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Mapping numerical magnitudes onto symbols: The numerical distance effect and individual differences in children's mathematics achievement

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ABSTRACT

Although it is often assumed that abilities that reflect basic numerical understanding, such as numerical comparison, are related to children's mathematical abilities, this relationship has not been tested rigorously. In addition, the extent to which symbolic and nonsymbolic number processing play differential roles in this relationship is not yet understood. To address these questions, we collected mathematics achievement measures from 6- to 8-year-olds as well as reaction times from a numerical comparison task. Using the reaction times, we calculated the size of the numerical distance effect exhibited by each child. In a correlational analysis, we found that the individual differences in the distance effect were related to mathematics achievement but not to reading achievement. This relationship was found to be specific to symbolic numerical comparison. Implications for the role of basic numerical competency and the role of accessing numerical magnitude information from Arabic numerals for the development of mathematical skills and their impairment are discussed.

Introduction

The understanding and processing of numerical quantity is crucial for success in education and employment. Sensitivity to numerical magnitude has been demonstrated in other species (Brannon, 2006; Dehaene, Dehaene-Lambertz, & Cohen, 1998) and emerges early in human development (Xu, 2003; Xu & Spelke, 2000). This awareness of numerical magnitude is thought to serve as a foundation

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on which competence in higher level processing such as mathematical calculation is built (Butterworth, 2005). If basic numerical magnitude processing does serve as such a foundation, a relationship should exist between mathematics achievement and tasks that reflect basic numerical processing. Understanding high-level processes through characterizing their connection to more basic cognitive abilities has proven to be fruitful in other cognitive domains such as reading, where basic skills (e.g., phonemic and phonological awareness) have been shown to be crucial for an individual's ability to read (Snowling & Hulme, 1994; Stanovich, Siegel, & Gottardo, 1997; Wagner et al., 1997). Against this background, we questioned whether similar developmental relationships exist between basic and higher level skills in the domain of numerical cognition.

The two basic abilities thought to index numerical magnitude processing are numerical estimation and numerical comparison. Recently, Booth and Siegler demonstrated that developmental changes in the performance of numerical estimation—and in particular number line estimation—are related to children's mathematics achievement and their ability to solve novel arithmetic problems (Booth & Siegler, 2006, in press; Siegler and Booth, 2004). These findings demonstrated how a basic skill, such as accurately placing a target number onto a number line, is related to success in higher level numerical and mathematical skills.

However, number line estimation is a task that reflects only some aspects of basic numerical competency. For example, although recent research has begun to incorporate nonsymbolic stimuli into number line estimation tasks (Ebersbach, Luwel, Frick, Onghena, & Verschaffel, 2008), an important facet of the relationship between basic numerical understanding and mathematics performance that has not been addressed through the use of number line estimation tasks is the role of nonsymbolic number processing. The representation and processing of nonsymbolic numerical magnitude has been previously suggested as a foundation on which language-based symbolic instruction in mathematics should be built (Barth, La Mont, Lipton, & Spelke, 2005). Understanding the role of nonsymbolic number processing is important because it can help to clarify whether the relationship between mathematical skills and basic numerical magnitude processing is due to individual differences in the format-independent mental representation of numerical magnitude or to the ability to access numerical magnitudes from abstract symbols such as Arabic numerals.

In other words, a relationship between basic numerical magnitude processing and mathematical ability that holds across both symbolic and nonsymbolic numerical processing would support the hypothesis that individual differences in the representational features of numerical magnitude are an important predictor of mathematics achievement. In contrast, a relationship between mathematical skills and basic numerical understanding found only for symbolic number processing may suggest that the relationship reflects something about symbolic processing rather than representation of numerical magnitude per se. A numerical magnitude comparison task can be an effective way to address this question because this type of task can use both symbolic and nonsymbolic stimuli.

In this context, it is important to clarify how numerical comparison tasks tap into mental representations of numerical magnitude. When adults and school-aged children compare symbolic or nonsymbolic numerical stimuli for their relative magnitude, the numerical distance effect (NDE) is obtained; participants are faster and more accurate at making responses when the numerical distance separating two numbers is relatively large, such as 7 (2 vs. 9), than when it is small, such as 2 (8 vs. 6) (Dehaene, Dupoux, & Mehler, 1990; Moyer & Landauer, 1967). The NDE is thought to arise from noisy mapping between external and internal representations of numerical magnitude. Specifically, numerical magnitudes that are closer on the number line are thought to share more mental representational features than are those that are farther apart. As a result, distinguishing between two numerical magnitudes is more difficult for quantities that are numerically closer together. Individuals with larger distance effects, therefore, are thought to have less distinct representations of numerical magnitude.

