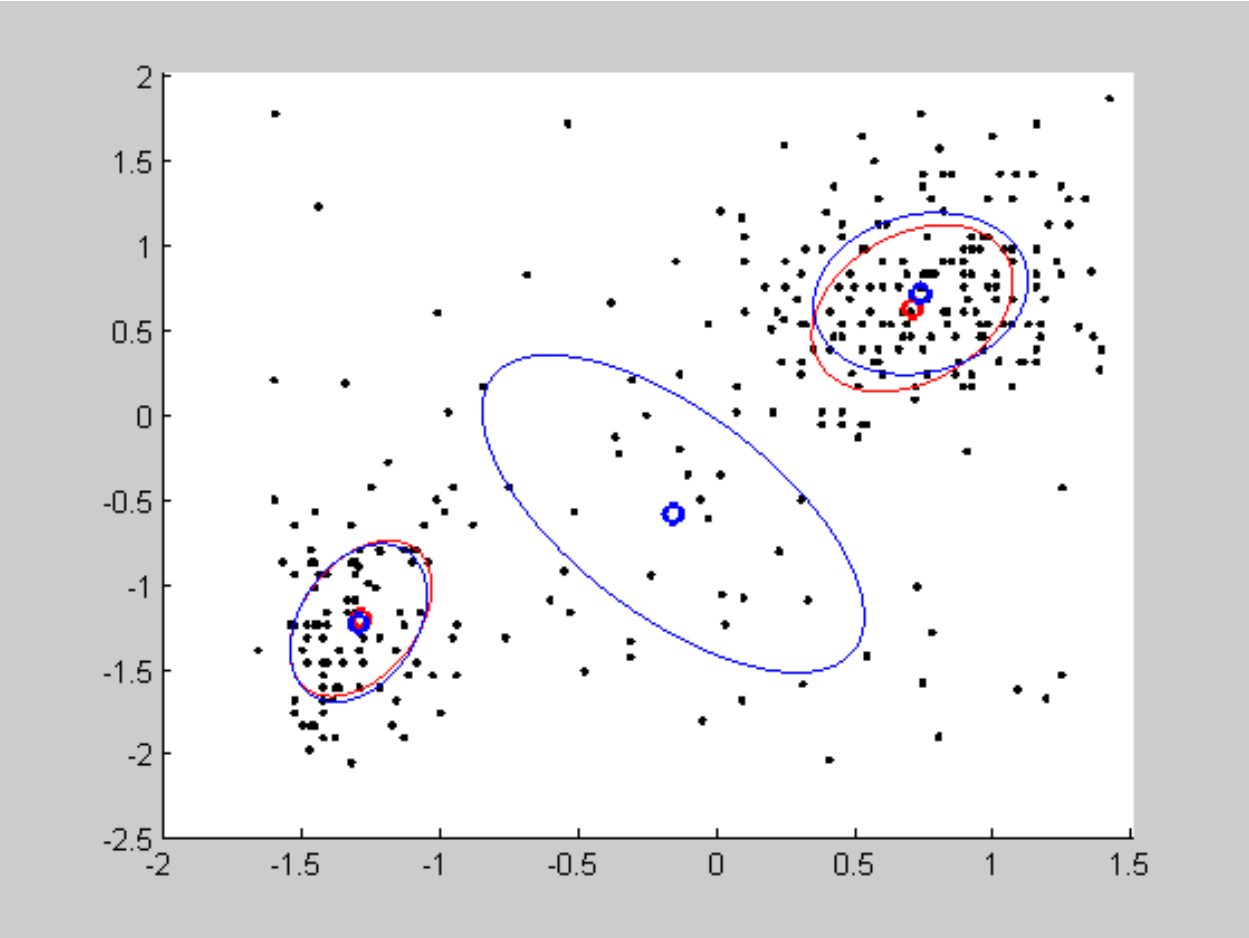
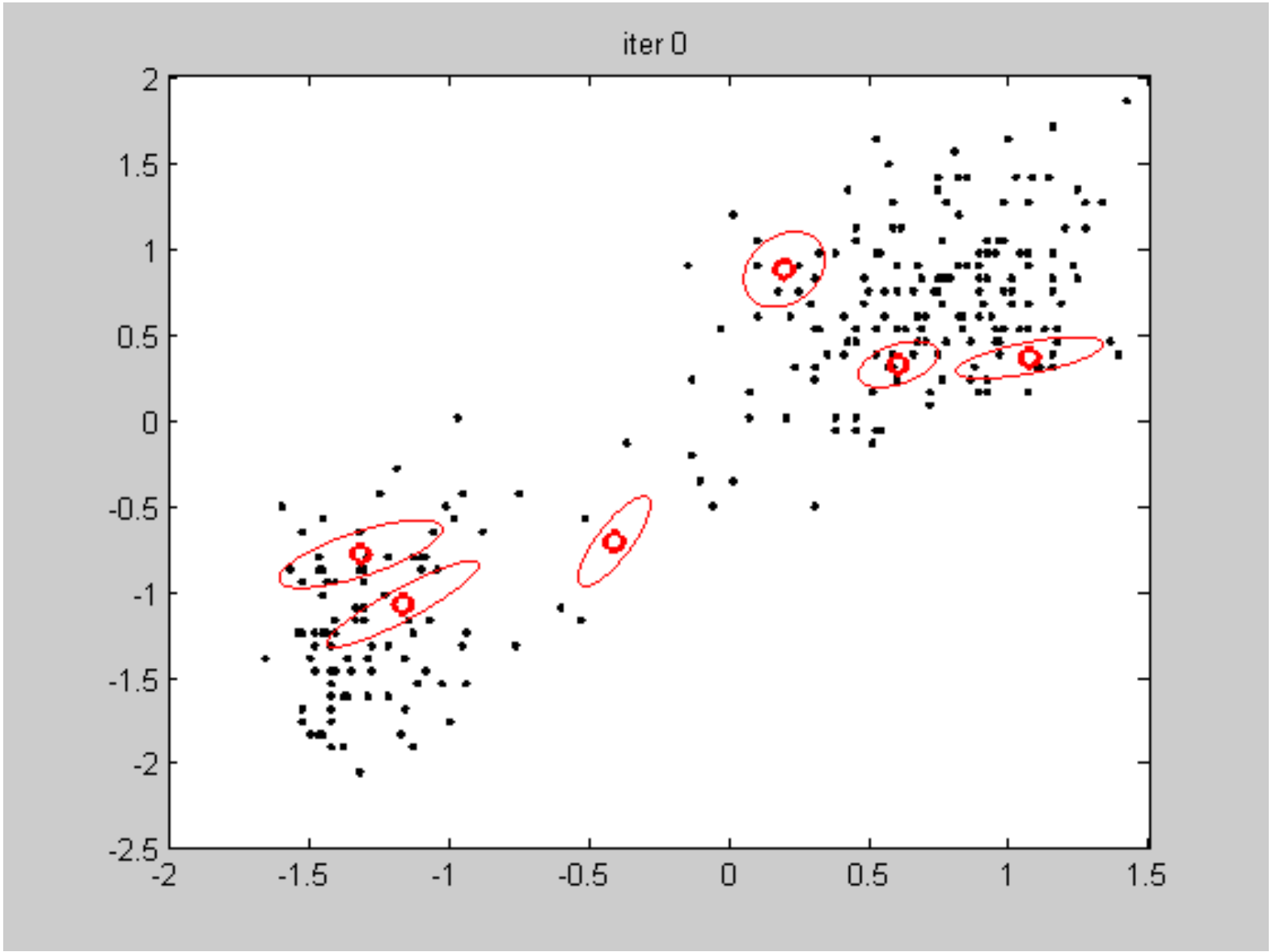


