1 batch fermentation

In a batch fermentation, the entire substrate is presented in the beginning and no medium is removed during the process.

Using Monod kinetics to describe growth of microorganisms:

$$\mu = \mu_{max} \cdot \frac{S}{S + K_s}$$

with μ as growth rate [1/h], S as substrate concentration [g/L] and K_s as Monod-constant [g/L], we can describe growth of biomass X [g/L] in batch fermentation by using following differential equation:

$$\frac{dX}{dt} = X \cdot \mu$$

By introducing $Y_{X/S}$ as biomass/substrate yield [g/g], we can also define change in substrate concentration:

$$\frac{dS}{dt} = -\frac{1}{Y_{X/S}} \cdot \mu \cdot X$$

If we are using batch fermentation for production of secondary metabolites (meaning that X is not our product), we can use the following equation to describe product formation rate q_p and change in product concentration:

$$q_p = q_{pmax} \cdot \frac{S}{K_p + S}$$

$$\frac{dP}{dt} = q_p \cdot X$$

We also have to change the differential equation for change in substrate concentration as the product formation also consumes substrate:

$$\frac{dS}{dt} = -\frac{1}{Y_{X/S}} \cdot \mu \cdot X - \frac{1}{Y_{P/S}} \cdot q_p \cdot X$$

2 fed-batch fermentation

In a fed-batch fermentation, substrate is added continuously at a certain point of time, which dilutes the components in the reactor.

We are also using the Monod kinetics from the batch fermentation to create differential equation to describe the change in biomass, product and substrate concentration. Since we have to take in mind the flow rate which dilutes the components in the reactor, we have to establish mass balances with $F = \frac{dV}{dt}$ and S_f as substrate concentration in the feed:

$$\frac{d(X \cdot V)}{dt} = X \cdot \mu \cdot V$$

$$V \cdot \frac{dX}{dt} + X \cdot \frac{dV}{dt} = X \cdot \mu \cdot V$$

$$\frac{dX}{dt} = \left(\mu - \frac{F}{V}\right) \cdot X$$

The same procedure can be applied to change in substrate and product concentration:

$$\frac{dS}{dt} = -\frac{1}{Y_{X/S}} \cdot \mu \cdot X + \frac{F}{V} \cdot (S_f - S) - \frac{1}{Y_{P/S}} \cdot q_p \cdot X$$

$$\frac{dP}{dt} = q_p \cdot X - \frac{F}{V} \cdot P$$

3 continuous fermentation

In a continuous fermentation, substrate is added and medium is removed during the whole fermentation. The microorganisms experience a substrate limitation, hence you can impose a certain growth rate on them by regulating the dilution rate (= flow rate). The reactor reaches a steady-state, in which all concentrations remain constant and do not change over time. Therefore, we have to define equations for the instationary and stationary state:

3.1 instationary

$$\frac{dX}{dt} = (\mu - D) \cdot X$$

$$\frac{dS}{dt} = -\frac{1}{Y_{X/S}} \cdot X \cdot \mu + D \cdot (S_f - S) - \frac{1}{Y_{P/S}} \cdot q_p \cdot X$$

$$\frac{dP}{dt} = q_p \cdot X - D \cdot P$$

3.2 stationary

With $\frac{dX}{dt} = \frac{dS}{dt} = 0$ we can define equations for the steady-state with a certain dilution rate. STY is the space-time-yield [g/L*h]. To simplify it further, we assume that the biomass is our product:

$$\mu = D$$

$$X = Y_{X/S} \cdot \left(S_f - \frac{K_s \cdot D}{\mu_{max} - D}\right)$$

$$STY = X \cdot D$$