Several models have been proposed to relate the NDE to numerical representation: the "accumulator" model (Cordes, Gelman, Gallistel, & Whalen, 2001), the "compressed number line" model (Dehaene, 1992), and the "numerosity code" model (Zorzi & Butterworth, 1999). Although these models differ in their characterization of the mental representation of quantity, they converge on the notion that the NDE is an important metric for modeling representations of numerical magnitude. Accordingly, the NDE frequently has been used in behavioral paradigms to quantify mental representation of numerical magnitude in adults and children (Buckley & Gillman, 1974; Butterworth, 2005; Duncan

& McFarland, 1980; Girelli, Lucangeli, & Butterworth, 2000; Holloway & Ansari, in press; Landerl, Bevan, & Butterworth, 2004; Moyer & Landauer, 1967; Rousselle & Noël, 2007; Rousselle, Palmers, & Noël, 2004; Rubinsten, Henik, Berger, & Shahar-Shalev, 2002; Sekuler & Mierkiewicz, 1977; Verguts & Van Opstal, 2005) as well as in neuroimaging paradigms to explore the neural representation of quantity (Ansari, Garcia, Lucas, Hamon, & Dhital, 2005; Dehaene et al., 1998; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Pinel, Dehaene, Riviere, & Le Bihan, 2001; Price, Holloway, Rasanen, Vesterinen, & Ansari, 2007; Temple & Posner, 1998). Against this background, the NDE is a paradigm that is well suited to explore the relationships between basic processing of numerical magnitude and higher level numerical and mathematical skills.

Indirect evidence for an association between basic numerical magnitude processing (as measured by number comparison) and mathematical skills has been revealed in studies of individuals with known mathematical disabilities resulting from developmental dyscalculia or genetic disorders. Dyscalculic children have been shown to perform differently on tasks of basic numerical magnitude processing compared with typically developing children (Landerl et al., 2004). Atypical performance on number comparison tasks has also been demonstrated in individuals with Williams syndrome (Paterson, Girelli, Butterworth, & Karmiloff-Smith, 2006) and chromosome 22q.11.2 deletion (Simon, Bearden, Mc-Ginn, & Zackai, 2005). These findings indicate that basic processing of numerical magnitude is affected in children who present with mathematical difficulties; therefore, the origins of such deficits may lie in these foundational competencies. However, evidence from a clinical population should be understood as suggestive rather than conclusive evidence for the importance of basic representations of numerical magnitude for the development of mathematical skills among typically developing children. Thus, an examination of the typically developing population is necessary in an effort to evaluate the generality of a link between numerical comparison and mathematical competency in both typically and atypically developing children.

It has been demonstrated that the size of the NDE decreases over developmental time in young school-aged children, and several potential sources for this change have been proposed (Duncan & McFarland, 1980; Holloway & Ansari, in press; Sekuler & Mierkiewicz, 1977). The early elementary school years, while proving to be a time during which the NDE is quite malleable, are also a time during which formal mathematics is first learned. Therefore, this period provides an appropriate window during which to investigate how individual differences in basic numerical magnitude processing relate to mathematical knowledge. Against this background, it can be hypothesized that individual differences in the size of the NDE might be related to between-participant variability in mathematical competence.

In light of the previous discussion, the purpose of the current study was twofold. First, we sought to conduct a direct examination in the typically developing population of the relationship between individual differences in the NDE and children's mathematical competence as measured by standardized tests. Second, we sought to examine whether symbolic and nonsymbolic comparison differentially predicted children's mathematics performance. To achieve these aims, children's reaction times for both symbolic (Arabic numerals) and nonsymbolic (arrays of squares) numerical comparisons, as well as standardized measures of their mathematical skills, were obtained. We also collected standardized measures of participants' reading scores to help assess the specificity of any potential relationships between the distance effect and mathematics achievement.

Method

Participants

A total of 87 children, ages 6 years ($n = 29$, 11 boys and 18 girls, range = 6 years 1 month to 6 years 11 months), 7 years ($n = 31$, 15 boys and 16 girls, range = 7 years 0 months to 7 years 11 months), and 8 years ($n = 27$, 13 boys and 14 girls, range = 8 years 0 months to 8 years 11 months) participated in the study. Children were recruited from elementary schools in New Hampshire and Vermont in the northeastern United States. Children were primarily of White (non-Hispanic) ethnic background and attended schools in an area of New Hampshire and Vermont composed primarily of middle- to upper middle-class families. The procedure was approved by the Committee for the Protection of Human Subjects at Dartmouth College.

Stimuli and procedure

Children were tested in their school using a 15-inch touch screen monitor (Planar Systems, Beaverton, OR, USA) connected to a Dell D810 laptop computer. Stimuli were presented using E-prime software (Psychological Software Tools, Pittsburgh, PA, USA). Children touched the side of the screen displaying the larger of two single-digit Arabic numerals or the more numerous of two arrays of squares. Children were instructed to respond as quickly and as accurately as possible. Stimuli remained on the screen until children responded. Between trials, a central fixation dot appeared for 1000 ms. In each condition, children were presented with a total of 72 trials consisting of combinations of one to nine squares or the Arabic numerals 1 to 9. The numerical distance between stimuli ranged from 1 to 6, with 12 comparison trials per distance. There was an equal number of pairs for each distance. Reaction times were removed for trials on which children made erroneous responses. In the nonsymbolic condition, the individual area, total area, and density of the squares were systematically varied to ensure that children could not reliably use nonnumerical cues to make a correct decision. Specifically, in 4 of each 12 distance pairings, the smaller numerosity was presented with the larger overall area. In another 4 exemplars of a particular distance, the smaller numerosity had a smaller overall area. The final 4 pairings presented both the small and large numerosities with equal overall areas. Within each group of 4 pairs, 2 of the small numerosity stimuli had larger density and 2 had smaller density compared with the large numerosity with which each was paired. In addition, individual square sizes were varied over all stimuli. These variations ensured that numerosity could not be reliably predicted from continuous variables. A vertical black line replaced the fixation dot in the center of the screen to assist participants in understanding which individual squares belonged with which group.