Figure: GMM (blue) vs. SMM(red) fit with VB with 20% added noise

Demo - Automatic pruning of components



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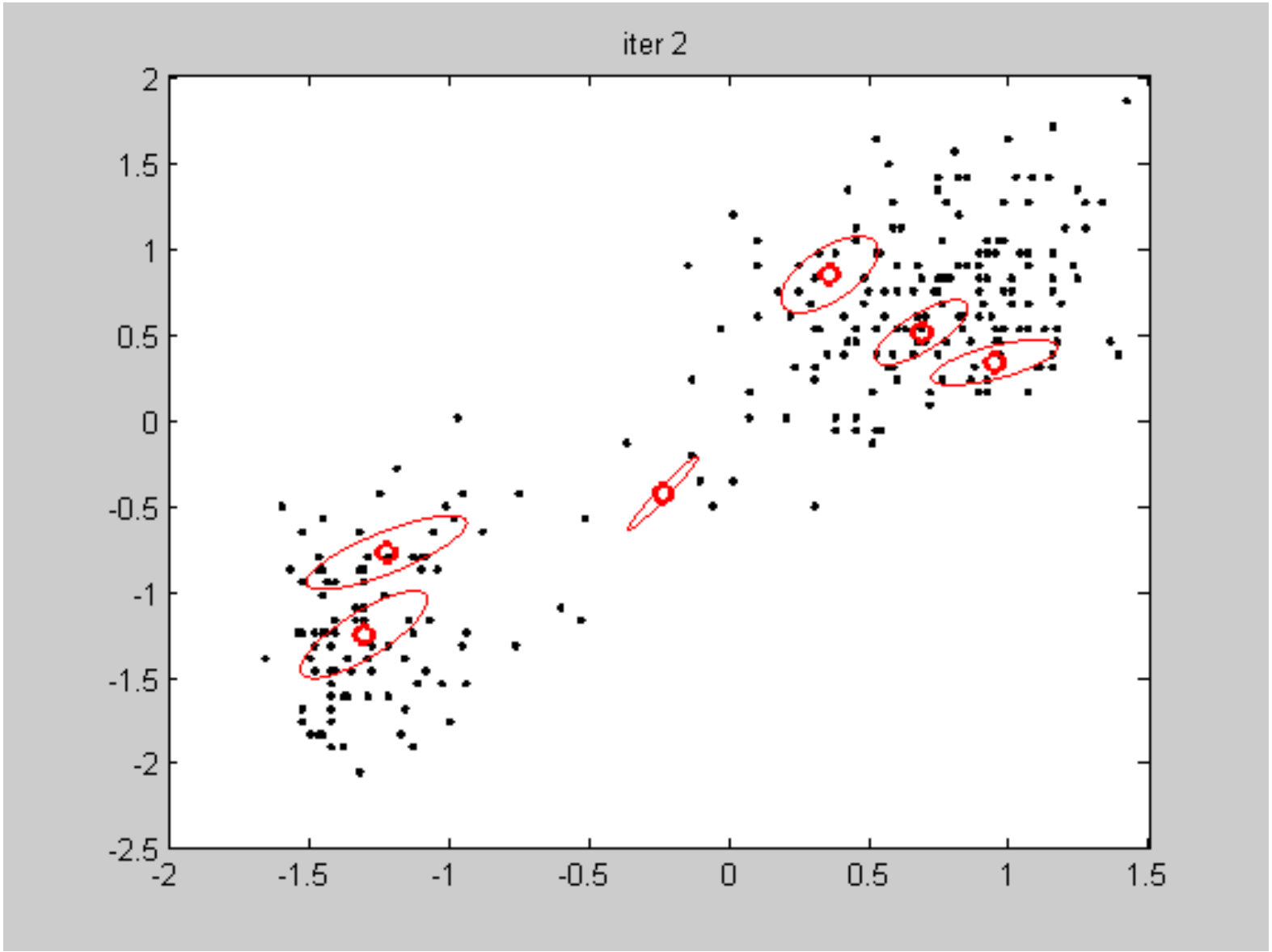

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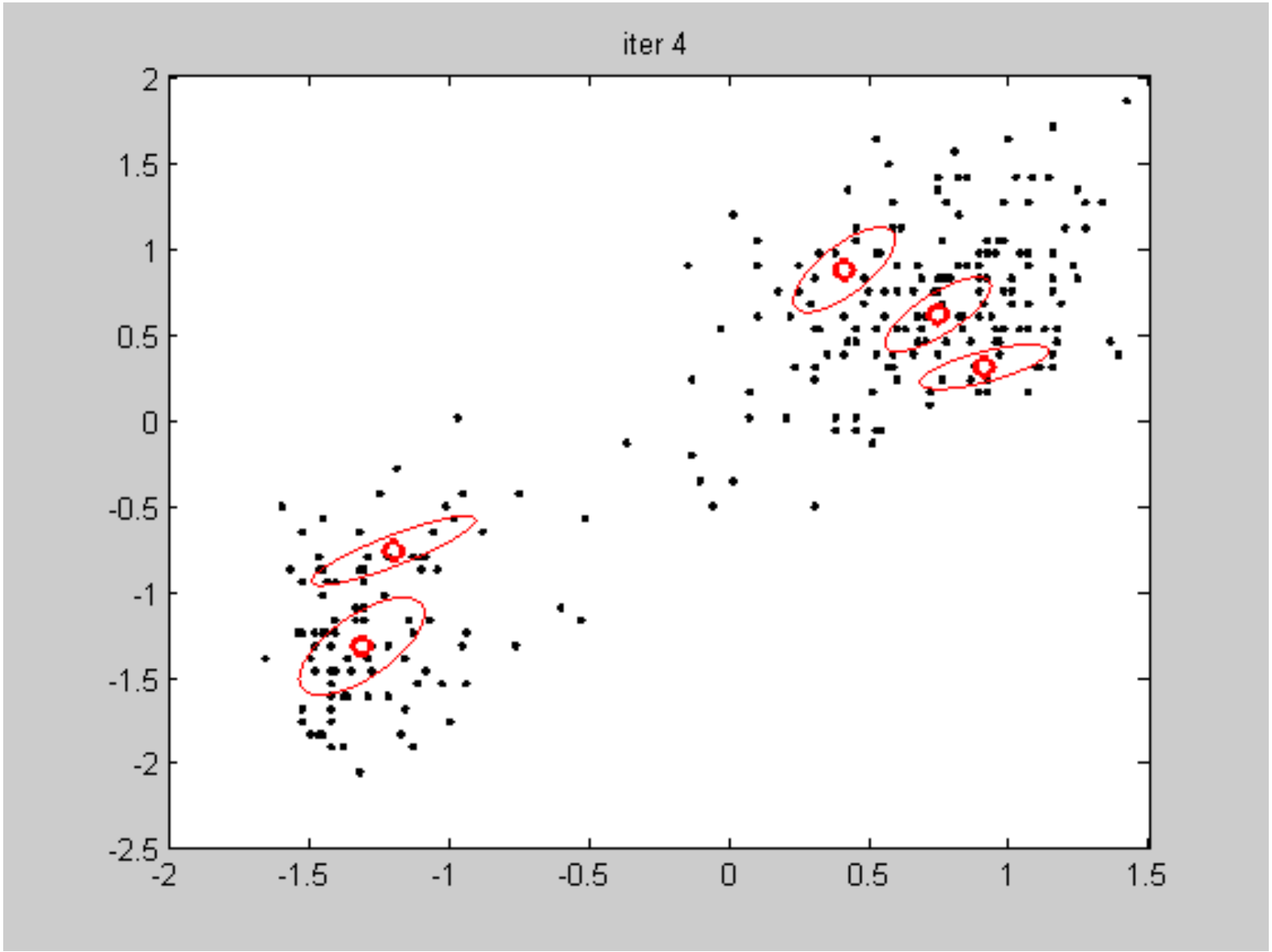

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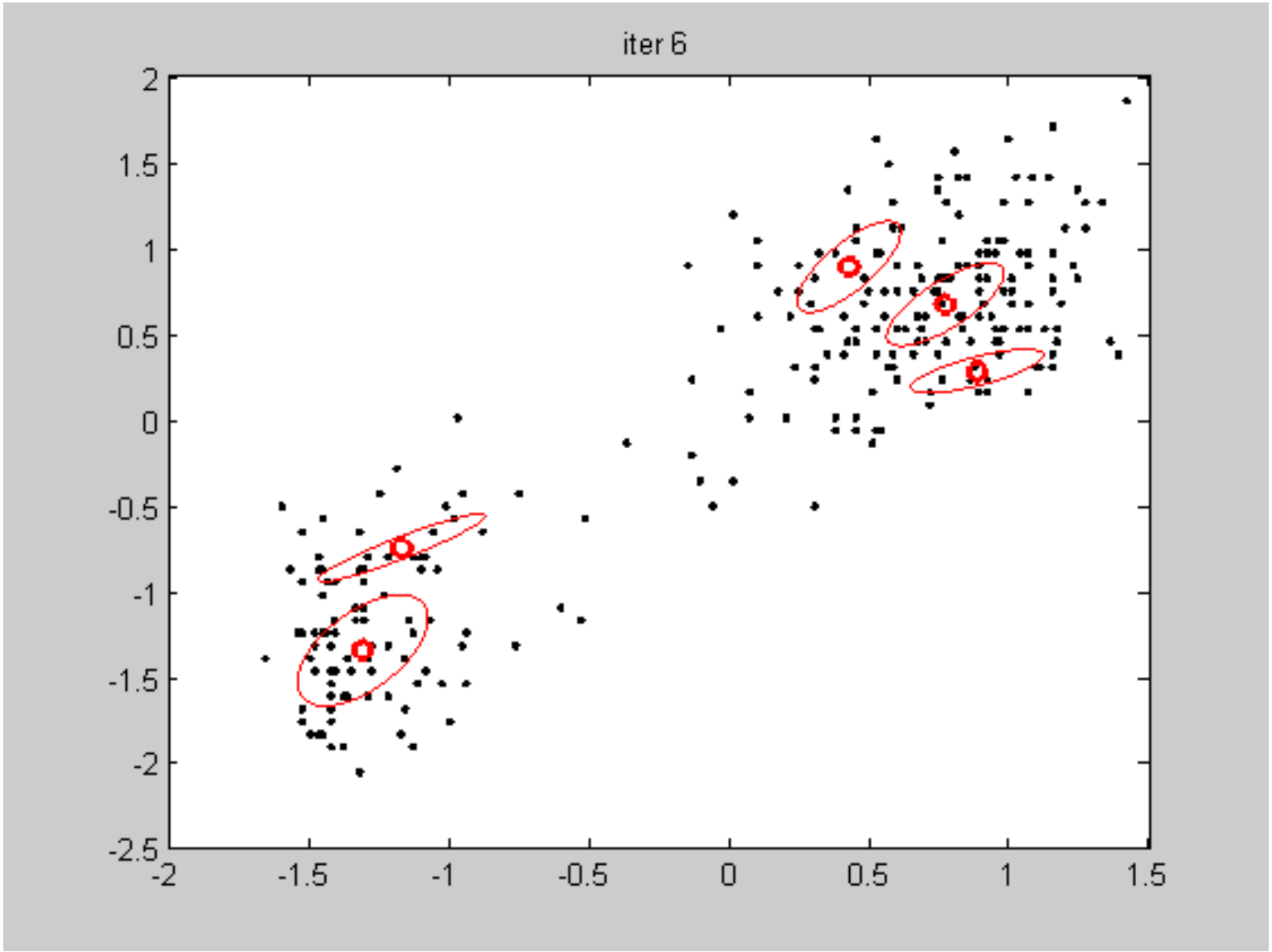

