Understanding factor analysis [1]

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Introduction

Thousands of variables have been proposed to explain or describe the complex variety and interconnections of social and international relations. Perhaps an equal number of hypotheses and theories linking these variables have been suggested.

The few basic variables and propositions central to understanding remain to be determined. The systematic dependencies and correlations among these variables have been charted only roughly, if at all, and many, if not most, can be measured only on presence–absence or rank order scales. And to take the data on any one variable at face value is to beg questions of validity, reliability, and comparability.

Confronted with entangled behavior, unknown interdependencies, masses of qualitative and quantitative variables, and bad data, many social scientists are turning toward factor analysis to uncover major social and international patterns.[2] Factor analysis can simultaneously manage over a hundred variables, compensate for random error and invalidity, and disentangle complex interrelationships into their major and distinct regularities.

Factor analysis is not without cost, however. It is mathematically complicated and entails diverse and numerous considerations in application. Its technical vocabulary includes strange terms such as *eigenvalues*, *rotate*, *dimensions*, *orthogonal*, *loadings*, and *communality*. Its results usually absorb a dozen or so pages in a given report, leaving little room for a methodological introduction or explanation of terms. Add to this the fact that students do not ordinarily learn factor analysis in their formal training, and the sum is the major cost of factor analysis: most laymen, social scientists, and policy-makers find the nature and significance of the results incomprehensible.

The problem of communicating factor analysis is especially crucial for peace research. Scholars in this field are drawn from many disciplines and professions, and few of them are acquainted with the method.

As our empirical knowledge of conflict processes, behavior, conditions, and patterns become increasingly expressed in factor-analytic terms, those who need this knowledge most in order to make informed policy decisions may be those who are most deterred by the packaging. Indeed, they

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[2] For a bibliography of applications of factor analysis in the social sciences (excluding psychology), see Rummel (1968). A bibliography of applications to conflict and international relations is given in the appendix below.

are unlikely to know that this knowledge exists.[3]

A conceptual map, therefore, is needed to guide the consumers of findings in conflict and international relations through the terminological obstacles and quantitative obstructions presented by factor studies. The aim of this paper is to help draw such a map. Specifically, the aim is to enhance the understanding and utilization of the results of factor analysis. Instead of describing how to apply factor analysis or discussing the mathematical model involved, I shall try to clarify the technical paraphernalia which may conceal important substantive data, propositions, or scientific laws.

By way of orientation, the first section of this paper will present a brief conceptual review of factor analysis. In the second section the scientific context of the method will be discussed. The major uses of factor analysis will be listed and its relation to induction and deduction, description and inference, causation and explanation, and classification and theory will be considered. To aid understanding, the third section will outline the geometrical and algebraic factor models, and the fourth section will define the factor matrices and their elements—the vehicles for presenting factor results. Since comprehending factor rotation is important for interpreting the findings, the fifth and final section is devoted to clarifying its significance.

A bibliography of factor analysis texts and applications to conflict and international relations is given in an appendix.

[3] How many readers know that over a decade ago Raymond Cattell (1949) gave us the first comprehensive findings on the extent to which foreign and domestic conflict behaviors have been correlated with many socioeconomic and political characteristics of nations?

1. Conceptual Overview

Factor analysis is a means by which the regularity and order in phenomena can be discerned. As phenomena co-occur in space or in time, they are patterned; as these co-occurring phenomena are independent of each other, there are a number of distinct patterns. Patterned phenomena are the essence of workaday concepts such as "table," "chair," and "house," and—at a less trivial level—patterns structure our scientific theories and hypotheses. We associate a pattern of attitudes, for example, with businessmen and another pattern with farmers. "Economic development" assumes a pattern of characteristics, as does the concept of "Communist political system." The notion of conflict itself embodies a pattern of elements, i.e., two or more parties and a perception of mutually exclusive or contradictory values or goals. And to mention phenomena that everyone talks about, weather also has its patterns.

What factor analysis does is this: it takes thousands and potentially millions of measurements and qualitative observations and resolves them into distinct patterns of occurrence. It makes explicit and more precise the building of fact-linkages going on continuously in the human mind.

Let us look at a concrete example. Table 1 presents information on fourteen nations for ten characteristics. The nations are selected to reflect major regional, political, economic, and cultural groupings; the characteristics reflect different facets of each nation, including domestic instability and foreign conflict. The table thus contains 14×10, or 140 pieces of information for 1955. Factor analysis addresses itself to this question: "What are the patterns of relationship among these data?"

These patterns can be viewed from two perspectives. One can look at the pattern

TABLE 1
SELECTED SAMPLE DATA FOR 1955

Nation	GNP per capita ($)	Trade (million $)	Power rank[a]	Stability[b]	Freedom of group oppos.[c]	Foreign conflict[d]	Agreement with US in UN[e]	Defense budget (million $)	% GNP for defense	Acceptance of internat'l law[f]
Brazil	91	2,729	7	0	2	0	69.1	148	2.8	0
Burma	51	407	4	0	1	0	− 9.5	74	6.9	0
China	58	349	11[g]	0	0	1	−41.7[j]	3,054	8.7	0
Cuba	359	1,169	3	0	1	0	64.3	53	2.4	0
Egypt	134	923	5	1	1	1	−15.4	158	6.0	1
India	70	2,689	10	0	2	0	−28.6	410	1.9	1
Indonesia	129	1,601	8	0	1	0	−21.4	267	6.7	0
Israel	515	415	2	1	2	1	42.9	33	2.7	1
Jordan	70	83	1	0	1	1	8.3[h]	29	25.7	0
Netherlands	707	5,395	6	1	2	0	52.3	468	6.1	1
Poland	468	1,852	9	0	0	1	−41.7	220	1.5	0
USSR	749	6,530	13	1	0	1	−41.7	34,000[i]	20.4[i]	0
UK	998	18,677	12	1	2	1	69.0	3,934	7.8	0
US	2,334	26,836	14	1	2	1	100.0	40,641	12.2	1

[a] Ranking based on product of population times energy production, with 1 as lowest rank.
[b] 0 = unstable as indicated by extensive rioting, guerilla warfare, coups, purges, or frequent general strikes, 1955–57. 1 = stable.
[c] 0 = political opposition not permitted; 1 = restricted opposition permitted, but cannot campaign for control of government; 2 = unrestricted.
[d] 1 = intensive foreign conflict as evidenced by frequent threats, severance of diplomatic relations, protests, or military action, 1955–57; 0 = little, if any, foreign conflict.
[e] Percent of votes in agreement minus percent in disagreement. Voting data are for the tenth session.
[f] 0 = does not subscribe to statute of International Court of Justice; 1 = subscribes with reservations.
[g] Both population and energy production are based on best available estimates.
[h] Estimated to be the same as Iran's vote.
[i] Geometric mean of several modes of computation. [j] Estimate.

of *variation of nations* across their characteristics, and then group the nations by their profile similarity. One might group together nations which are all high on GNP per capita, low on trade, high on power, etc. When applied to discern patterns of profile similarity of individuals, groups, or nations, the analysis is called Q-factor analysis.[4]

The regularity in the data of Table 1 can be looked at from a second perspective, however. The focus now is the patterns of *variation of characteristics*. In Table 1, for example, nations high on GNP per capita also appear low on trade and power. There is a regularity, therefore, in the nation values on these three characteristics, and this regularity is described as a pattern of variation. Many of our social concepts define such patterns. For example, the concept of "economic development" involves (among other things) GNP per capita, literacy, urbanization, education, and communication; it is a pattern because these characteristics are highly intercorrelated with each other. Factor analysis applied to delineate patterns of variation in characteristics is called R-factor analysis.[5]

[4] For a Q-factor analysis of UN voting, see Russett (1966). A Q-factor analysis of nations on many of their characteristics has been reported by Banks and Gregg (1965).

[5] Most factor analysis done on nations has been R-factor analysis. As one example out of many, see Tanter (1966). R- and Q-factor analyses do not exhaust the kinds of patterns that may be considered. Other possible patterns of variation are those in characteristics over

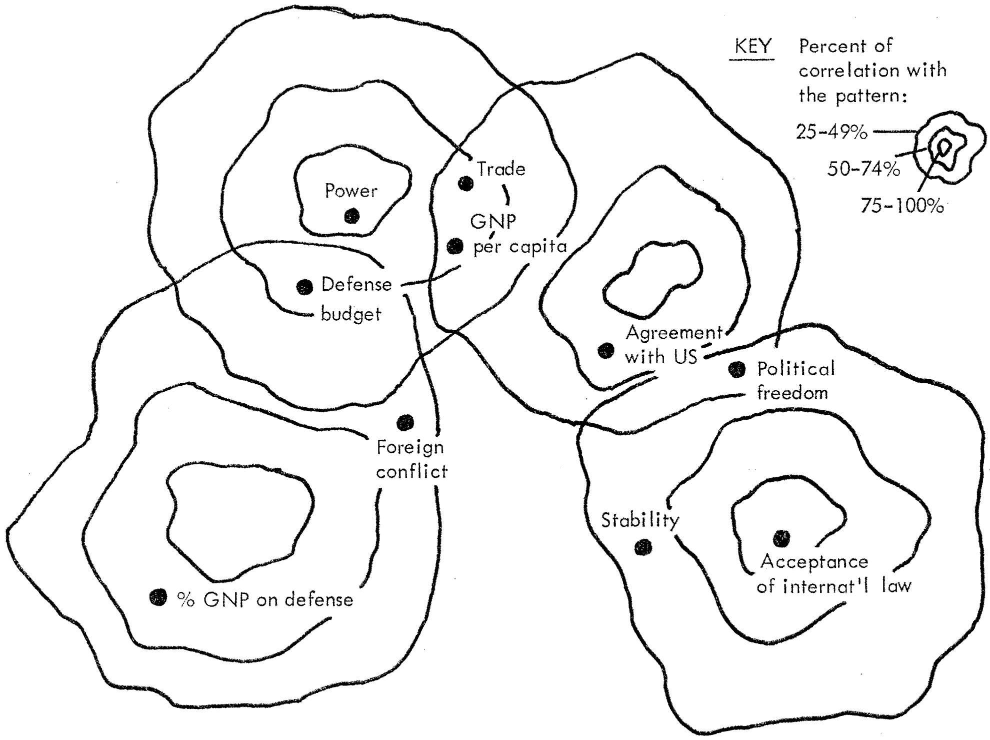

FIG. 1. The four major kinds of regularity in the interrelationships between the characteristics *power, US voting agreement, foreign conflict,* and *international law* (see Table 1).

What actual patterns of characteristics are revealed for the data in Table 1 by factor analysis? Figure 1 displays the four major kinds of regularity in the interrelationships between the characteristics: *power, US voting agreement, foreign conflict,* and *international law.* They involve, respectively, 27.6, 21.0, 16.2, and 15.3 percent of the variation[6] in the 140 pieces of information in Table 1; added together, these patterns indicate that 80.1 percent of this information has an underlying regularity.

Each pattern in Figure 1 is laid out in three isobars. The central isobar includes characteristics with at least 75 percent of their variation involved in the pattern. These are most central to interpreting the pattern. The two remaining isobars define characteristics related to the pattern in the range of 50–74 percent and 25–49 percent of their variation, respectively. These groups of isobars show (1) what patterns

time units for a specified nation (this identifies similar time periods); in nations over time units for a characteristic (this identifies nations similarly changing on a characteristic); and in time units over nations for a characteristic (this identifies similar time periods for nations changing on a characteristic). For a discussion of these varieties of analysis, see Rummel (1968), ch. 8.

[6] Section 4.2, below, discusses how such percentage figures are derived from the factor results.

exist in the data and how they overlap, (2) what characteristics are involved in what pattern and to what degree, and (3) what characteristics are involved in more than one pattern.

To display another perspective, Figure 2 plots these four patterns as profiles for the nations in Table 1. On the horizontal axis, nations are ordered from low to high power-pattern values. Magnitudes on the vertical axis are in standard scores. The average score is zero and 95.5 percent of the fourteen nations will (if normally distributed) fall between scores of +2.00 and –2.00; 68.3 percent of them will fall between scores of +1.00 and –1.00. Each pattern has a different shape, which illustrates what is meant by saying that factor analysis divides the regularity in the data into its distinct patterns. If each of the ten characteristics in Table 1 were plotted as was done for the patterns in Figure 2, and those characteristics with similarly-shaped plots were grouped together, there would be four major groups, and the modal plot within each group would correspond to each of the patterns shown. Figures 1 and 2 are alternative *representations* of the results of factoring Table 1.