Children's mathematical skills were assessed using the Mathematics Fluency and Calculation subtests of the Woodcock–Johnson III Tests of Achievement, which were combined to form a Composite Mathematics score of mathematics achievement (Woodcock, McGrew, & Mather, 2001). The Mathematics Fluency subtest requires participants to answer as many single-digit addition, subtraction, and multiplication problems as possible within a 3-min period. In the Calculation subtest, participants solve increasingly difficult calculation problems without a time limit. Importantly, neither of the subtests contains a numerical comparison component, thereby excluding the possibility that any of the correlations presented below are driven by correlations between two different measures of numerical magnitude comparison.

It is possible that potential relationships between the distance effect and mathematics achievement could be related to more general cognitive differences between individuals. Such general cognitive differences would presumably be related to both mathematical skill and other crucial cognitive abilities such as reading. Therefore, we also collected measures of children's reading ability to help assess the specificity of the relationships between basic numerical abilities and mathematics achievement and to rule out the possibility that any relationships are related to individual differences in more general cognitive processes rather than to differences in the processing and representation of numerical magnitude. Reading skills were tested using the Letter–Word Identification and Word Attack subtests, which were combined to form a Composite Reading score of basic reading ability. During the Letter–Word Identification task, participants needed to correctly read real words aloud to the experimenter. Similarly, the Word Attack task required participants to correctly pronounce pseudowords to the experimenter. Neither test was timed. Standardized test scores in Composite Reading (M = 110.7, SD = 12.58, range = 80–148), Mathematics Fluency (M = 102.3, SD = 10.66, range = 78–138), and Calculation (M = 105.2, SD = 12.35, range = 83–144) were within the normal (corresponding to standard scores of 85–115) to above normal range for all but five children, with two scoring 80 and 81 in Composite Reading, two scoring 83 and 84 in Calculation, and one scoring 78 in Mathematics Fluency. In an effort to avoid the inclusion of children with extremely high or low standard scores on the tests of mathematical competence and reading, we included only those scores that were within 2 standard deviations of the population mean, that is, scores between 70 and 130. This resulted in the scores of some children being excluded as outliers. One child was removed from the Mathematics Fluency correlational analyses (standard score = 138). Two children were removed from the Calculation correlational analyses (standard scores = 139 and 144). Three children were removed from the Composite Mathematics correlational analyses (standard scores = 131, 135, and 141). Finally, five children were removed from the correlational analyses using Composite Reading standard scores (132, 136, 138, 138, and 148).

Results

Reaction time

A mixed design analysis of variance (ANOVA) using distance (six levels: mean reaction time of Distances 1–6) and task (two levels: symbolic and nonsymbolic) as within-participants variables and age (three levels) as a between-participants variable was conducted on participants' reaction times. Because the assumption of sphericity was violated, all within-participants effects are reported using the Greenhouse–Geisser adjustment. Means and standard deviations of reaction time for each distance can be found in Table 1.

We found a significant main effect of distance, with smaller distances having longer reaction times than larger distances, $F(3, 179) = 123.35$, $p < .001$, $\eta^2 = .59$, power = 1. More specifically, Bonferroni-corrected t tests demonstrated that a smaller distance always took longer to compare than a larger distance with the exception of Distances 4 and 5, whose reaction times were not significantly different. We found a main effect of task, characterized by longer reaction times in nonsymbolic comparison, $F(1, 84) = 6.08$, $p < .05$, $\eta^2 = .06$, power = .68. We also found a main effect of age, $F(2, 84) = 17.78$, $p < .001$, $\eta^2 = .29$, power = 1, which reflects the finding that the 7- and 8-year-olds did not significantly differ in overall reaction time but were significantly faster than the 6-year-olds, as demonstrated by post hoc Bonferroni-corrected t tests.

In addition, we found a significant Distance $\times$ Task interaction, $F(3, 179) = 9.29$, $p < .001$, $\eta^2 = .10$, power = .99. To examine the nature of this interaction, Bonferroni-corrected t statistics compared reaction times of both tasks at each level of distance. The critical t statistic for significance at the .05 alpha level adjusted for six comparisons was 2.45. The latency required to compare the numerical distance of 1 was greater in the nonsymbolic task than in the symbolic task, $t(86) = 3.59$, $p < .05$. However, the tasks did not differ significantly in latency at Distances 2 through 6, $ts(86) < 2.32$. Finally, we found an interaction between age and distance that approached significance, $F(5, 179) = 2.28$, $p = .06$, $\eta^2 = .05$, power = .67. Although not significant, an inspection of the means in Table 1 suggests greater differences in reaction times between the levels of distance for the 6-year-olds than for the 7- and 8-year-olds. Therefore, the slope relating distance and reaction time seems to be greater for the 6-year-olds compared with the two older groups of children. Age did not interact with the main effect of task or the Task $\times$ Distance interaction.