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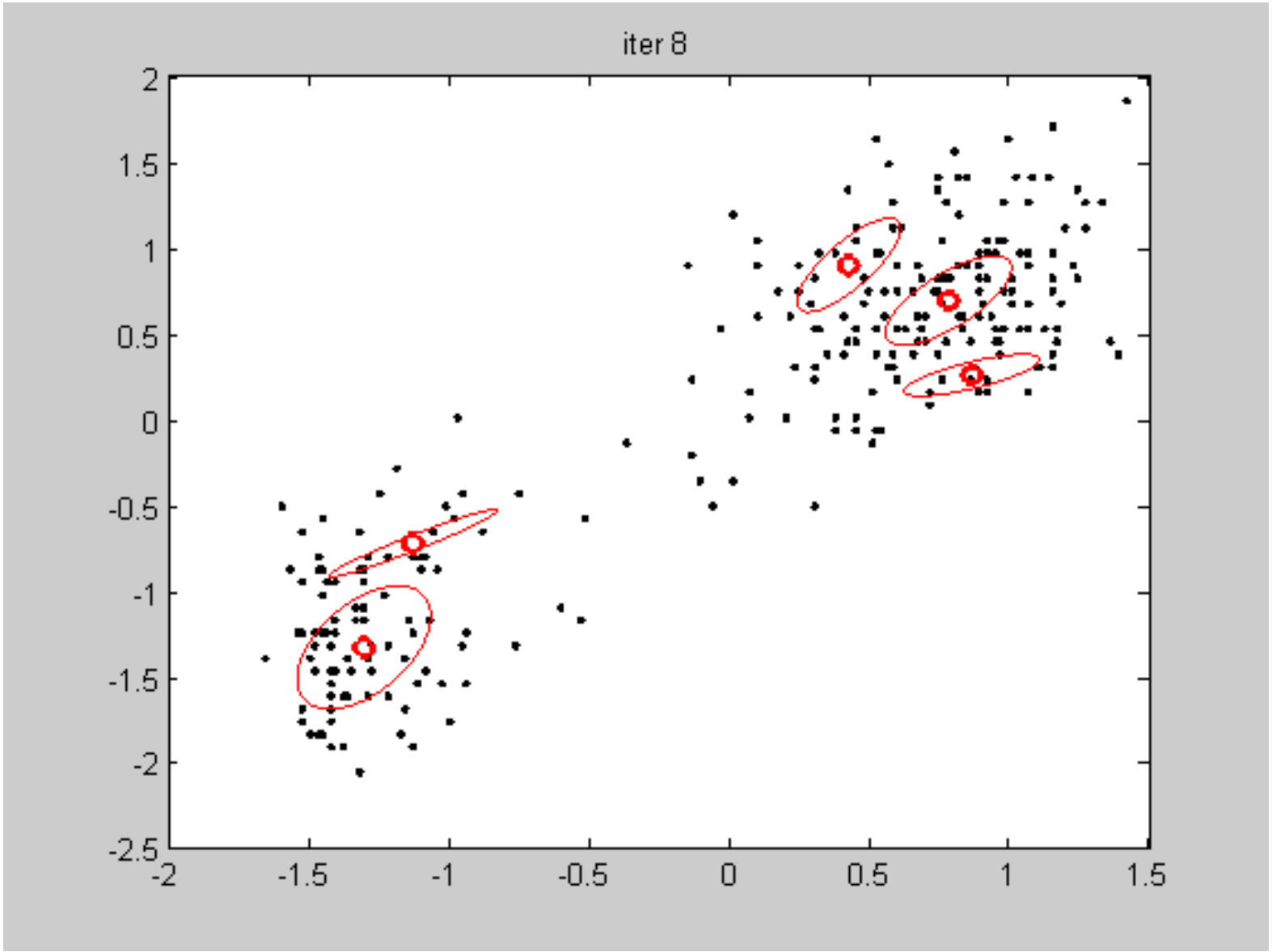

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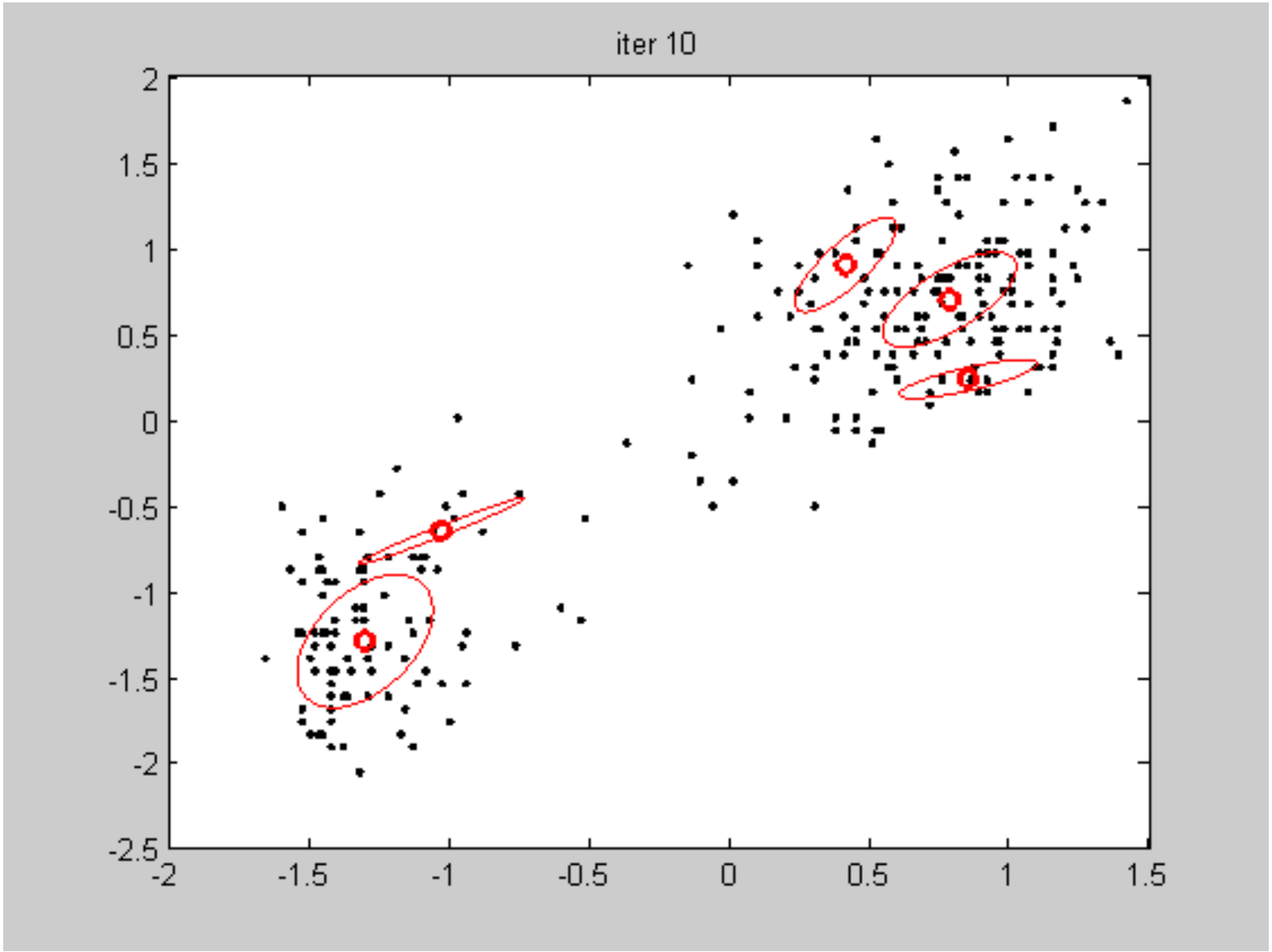