2. Factor Analysis and Scientific Method

Factor analysis can be applied in order to explore a content area, structure a domain, map unknown concepts, classify or reduce data, illuminate casual nexuses, screen or transform data, define relationships, test hypotheses, formulate theories, control variables, or make inferences. Our consideration of these various overlapping usages will be related to several aspects of scientific method: induction and deduction; description and inference; causation, explanation, and classification; and theory.

2.1 USES OF FACTOR ANALYSIS

This section will outline factor-analysis applications relevant to various scientific and policy concerns. Many of the uses described below overlap. My aim is not to avoid redundancy but explicitly to relate factor analysis to the diverse interests of readers.

Interdependency and pattern delineation. If a scientist has a table of data—say, UN votes, personality characteristics, or answers to a questionnaire—and if he suspects that these data are interrelated in a complex fashion, then factor analysis may be used to untangle the linear relationships into their separate patterns. Each pattern will appear as a factor delineating a distinct cluster of interrelated data.

Parsimony or data reduction. Factor analysis can be useful for reducing a mass of information to an economical description. For example, data on fifty characteristics for 300 nations are unwieldy to handle, descriptively or analytically. The management, analysis, and understanding of such data are facilitated by reducing them to their common factor patterns. These factors concentrate and index the dispersed information in the original data and can therefore replace the fifty characteristics without much loss of information. Nations can be more easily discussed and compared on economic development, size, and totalitarianism dimensions, for example, than on the hundreds of characteristics each dimension involves.

Structure. Factor analysis may be employed to discover the basic structure of a domain. As a case in point, a scientist may want to uncover the primary independent lines or dimensions—such as size, leadership, and age—of variation in group characteristics and behavior. Data collected on a

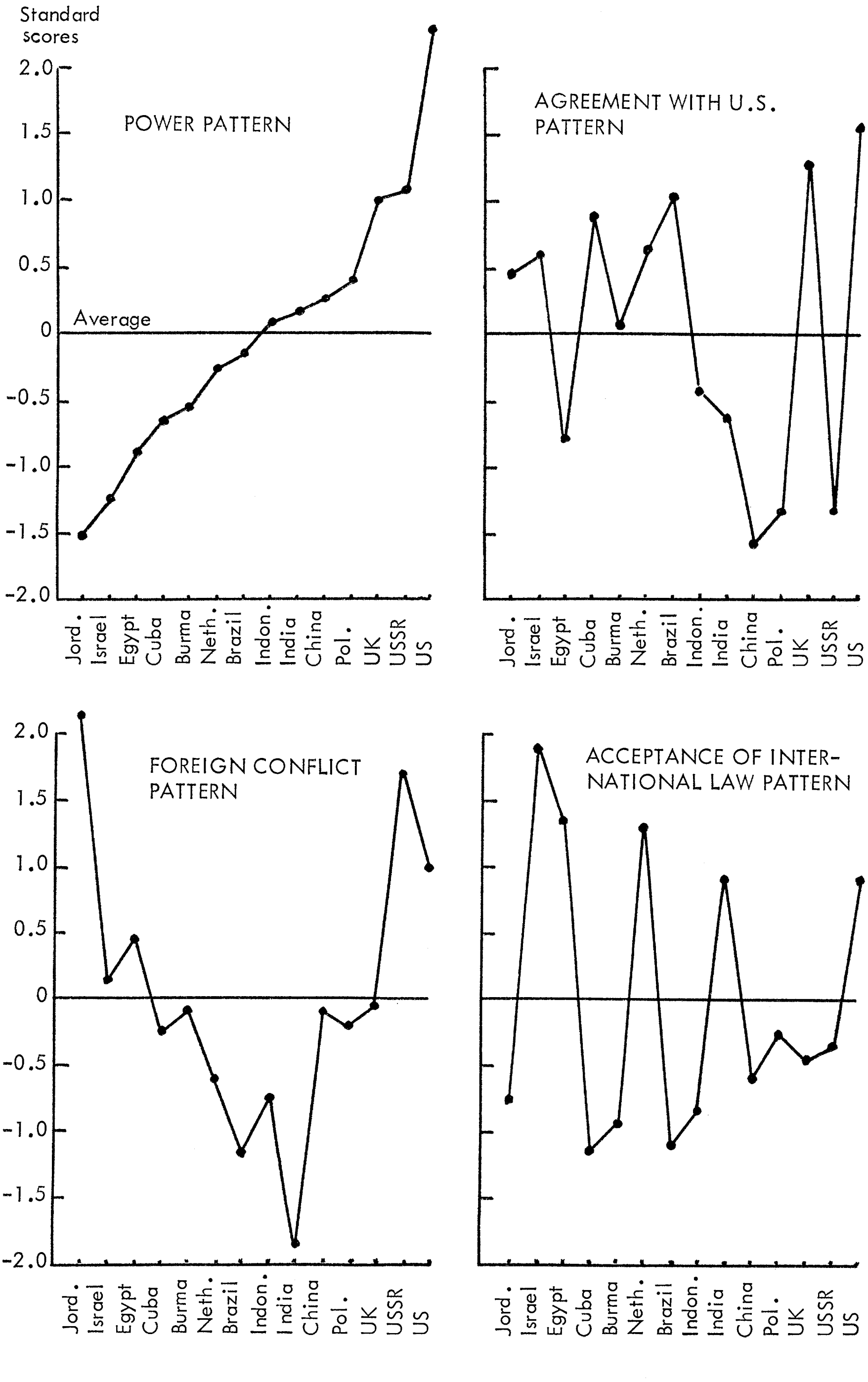

FIG. 2. Nation profiles: a second way of representing the results of factoring Table 1 data. The nations on the horizontal axes are ordered in each case by their power pattern scores.

large sample of groups and factor-analyzed can help disclose this structure.

Classification or description. Factor analysis is a tool for developing an empirical typology.[7] It can be used to group interdependent variables into descriptive categories, such as ideology, revolution, liberal voting, and authoritarianism. It can be used to classify nation profiles into types with similar characteristics or behavior. Or it can be used on data matrices of a transaction type or a social-choice type to show how individuals, social groups, or nations cluster on their transactions with or choices of each other.

Scaling. A scientist often wishes to develop a scale on which individuals, groups, or nations can be rated and compared. The scale may refer to such phenomena as political participation, voting behavior, or conflict. A problem in developing a scale is to weight the characteristics being combined. Factor analysis offers a solution by dividing the characteristics into independent sources of variation (factors). Each factor then represents a scale based on the empirical relationships among the characteristics. As additional findings, the factor analysis will give the weights to employ for each characteristic when combining them into the scales. The factor score results (see section 4.5 below) are actually such scales, developed by combining characteristics in terms of these weights.

Hypothesis testing. Hypotheses abound regarding dimensions of attitude, personality, group, social behavior, voting, and conflict. Since the meaning usually associated with "dimension" is that of a cluster or group of highly intercorrelated characteristics or behavior, factor analysis may be used to test for their empirical existence. Which characteristics or behavior should, by theory, be related to which dimensions can be postulated in advance and statistical tests of significance can be applied to the factor-analysis results.

Besides those relating to dimensions, there are other kinds of hypotheses that may be tested. To illustrate: if the concern is with a relationship between economic development and instability, *holding other things constant,* a factor analysis can be done of economic and instability variables along with other variables that may affect (hide, mediate, depress) their relationship. The resulting factors can be so defined (rotated) that the first several factors involve the mediating measures (to the maximum allowed by the empirical relationships). A remaining independent factor can be calculated to best define the postulated relationships between the economic and instability measures. The magnitude of involvement of both variables in this pattern enables the scientist to see whether an economic development–instability pattern actually exists when other things are held constant.

Data transformation. Factor analysis can be used to transform data to meet the assumptions of other techniques. For instance, application of the multiple regression technique assumes (if tests of significance are to be applied to the regression coefficients) that predictors—the so-called independent variables—are statistically unrelated (Ezekiel and Fox, 1959, pp. 283–84). If the predictor variables are correlated in violation of the assumption, factor

[7] For example, see Borgatta and Cottrell's classificatory work on groups (1955) and Schuessler and Driver's on tribes (1956). Selvin and Hagstrom (1963) show, through an example, how to use factor analysis to develop a classification of groups. Using factor analysis, Russett classifies nations into their regional groups (1967) and their UN voting blocs (1966).

analysis can be employed to reduce them to a smaller set of *uncorrelated* factor scores. The scores may be used in the regression analysis in place of the original variables with the knowledge that the meaningful variation in the original data has not been lost.[8] Likewise, a large number of dependent variables also can be reduced through factor analysis.

Exploration. In a new domain of scientific interest like peace research, the complex interrelations of phenomena have undergone little systematic investigation. The unknown domain may be explored through factor analysis. It can reduce complex interrelationships to a relatively simple linear expression and it can uncover unsuspected, perhaps startling, relationships. Usually the social scientist is unable to manipulate variables in a laboratory but must deal with the manifold complexity of behaviors in their social setting. Factor analysis thus fulfills some functions of the laboratory and enables the scientist to untangle interrelationships, to separate different sources of variation, and to partial out or control for undesirable influences on the variables of concern.[9]

Mapping. Besides facilitating exploration, factor analysis also enables a scientist to map the social terrain. By mapping I mean the systematic attempt to chart major empirical concepts and sources of variation. These concepts may then be used to describe a domain or to serve as inputs to further research. Some social domains, such as international relations, family life, and public administration, have yet to be charted. In some other areas, however, such as personality, abilities, attitudes, and cognitive meaning, considerable mapping has been done.

Theory. As will be discussed in section 2.5 below, the analytic framework of social theories or models can be built from the geometric or algebraic structure of factor analysis.

2.2 INDUCTION AND DEDUCTION

The use of "and" rather than "versus" in the headings of this and the following subsection emphasizes that these different ways of interpreting or using factor analysis are not mutually exclusive. They are different sides of the same coin. The side evident in a particular set of results depends upon the interpretation.

Factor analysis is most familiar to researchers as an *exploratory* tool for unearthing the basic empirical concepts in a field of investigation. Representing patterns of relationship between phenomena, these basic concepts may corroborate the reality of prevailing concepts or may be so new and strange as to defy immediate labeling. Factor analysis is often used to discover such concepts reflecting unsuspected influences at work in a domain. The delineation of these interrelated phenomena enables generalizations to be made and hypotheses posed about the underlying influences bringing about the relationships. For example, if a political scientist were to factor the attributes and votes of legislators and were to find a pattern involving urban constituencies and liberal votes, he could use this finding to develop a theory linking urbanism and liberalism. The ability to *relate* data in a meaningful fashion is a

[8] For practical applications of this two-step design, see Buckatzsch (1947) and Berry (1960).

[9] On this and related points, see the particularly excellent chapters 19 and 20 in Cattell (1952). Cattell (1966) has recently elaborated the position that factor analysis is, among other things, an experimental method.

prime aspect of induction and, for this, factor analysis is useful and efficient.

Factor analysis may also be employed deductively, in two ways. One way is to elaborate the geometric or algebraic structure of factor analysis as part of a theory. Within the theory the factor analysis *model* can then be used to arrive at deductions about phenomena. This approach is described more fully in section 2.5 below.

The second deductive approach is to *hypothesize* the existence of particular dimensions and then to factor-analyze the data to see whether these dimensions emerge.[10] Although factor analysis is not often used this way, the restraint is not due to methodology but to research tradition. If, as an example, scholars believe that ideology, power, and trade are the primary patterns of international behavior, then this proposition can be tested. Data can be collected on those variables that index international relations in its greatest diversity, and those specific variables distinguishing (by theory) the ideology, power, and trade patterns should be defined. To test whether these patterns actually exist is the factor-analysis task.

2.3 DESCRIPTION AND INFERENCE

A data matrix alone may be of primary interest. Research is then centered on *describing* the regularities in these data. Statistical problems like the type of underlying frequency distribution, sample size, and randomness of selection are not part (and need not be part) of the research design. As cases in point, all roll call votes in a UN General Assembly session can be analyzed to describe the voting patterns of nations for that session, as did Alker (1964), or the voting blocs into which nations were grouped, as did Russett (1966).