To quantify individual differences in the size of the NDE, reaction times for comparisons with large numerical distances (mean of Distances 5 and 6) were subtracted from those with small numerical distances (mean of Distances 1 and 2) separately for the symbolic and nonsymbolic tasks. These values were then divided by the reaction times for large distance comparisons (mean of Distances 5 and 6) for each child to yield a measure that quantifies the increase in reaction time from large to small distance

Table 1
Means and standard deviations of reaction times for each level of distance of both symbolic and nonsymbolic tasks for each age group

	Symbolic comparison reaction times (ms)				Nonsymbolic comparison reaction times (ms)			
	6-year-olds	7-year-olds	8-year-olds	Overall	6-year-olds	7-year-olds	8-year-olds	Overall
1	1587 (420)	1272 (278)	1143 (160)	1337 (356)	1742 (641)	1489 (470)	1286 (348)	1511 (531)
2	1480(344)	1211 (264)	1092 (148)	1264 (309)	1476 (375)	1347 (363)	1174 (233)	1336 (351)
3	1393 (222)	1127 (224)	1052 (127)	1192 (244)	1345 (248)	1196 (257)	1043 (184)	1198 (261)
4	1307 (279)	1064 (214)	967 (153)	1115 (262)	1333 (256)	1132 (252)	1015 (151)	1163 (260)
5	1287 (280)	1074 (200)	961 (106)	1110 (247)	1273 (243)	1093 (217)	993 (144)	1122 (235)
6	1231 (254)	1024 (184)	931 (115)	1064 (229)	1224 (242)	1033 (222)	979 (134)	1080 (229)

Note. Standard deviations are in parentheses.

while accounting for the individual differences in reaction time. Finally, although we are testing directional hypotheses, significance values of all correlational analyses come from a two-tailed distribution.

A comparison of this measure of the distance effect between formats revealed that the symbolic distance effect was, on average, significantly smaller than the distance effect for nonsymbolic comparison, $t(86) = -3.97$, $p < .001$. A correlational analysis across all age groups (see Table 2 and Figs. 1 and 2) revealed a negative correlation between Calculation scores and the size of the symbolic NDE, $r(85) = -.222$, $p < .05$. In addition, this analysis revealed a significant negative correlation between the symbolic NDE and the Mathematics Fluency subtest standard scores, $r(86) = -.339$, $p < .01$, as well as a trend toward significance for the Composite Mathematics standard scores and the symbolic NDE, $r(84) = -.188$, $p = .087$. No significant correlation was found between the symbolic NDE and Composite Reading scores, $r(80) = .006$, *ns*. In contrast to the symbolic distance effect, the nonsymbolic distance effect was found to be uncorrelated with both mathematics achievement and the symbolic distance effect, although it was significantly correlated with the Composite Reading scores, $r(77) = .249$, $p < .05$.

To investigate the extent to which the symbolic distance effect explains unique variance in Mathematics Fluency and Calculation, hierarchical regression analyses were conducted. Five steps (see Table 3) were sequentially included in these analyses to determine whether the symbolic distance effect explained variance within Mathematics Fluency and Calculation over and above age differences (Step 1), processing speed required for symbolic comparison (Step 2), nonsymbolic numerical representation (Step 3), and more general cognitive capacities that might also be reflected in other cognitive skills such as reading (Step 4). Results from this model demonstrate that even after controlling for

Table 2
Correlations between standardized test scores and the distance effect calculated from reaction time scores

Measure	1	2	3	4	5	6	7	8
1. Symbolic distance effect	–	.151	.327**	.254*	−.339**	−.222*	−.188	.006
2. Nonsymbolic distance effect		–	.177	.526***	.037	−.073	−.015	.249*
3. Mean RT symbolic comparison			–	.728***	−.365**	−.071	−.045	.062
4. Mean RT nonsymbolic comparison				–	−.213*	−.098	−.06	.142
5. Mathematics Fluency					–	.545***	.659***	.286*
6. Calculation						–	.964***	.341**
7. Composite Mathematics							–	.436***
8. Composite Reading								–

Note. RT, response time.
* $p < .05$.
** $p < .01$.
*** $p < .001$.

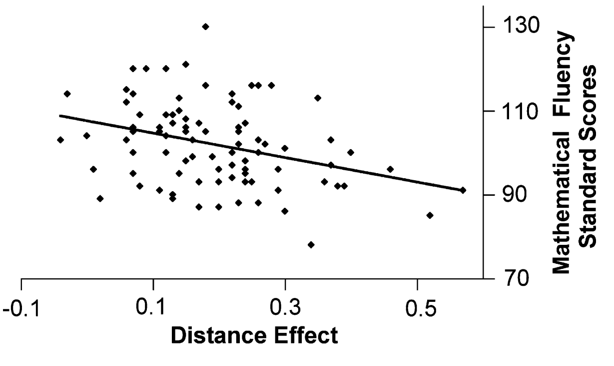

Fig. 1. Scatterplot showing significant correlation between symbolic NDE and standard scores on the Mathematics Fluency subtest of the Woodcock–Johnson III battery for all participants. The solid line represents the linear regression line for this relationship.