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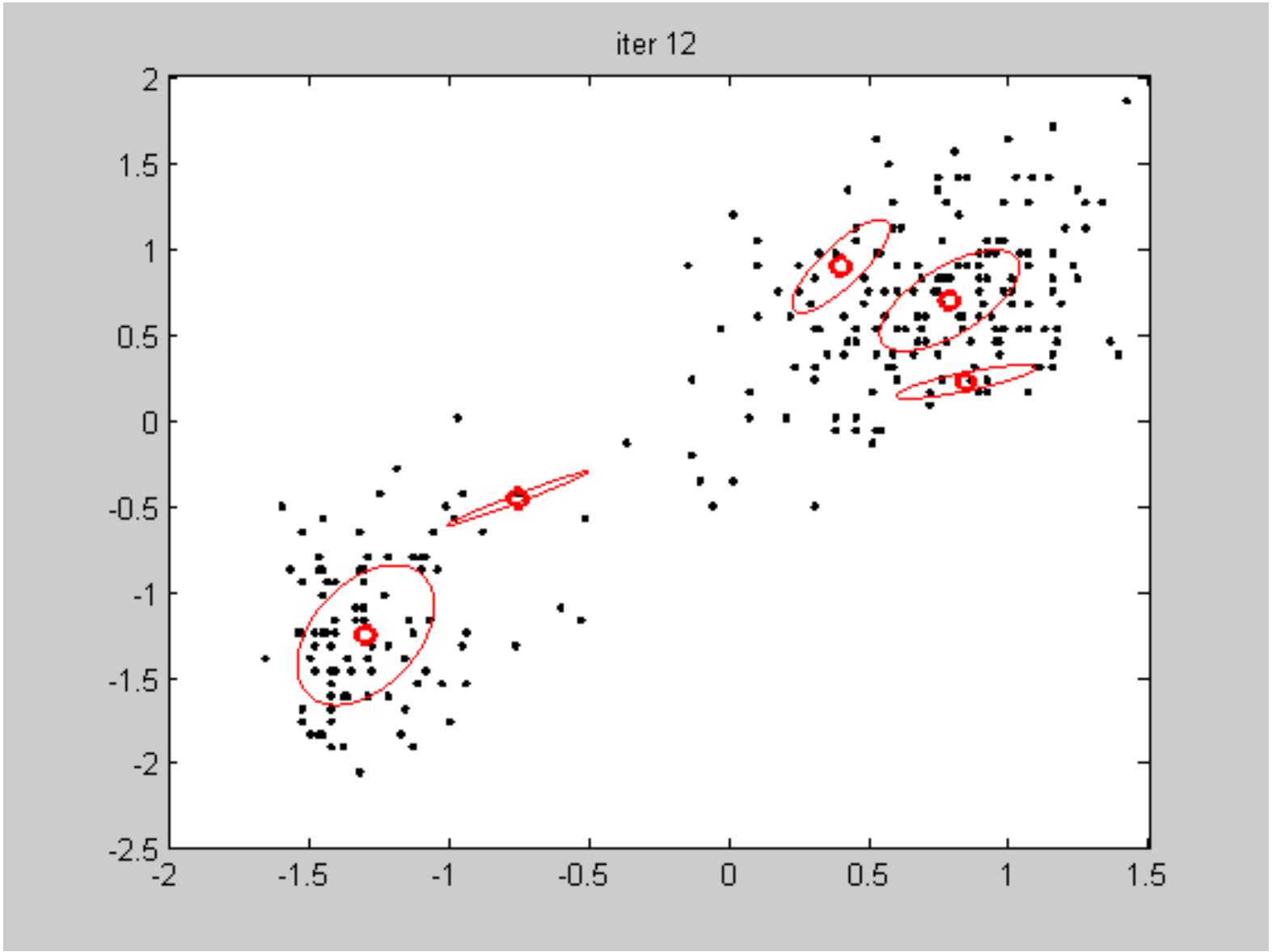

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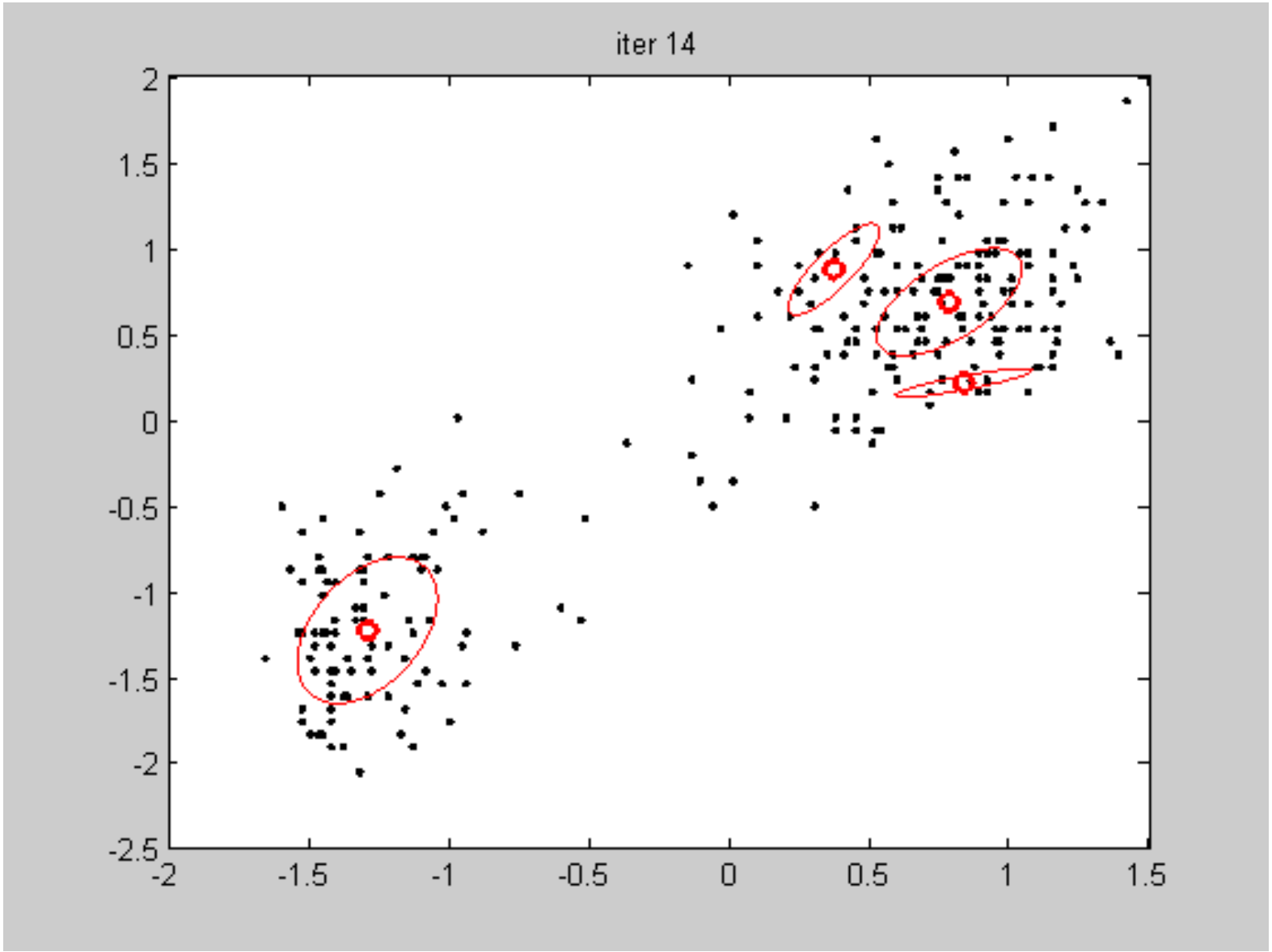

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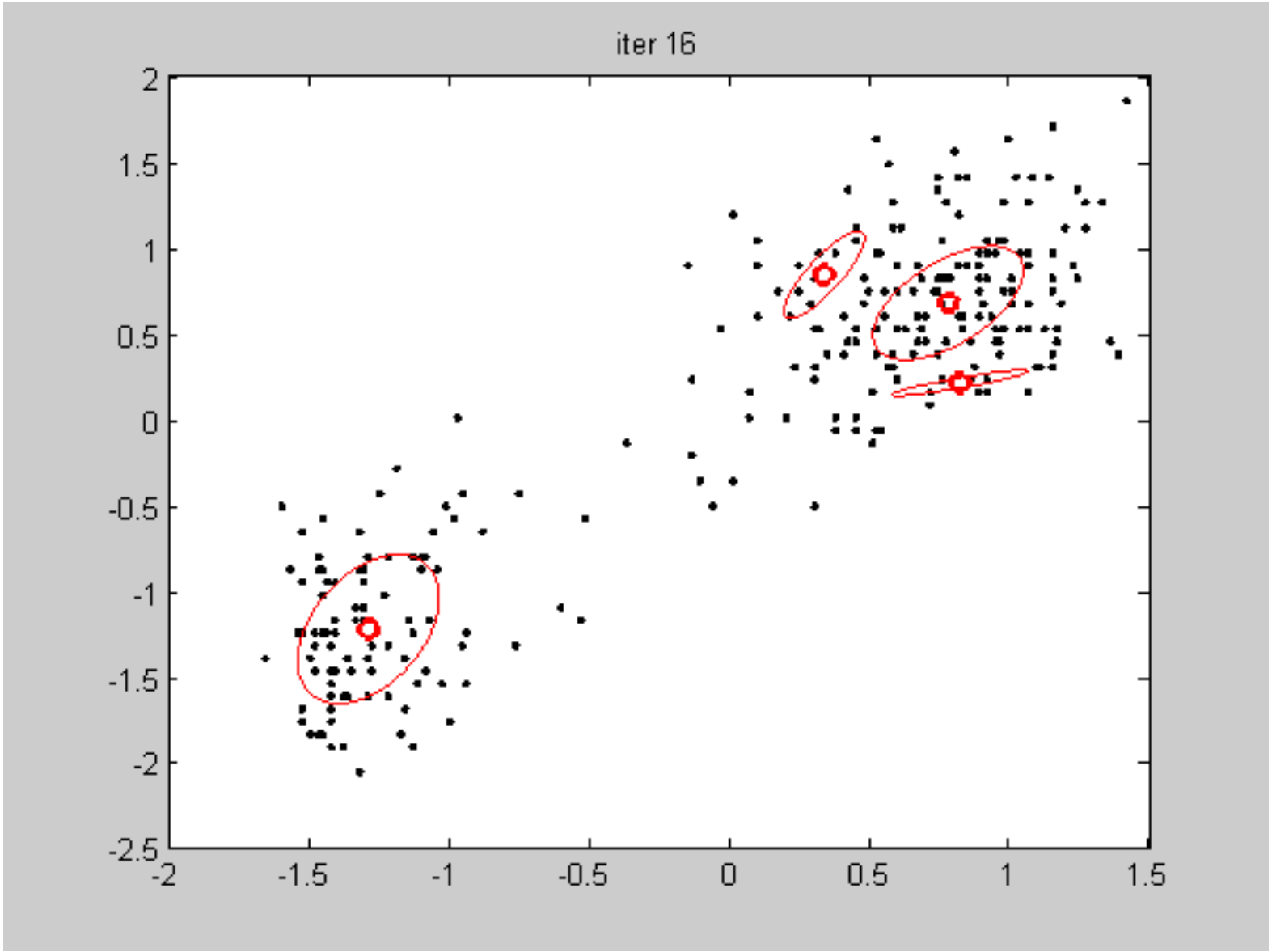