Description may be only an intermediate goal, however. The ultimate goal may be to connect a number of descriptive studies to make *generalizations* about what patterns exist for such phenomena as, say, legislative voting, foreign conflict, political systems, personality, or role behavior.[11] Although generalization from a number of descriptive studies is a form of inference, it need not be statistical inference in the sense that some statistical test of significance is applied. In fact, factor analysis is seldom employed for statistical inference, although many social scientists consider it a statistical method. The statistical requirement of a representative sample is usually met by the research design, but the additional statistical assumptions such as a normal frequency distribution are seldom satisfied. Indeed, the canonical factor model (section 3 below) which has been formulated to allow statistical inference is seldom used, and tests of significance for factor loadings are virtually unknown in the applied literature.

Description, then, and generalization from a number of descriptive studies have been the tradition in applied factor analysis. Although tests of significance can be determined for the factors and loadings of a particular sample, factor analysis itself does

[10] See the discussion on the relationship between hypotheses and factor analysis in Cattell (1952, pp. 13–14). For an application of factor analysis to test a hypothesis about the supposed dimensions of urban areas, see van Arsdol, Camilleri, and Schmid (1958).

[11] An extended discussion of description and explanation with regard to factor analysis in psychology is given by Henrysson (1957). Thurstone (1947, ch. 6) discusses factors as explanatory concepts in terms of a demonstration problem involving the dimensions of cylinders. His illustration of this problem is helpful for understanding factor analysis in practice.

not require such tests.[12] Factor analysis is a mathematical tool as is the calculus, and not a statistical technique like the chi-square, the analysis of variance, or sequential analysis.

2.4 CAUSATION, EXPLANATION, AND CLASSIFICATION

The idea of "cause" has had a strange fascination for scientists. Books and scholarly papers have been devoted to just the meaning and usage of the term.[13] No wonder, then, that the relationship between causation and factor analysis has been controversial in factor-analysis literature. The issue centers on whether a factor pattern represents a causal nexus.[14]

Modern science conceives of causation as a temporal regularity of phenomena or, more precisely, a functional (mathematical) relationship between phenomena. The term "cause" is then simply an expression of uniform relationships, that is, of a generally observed concurrence or concomitance of phenomena. Even though this interpretation drops out interesting connotations like "to bring about," or "to influence," it removes a fuzziness from the concept and gives it a denotation consonant with scientific method and philosophy.

Does factor analysis define factors, then, that can be called causes of the patterns they represent? The answer must be yes.[15] Each of the variables analyzed is mathematically related to the factor patterns. The regularities in the phenomena are described by these patterns, and it is these regularities that indicate a causal nexus. Just as the pattern of alignment of steel filings near a magnet can be described by the concept of magnetism, for example, so the concept of magnetism can be turned around and be said to cause the alignment. Likewise, an economic development pattern delineated by factor analysis can be called a cause. In this sense, an authoritarianism factor causes certain attitudes, a turmoil factor causes riots, and an urbanism factor causes liberal voting.

The term *explanation* adds nothing to the term *cause*. Although laden in the social sciences with a surplus meaning associated with *verstehen*, a feeling of understanding or getting the sense of something,[16] the explanation of phenomena is nothing more than *being able to predict or mathematically relate* phenomena. To explain an event is to be able to predict it (see Hempel, 1965, ch. 12 and, for contrast, Hanson, 1959). To explain that the Roman Empire fell because of disunity and moral decay is to say that, given the presence of

[12] The distinction being drawn here is between descriptive and inferential statistics, not between description and statistics.

[13] Some of the more excellent treatments are those by Frank (1955, ch. 1), Kaufmann (1958, ch. 6), the essays by Russell, Feigl, and Nagel in Part V of Feigl and Brodbeck (1953), and Nagel (1961).

[14] "It would seem that in general the variables highly loaded in a factor are likely to be the causes of those which are less loaded, or at least that the most highly loaded measures—the factor itself—is causal to the variables that loaded on it" (Cattell, 1952, p. 362). Cattell and Sullivan (1962) conducted a demonstration experiment by factoring data on cups of coffee to determine whether patterns corresponding to known causal influences could be delineated. They found a strong correlation between the known patterns of influences and those defined by the factor analysis. With like results a similar experiment was conducted on the dynamics of balls (Cattell and Dickman, 1962). These artificial experiments are helpful in understanding applied factor analysis.

[15] Positive empirical evidence for this view is referred to in note 14.

[16] See the clear and explicit analysis by Abel (1953) of the operation of *verstehen* in the social sciences.

these two elements in an empire with the characteristics of the Roman Empire, the empire will break up or be conquered.

Prediction itself is based on the identification of causal relations, i.e., regularity. Therefore, if a factor can be called a cause, it can be called an explanation.

If one wants to avoid controversy over causation, on the other hand, factor patterns may be treated as purely descriptive or classificatory. A factor name like "turmoil" will then be a noun describing phenomena sharing one characteristic: appearance in time or space with a certain uniformity. "House," "horse," "social group," "legislature," and "nation" are such nouns, and factors may be conceived likewise. "Economic development" or "size," as factors actually delineated through factor analysis (Rummel *et al.*, 1968), can be descriptive categories subsuming a pattern of telephones per capita, GNP per capita, and vehicles per capita as distinct from a pattern of population, area, and national income.

2.5 THEORY

The aim of science is theory. Facts or data are meaningless in themselves. They must be linked through propositions which confer meaning. Were man unable to perceive such relationships, his capacity to manipulate, process, or understand facts would be overwhelmed. Relationships recurring with high probability become scientific laws that may be incorporated into a theory covering the domain in which they are applicable.

A scientific theory consists of two components:[17] *analytic* and *empirical*. The analytic component is the linking of symbolic statements through chains of reasoning that obey logical or mathematical rules but that have little or no operational-empirical content. The symbols involved may refer to line, atom, dimension, force, power (mechanical or social), group, or ideology. Statements involving these symbols may be associated through verbal reasoning, symbolic logic, or mathematics. Whatever the symbols or mode of reasoning, this analytic component of theories can be the creation of the scientist's imagination, the distillation of a scholar's experience with the subject matter, or a tediously built structure slowly erected on a foundation of numerous experiments, investigation, and findings.

The empirical component of theories is operational. It fastens the abstract analytic part of a theory to the facts. While the analytic part need have no empirical interpretation, the empirical component must verifiably link to data for a theory to apply to "reality."

A confusion between the empirical and analytic parts of a theory may have militated against a more theoretical use of factor analysis in the literature. The geometric or algebraic nature of the factor model can structure the analytic framework of theory. The factors themselves can be postulated. From them, operational deductions with empirical content can be derived and tested.[18] The factor model represents a mathematical formalism departing from the calculus functions of classical physics. The

[17] One of the best discussions of theory is given by Nagel (1961, ch. 6). That theory construction consists of two parts is argued by Einstein. See the essays on Einstein's philosophy by Frank, Lenzen, and Northrup in Schilpp (1949).

[18] An exciting theoretical use of factor analysis has been published by Cattell (1962). He describes a role behavior model potentially rooted in empirical data, tying together personality, structure, and syntal group dimensions. A theoretical embodiment of factor analysis to relate the attributes and behavior of social units is described in Rummel (1965).

analytic part of the factor model is akin to that of quantum theory.[19] Vectors and their position, linear operators, and the dimensions (factors) of a system are the focus of concern.

Since factor analysis incorporates analytic possibilities as a theory and empirical techniques for connecting the theory to social phenomena, its potentiality promises much theoretical development for the social sciences. Looking ahead for a century, I suggest that factor analysis and the complementary multiple regression model are initiating a scientific revolution in the social sciences as profound and far-reaching as that initiated by the development of the calculus in physics.

3. *The Factor Model*

In application, there are not one but several factor models which differ in significant respects. A model most often applied in psychology is called *common factor analysis.* Indeed, psychologists usually reserve the term "factor analysis" for just this model. Common factor analysis is concerned with defining the patterns of *common* variation among a set of variables. Variation unique to a variable is ignored. In contrast, another factor model called *component factor analysis* is concerned with patterning *all* the variation in a set of variables, whether common or unique.

Other factor models are *image analysis, canonical analysis,* and *alpha analysis.* Image analysis has the same purpose as common factor analysis, but more elegant mathematical properties. Canonical analysis defines common factors for a *sample of cases* that are the best estimates of those for the population; it enables tests of significance. Alpha analysis defines common factors for a *sample of variables* that are the best estimates of those in a universe of content.

It would be beyond the purpose of this paper to discuss these models in any detail. (For such a discussion, see Rummel, 1968, ch. 5.) In the following sections only their general mathematical properties will be outlined. These properties clearly distinguish the factor-analysis models from others used in the social sciences, such as analysis of variance and multiple regression, and justify our consideration of these properties in reference to a generalized factor model.

3.1 GEOMETRIC MODEL

An understanding of the patterns defined by factor analysis can be enhanced through a geometric interpretation. Each nation of Table 1 can be thought of as defining a coordinate axis of a geometric space. For example, the US, the UK, and the USSR can define a three-dimensional space as given in Figure 3. Imagine that the axis for the UK is projecting at right angles from the paper. Although pictorially constrained to three dimensions, the space can be analytically extended to fourteen dimensions at right angles to each other and thus represent the fourteen nations in Table 1.

Now, in this space each characteristic can be considered a point located according to its value for each nation. Such a plot is shown in Figure 3 for the GNP per capita and trade values of the US, UK, and USSR. To make the plot explicit, projections for

[19] The relationship between classical physics and quantum theory, or between Cartesian analysis and Hilbertian analysis as related to factor analysis, is discussed by Ahmavaara (Ahmavaara and Markkanen, 1958, pp. 48–63). This analysis is the most refreshing and provocative the author has read on the subject. See also footnote 25, below.

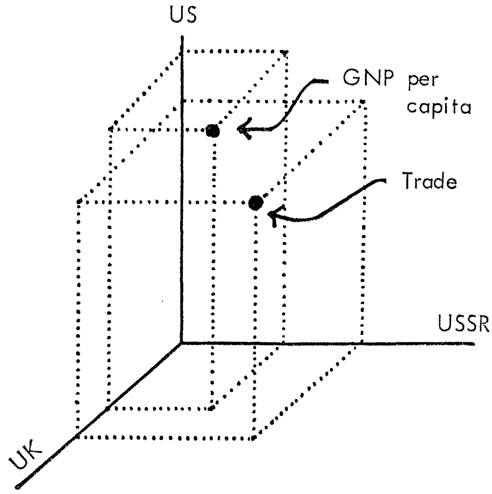

FIG. 3. Three-dimensional representation of the patterns defined by factor analysis for three nations and two characteristics.

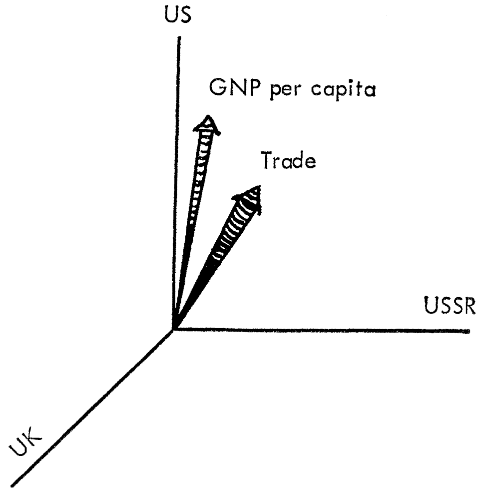

FIG. 4. Three-dimensional representation, as in Figure 3; the two characteristics of the three nations have now been drawn in as vectors rather than points.

each point are drawn as dotted lines to each axis.[20]

If for each point in Figure 3 we draw a line from the origin to the point and top the line off with an arrowhead as shown in Figure 4, then we have a *vector* representation of the data. The ten characteristics of Table 1 similarly plotted as vectors in an imaginary space of the fourteen nations (dimensions) would describe a *vector space.* In this space, consider two vectors representing any two of these characteristics for the fourteen nations. *The angle between these vectors measures the relationship between the two characteristics for the fourteen nations.* The closer to 90° the angle is, the less the relationship is. If two vectors are at a right angle, the characteristics they represent are uncorrelated: they have no relationship to each other. In other words, some nations will be high on one characteristic, say GNP per capita, and low on the other, say trade; some nations will be low on GNP per capita and high on trade; some nations will be high on both, and some will be low on both. No regularity exists in their covariation.

The closer the angle between the vectors is to zero, the stronger the relationship between the characteristics. An angle of zero means that nations high or low on one characteristic are proportionately high or low on the other. Obtuse angles mean a negative relationship. At the extreme, an angle of 180° between two vectors means that the two characteristics are inversely related: a nation high on one characteristic is proportionately low on the other.[21]

[20] Prior to plotting, the data would have to be made comparable through some standardization procedure.