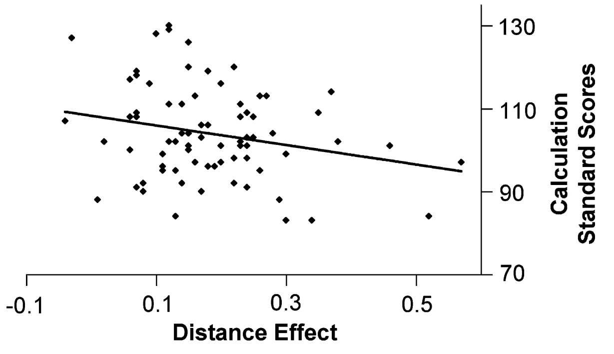

Fig. 2. Scatterplot showing significant correlation between symbolic NDE and standard scores on the Calculation subtest of the Woodcock–Johnson III battery for all participants. The solid line represents the linear regression line for this relationship.

Table 3
Hierarchical regression analysis predicting Mathematics Fluency and Calculation

Step	Mathematics Fluency				Calculation			
	Predictor	β	R^2	ΔR^2	Predictor	β	R^2	ΔR^2
1	Age	−.147	.009	.009	Age	−.174	.008	.008
2	Symbolic mean RT	−.401**	.129	.142**	Symbolic mean RT	−.108	.029	.021
3	Nonsymbolic NDE	.063	.129	.011	Nonsymbolic NDE	−.126	.032	.003
4	WJ reading SS	.294**	.209	.087**	WJ reading SS	.374**	.170	.125**
5	Symbolic NDE	−.236*	.251	.049*	Symbolic NDE	−.190	.202	.032

Note. RT, response time; NDE, numerical distance effect; WJ, Woodcock–Johnson III battery; SS, standard score.
* $p < .05$.
** $p < .01$.

these other factors, the symbolic distance effect explained a significant amount of unique variance in Mathematics Fluency, $\Delta R^2 = .049$, $F(1, 72) = 5.07$, $p < .05$, as well as a marginally significant amount of unique variance in Calculation, $\Delta R^2 = .03$, $F(1, 72) = 2.85$, $p = .09$. In a complementary set of analyses (see Table 4), Steps 3 and 5 were switched to determine whether individual differences in the nonsymbolic distance effect could explain variance over and above age, mean reaction time, reading ability, and the symbolic distance effect. This analysis revealed that the nonsymbolic distance effect did not explain significant variance in either Mathematics Fluency scores, $\Delta R^2 = .004$, $F(1, 72) = 0.38$, *ns*, or Calculation scores, $\Delta R^2 = .014$, $F(1, 72) = 1.30$, *ns*.

Further analysis was conducted on the significant correlations between the distance effect and mathematics achievement measures to examine the relationship between the symbolic NDE and test

Table 4
Hierarchical regression analysis predicting Mathematics Fluency and Calculation

Step	Mathematics Fluency				Calculation			
	Predictor	β	R^2	ΔR^2	Predictor	β	R^2	ΔR^2
1	Age	−.147	.009	.009	Age	−.174	.008	.008
2	Symbolic mean RT	−.401**	.151	.142**	Symbolic mean RT	−.108	.029	.021
3	Symbolic NDE	−.236*	.200	.049*	Symbolic NDE	−.190	.068	.039
4	WJ reading SS	.294**	.295	.095**	WJ reading SS	.374**	.187	.143**
5	Nonsymbolic NDE	.063	.299	.004	Nonsymbolic NDE	−.126	.202	.014

Note. RT, response time; NDE, numerical distance effect; WJ, Woodcock–Johnson III battery; SS, standard score.
* $p < .05$.
** $p < .01$.

scores for each age group. For the 6-year-olds, we found significant correlations between the NDE and Mathematics Fluency scores, $r(29) = -.373$, $p < .05$, Calculation scores, $r(28) = -.436$, $p < .05$, and Composite Mathematics standard scores, $r(27) = -.454$, $p < .05$. For the 7-year-olds, the NDE was significantly related to Mathematics Fluency, $r(31) = -.459$, $p < .01$, but was not significantly related to either Calculation, $r(31) = -.149$, *ns*, or Composite Mathematics, $r(31) = -.03$, *ns*. No significant relationships between the NDE and either Mathematics Fluency, $r(31) = -.171$, *ns*, Calculation, $r(31) = -.113$, *ns*, or Composite Mathematics, $r(31) = -.096$, *ns*, were found in 8-year-olds.

We then examined whether these age-related differences in the strength of the correlations between the symbolic NDE and Mathematics Fluency, Calculation, and Composite Mathematics scores were statistically significant. Correlations were first transformed into Fisher's z statistics and then compared using a z test. For the relationship between the symbolic NDE and Mathematics Fluency scores, the correlation for the 6-year-olds was not significantly different from that for either the 7-year-olds, $z = 0.37$, *ns*, or the 8-year-olds, $z = -0.77$, *ns*. The difference between the correlations for the 7- and 8-year-olds was also not significant, $z = -1.13$, *ns*. Similarly, for the relationship between the symbolic NDE and Calculation scores, the correlation for the 6-year-olds was again not significantly different from the correlation for either the 7-year-olds, $z = -1.12$, *ns*, or the 8-year-olds, $z = -1.22$, *ns*. In addition, the correlation for the 7-year-olds did not differ significantly from the correlation for the 8-year-olds, $z = -0.14$, *ns*. Finally, no significant differences were found between the strength of the correlation between the symbolic NDE and Composite Mathematics scores of the 6-year-olds and either the 7-year-olds, $z = -1.65$, *ns*, or the 8-year-olds, $z = -1.37$, *ns*. Likewise, no difference was found between the correlations for the 7-year-olds and the 8-year-olds, $z = 0.21$, *ns*.

