University of Mars
Institute of Intergalactic Travel

Linear Maps

Flower

Linear Algebar

A. The Vector Space of Linear Maps

Problem 1

假设 $T \in \mathcal{L}(\mathbb{F}^n, \mathbb{F}^m)$.证明存在$A_{j,k} \in \mathbb{F}$，其中 $j = 1, ..., m \quad k = 1, ..., n$，使得

$$T(x_1, ..., x_n) = (A_{1,1}x_1 + ... + A_{1,n}x_n, ..., A_{m,1}x_1 + ... + A_{m,n}x_n)$$

对于每一个 $(x_1, ..., x_n) \in \mathbb{F}^n$都成立.

Proof: 对于任意的 $x \in \mathbb{F}^n$，我们可以写

$$x = x_1e_1 + ... + x_ne_n,$$

其中 $e_1, ..., e_n$ 是 $\mathbb{F}^n$ 的标准基. 因为 T 是线性的，我们有

$$\begin{aligned} Tx &= T(x_1e_1 + ... + x_ne_n) \\ &= x_1Te_1 + ... + x_nTe_n. \end{aligned}$$

现在对于 $Te_k \in \mathbb{F}^m$, 其中 $k = 1, ..., n$, 都存在 $A_{1,k}, ..., A_{m,k} \in \mathbb{F}$ 使得

$$\begin{aligned} Te_k &= A_{1,k}e_1 + ... + A_{m,k}e_m \\ &= A_{1,k}, ..., A_{m,k} \end{aligned}$$

因此

$$x_kTe_k = (A_{1,k}x_k, ..., A_{m,k}x_k).$$

所以我们有

$$\begin{aligned} Tx &= \sum_{k=1}^{n}(A_{1,k}x_k, ..., A_{m,k}x_k) \\ &= \left(\sum_{k=1}^{n}A_{1,k}x_k, ..., \sum_{k=1}^{n}A_{m,k}x_k\right), \end{aligned}$$

就证得存在$A_{j,k} \in \mathbb{F}$，其中 $j = 1, ..., m$ 并且 $k = 1, ..., n$ 使得等式成立. □

Problem #

假设 $T \in \mathcal{L}(\mathbb{F}^n, \mathbb{F}^m)$. 证明存在$A_{j,k} \in \mathbb{F}$，其中 $j = 1, ..., m \quad k = 1, ..., n$，使得

Proof: 对于任意的 $x \in \mathbb{F}^n$，我们可以写

$$x = x_1e_1 + ... + x_ne_n,$$

其中 $e_1, ..., e_n$ 是 $\mathbb{F}^n$ 的标准基. 因为 T 是线性的，我们有

$$\begin{aligned} Tx &= T(x_1 e_1 + ... + x_n e_n) \\ &= x_1 T e_1 + ... + x_n T e_n. \end{aligned}$$

现在对于 $T e_k \in \mathbb{F}^m$, 其中 $k = 1, ..., n$, 都存在 $A_{1,k}, ..., A_{m,k} \in \mathbb{F}$ 使得

$$\begin{aligned} T e_k &= A_{1,k} e_1 + ... + A_{m,k} e_m \\ &= A_{1,k}, ..., A_{m,k} \end{aligned}$$

因此

$$x_k T e_k = \left(A_{1,k} x_k, ..., A_{m,k} x_k\right).$$

所以我们有

$$\begin{aligned} Tx &= \sum_{k=1}^{n} \left(A_{1,k} x_k, ..., A_{m,k} x_k\right) \\ &= \left(\sum_{k=1}^{n} A_{1,k} x_k, ..., \sum_{k=1}^{n} A_{m,k} x_k\right), \end{aligned}$$

就证存在$A_{j,k} \in \mathbb{F}$，其中 $j = 1, ..., m$ 并且 $k = 1, ..., n$ 使得等式成立. It is’t right. □

A.1. The Vector Space of Linear Maps

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