[21] The cosine of this angle between vectors is, with minor qualifications, equal to the product moment correlation coefficient between the characteristics represented by the vectors. Thus, a correlation of 1.00 between two variables on twenty cases means that the angle is zero between the two vectors (variables) plotted in the space of twenty dimensions (cases).

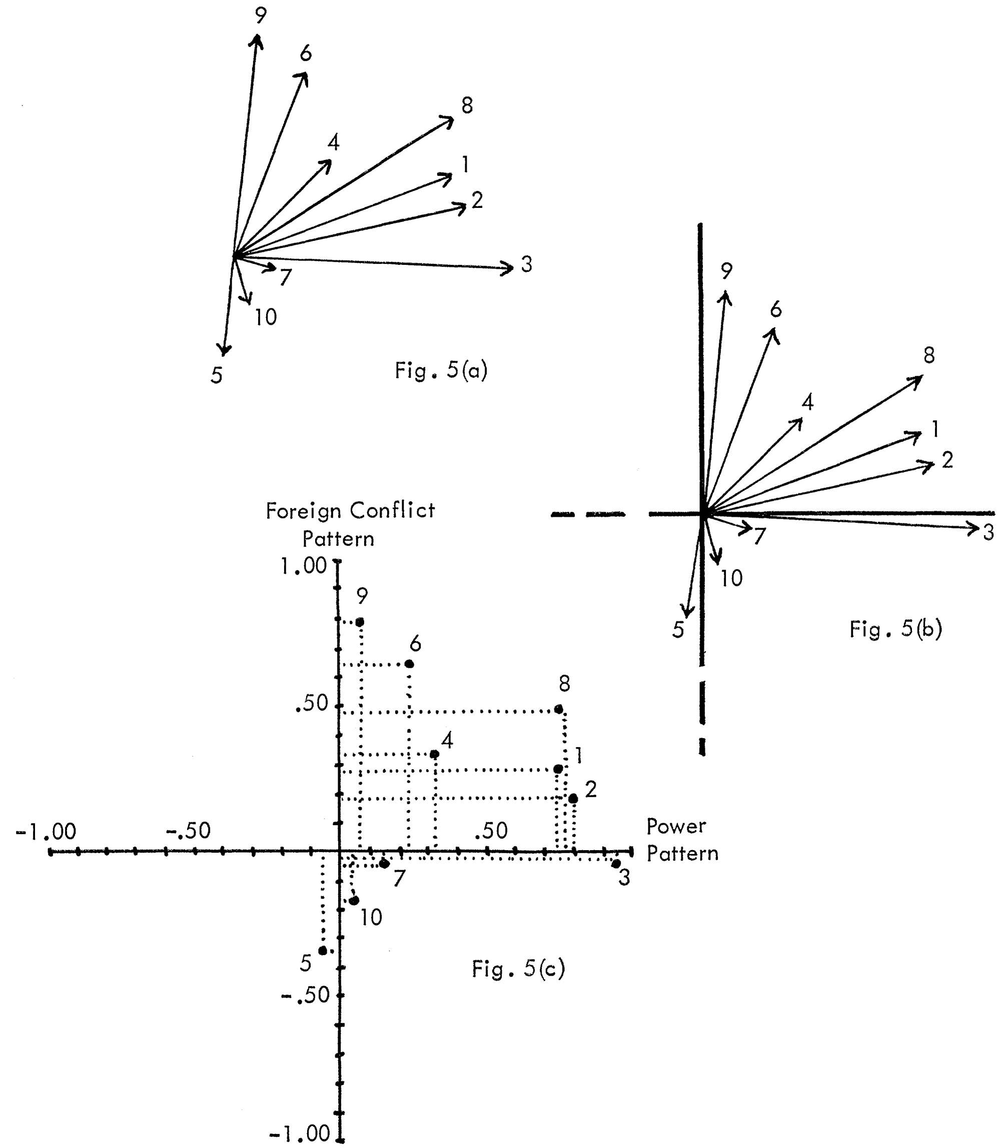

FIG. 5. (a) Projection of the ten characteristics in Table 1 as in a fourteen-dimensional space (for the fourteen nations). (b) Axes projected through each cluster of vectors. (c) Actual factor axes identified and loadings of each characteristic indicated.

Let the ten characteristics of Table 1 be projected in the fourteen-dimensional space defined by the fourteen nations as suggested in Figure 5(a). The *configuration* of vectors will then reflect the data interrelationships. Characteristics that are highly interrelated will cluster together; characteristics that are unrelated will be at right angles to each other. By inspecting the configuration we can discern the distinct clusters of vectors (if such clusters exist), and *these clusters index the patterns of relationship in the data: each cluster is a pattern.*

Were we dealing with characteristics of two or three nations, patterns could be found by simply plotting the characteristics as vectors. What factor analysis does *geometrically* is this: it enables the clusters of vectors to be defined when the number of cases (dimensions) exceeds our graphical limit of three. Each *factor* delineated by factor analysis defines a distinct cluster of vectors.[22]

Consider Figure 5(a) again. Factor analysis would mathematically lay out such a plot and then project an axis through each cluster as shown in Figure 5(b). This is analogous to giving each vector point in a cluster a mass of one and letting the factor axes fall through their center of gravity.[23] The *projection* of each vector point on the factor axes defines the clusters. These projections are called *loadings* and the factor axes are often called *factors* or *dimensions.*

Figure 5(c) pictures the power and foreign conflict patterns of Table 1. For simplicity, the configuration of points is shown, rather than vectors, and the two factor axes are indicated (as actually derived from a factor analysis). The loadings of each characteristic (i.e., each point in space) on each axis are also displayed. This figure may clarify how factor loadings as a set of numbers can define (1) a pattern of relationships and (2) the association of each characteristic with each pattern. We will consider this geometrical perspective again when the factor matrices are described.

3.2 ALGEBRAIC MODEL

For the more symbolically-oriented reader it may be helpful to present the algebraic model involved. (Others may wish to skip this section.)

A traditional approach to expressing relationships is to establish the mathematical function $f(X, W, Z)$ connecting one variable, Y, with the set of variables X, W, and Z. Such a function might be $Y = 2X + 3Z - 2W$, or $Y = 4XW/Z$. The variables on both the right and the left side of the equation are known, data are available, and it is only a question of determining the best function for describing the relationships.[24]

Let us say, however, that we have a number of variables, Y_1, Y_2, Y_3, and so on, but that we know neither the variables to enter in on the right side of the equation nor the functions involved. This might be the situation with UN voting, for example. We may know the votes of nations on one roll-call (Y_1), a second roll-call (Y_2), etc., but *not* know what nation characteristics are related to what roll-calls in what way. Moreover, we may not be able to measure well the characteristics, like nationalism,

[22] I am referring to the results of the factor-analysis research design, which include the application of a factoring technique plus simple structure rotation. (See section 5.2, below.) For those familiar with linear algebra, it may be helpful to know that a factor analysis defines a *set of basis dimensions* for the column vectors of a data matrix. Each basis dimension of a *rotated* set uniquely *generates* an independent subset of the original vectors. The basis dimensions of an unrotated set are ordered by their contributions to generating all the vectors.

[23] The configuration of vectors in Figure 5 is four-dimensional. Therefore, although the placement of the two independent axes is the best (orthogonal) definition of the two clusters in four-dimensional space, the two-dimensional figure can only display this fit imperfectly.

[24] This is where curve-fitting techniques like multiple linear and curvilinear regression analysis are helpful.

ideology, and democracy, that we feel might be most related to UN voting. In other words, we have data that we wish to explain mathematically but the variables that would give us this explanation are unknown or unmeasurable. We are then in the same dilemma the nuclear physicist was in, decades ago, in describing quantum phenomena; and like him, we resort to an untraditional mathematical approach.[25]

Let us assume that our Y variables are related to a number of functions operating linearly. That is,

$$
\begin{aligned}
Y_1 &= \alpha_{11}F_1 + \alpha_{12}F_2 + \ldots + \alpha_{1m}F_m, \\
Y_2 &= \alpha_{21}F_1 + \alpha_{22}F_2 + \ldots + \alpha_{2m}F_m, \\
Y_3 &= \alpha_{31}F_1 + \alpha_{32}F_2 + \ldots + \alpha_{3m}F_m, \\
&\;\;\vdots \\
Y_n &= \alpha_{n1}F_1 + \alpha_{n2}F_2 + \ldots + \alpha_{nm}F_m,
\end{aligned}
\qquad (1)
$$

where: Y = a variable with known data
α = a constant
F = a function, $f(\ )$, of some unknown variables.

It is crucial in understanding factor analysis to remember that *F stands for a function of variables and not a variable.* For example, the functions might be $F_1 = XW + 2Z$, and $F_2 = 3X^2Z/\sqrt{W}$. The unknown variables entering into each function, F, of equations (1) are related in unknown ways, although the equations relating the functions *themselves* are linear.[26] To take our UN voting example again, two functions, F, related to voting behavior may be ideology and nationalism. But each of these functions itself may be the result of a complex *interaction* between socioeconomic and political variables.

Within this algebraic perspective, what does factor analysis do? By application to the known data on the Y variables, *factor analysis defines the unknown F functions.* The loadings emerging from a factor analysis are the α constants. The factors are the F functions. The size of each loading for each factor measures how much that specific function is related to Y. For any of the Y variables of equations (1) we may write

$$Y = \alpha_1F_1 + \alpha_2F_2 + \alpha_3F_3 + \ldots + \alpha_mF_m, \quad (2)$$

with the F's representing factors and the α's representing loadings. We may find that some of the F functions are common to several variables. These are called *group factors* and their delineation is often the goal of factor analysis. For UN voting with each Y variable being a UN roll-call, for example, Alker and Russett (1965) found "supernationalism" and "cold war" as group factors, among others, related to voting.

Besides determining the loadings, α, factor analysis will also generate data (scores) for each case (individual, group, or nation) on each of the F functions uncovered. These derived values for each case are called *factor scores.* They, along with the data on Y and the equations (1), give a mathematical relationship among data as useful and important as the classical equations like $Y = 2X + 3Z$.

Let us look at the data of Table 1 in the context of this section. The table lists data on ten variables representing characteristics of fourteen nations. A factor analysis of these data brought out four functions, F, as linearly related to two or more variables. These results enable us to give content to equations (1). Leaving out those

[25] The factor analysis model has much in common with quantum theory. This is one reason I have argued, as I do in section 2.5 above, that factor analysis is a theoretical structure as well as a data analysis technique. See Margenau (1950, ch. 17) for a clear and simple description of quantum theory. Burt (1941) and Ahmavaara in Ahmavaara and Markkanen (1958) have also drawn the comparison of factor analysis with quantum theory.

[26] Confusion on this score has caused much unfounded criticism of factor analysis as delineating only linear relationships.

TABLE 2
CORRELATION MATRIX,* SELECTED SAMPLE DATA

Characteristic	1	2	3	4	5	6	7	8	9	10
1. GNP per capita	*.97*									
2. Trade	(.93)	*.97*								
3. Power	(.55)	(.66)	*.89*							
4. Stability	(.62)	(.55)	.25	*.63*						
5. Freedom	.31	.40	–.10	.32	*.91*					
6. Foreign Conflict	.36	.30	.25	.46	–.32	*.61*				
7. US Agreement	(.58)	(.59)	–.07	.36	(.75)	.11	*.89*			
8. Defense Budget	(.79)	(.71)	(.66)	.49	–.07	–.38	–.18	*.90*		
9. % GNP for Defense	.17	.17	.06	.15	–.28	–.44	.11	.47	*.73*	
10. Accept. of Inter'l Law	.34	.22	–.02	(.56)	(.57)	–.04	–.24	.14	–.24	*.82*

* These are product moment correlation coefficients. The data for the characteristics are given in Table 1. Elements in the principal diagonal are the squared multiple correlation coefficient of the variable with all the others. Correlations greater than or equal to .50 are shown in parentheses.

functions, F, that are multiplied by small or near-zero loadings, α, the findings are:

$$\begin{array}{lllll}
\text{Power} & = .92F_1 & & & \\
\text{Defense budget} & = .75F_1 & & & \\
\text{GNP per capita} & = .73F_1 & & & \\
\text{Trade} & = .79F_1 & + .51F_2 & & \\
\text{US agreement} & = & + .93F_2 & & \\
\text{Freedom} & = & + .77F_2 & & \\
\text{\% GNP for defense} & = & & + .79F_3 & \\
\text{Foreign conflict} & = & & + .64F_3 & \\
\text{Accept. internat'l law} & = & & & + .87F_4 \\
\text{Stability} & = & & & + .63F_4
\end{array} \qquad (3)$$

When the results are arranged in this fashion the patterns of relationship are well brought out; a pattern is now defined as a number of variables similarly related to the same F function.

