Comparison: Original Cauchy vs Corrected Cauchy vs Welsch
Top row: φ(w) = effective gradient weight | Bottom row: f(w) = IS-Reshape function

Original Cauchy (centered at w=0)
$\varphi = (1+\lambda)w / (1 + \lambda w^2)$

Corrected Cauchy (centered at w=1)
$\varphi = (1+\lambda)w / (1 + \lambda(w-1)^2)$

Welsch (centered at w=1)
$\varphi = \exp(-0.5\lambda(w-1)^2) \times w$

Original Cauchy f(w)
$f = (1+\lambda)/\sqrt{\lambda} \cdot \arctan(\sqrt{\lambda} \cdot w)$

Corrected Cauchy f(w)
$f = (1+\lambda)/(2\lambda) \cdot \ln(1+\lambda(w-1)^2) + (1+\lambda)/\sqrt{\lambda} \cdot \arctan(\sqrt{\lambda}(w-1))$

Welsch f(w)
$f = \int \exp(-0.5\lambda(w-1)^2) \cdot w \, dw$