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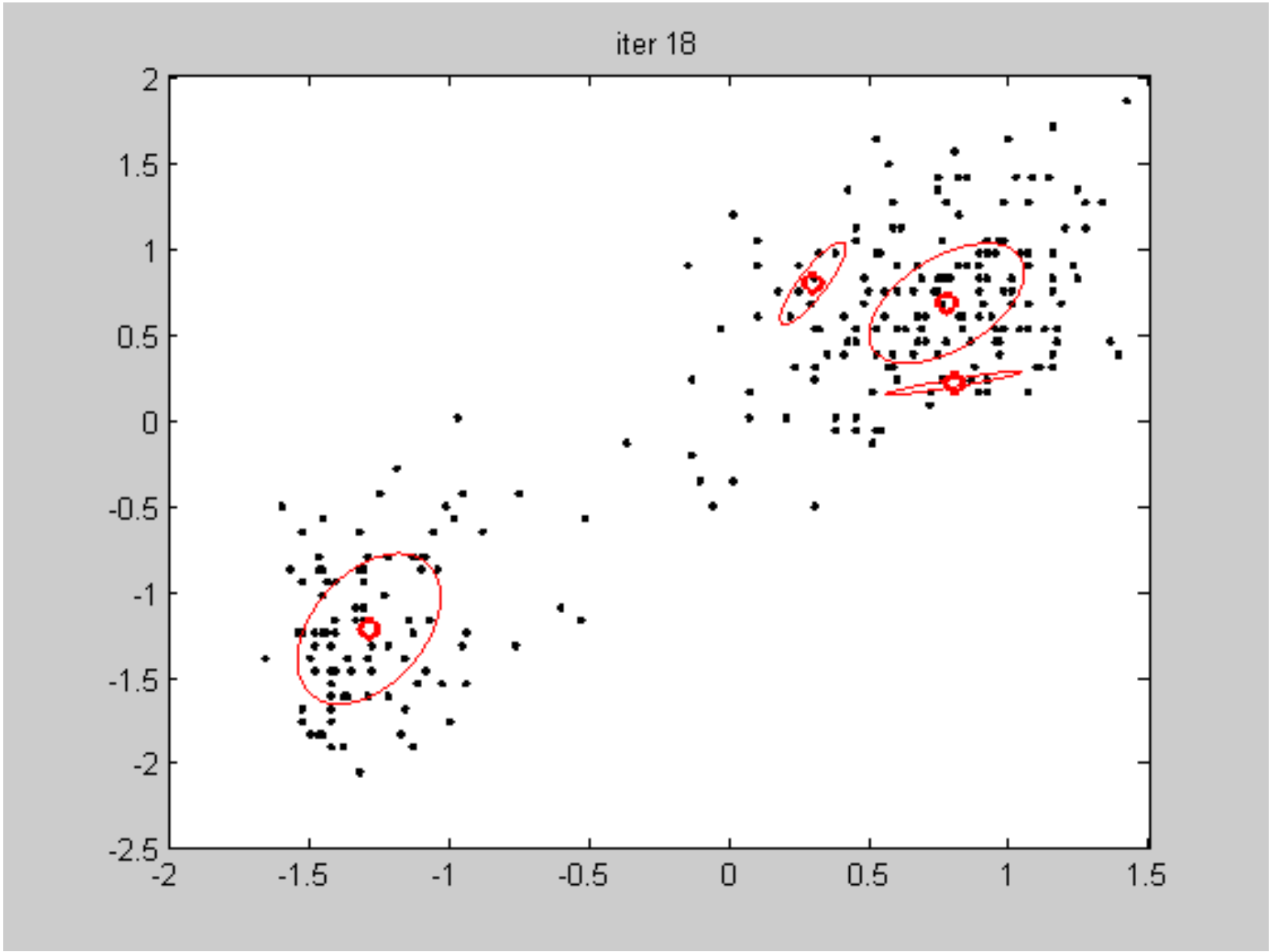

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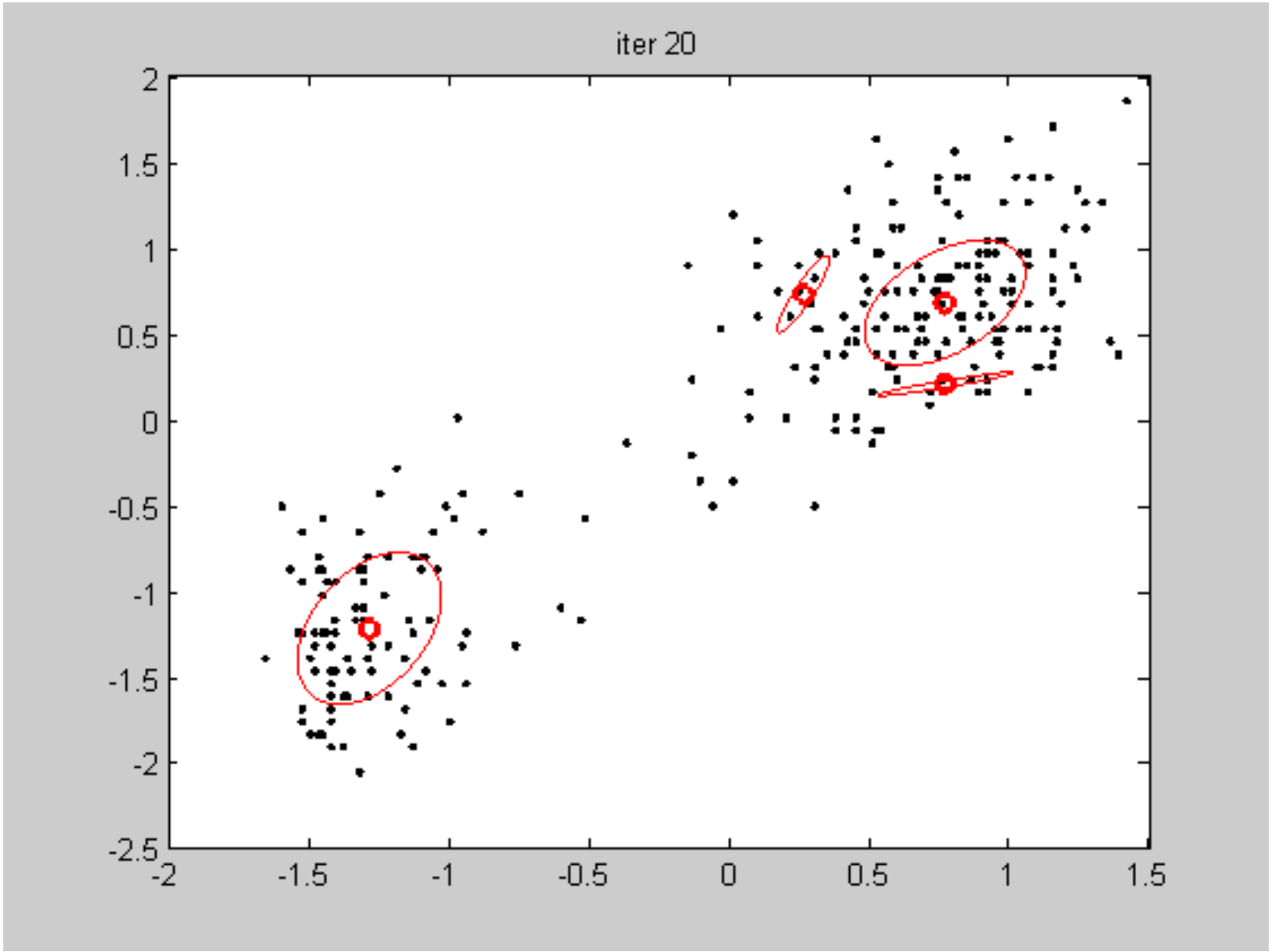