4. *Interpreting Factor Tables*

Factor results are usually displayed in one or more tables. These tables consolidate more information than the length of a research report may allow to be discussed or highlighted. When a factor analysis is reported for, say, fifty variables for ninety nations, none but the results most salient to the purpose of the analysis can be evaluated. Often this only consists of describing the distinct patterns that have been found. The reader, however, may have other interests. He may wish to know how a particular variable (say, GNP per capita or riots) relates to these patterns; how two particular variables (say, trade and mail) interrelate; or how two nations (say, France and Britain) compare on their pattern profiles. This section, therefore, will describe the format and aspects of typical tables containing factor results, so that the reader may interpret those findings of most concern to him.

4.1 CORRELATION MATRIX

The most often employed techniques of factor analysis—centroid and principal axis—are applied to a matrix of correlation coefficients among all the variables. The matrix is analogous to a between-city mileage table, except that for cities we substitute variables, and for mileage we have a coefficient of correlation. Such a matrix for the data in Table 1 is shown in Table 2.

The full correlation matrix involved in the factor analysis is usually shown if the number of variables analyzed is not overly large. Often, however, the matrix is presented without comment. The factor analysis and not the correlation matrix is the aim, and it is on the factors that the discussion will focus. Nevertheless, the correlation matrix contains much useful knowledge and the reader can peruse it for relationships between pairs of variables. Specifically, the correlation matrix has the following features:

(1) The coefficients of correlation express the degree of linear relationship between the row and column variables of the matrix. The closer to zero the coefficient, the less the relationship; the closer to one, the greater the relationship. A negative sign indicates that the variables are inversely related.[27]

(2) To interpret the coefficient, square it and multiply by 100. This will give the *percent variation in common* for the data on the two variables. Thus, in Table 2, the correlation of .36 between GNP per capita and foreign conflict means that $(.36)^2 \times 100 = 13$ percent of the variation of the fourteen nations in Table 1 on these two characteristics is in common. In other words, if one knows the nation values on one of the two variables one can produce (predict, account for, generate, or explain) 13 percent of the values on the other variable. Consider the correlation of .62 between GNP per capita and stability as another example. This correlation implies that 38.4 percent $(62^2 \times 100)$ of the stability of these fourteen nations can be predicted from their GNP per capita. Assuming that the sample of nations is random, if a fifteenth nation were randomly added to the sample and only its GNP per capita were known, then its foreign conflict could be predicted within 13.0 percent and its stability within 38.4 percent of the true value.

(3) The correlation coefficient between two variables is the cosine of the angle between the variables as vectors plotted on the cases (coordinant axes). Thus, the correlation of .93 between GNP per capita and trade in Table 2 can be interpreted as a cosine of .93 (an angle of 21.3°) for the two vectors plotted on the fourteen-nation coordinate axis. (This assumes that the data are standardized.) Section 3.1, above, discusses the geometry of this interpretation.

(4) In Table 2 the principal diagonal of the correlation matrix is indicated in italics. The principal diagonal usually contains the correlation of a variable within itself, which is always 1.0. Often, however, when the correlation matrix is to be factored (using the common factor analysis model), the principal diagonal will contain *communality estimates* instead. These measure the variation of a variable in common with all the others together.

One estimate commonly employed for the communality measure is the *squared multiple correlation coefficient* (SMC) of one variable with all the others. The SMC multiplied by 100 measures the percent of variation that can be produced (predicted, accounted for, generated, or explained) for one variable from all the others. To refer to our example again: Table 2 has SMC values in the principal diagonal. For foreign conflict this is .61. This means that 61 percent of the foreign conflict data in Table 2 can be predicted from (is dependent upon) data on the remaining nine charac-

[27] The idea of a correlation coefficient gives another perspective on factor analysis. The patterns discovered by a factor analysis consist of those variables highly *inter*correlated. Thus, if variable A is highly correlated with both B and C, and if B and C are highly correlated with each other, then A, B, and C form a correlation cluster. If A, B, and C are not correlated with other variables, then they form an independent pattern that factor analysis will delineate.

A loading: degree and direction of relationship of the variables with this pattern

Separate patterns of relationships between the variables

The communality: proportion of variation of each variable involved in the patterns; sum of squared factor loadings

VARIABLES	FACTORS 1	2	3	4	h^2
1. GNP per cap	.96	-.02	-.08	-.04	.93
2. Trade	.94	.00	-.26	-.05	.95
3. Power	.58	-.42	-.42	.43	.87
4. Stability	.69	.07	.41	.08	.65
5. Freedom	.39	.84	-.03	-.07	.86
6. Foreign Conflict	.38	-.49	.41	-.04	.55
7. U.S. Agreement	.56	.61	-.17	-.42	.89
8. Defense Budget	.79	-.44	-.04	.00	.82
9. % GNP for Defense	.22	-.57	.25	-.48	.67
10. Accept. of Inter'l Law	.41	.50	.49	.40	.82
Percent Total Variance	40.9	22.5	9.1	7.6	80.1
Percent Common Variance	50.9	28.1	11.4	9.6	
Eigenvalues	4.09	2.25	.91	.76	

Percent of variation among all the variables involved in the patterns = H

Percent of variation among all the variables involved in the particular patterns = PTV

Sum of column of squared factor loadings: algebraic roots of a characteristic equation

Variation among all the variables involved in a particular pattern as a percent of that involved in all the patterns = PTV/H

FIG. 6. Unrotated factor matrix (diagrammed) from data in Table 1. (Principal axes technique. Factoring stopped at eigenvalues less than .50.)

teristics. By knowing a nation's data on the nine characteristics we could determine the incidence of foreign conflict behavior for that nation within 61 percent of the true value, on the average.

With an understanding of the key interpretations just given, the reader should be able to consult a correlation matrix and test a number of hypotheses and theories. Many of our social hypotheses involve relations between two variables, and it is in the correlation matrix that such empirical relations are described.

4.2 UNROTATED FACTOR MATRIX

Two different factor matrices are often displayed in a report on a factor analysis. The first is the *unrotated* factor matrix; it is usually given without comment. The second is the *rotated* factor matrix; it is generally the object of interpretation. The rotated matrix will be considered in the next section (4.3).

Figure 6 displays the format of an unrotated factor matrix. The columns define the factors; the rows refer to variables. In the intersection of row and column is given the *loading* for the row variable on the column factor. The h^2 column on the right of the table, and the rows beneath the table for total and common variance and eigenvalues, give additional information to be described below. The features of the matrix which are useful for interpretation are as follows:

(1) The *number* of factors (columns) is the number of substantively meaningful independent (uncorrelated) patterns of relationship among the variables.[28] Again considering the ten national characteristics, for which Figure 6 presents the unrotated matrix: as can be seen from the number of factors, there are *four* independent patterns of relationship in the data. These may be thought of as evidencing four different kinds of influence (causes) on the data, as presenting four categories by which these data may be classified, or as illuminating four empirically different concepts for describing national characterics.

(2) The loadings, α, measure which variables are involved in which factor pattern and to what degree (see equations 1 and 2, section 3.2).[29] They can be interpreted *like correlation coefficients* (see section 4.1, above). The square of the loading multiplied by 100 equals the percent variation that a variable has in common with an unrotated pattern.

One can look at the percent figure as the percent of data on a variable that can be produced or predicted by knowing the values of a case (such as a nation) on the pattern or on the other variables involved in the same pattern. Another perspective is that the percent figure is the reliability of prediction of a variable from the pattern or from the other variables in the same pattern. By comparing the factor loadings for all factors and variables, those particular variables involved in an independent pattern can be defined, and those variables most highly related to a pattern can also be seen.

For example, consider the unrotated factor loadings for the ten characteristics as shown in the first section of Table 3. Let a pattern be limited to those variables with 25 percent or more of their variation involved in a pattern (loading of .50, squared and multiplied by 100). Then the first pattern of interrelationships[30] in Table 3 involves high GNP per capita (.96), trade (.94), power (.58), stability (.69), US agreement (.56), and defense budgets (.79).

(3) The *first* unrotated factor pattern delineates the *largest pattern* of relationships in the data; the *second* delineates the next largest pattern that is independent of (uncorrelated with) the first; the *third* pattern delineates the third largest pattern that is independent of the first and second; and so on. Thus the amount of variation

[28] There is some question about the criteria for determining the exact number of factor patterns for a set of data. Variation in the number of patterns defined by different criteria is usually small and, at any rate, normally concerns the minor patterns. The larger patterns, involving many variables with high loadings, will ordinarily be found and reported regardless of the criteria employed.

[29] Note how the organization of a factor matrix is like the layout of equations 1 in section 3.2. Rather than explicitly organizing the factor results in equations, factor analysts use the matrix format, where the first column refers to the F_1 function, the second column to the F_2 function, etc., and the elements (loadings) of the matrix are the constants, α, that have been found by the analysis.

[30] These patterns differ from those given as examples for these variables in section 1. This is because we are now discussing *unrotated* patterns. The reason for the differences will be discussed below.

TABLE 3
Factor Matrices,* Selected Sample Data

	Unrotated factors[a]					Orthogonally rotated factors[b]				Primary oblique factors: Pattern factors[c]				Primary oblique factors: Structure factors[c]			
Variables	1	2	3	4	h^2	1	2	3	4	1	2	3	4	1	2	3	4
1. GNP per capita	(.96)	–.02	–.08	–.04	.93	(.73)	.47	.29	.30	(.64)	.46	.21	.19	(.78)	(.57)	.42	.46
2. Trade	(.94)	.00	–.26	–.05	.95	(.79)	(.51)	.19	.16	(.73)	(.52)	.07	.04	(.81)	(.60)	.33	.34
3. Power	(.58)	–.42	–.42	.43	.87	(.92)	–.17	–.03	–.01	(.98)	–.15	–.17	–.07	(.90)	–.08	.14	.08
4. Stability	(.69)	.07	.41	.08	.65	.32	.25	.34	(.63)	.17	.16	.31	(.60)	.40	.35	.39	(.69)
5. Freedom	.39	(.84)	–.03	–.07	.86	–.02	(.77)	–.34	.40	–.07	(.73)	–.34	.32	–.04	(.81)	–.34	.49
6. Foreign conflict	.38	–.49	.41	–.04	.55	.25	–.19	(.64)	.23	.12	–.23	(.62)	.26	.35	–.14	(.67)	.25
7. US agreement	(.56)	(.61)	–.17	–.42	.89	.13	(.93)	–.03	.11	.05	(.94)	–.03	–.01	.12	(.94)	–.01	.27
8. Defense budget	(.79)	–.44	–.04	.00	.82	(.75)	.10	.48	.12	(.67)	.09	.39	.06	(.82)	.17	(.61)	.24
9. % GNP for defense	.22	(–.57)	.25	–.48	.67	.07	–.03	(.77)	–.17	–.05	.00	(.82)	–.18	.17	–.05	(.79)	–.15
10. Accept. IR law	.41	.50	.49	.40	.82	.03	.18	–.13	(.87)	–.06	.06	–.15	(.89)	.08	.30	–.13	(.89)
Percent total variance	40.9	22.5	9.1	7.6	80.1	27.6	21.0	16.2	15.3								
Percent common variance	50.9	28.1	11.4	9.6		34.7	26.5	20.4	19.4								
Sum of squares										2.41	2.01	1.52	1.39				

* Loadings greater than an absolute value of .50 shown in parentheses.
[a] From Figure 6.
[b] Varimax rotation.
[c] Biquartimin rotation at 12 major cycles and 712 iterations.

in the data described by each pattern decreases successively with each factor; the first pattern defines the greatest amount of variation, the last pattern the least. Note that unrotated factor patterns are uncorrelated with each other.[31]

(4) The column headed "h^2" displays the *communality* of each variable. This is the proportion of a variable's total variation that is involved in the patterns. The coefficient (communality) shown in this column, multiplied by 100, gives the percent of variation of a variable in common with each pattern.