Accuracy

Like the reaction time data, accuracy (Table 5) was also analyzed using a mixed design ANOVA using distance (six levels: mean proportions of correct trials of Distances 1–6) and task (two levels: symbolic and nonsymbolic) as within-participants variables and age (three levels) as a between-participants variable. Again, because the assumption of sphericity was violated, the Greenhouse–Geisser adjustment was used on all within-participants effects and interactions. A main effect of distance was found, $F(4, 308) = 161.91$, $p < .001$, $\eta^2 = .66$, power = 1. Bonferroni-corrected t statistics indicated that accuracy at each level of distance was significantly different from that at every other level of distance. Moreover, smaller distances were always associated with less accuracy than were larger distances. There were two exceptions to this pattern: Accuracy in Distance 4 was not significantly different from accuracy in Distance 5, and accuracy in Distance 5 was not significantly different from accuracy in Distance 6. There was also a significant Distance × Task interaction, $F(4, 300) = 7.73$, $p < .001$, $\eta^2 = .08$, power = .99. This interaction was further characterized with Bonferroni-corrected t statistics used to compare accuracy of both tasks at each level of distance. Again, the critical t statistic for significance at the .05 alpha level adjusted for six comparisons was 2.45. Consistent with the reaction time data, accuracy in the nonsymbolic task was significantly worse than accuracy in the symbolic task at Distance 1, $t(86) = 3.34$, $p < .05$. Furthermore, as with the reaction time data, no significant differences between the two tasks for Distances 2 through 6 were found, $ts(86) < 2.33$.

Table 5
Means and standard deviations of accuracy scores (percentage correct) for each level of distance of both symbolic and nonsymbolic tasks for each age group

	Symbolic comparison (% correct)				Nonsymbolic comparison (% correct)			
	6-year-olds	7-year-olds	8-year-olds	Overall	6-year-olds	7-year-olds	8-year-olds	Overall
1	83 (12)	81 (12)	81 (15)	82 (13)	73 (15)	77 (13)	73 (13)	75 (13)
2	85 (11)	96 (11)	90 (10)	87 (11)	81 (13)	88 (9)	84 (13)	85 (12)
3	91 (12)	91 (9)	94 (8)	92 (10)	90 (10)	94 (8)	92 (9)	92 (9)
4	93 (9)	95 (8)	95 (8)	94 (8)	92 (11)	97 (6)	96 (6)	95 (8)
5	94 (9)	94 (9)	96 (9)	95 (9)	97 (5)	97 (6)	98 (5)	97 (5)
6	95 (10)	97 (8)	98 (7)	97 (8)	96 (7)	99 (5)	97 (6)	98 (6)

Note. Standard deviations are in parentheses.

Table 6
Correlations between standardized test scores and the distance effect calculated from accuracy scores

Measure	1	2	3	4	5	6	7	8
1. Symbolic distance effect	–	.021	.351**	.220*	−.033	−.192	−.169	−.016
2. Nonsymbolic distance effect		–	−.116	.654***	−.129	−.124	−.100	−.100
3. Mean RT symbolic comparison			–	.015	.115	−.088	−.031	−.067
4. Mean RT nonsymbolic comparison				–	−.103	−.158	−.128	.166
5. Mathematics Fluency					–	.476***	.659***	.286*
6. Calculation						–	.964***	.365**
7. Composite Mathematics							–	.436***
8. Composite Reading								–

Note. RT, response time.
* $p < .05$.
** $p < .01$.
*** $p < .001$.

An identical correlational analysis across age groups was also conducted using the distance effect calculated from accuracy scores (Table 6). The distance effect from accuracy scores was calculated in the same manner as was the distance effect from reaction time scores. Although the symbolic distance effect was again smaller than the nonsymbolic distance effect, $t(86) = 6.67$, $p < .001$, no significant correlations were found between any of the mathematics achievement measures and the distance effect calculated from accuracy data. Note that the *t* statistics comparing the symbolic and nonsymbolic distance effects in the reaction time and accuracy data are in opposite directions. This is due to the fact that the scales of these two variables are reversed; that is, smaller reaction times and larger accuracies reflect better performance.

Discussion

It has long been assumed that components of basic numerical understanding serve as a cognitive scaffold for the development of more complex mathematical skills (Dehaene, 1997). Although suggested in a previous study (Durand, Hulme, Larkin, & Snowling, 2005), this relationship has so far not been investigated systematically. In addition, although other tasks such as number line estimation (Booth & Siegler, 2006, in press; Siegler and Booth, 2004) have shown a relationship between basic processing and higher mathematical skills, the origin and nature of this relationship remain unclear.

To address this outstanding question, the current study examined whether the ability to compare symbolic and nonsymbolic numerical magnitudes is related to individual differences in children's standardized mathematics achievement. Initial analysis across age groups revealed a relationship between children's mathematical competence and the size of their symbolic NDE calculated from reaction time; large distance effects are associated with relatively lower scores on the mathematics tests. This finding suggests that the basic ability to discriminate the relative magnitude of a number is associated with an individual's ability to perform simple arithmetic. Children with relatively larger distance effects on reaction time may have noisier mappings between Arabic numerals and the magnitudes they represent. These noisy mappings could then lead to greater difficulty in efficiently accessing exact representations of symbolic numerical magnitudes in the process of calculation.