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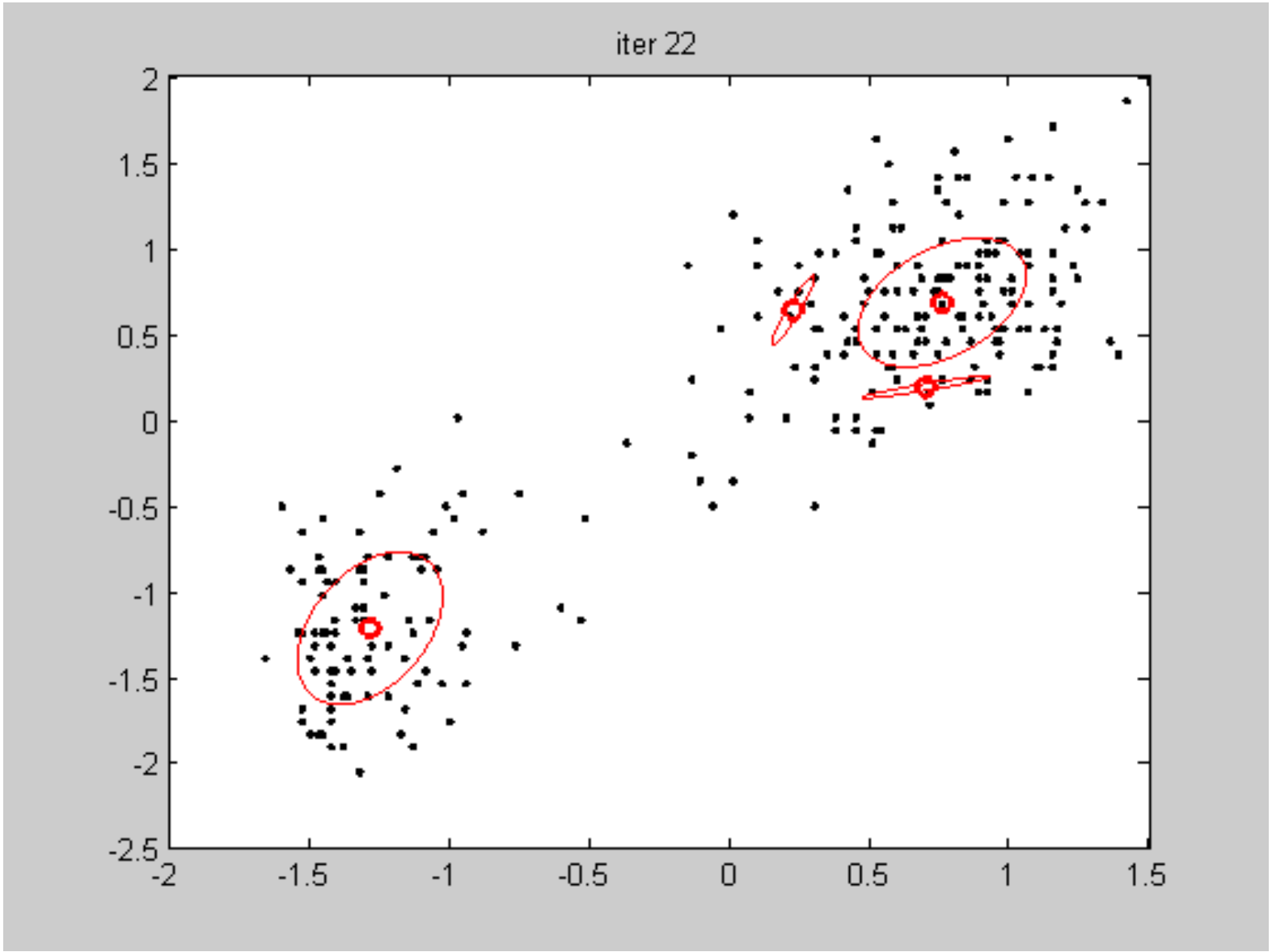

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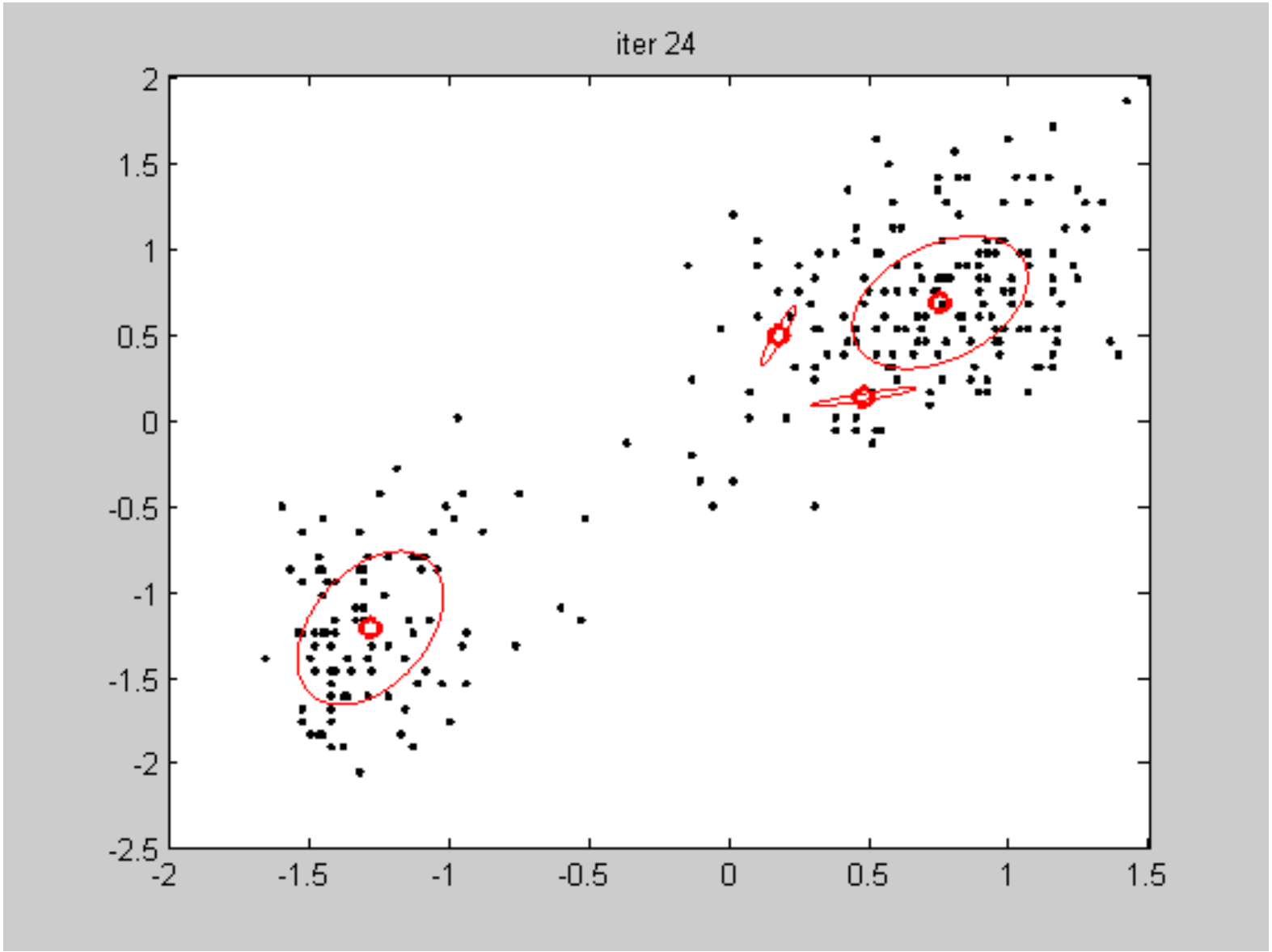

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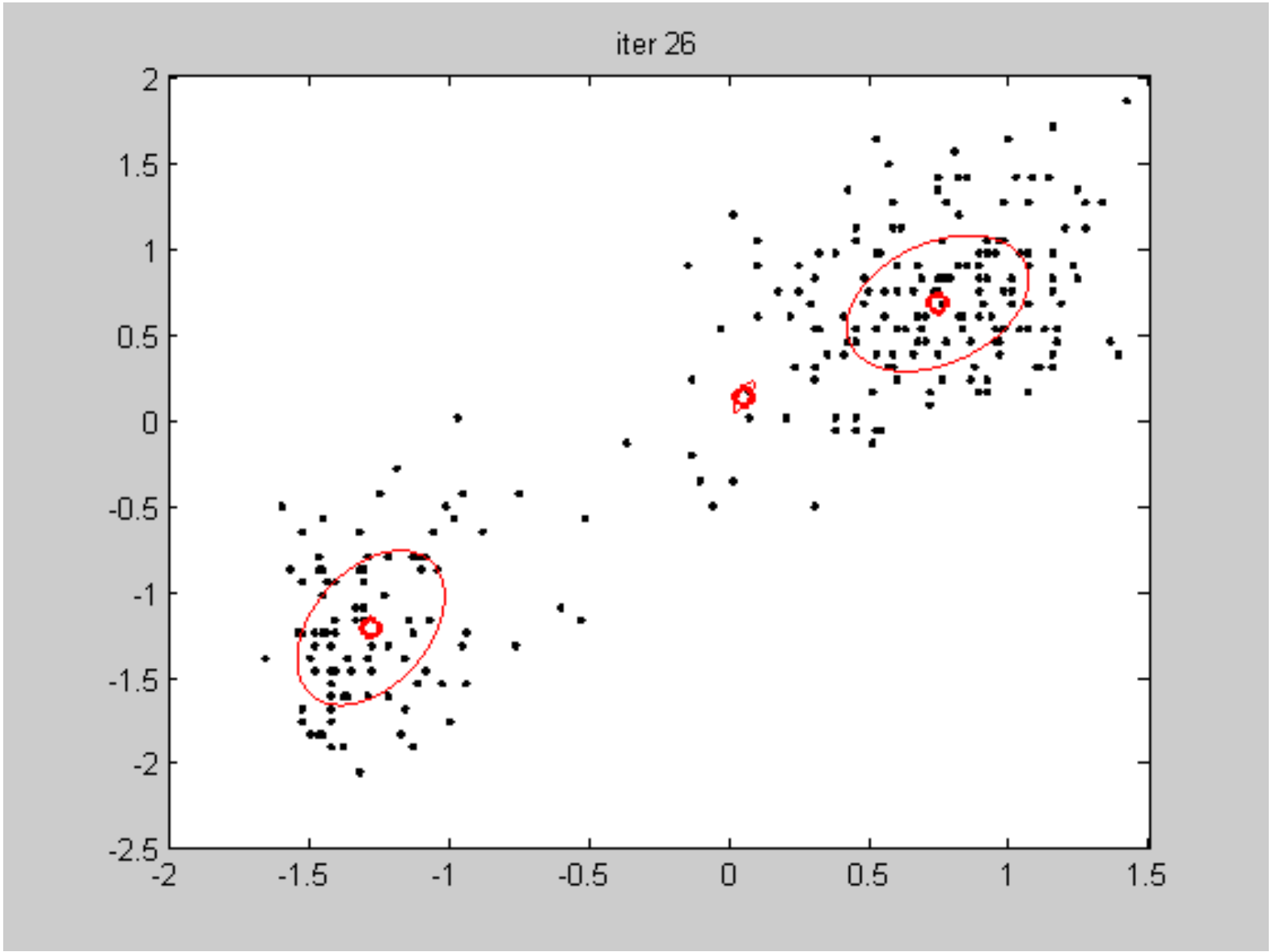