This communality may also be looked at as a measure of *uniqueness.* By subtracting the percent of variation in common with the patterns from 100, the uniqueness of a variable is determined. This indicates to what degree a variable is unrelated to the others—to what degree the data on a variable cannot be derived from (predicted from) the data on the other variables. In Table 3, for example, foreign conflict has a communality of .55. This says that 55 percent of the foreign conflict behavior as measured for the fourteen nations can be predicted from a knowledge of nation values on the four patterns; and that 45 percent of it is unrelated to the other nine characteristics.

The h^2 value for a variable is calculated by summing the squares of the variable's loadings. Thus for power in Table 3 we have $(.58)^2 + (-.42)^2 + (-.42)^2 + (.43)^2 = .87$, the h^2 value.

(5) The ratio of the sum of the values in the h^2 column to the number of variables, multiplied by 100, equals the percent of total variation in the data that is patterned. Thus it measures the order, uniformity, or regularity in the data. As can be seen in Table 3, for the ten national characteristics the four patterns involve 80.1 percent of the variation in the data. That is, we could reproduce 80.1 percent of the data for fourteen nations on these ten characteristics by knowing the nation scores on the four patterns.

(6) At the foot of the factor columns in Figure 6 (above), the *percent of total variance* figures show the percent of total variation among the variables that is related to a factor pattern. This figure thus measures the amount of data in the original matrix that can be reproduced by a pattern: it measures a pattern's comprehensiveness and strength.

The sum of these figures for each pattern equals the sum of the column of h^2 multiplied by 100. Looking along the row of percent of total variance figures and up the column of h^2, one can see how the order in the data is divided by pattern and by variable.

The percent of total variance figure for a factor is determined by summing the column of squared loadings for a factor, dividing by the number of variables, and multiplying by 100.

(7) The *percent of common variance* figures indicate how whatever regularity exists in the data is divided among the factor patterns. The percent of *total* variance figures, discussed above, measure how much of the data *variation* is involved in a pattern; the percent of *common* variance figures measure how much of the *variation* accounted for by *all* the patterns is involved in *each* pattern. These latter figures are calculated in the same way as the percent of total variance, except that the divisor is

[31] To say the factors are uncorrelated means that the factor *scores* (to be discussed in section 4.5) on the factor patterns are uncorrelated, and not necessarily the factor loadings. Factor loadings are, however, independent (orthogonal).

now the sum of the column of h^2 values, which measures the common variation among the data.

(8) The *eigenvalues* equal the sum of the column of squared loadings for each factor. They measure the amount of variation accounted for by a pattern. Dividing the eigenvalues either by the number of variables or by the sum of h^2 values and multiplying by 100 determines the percent of either total or common variance, respectively. Often only the eigenvalues are displayed at the foot of factor tables. [32]

Not all factor studies present the h^2 values or the percent of common or total variance. From the points just made, however, the reader should be able to calculate them himself. In conjunction, information on the factor loadings and on communalities should enable the reader to relate the findings in an unrotated matrix to his particular concerns.

4.3 ROTATED FACTOR MATRIX

The rotated factor matrix should not differ in format from the unrotated factor matrix, except that the h^2 may not be given and eigenvalues are inappropriate.[33]

The unrotated factors successively define the most general patterns of relationship in the data. Not so with the rotated factors. They delineate the distinct *clusters* of relationships, if such exist. This is mentioned here to alert the reader to this difference. The distinction is clarified with illustrations in section 5 below.

The following features characterize the rotated matrix:

(1) If the rotated matrix is *orthogonal,* that is mentioned in the title of the matrix (e.g., "orthogonally rotated factors"), or else the word *varimax* or *quartimax* appears in the title (these are techniques for orthogonal rotation). An orthogonally rotated matrix appears in the second section of Table 3, for the ten national characteristics of Table 1. The unrotated factor matrix from Figure 6 is also given for comparison (first section of Table 3). For an orthogonally rotated matrix the following aspects should be noted:

(1.1) Several features of the unrotated matrix are preserved by the orthogonally rotated matrix. These are the features described in section 4.2, above, under point (1) on the number of factors indicating the number of patterns, point (2) on interpreting loadings, point (6) on the percent of total variance, and point (7) on the percent of common variance.

(1.2) The h^2 values given for the unrotated factors do not change with orthogonal rotation. Hence they may be given with either the unrotated or the rotated factor matrix.

(1.3) In the unrotated matrix, factor patterns are ordered by the amount of data variation they account for, with the first defining the greatest degree of relationship in the data. In the orthogonally rotated

---

[32] The eigenvalues are extracted only if the principal axes method of factor analysis is used. An eigenvalue is the root of the characteristic equation $[R - \lambda I] = 0$, where R is the correlation matrix, λ is an eigenvalue, I is an identity matrix, and the brackets mean that the determinant is being computed. Let X be an orthogonal matrix with columns determined such that $XR = \lambda X$. Then the various roots, λ, are the eigenvalue solutions to the equation and X is the matrix of eigenvectors. The factor matrix is equal to the eigenvectors times the reciprocal square root of their associated eigenvalues.

[33] Although equal to the sum of squared factor loadings, the eigenvalue is technically a solution of the characteristic equation (see footnote 32) for the *unrotated* factors. The rotated factors are derived from these by transformation (rotation).

matrix, no significance is attached to factor order.

(1.4) Factors are uncorrelated (refer back to footnote 31). For example, in Table 3, the first orthogonally rotated pattern—which might be labeled a power pattern—is uncorrelated with the second pattern, that of UN agreement with the US.

(2) If the rotated matrix is *oblique* rather than orthogonal, the title or description of the matrix will indicate this. The title may also contain strange terms like *covarimin, quartimin,* or *biquartimin*. These refer to various criteria for the rotation and need not trouble us here.

Oblique rotation means that the best definition of the uncorrelated *and* correlated cluster patterns of interrelated variables is sought. Orthogonal rotation defines only *uncorrelated* patterns; oblique rotation has greater flexibility in searching out *patterns regardless of their correlation.* This difference is elaborated with geometric illustrations in section 5 below.

Oblique rotation takes place in one of two coordinant systems: either a system of *primary* axes or a system of *reference* axes. The reference axes give a slightly better definition of the clusters of interrelated variables than do the primary ones. For each set of axes there are two possible matrices: factor *structure* and factor *pattern* matrices. It is irrelevant to the consumer of factor results whether oblique primary or reference factors are given. There is an important difference, however, between the pattern matrix and the structure matrix.

(2.1) The *primary factor pattern* matrix and the *reference factor structure* matrix delineate the oblique patterns or clusters of interrelationship among the variables. Their loadings define the separate patterns and degree of involvement in the patterns for each variable. Unlike the unrotated or the orthogonally rotated factors, however, their loadings cannot be strictly interpreted as the correlation of a variable with a pattern, and the squared loadings do not precisely give the percent of variation of a variable involved in a pattern. Nevertheless, as in the orthogonal factor matrix, their loadings are zero when a variable is not involved in a pattern, and close to 1.0 when a variable is almost perfectly related to a factor pattern.[34] The less correlated the oblique patterns are with each other, the more their loadings are like correlations of variables with patterns. With this understanding in mind, the reader might *roughly* interpret the primary pattern matrix or reference structure matrix loadings as correlations. By squaring them and multiplying by 100 to get an idea of the *approximate* percent of variation involved, the reader will have a conceptual anchor for understanding the configuration of loadings.

The third section of Table 3 displays the (primary) oblique pattern factor matrix for the ten national characteristics. These may be compared with the orthogonally rotated factors shown in the second section. Note how much more distinct the patterns are when defined by oblique rotation (the pattern matrix) than by orthogonal rotation. There are fewer moderate loadings and more high and low loadings, thus giving a better definition of the pattern of relationships.

(2.2) The *primary factor structure* matrix and the *reference factor pattern* matrix give the correlation of each variable with each pattern. The loadings are strictly interpretable as correlations. They can be squared and multiplied by 100 to measure the per-

[34] The pattern matrix loadings are best understood as regression coefficients of the variables on the patterns.

TABLE 4
IMPORTANT DIFFERENCES AMONG ROTATED FACTOR MATRICES

		Primary axes		Reference axes	
	Orthogonal	Pattern	Structure	Pattern	Structure
Characteristics of loadings:					
Loadings distinguish clusters of interrelated variables	yes	yes	no	no	yes
Loadings measure correlation between cluster and variable	yes	no	yes	yes	no

cent of variation of a variable accounted for by a pattern. The last section of Table 3 shows the (primary) oblique structure factors matrix for the ten national characteristics. The basic difference between the primary *structure* and *pattern* matrices (or reference pattern and structure matrices) relevant for interpretation is that the primary pattern loadings best show what variables are highly involved in what clusters. The primary pattern loadings distinctly display the patterns. The primary structure loadings, however, do not display them well; instead, they measure the correlation of variables with the patterns. Note in Table 3 how much better the patterns among the ten national characteristics are differentiated by the pattern matrix loadings than by the structure matrix.

By this time, the many distinctions mentioned may have created more confusion than understanding. Table 4 shows the important differences for the several matrices considered. The difference between primary and reference matrices is one of geometric perspective. Reference matrices give a slightly better definition of the oblique patterns and are preferred by psychologists. Because of a simpler geometrical representation, however, I often use the primary matrices.

(2.3) The oblique factors will have a correlation among them as shown in a factor correlation matrix. This matrix is discussed in section 4.4, below.

(2.4) Figures for percent of common variance and percent of total variance are not given for the oblique factors. In order to get some measure of the strength of the separate oblique factor patterns, the sum of a column of squared factor loadings may be computed. This has been done in Table 3 for the oblique factors for the ten national characteristics.

4.4 FACTOR CORRELATION MATRIX

This is a correlation matrix between oblique factor patterns found through oblique rotation. Some studies may call this a matrix of factor cosines. The cosines, however, can be read as correlations between patterns, and vice versa. The characteristics of a correlation matrix described in section 4.1 apply equally well here.

What does a nonzero correlation between two factors mean? It means that the data patterns themselves have a relationship, to the degree measured by the factor correlations. The idea that patterns can be related is not strange, since we continually deal with such notions in social theorizing. Weather patterns are related to transportation patterns, for example, and a modernization pattern is related to cultural patterns. Factor analysis makes these links explicit through oblique rotation and the factor correlation matrix.

TABLE 5
FACTOR CORRELATIONS, SELECTED SAMPLE DATA

Factors	Factors 1	2	3	4
1. Power	1.00			
2. US Agreement	.09	1.00		
3. Foreign Conflict	.31	.00	1.00	
4. Accept. of Inter'l Law	.20	.28	.04	1.00

Table 5 presents the factor correlations for the oblique factors shown in Table 3. From Table 5 it can be seen that voting agreement with the US and foreign conflict patterns are in fact orthogonal (uncorrelated) to each other. The foreign conflict pattern does have some positive relationship (.31) to the power pattern, however.

Sometimes the factor correlation matrix can itself be factor-analyzed, as was the variable correlation matrix. This will uncover the pattern of relationships among the factors; the interpretation of these patterns does not differ from those found for the variable correlations. The reduction of factor interrelationships to their patterns is called *higher order* factor analysis.

4.5 FACTOR SCORE MATRIX

The factor matrix presents the loadings, α, by which the existence of a pattern for the variables can be ascertained. The *factor score matrix* gives a score for each case (such as a nation) on these patterns.

The factor scores are derived in the following way: Each variable is weighted proportionally to its involvement in a pattern; the more involved a variable, the higher the weight. Variables not at all related to a given pattern—like the case of defense budget as percent of GNP, a variable unrelated to the orthogonally rotated first pattern in Table 3—would be weighted

TABLE 6
SELECTED SAMPLE FACTOR SCORES*

Nations	Orthogonally rotated factors 1 (Power)	2 (Agree US)	3 (For. conflict)	4 (Inter'l law)
Brazil	– .389	1.053	–1.227	–1.070
Burma	– .584	.010	– .097	– .955
China	.325	–1.601	– .083	– .641
Cuba	– .662	.859	– .325	–1.183
Egypt	– .716	– .761	.448	1.331
India	.182	– .639	–1.807	.909
Indonesia	.027	– .480	– .712	– .757
Israel	–1.275	.518	.097	1.897
Jordan	–1.577	.426	2.018	– .719
Netherlands	–0.315	.570	– .638	1.292
Poland	.410	–1.382	– .296	– .267
USSR	1.129	–1.336	1.726	– .304
UK	1.081	1.178	– .024	– .404
US	2.365	1.586	.919	.906

* These are standardized regression estimates.

near zero. To determine the score for a case on a pattern, then, the case's data on each variable is multiplied by the pattern weight for that variable. The sum of these weight-times-data products for all the variables yields the factor score. Cases will have high or low factor scores as their values are high or low on the variables entering a pattern.[35] For an economic development pattern involving GNP per capita, telephones per capita, and vehicles per capita, for example, the factor scores derived from the weighted summation of data of nations on these variables would place the United States as the highest, Japan as moderate, and Yemen near the bottom.

