

Linear Maps

Flower

Linear Algebra

A. The Vector Space of Linear Maps

Problem 1

假设 $T \in \mathcal{L}(\mathbb{F}^n, \mathbb{F}^m)$. 证明存在 $A_{j,k} \in \mathbb{F}$, 其中 $j = 1, \dots, m$ $k = 1, \dots, n$, 使得

$$T(x_1, \dots, x_n) = (A_{1,1}x_1 + \dots + A_{1,n}x_n, \dots, A_{m,1}x_1 + \dots + A_{m,n}x_n)$$

对于每一个 $(x_1, \dots, x_n) \in \mathbb{F}^n$ 都成立.

Proof: 对于任意的 $x \in \mathbb{F}^n$, 我们可以写

$$x = x_1e_1 + \dots + x_ne_n,$$

其中 e_1, \dots, e_n 是 \mathbb{F}^n 的标准基. 因为 T 是线性的, 我们有

$$\begin{aligned} Tx &= T(x_1e_1 + \dots + x_ne_n) \\ &= x_1Te_1 + \dots + x_nTe_n. \end{aligned}$$

现在对于 $Te_k \in \mathbb{F}^m$, 其中 $k = 1, \dots, n$, 都存在 $A_{1,k}, \dots, A_{m,k} \in \mathbb{F}$ 使得

$$\begin{aligned} Te_k &= A_{1,k}e_1 + \dots + A_{m,k}e_m \\ &= A_{1,k}, \dots, A_{m,k} \end{aligned}$$

因此

$$x_kTe_k = (A_{1,k}x_k, \dots, A_{m,k}x_k).$$

所以我们有

$$\begin{aligned} Tx &= \sum_{k=1}^n (A_{1,k}x_k, \dots, A_{m,k}x_k) \\ &= \left(\sum_{k=1}^n A_{1,k}x_k, \dots, \sum_{k=1}^n A_{m,k}x_k \right), \end{aligned}$$

就证得存在 $A_{j,k} \in \mathbb{F}$, 其中 $j = 1, \dots, m$ 并且 $k = 1, \dots, n$ 使得等式成立. \square

Problem

假设 $T \in \mathcal{L}(\mathbb{F}^n, \mathbb{F}^m)$. 证明存在 $A_{j,k} \in \mathbb{F}$, 其中 $j = 1, \dots, m$ $k = 1, \dots, n$, 使得

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$$\begin{aligned} Tx &= T(x_1 e_1 + \dots + x_n e_n) \\ &= x_1 T e_1 + \dots + x_n T e_n. \end{aligned}$$

现在对于 $T e_k \in \mathbb{F}^m$, 其中 $k = 1, \dots, n$, 都存在 $A_{1,k}, \dots, A_{m,k} \in \mathbb{F}$ 使得

$$\begin{aligned} T e_k &= A_{1,k} e_1 + \dots + A_{m,k} e_m \\ &= A_{1,k}, \dots, A_{m,k} \end{aligned}$$

因此

$$x_k T e_k = (A_{1,k} x_k, \dots, A_{m,k} x_k).$$

所以我们有

$$\begin{aligned} Tx &= \sum_{k=1}^n (A_{1,k} x_k, \dots, A_{m,k} x_k) \\ &= \left(\sum_{k=1}^n A_{1,k} x_k, \dots, \sum_{k=1}^n A_{m,k} x_k \right), \end{aligned}$$

就证存在 $A_{j,k} \in \mathbb{F}$, 其中 $j = 1, \dots, m$ 并且 $k = 1, \dots, n$ 使得等式成立. It is't right. \square

A.1. The Vector Space of Linear Maps

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Problem 2

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Proof: Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magnam aliquam quaerat voluptatem. Ut enim aequae doleamus animo, cum corpore dolemus, fieri tamen permagna accessio potest, si aliquod aeternum et infinitum impendere malum nobis opinemur. Quod idem licet transferre in voluptatem, ut postea variari voluptas distinguere possit, augeri amplificarique non possit. At etiam Athenis, ut e patre audiebam facete et urbane Stoicos irridente, statua est in quo a nobis philosophia

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□

Problem 2

Adding `rbx` to `rcx` gives the desired result. What is `fn main()` in Rust would be `int main()` in C.

```
fn main() {  
    println!("Hello World!");  
}
```

This has ``backticks`` in it (but the spaces are trimmed). And here the leading space is also trimmed.

Problem 999

In this report, we will explore the various factors that influence fluid dynamics in glaciers and how they contribute to the formation and behaviour of these natural structures.

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Glaciers form an important part of the earth's climate system.