The specificity of the relationship for Arabic numerals, as revealed by the absence of similar results for the nonsymbolic NDE indicates that symbolic and nonsymbolic distance effects index systems that relate differentially to higher level mathematical skills. This specificity suggests that the relationship between mathematics achievement and the NDE is driven by processes involved in accessing the meaning of Arabic numerals. It is possible that the process of connecting a symbol with a quantity presumably makes the representation of that quantity more discrete than presymbolic representations. The increase in distinctness of representation gained through this symbolic mapping could, therefore, make the process of comparing symbolically represented quantities easier. Indeed, recent neuroimaging evidence in adults has shown that the neural representation of symbolic numerical magnitudes is more finely tuned than that of nonsymbolic numerical magnitudes (Piazza, Pinel, Le Bihan, & Dehaene, 2007). By virtue of providing more discrete representations, a strong connection between symbol and referent could result in a relatively smaller distance effect by reducing the relative overlap between the two symbolic quantities being compared. Better mapping between numerals and numerical magnitudes would also result in decreased latencies for numerical comparison across all levels of distance. At the same time, stronger connections between numerical symbols and the quantities they represent could also afford more fluent manipulation of these quantities when completing arithmetic problems such as those tested in the current experiment by the standardized mathematics achievement measures. In this way, children who have developed more distinct and automatic connections between numerals and their meanings will be able to compare two numbers more quickly and with smaller distance effects as well as perform better on standardized achievement measures than their peers whose symbol-to-referent connections are less strong.

Our data reflecting a relationship between symbolic, but not nonsymbolic, numerical comparison and mathematics performance are consistent with the findings of Rousselle and Noël (2007), who showed that performance is atypical in children with mathematical learning disabilities when making symbolic, but not nonsymbolic, comparisons. It is important to note a difference between our results and the findings of Rouselle and Noël in the manner in which the distance effect relates to mathematical abilities. In their study, children with mathematics-specific learning disabilities, who were defined by greatly impaired performance on standardized tests, were shown to have smaller distance effects than were children with typical performance on tests of mathematical competence. This pattern is the opposite of the relationship exposed in our study, where children with lower mathematics achievement were shown to have larger distance effects. Rouselle and Noël suggested that their findings might be due to the mathematically disabled children's use of atypical strategies during numerical comparison tasks to compensate for their impaired ability in extracting the meaning from numerical symbols. Here we suggest that children with less efficient, but not impaired, mappings between numerical symbols and their meanings show relatively lower Mathematics Fluency scores as well as larger distance effects. Thus, our data converge with the argument put forth by Rouselle and Noël—that mathematics performance and its impairment are related to the efficiency with which children can access and use the meaning of a symbolic numeral.

Our data also question the commonly held assumption that the processing of nonsymbolic magnitude serves as a precursor to the symbolic representation of magnitude (Barth et al., 2005) and that symbolic magnitudes are acquired through a process of mapping abstract symbols onto those representations that are tapped by nonsymbolic stimuli (Verguts & Fias, 2004). The absence of a correlation between nonsymbolic numerical magnitude comparison and standardized test scores (with the exception of the single correlation described above), as well as the nonsignificant correlation between symbolic and nonsymbolic distance effects, suggests that different developmental trajectories may underlie the processing and representation of symbolic and nonsymbolic numerical magnitude. In addition, the lack of any significant correlations between the symbolic and nonsymbolic distance effects challenges the hypothesis that symbolic numerical representation and nonsymbolic numerical representation are strongly related (Dehaene, 1997) and suggests the existence of different underlying representations for symbolic and nonsymbolic numerical magnitude. Thus, these data, in concert with those reported by Rousselle and Noël (2007), should encourage more empirical work to clarify the extent to which symbolic and nonsymbolic number representation are related.

Although it has been shown previously that the NDE decreases over developmental time (Duncan & McFarland, 1980; Holloway & Ansari, in press; Sekuler & Mierkiewicz, 1977), the current use of mathematics achievement measures that are standardized across age (i.e., variability in the scores that is due to age is accounted for) precludes the possibility that our results reflect a purely age-related phenomenon. However, although the relationship between the size of the NDE and individual differences in mathematics achievement cannot be explained by age-related changes in both of these variables, the data may suggest that the relationship changes over developmental time. Specifically, the results of the correlations separated by age group indicate that the relationship between mathematics achievement and the NDE exists primarily in the 6-year-olds and is diminished by 8 years of age. This could suggest that basic numerical understanding is particularly predictive of mathematics performance at an age when children are first being introduced to formal mathematics.

The fact that the correlations between mathematical skills and the symbolic distance effect were not found to differ significantly between the age groups could potentially be explained in several different ways. As suggested above, the particularly strong correlations in the youngest group of children may indicate that the symbolic distance effect is particularly predictive of mathematical skills in children who are first being exposed to formal training in symbolic number use but that the differences between the age groups did not reach significance due to the relatively small sample sizes. On the other hand, the lack of significant differences between the age groups may indeed reflect an age-invariant relationship between individual differences in the symbolic NDE and mathematics achievement scores. This could mean that a similar correlation between the symbolic NDE and mathematical skills potentially does exist in the older children or that the relationship we found in the younger groups is spurious and no consistent relationship exists. This latter option seems unlikely on consideration of the significant correlations across the whole group and following the examination of the correlations for each group separately. Although they did not reach significance, the correlations of the older children—Mathematics Fluency and Calculation in 8-year-olds and Calculation in 7-year-olds—all were negative and, thus, in the same direction as those for the youngest group of children. This suggests that if the correlations we found were relatively good estimates of the population values, an increase in sample size would reveal that the relationship between the symbolic distance effect and the standardized tests would be present in all age groups but would be significantly stronger in the 6-year-olds.