How are factor scores to be interpreted? Simply as data on any variable are interpreted. GNP as a variable, for example, is a composite of such variables as hog

[35] These factor scores then give values for each case on the functions, F, of equations 1 through 3 in section 3.2. With the constant, α, defined by the factor matrix and the factor scores defining the value of the function, F, the factor equations are completely specified.

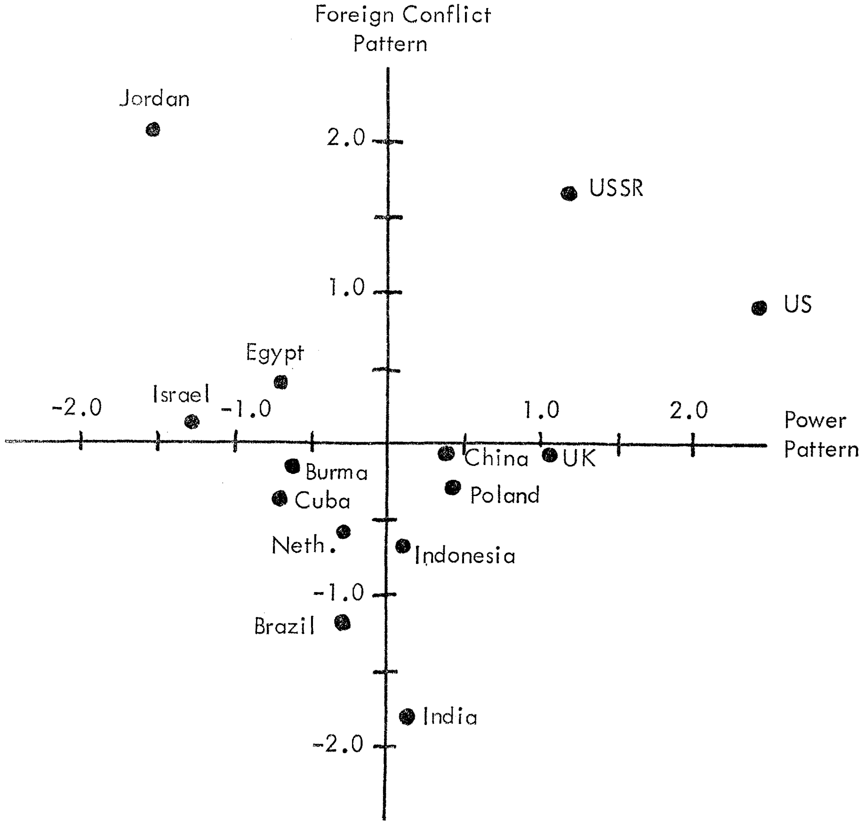

FIG. 7. Factor scores for the fourteen nations on two patterns, *power* and *foreign conflict*.

production, steel production, and vehicle production. Similarly, population is a composite of population subgroups. In the same fashion, factor scores on, say, an economic development pattern are a composite. The composite variables represented by factor scores can be used in other analyses or as a means of comparing cases on the patterns. But the factor scores have one feature that may not be shared by many other variables. They embody phenomena with a functional unity: the phenomena are highly interrelated in time or space.

Table 6 displays the factor scores for the fourteen nations on the (orthogonally rotated) four patterns of Table 3. These scores are standardized, which means they have been scaled so that they have a mean of zero and about two-thirds of the values lie between +1.00 and –1.00. Those scores greater than +1.00 or less than –1.00, therefore, are unusually high or low. Figure 2 (earlier) plots these factor scores for the four patterns separately, and Figure 7 plots scores on the power and foreign conflict patterns against each other.

4.6 INTERPRETING FACTORS

The loadings and factor scores describing the patterning of the data are found by the analysis. Once the patterns are determined, the scientist will study them and attach an appropriate label. These labels facilitate the communication and discussion of the results; they also serve as instrumental tags for further manipulation, mnemonic recall, and research. The scientist may label the patterns in any one of three ways: *symbolically, descriptively,* or *causally.*

Symbolic labels are simply any symbols without substantive meaning of their own. Their purpose is merely to denote the patterns. Three factor patterns, for example, may be labeled D_1, D_2, and D_3, or A, B, and C. A label such as D_1 can be made equivalent to a given pattern without fear of adding surplus meaning. Alternatively, to name a pattern "economic development" or "totalitarianism" might have different connotations for different people.

Although symbolic tags are precise and help avoid confusion, they also create problems in communicating research findings and comparing studies. At the present stage of research in the social sciences, symbolic tags have yet to acquire agreed-on meanings reflecting a well-tested set of patterns, as has happened with vitamins.

By contrast, *descriptive* labels like "agreement with the US," once defined, can be easily remembered and referred to without redefinition. They are clues to factor content perhaps similar to those found by other studies. A descriptive interpretation of a pattern comprises selecting a concept that will reflect the nature of the phenomena involved. If, for example, a factor analysis of nations uncovers a pattern of intercorrelated data on total area, total population, total GNP, and magnitude of resources, the pattern might be named "size." The descriptive label is meant to categorize the findings.

In *causal* naming of patterns, the scientist reasons from the discovered patterns to the *underlying* influences causing them. The causal tag is a capsule explanation of why a pattern involves particular variables. For example, a factor pattern comprising coups and purges may be symbolically labeled "C_1," descriptively named "revolution," or causally termed "modernization." In the last case, the scientist may believe that the occurrence and intercorrelation among these revolutionary actions results from the social disruption of a rapid shift from a traditional society to a modern industrial nation. As another example, a factor analysis of Congressional roll call votes may uncover a highly intercorrelated pattern of foreign policy issues. A descriptive label could be "foreign policy" pattern. Causally, however, it might be called an "isolationist" pattern by reasoning that a common isolationist attitude underlies the uniformity in foreign policy voting.

The approach to the interpretation of factor patterns is a matter of personal taste, communication, and long-run research strategy. The scientist may wish to use concepts that are congenial to the interests of the reader to facilitate communication, encourage thought about the findings, and make their use easier. There is always the danger, however, of the fallacy of misplaced concreteness. The interpretations of the findings within the research and lay community may be as much a result of the tag itself as of what the tag denotes.

5. *Factor Rotation*

Almost all published factor analyses employ rotation. A section on rotation, therefore, may aid in comprehending the nature of the results. The character of the unrotated solution will be discussed first, and after that the nature and rationale for rotation.

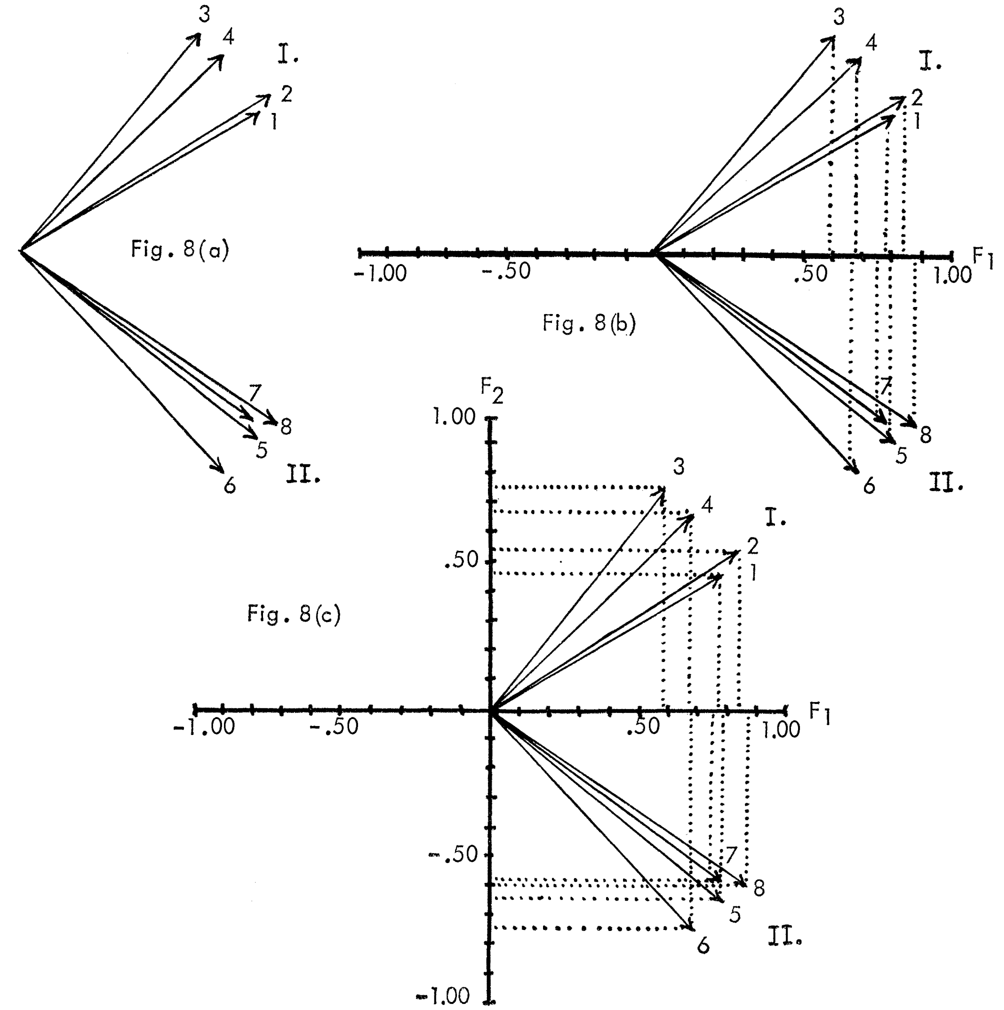

FIG. 8. Representation of eight hypothetical variables plotted according to (say) 50 cases. (a) The eight variables in two clusters, I and II. (b) The first factor, F_1, falls between the two clusters. (c) The second factor added, orthogonal to the first.

5.1 CHARACTER OF UNROTATED FACTORS

For the most popular factor-analysis techniques (centroid and principal axes), the factor patterns define decreasing amounts of variation in the data. Each pattern may involve all or almost all the variables, and the variables may therefore have moderate or high loadings for several factor patterns. To uncover the first pattern, a factor is fitted to the data to account for the greatest regularity; each successive factor is fitted to best define the remaining regularity. *The result of this is that the first unrotated factor may be located between independent clusters of interrelated variables.* These clusters cannot be distinguished

TABLE 7

UNROTATED AND ROTATED FACTOR LOADINGS*
(HYPOTHETICAL CASE)

	Unrotated		Rotated	
	F_1	F_2	F_1	F_2
1	(.76)	.48	.25	(.85)
2	(.83)	(.51)	.27	(.92)
3	(.59)	(.73)	–.02	(.95)
4	(.63)	(.66)	.01	(.91)
5	(.77)	(–.60)	(.98)	.08
6	(.64)	(–.71)	(.97)	–.08
7	(.72)	(–.53)	(.91)	.09
8	(.81)	(–.58)	(.96)	.13

* Loadings greater or equal to an absolute value of .50 are shown in parentheses.

in terms of their loadings on the first factor, although they will have loadings different in sign on the second and subsequent factors.

This situation may be illustrated in a two-factor, eight-variable case by Figure 8. Part (a) of this figure shows the eight hypothetical variables plotted according to their data for, say, 50 cases. These fifty cases are the coordinant axes of the space.[36] As shown in Figure 8(b), the first factor, F_1, falls between the two clusters of interdependent variables labeled I and II. In this position F_1 maximally reflects the variation of (i.e., has maximum loadings for) *all* eight variables. Another way of saying this is that the first factor lies along the center of gravity of all the points representing the variables. Observe that the separate loadings (dotted lines) of these variables on the first factor does not enable the clusters to be distinguished. Table 7 gives the factor loadings for the eight variables on unrotated F_1. Figure 8(c) shows the variable loadings on the second factor, which is placed at right angles (orthogonal) to the first; Table 7 also gives these loadings on unrotated F_2.