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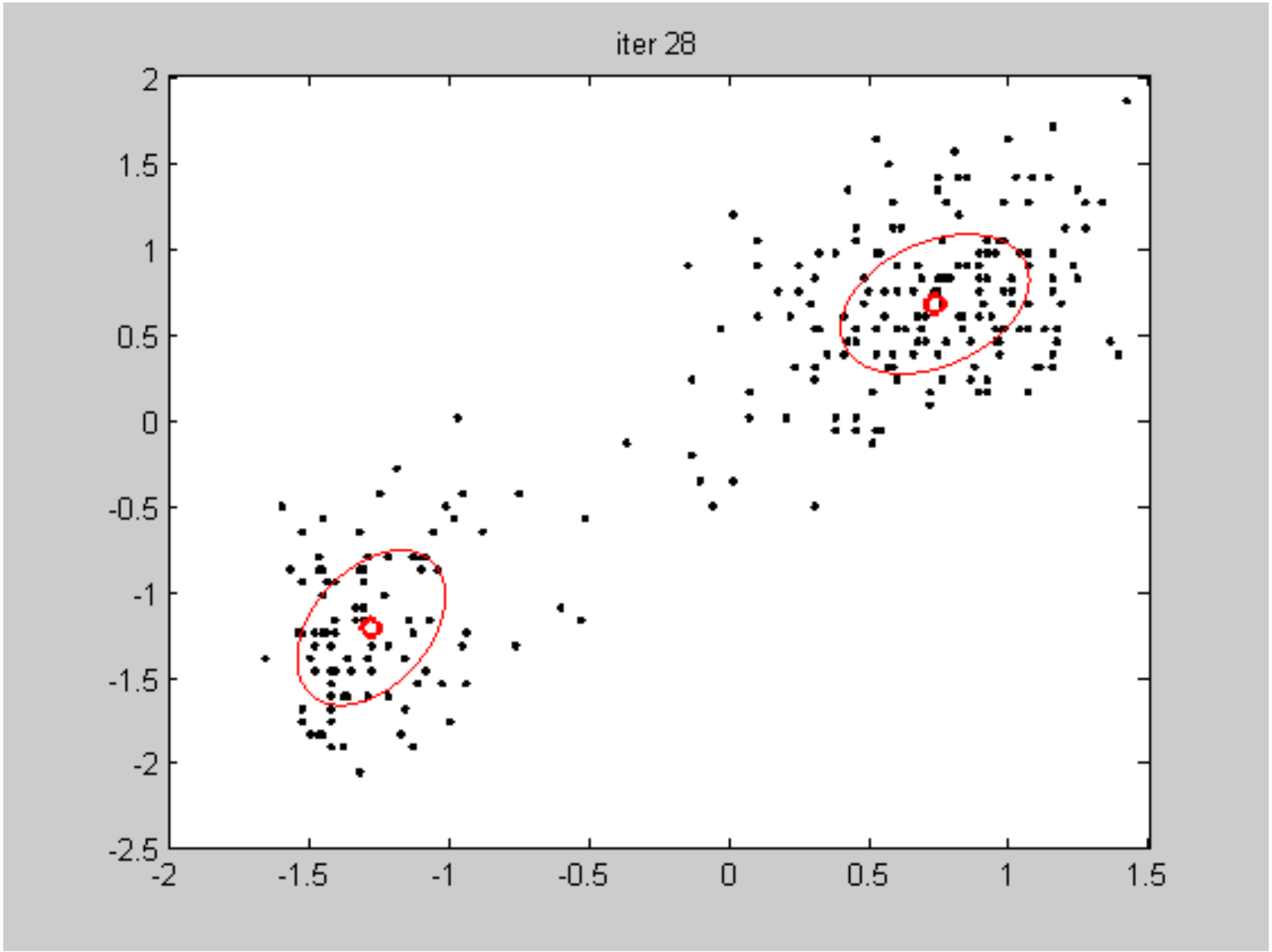

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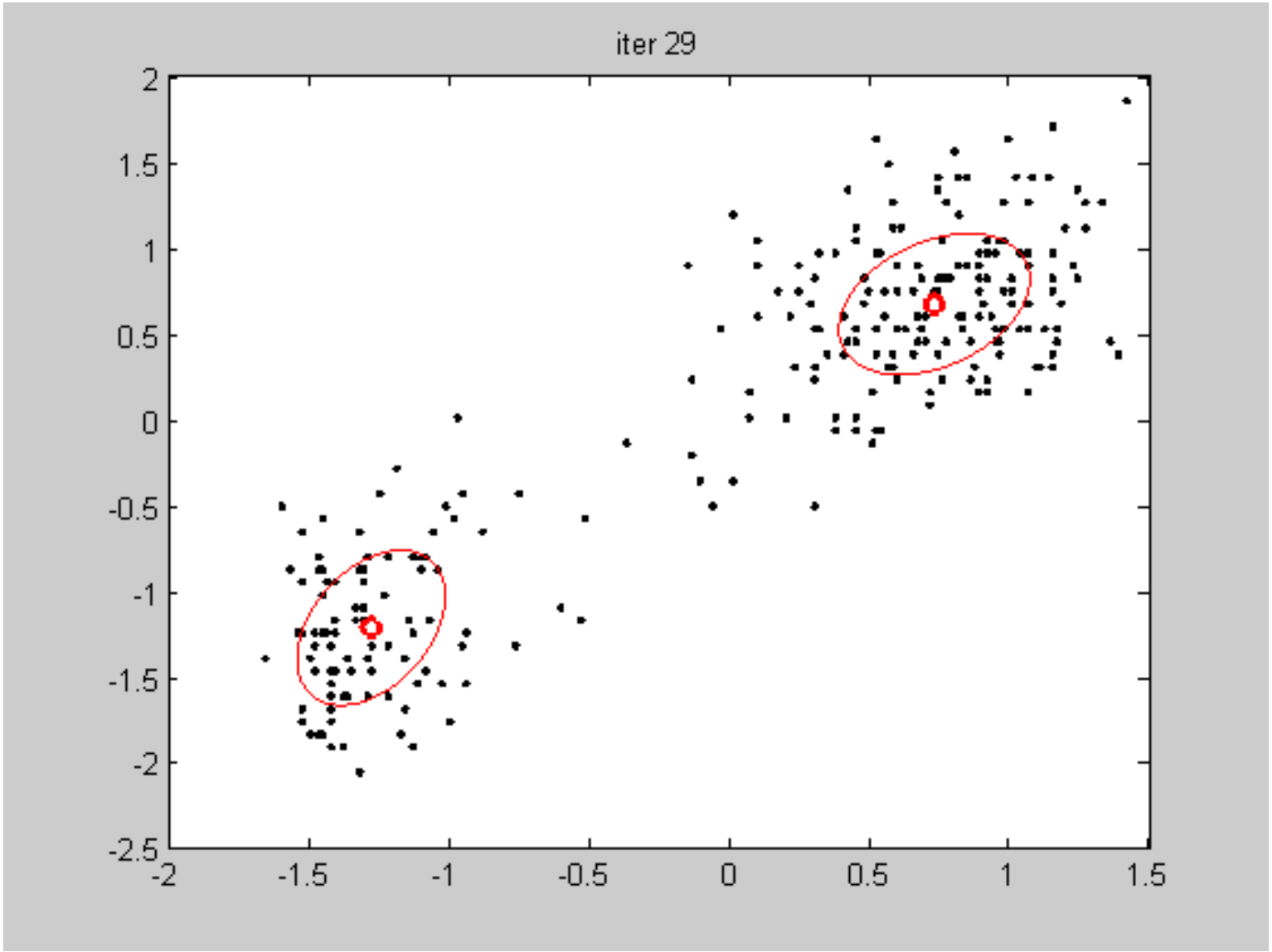

FlowVB is largely based on the algorithm presented in

C Archambeau and M Verleysen. Robust Bayesian Clustering. *Neural Networks*, 20(1):129–138, 2007.