One potential reason for the age-related trend in the correlations is our use of single-digit numerals in the symbolic comparison task. It is possible that the relationship between the symbolic distance effect and mathematics achievement in the 6-year-olds is revealed in the variability in mental representation of the numerals 1 through 9 but in older children can be adequately captured only with larger numerals. However, the fact that a skill as basic as comparing two single-digit numbers predicts variance in more complex mathematical tasks speaks to the importance of basic numerical processing for the development of mathematical understanding.

It must be noted that by using small numerosities (specifically 1–3) in the nonsymbolic comparison task, we cannot rule out that individual differences in children's subitizing abilities could have influenced a subset of the nonsymbolic comparison trials. This subset consists solely of comparison pairs 1–2 and 2–1 given that the total numerosity represented by all other nonsymbolic comparison pairs exceeded the subitizing range. Future research seeking to further characterize the relationship between the distance effect and mathematics achievement should employ larger numerosities to better understand how the two phenomena are related and to avoid conflating subitizing with enumeration.

As highlighted in the Method section, none of the standardized measures of mathematical ability contained number comparison or estimation. Hence, the correlations revealed in this study are not simply relationships between similar measures but instead indicate that similar processes are engaged during symbolic numerical magnitude comparison and tasks measuring mathematics achievement. One such common process used in both numerical comparison and the Mathematics Fluency task is speed of processing. Indeed, as seen in Table 3, individual differences in reaction time required to make comparisons (averaged across all distances) are significantly correlated with Mathematics Fluency. Thus, it could be argued that the correlation between the symbolic distance effect and Mathematics Fluency is due simply to both tasks being speeded. However, by using each child's reaction time to large distances as a baseline, we were able to calculate the NDE as a ratio that accounts for individual differences in reaction time. In addition, our regression analysis demonstrated that the symbolic distance effect explained a significant amount of variance in Mathematics Fluency even after individual differences in speed were taken into account. Furthermore, as can be seen in Table 3, a significant negative correlation between the symbolic NDE and Calculation was also found. Calculation is an untimed standardized test of children's mathematical competence. If the relationships reported here were fully accounted for by speed of processing, no correlation between the symbolic NDE and Calculation should have been found.

Nevertheless, the results show that the most consistent relationship between individual differences in symbolic number processing and between-participants variability in mathematical competence exists for the Mathematics Fluency test. One reason for this might be that both symbolic number comparison and Mathematics Fluency require fast and efficient access of the numerical magnitude represented by Arabic numerals. In this way, symbolic number comparison and Mathematics Fluency may share core processes associated with accessing numerical magnitude information from Arabic numerals rather than both being associated simply with general speed of processing.

In this context, it should also be acknowledged that the current findings are specific to the reaction time data in that no significant correlations between standardized tests and measures of accuracy were found. Accuracy was found to be high in all groups, and this may have reduced the sensitivity of this measurement in the correlational analyses.

In addition to individual differences in speed of processing, other more general individual differences in cognitive processes could also potentially be related to both numerical comparison and the distance effect and, thus, could explain their association in the current study. However, the absence of significant relationships between the symbolic NDE and reading scores suggests that the results are specific to mathematical ability rather than related more generally to individual differences in general cognitive functions.

We found an unexpected significant correlation between the nonsymbolic distance effect and Composite Reading scores. Individuals whose nonsymbolic distance effects in reaction time were relatively larger tended to have higher reading scores. Although this is an interesting finding, it was not predicted and deserves further investigation. At this point, it can only be speculated that this finding reflects processes related to visual attention that are engaged in both the nonsymbolic numerosity comparison task and the standardized test of reading. Previous studies have revealed a link between visual attention and disorders of reading (Facoetti, Paganoni, Turatto, Marzola, & Mascetti, 2000).

In sum, the current results demonstrate the existence of a relationship between individual differences in the NDE and mathematical competence. Our findings highlight the importance of efficient mappings between numerical symbols and their quantitative meaning for the development of mathematical abilities. Further research into this relationship that examines larger numerosities and a broader developmental age range could reveal a potential utility of measures of the symbolic NDE for the assessment of children's mathematical skills and provide empirical support for the use of numerical comparison for the diagnosis (Butterworth, 2003) and remediation (Griffin, 2004; Wilson et al., 2006) of mathematical disabilities such as dyscalculia. Furthermore, our results provide support for the importance of studying low-level numerical processing and the acquisition of symbolic representations of numerical magnitude to better understand the development of mathematical competence. Future research should seek to study the relationship among mathematical ability, numerical comparison, and other measures of basic symbolic and nonsymbolic numerical understanding longitudinally so as to fully characterize how these fundamental abilities are used by children as foundations on which mathematical competency is constructed.

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