The first unrotated factor delimits the most comprehensive classification, the widest net of linkages, or the greatest order in the data. For comparative political data, a first factor could be a "political institutions" pattern, and a second might define the democratic and totalitarian poles. For international relations, the first factor could be participation in international relations, and a second factor might reflect a polarization between cooperation and conflict. For variables measuring heat, the first factor could be temperature and a second might delineate the extremes of hot and cold. For physiological measurements on adults, the first factor could be size and a second might mirror a polarization between height and girth.

5.2 CHARACTER OF ROTATED FACTORS

A scientist may rotate factors to see if a hypothesized cluster of relationships exists. This can be done by postulating the loadings of a hypothetical factor matrix and then rotating the factors to a best fit with this matrix. The truth of the hypothesis is tested by the difference between the fitted and hypothesized factor loadings.

Alternatively, a scientist may rotate the factors to control for certain influences on the results. He may rotate the first factor to a variable or group of variables and then rotate the subsequent factors to be at right angles (uncorrelated) with the first. This removes the effects of variables highly loaded on the first factor and enables us to assess the patterns independent of them.

Most often, however, a scientist rotates his factors to a *simple structure* solution. When a factor matrix is entitled "rotated factors," this almost always means a simple structure rotation. That is, each factor has

[36] At this point the reader may find it helpful to review section 3.1.

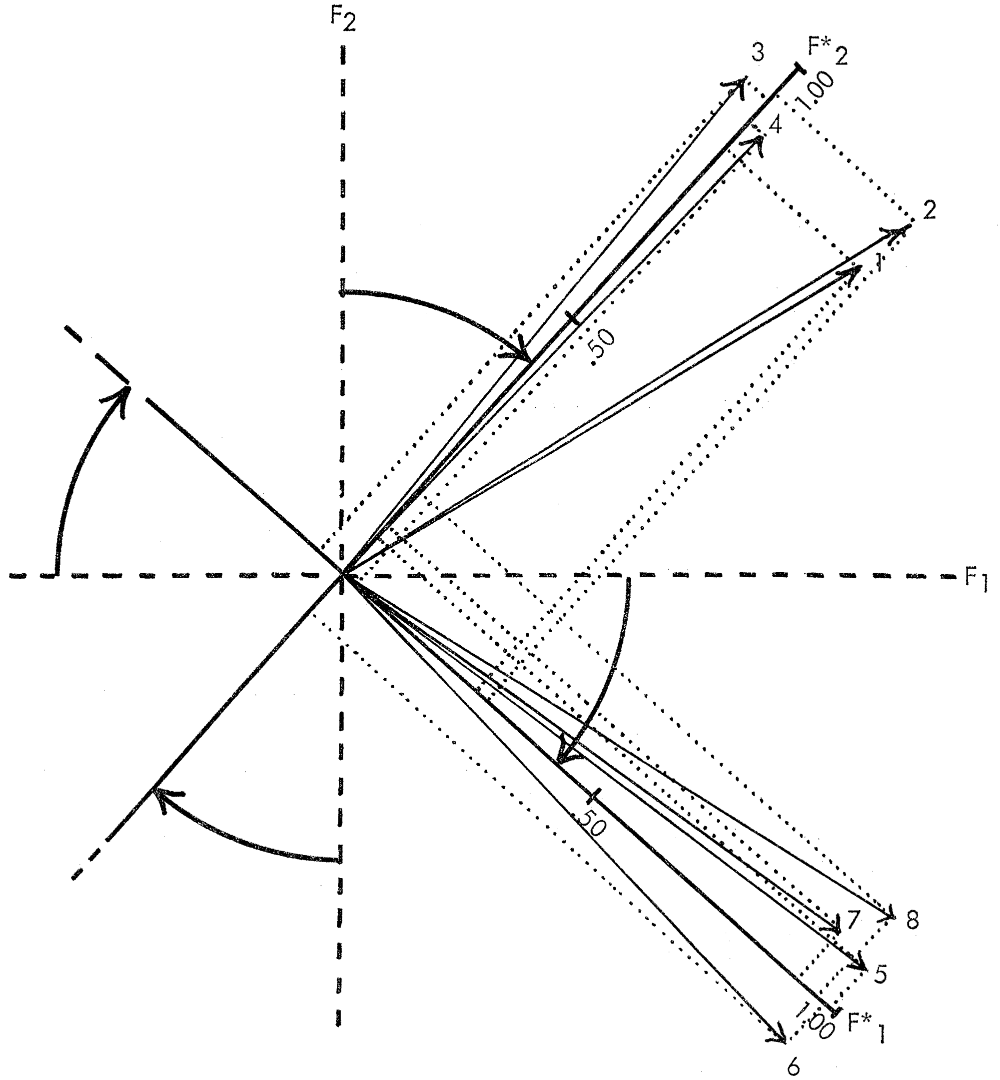

FIG. 9. A simple structure rotation of the unrotated factors shown in Figure 8(c). Arrows show direction and degree of rotation; F^*_1 and F^*_2 are the new positions of the factors.

been rotated until it defines a distinct cluster of interrelated variables. Through this rotation the factor interpretation shifts *from* unrotated factors delineating the most comprehensive data patterns *to* factors delineating the distinct groups of interrelated data.

Consider again the unrotated factors shown in Figure 8(c). A simple structure rotation would be equivalent to that shown in Figure 9. The new factor positions F^*_1 and F^*_2 now clearly distinguish the two clusters. This rotated factor matrix is shown in Table 7 alongside the unrotated factors.

TABLE 8
SIMPLE STRUCTURE TYPE OF MATRIX

Unrotated factors			Simple structure		
F_1	F_2	F_3	F^*_1	F^*_2	F^*_3
x	x		x		
x	x	x	x		
x	x	x	x	x	
x		x		x	
x	x	x		x	
x	x			x	x
x		x			x

Note: The letter "x" indicates a moderate or large factor loading.

A simple structure rotation has several characteristics of interest here:

(1) Each variable is identified with one or a small proportion of the factors. If the factors are viewed as explanations, causes, or underlying influences, this is equivalent to minimizing the number of agents or conditions needed to account for the variation of distinct groups of variables.

(2) The number of variables loading highly on a factor is minimized. This changes the unrotated factor patterns *from* being general to the largest number of variables *to* patterns involving separate groups of variables. The rotation attempts to define a small number of distinct clusters of interrelated phenomena. The simple structure type of matrix is illustrated in Table 8. The moderate and large factor loadings are indicated by x and small loadings are left blank.

(3) A major ontological assumption underlying the use of simple structure is that, whenever possible, our model of reality should be simplified. If phenomena can be described equally well using simpler factors, then the principle of *parsimony* is that we should do so. Simple structure maximizes parsimony by shifting from general factors involving all the variables to group factors involving different sets of variables.

(4) A goal of research is to generalize factor results. The unrotated factor solution, however, depends on all the variables. Add or subtract a variable from the study and the results are altered. The unrotated solution should be adjusted, then, so that the factors will be *invariant* of the variables selected. An invariate factor solution will delineate the same clusters of relationships regardless of the extraneous variables included in the analysis.

One of the chief justifications for simple structure rotation is that it determines invariant factors. This enables a comparison of the factor results of different studies. Very seldom do different scientists study exactly the same variables. But when variables overlap between studies and each study employs simple structure rotation, tests can be made to see if the same patterns are consistently emerging.

5.3 ORTHOGONAL SIMPLE STRUCTURE ROTATION

One important type of simple structure rotation is orthogonal simple structure. A second type is oblique simple structure (discussed in section 5.4 below). Factors rotated to orthogonal simple structure are usually reported simply as "orthogonal factors." Occasionally, the *varimax* or *quartimax* criteria for achieving the rotation are used to designate the factors.

Orthogonality is a restriction placed on the simple-structure search for the clusters of interdependent variables. The total set of factors is rotated as a rigid frame, with each factor immovably fixed to the origin at a right angle (orthogonal) to every other factor. This system of factors is rotated around the origin until the system is maximally aligned with the separate clusters of variables. If all the clusters are uncorrelated with each other, each orthogonal factor will be aligned with a distinct cluster. The more correlated the separate clusters are, however, the less clearly can orthogonal rotation discriminate them. Simple structure can then only be approximated, not achieved.

Whether or not uncorrelated *clusters* of relationship exist in the data, orthogonal rotation will still define uncorrelated *patterns* of relationships. These patterns may not completely overlap with the distinct clusters, but the delineation of these uncorrelated factors is useful. Results involving uncorrelated patterns are easier to communicate, and the loadings can be interpreted as correlations. Moreover, orthogonal factors are more amenable to subsequent mathematical manipulation and analysis.

5.4 OBLIQUE SIMPLE STRUCTURE ROTATION

Whereas in orthogonal simple structure rotation the final factors are necessarily uncorrelated, in oblique rotation the factors are allowed to become correlated. In orthogonal rotation the whole factor structure is moved around the origin as a rigid frame (like the spokes of a wheel around the hub) to fit the configuration of clusters of interrelated variables. In oblique rotation to simple structure, however, the factors are rotated individually to fit each distinct cluster. The relationship between the resulting factors then reflects the relationship between the clusters. Figure 10(a) shows a two-factor orthogonal simple structure rotation; for comparison, Figure 10(b) displays a simple structure oblique rotation to the same clusters.[37]

[37] Various mathematical criteria are employed to achieve oblique simple structure, and have such exotic names as *quartimin, covarimin, biquartimin, binormin, promax,* and *maxplane.* These may sometimes appear in the title of an oblique factor matrix.

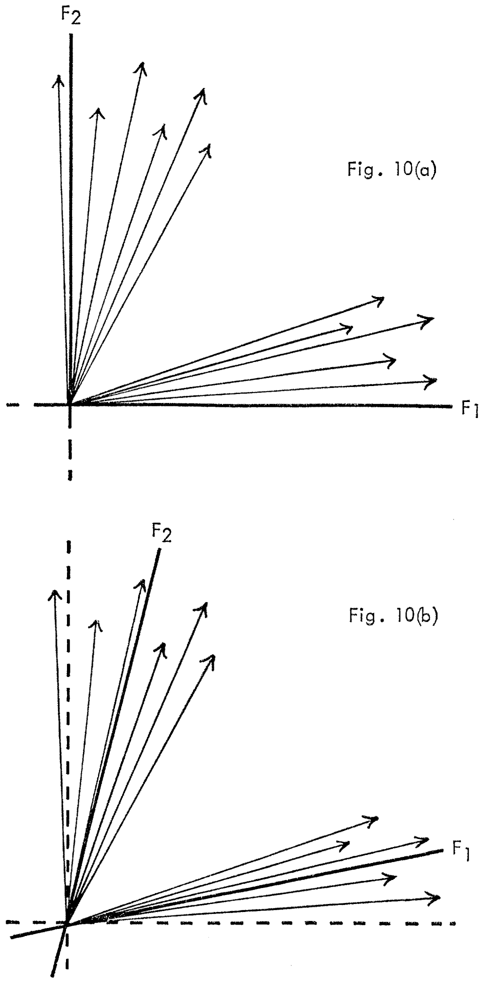

FIG. 10. (a) A two-factor orthogonal simple structure rotation. (b) A simple structure oblique rotation to the same clusters.

Orthogonal rotation is a subset of oblique rotations. If the clusters of relationships are *in fact uncorrelated,* then oblique rotation will result in orthogonal factors. Therefore, the difference between orthogonal and oblique rotation is not in discriminating uncorrelated or correlated factors but in determining whether this distinction is empirical or imposed on the data by the model.

Controversy exists as to whether orthogonal or oblique rotation is the better scientific approach. Proponents of oblique rotation usually advocate it on two grounds: First, it generates additional information; there is a more precise definition of the boundaries of a cluster, and the central variables in a cluster can be identified by their high loadings. Secondly, the correlations between the clusters are obtained, and these enable the researcher to gauge the degree to which his data approximate orthogonal factors.

Besides yielding more information, oblique rotation is justified on epistemological grounds. One justification is that the real world should not be treated as though phenomena coagulate in unrelated clusters. As phenomena can be interrelated in clusters, so the clusters themselves can be related. Oblique rotation allows this reality to be reflected in the loadings of the factors and their correlations. A second justification is that correlations between the factors now allow the scientific search for uniformity to be carried to the second order (see above, section 4.4). The factor correlations themselves may be factor-analyzed to determine the more general, the more abstract, the more comprehensive relationships and the more pervasive influences underlying phenomena.

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