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Neural Networks 20 (2007) 129–138

Neural Networks

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Robust Bayesian clustering

Cédric Archambeau*, Michel Verleysen[1]

Machine Learning Group, Université catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium

Received 25 October 2005; accepted 12 June 2006

Abstract

A new variational Bayesian learning algorithm for Student-t mixture models is introduced. This algorithm leads to (i) robust density estimation, (ii) robust clustering and (iii) robust automatic model selection. Gaussian mixture models are learning machines which are based on a divide-and-conquer approach. They are commonly used for density estimation and clustering tasks, but are sensitive to outliers. The Student-t distribution has heavier tails than the Gaussian distribution and is therefore less sensitive to any departure of the empirical distribution from Gaussianity. As

The latent variable model

A finite Student-t mixture model (SMM) is of the form

$$p(\boldsymbol{x}|\boldsymbol{\theta}_s) = \sum_{m=1}^{M} \pi_m \, \mathcal{S}(\boldsymbol{x}|\boldsymbol{\mu}_m, \boldsymbol{\Lambda}_m, \nu_m),$$

where

π_m	Mixing proportions, with $\sum_{m=1}^{M} \pi_m = 1$
$\boldsymbol{\mu}_m$	Expected value of component m
$\boldsymbol{\Lambda}_m$	Precision matrix of component m
ν_m	Degrees of freedom of component m
$\boldsymbol{\theta}_s$	Ensemble of parameters

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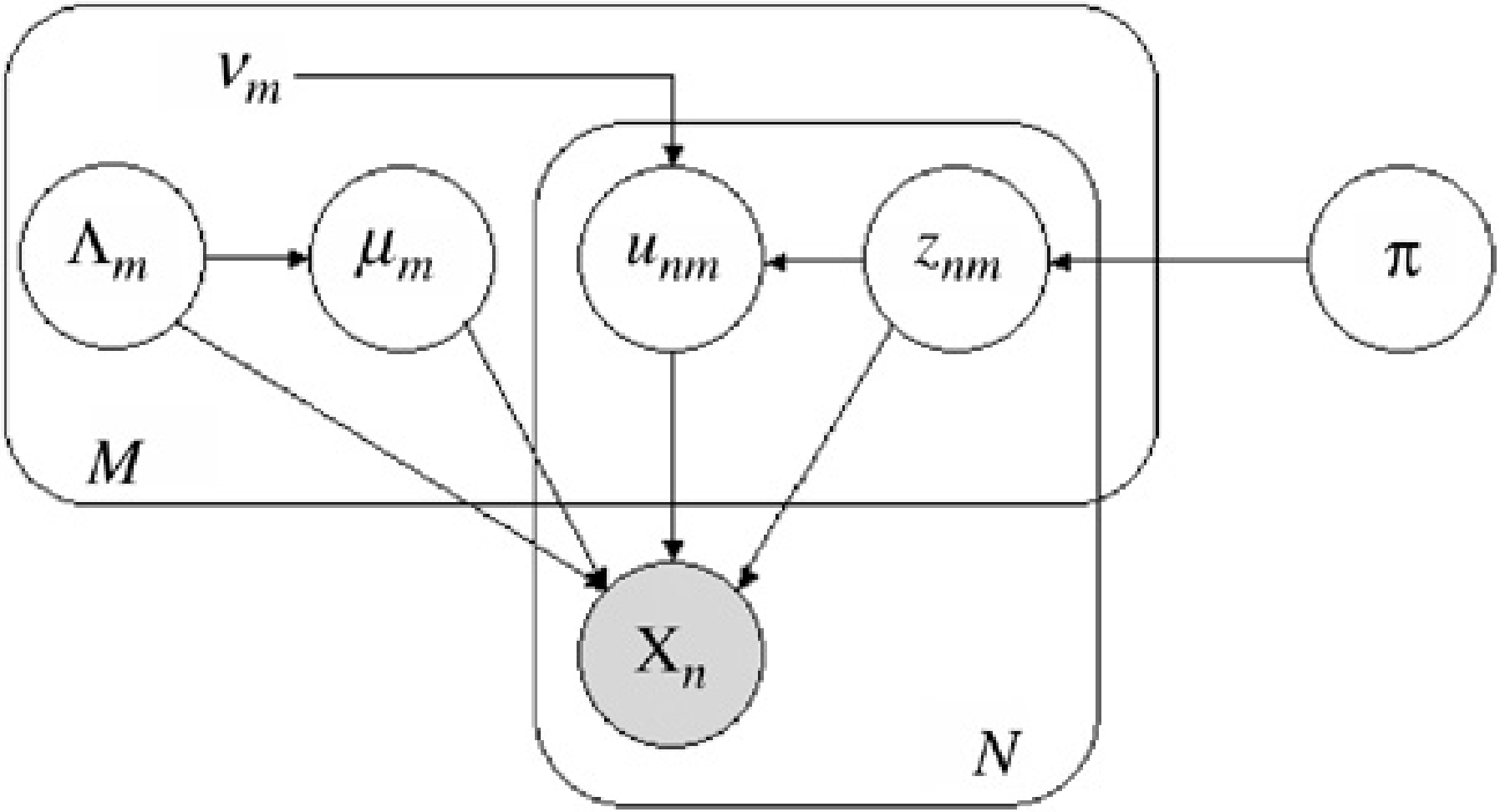
In contrast to GMM we need a further latent variable. Note that the Student-t density can be written as an *infinite scale mixture*:

$$\mathcal{S}(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda}, \nu) = \int_0^{+\infty} \mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu}, u\boldsymbol{\Lambda})\, \mathcal{G}\left(u \middle| \frac{\nu}{2}, \frac{\nu}{2}\right) \boldsymbol{d}u$$

We therefore have unobserved variables z_{nm} and u_{nm} for every observation and component.

Representation as Bayesian Graphical Model



from Archambeau & Verleysen (2007)

Variational Bayesian Inference

The evidence of the data

$$p(X|\mathcal{H}_M),$$

given the model structure $\mathcal{H}_M$, is untractable when using the real posterior

$$p(U, Z, \theta_s|X, \mathcal{H}_M).$$

Instead use an approximation and assume that it factorizes as

$$q(U, Z, \theta_s) = q(U, Z)\, q(\theta_s).$$

It can be shown that this leads to a *lower bound* for the real evidence.

Variational Bayesian Inference (cont.)

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During the iteration we remove clusters where π_m is close to zero.

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  - For every restart randomly select M points from this solution

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- Not so good results with HSCT, WNV, ND
- Found that some Pre- and Post-processing improved results with GvHD

Pre-processing

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Post-processing: FlowMerge

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- wrote our own implementation of FlowMerge

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- One-pass determination of number of mixture components
- Optional random restarts to improve results
- Improve performance by using PCA and flowMerge

Further developement

We are porting the algorithm to Python for further developement.

We are looking for additional developers and feedback from users.

Get involved:

Project page: **http://github.com/FlowVB/FlowVB**
Mailing list: **http://groups.google.com/group/flowvb**

Appendix

Variational Bayesian Inference

For a given model structure $\mathcal{H}_M$, the evidence is given by:

$$p(X|\mathcal{H}_M) = \int_{\theta_s} \int_U \sum_Z p(X, U, Z, \theta_s|\mathcal{H}_M) \boldsymbol{d}U \boldsymbol{d}\theta_s$$

This quantity is untractable, but it can be lower-bounded by

$$\begin{aligned} \log p(X|\mathcal{H}_M) \geq & \log p(X|\mathcal{H}_M) \\ & - KL[q(U, Z, \theta_s)||p(U, Z, \theta_s|X, \mathcal{H}_M)], \end{aligned}$$

where $KL(\cdot)$ is the Kullback-Leibler divergence, and $q(U, Z, \theta_s)$ is an approximation to $p(U, Z, \theta_s|X, \mathcal{H}_M)$.
By assuming that q factorizes as $q(U, Z, \theta_s) = q(U, Z)\, q(\theta_s)$, this quantity becomes tractable.

The latent variable model (cont.)

Putting the above together we get the following latent variable model:

$$p(\boldsymbol{z}_n|\boldsymbol{\theta}_s, \mathcal{H}_M) = \prod_{m=1}^{M} \pi_m^{z_{nm}}$$

$$p(\boldsymbol{u}_n|\boldsymbol{\theta}_s, \mathcal{H}_M) = \prod_{m=1}^{M} \mathcal{G}\left(u_{nm} \middle| \frac{\nu_m}{2}, \frac{\nu_m}{2}\right)^{z_{nm}}$$

$$p(\boldsymbol{x}_n|\boldsymbol{\theta}_s, \mathcal{H}_M) = \prod_{m=1}^{M} \mathcal{N}(\boldsymbol{x}_n|\boldsymbol{\mu}_m, u_{nm}\boldsymbol{\Lambda}_m)^{z_{nm}}$$

Priors

We introduce the following conjugate priors for our parameters:

Parameter	*Prior*
$\boldsymbol{\pi} = \{\pi_m\}_{m=1}^{M}$	$\mathcal{D}(\boldsymbol{\pi}\|\varkappa_0)$ (Dirichlet)
$\boldsymbol{\mu}_m, \boldsymbol{\Lambda}_m$	$\mathcal{NW}(\boldsymbol{\mu}_m, \boldsymbol{\Lambda}_m\|\boldsymbol{\theta}_{\mathcal{NW}_0}) = \mathcal{N}(\boldsymbol{\mu}_m\|\boldsymbol{m}_0, \eta_0\boldsymbol{\Lambda}_m)\mathcal{W}(\boldsymbol{\Lambda}_m\|\gamma_0, \boldsymbol{S}_0)$ (Joint Gaussian-Wishart)
ν_m	no prior, since no conjugate prior exists