π Pi Day Lecture

What is Pi?

π is a mathematical constant. It is a number but not just any number. It represents a ratio!!! It is accurately defined as the ratio of a circles circumference to its diameter. So we should conclude that because it is equal to the ratio of a circle's circumference to its diameter that it is a rational number. No!!! It's an irrational number and more specifically it is a transcendental number. It is approximately 3.14159.

or… 3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

Here is one way to think of it…

Take a circle and one rotation and the distance it has traveled on the ground is 3.14…

What type of number is it?

It is an irrational number, a number that can not be expressed as a fraction. Its decimal representation never ends and never repeats (at least as far as we know).

It is also a transcendental number which means there is no polynomial with rational coefficients of which Pi is a root. Don't worry about this but the significance of it being transcendent to Geometry is that is not constructible. It is impossible to construct a square whose area is equal to that of a circle.

Derivation of the word

The word Pi came from the greek word Περίμετρος – perimeter and

Fun facts

A Japanese psychiatric counselor named Akira Haraguchi has recited pi to 83,431 decimal places from memory
With the aid of computers, the number has been computed to more than a trillion digits

How many decimals must we use to be accurate?

This is a good and valid question. For example, lets use our calculators and determine the circumference of the earth (the equator), the radius of the earth is 6,357km, in two different ways

1) Let's use the approximation of 3.14
   2(3.14)(6,357km)
   <u>39,922km</u>

2) Let's be more exact by using the π on your calculator

$2\pi(6{,}357\text{km})$

39,942km

This is about 20km difference which is not significant for the ACCA pre-algebra class but is very significant for NASA or geographers, etc.
Does this mean we need to use lots of decimal places for our calculations?

No…

Going to 11 decimal places will calculate the circumference of the earth within 1 millimeter of its actual circumference. Do we need the trillion the computer has computed? No….
Going to 39 decimal places is enough to calculate with precision to the size of an atom any observable circle in the known universe…

Archimedes Geometric approach to calculating Pi

Pi was also known as Archimedes constant because he was the first to estimate π rigorously by developing a particularly brilliant method and he did this based on Geometry. Archimedes was a Greek mathematician, physicist, engineer, inventor and astronomer living between 287 BC – 212 BC. Many argue he was the greatest mathematician of antiquity! This would be to argue he is more brilliant than Euclid or Pythagoras. Bold claim!!!

Now we continue and describe his method of calculating π. If we know how to find the perimeter of a square inscribed in a circle and a square circumscribed about that same circle, we can be sure the circle's circumference is somewhere between these two values. Using this same line of reasoning, if we can find the perimeter of a regular pentagon inscribed in a circle and another regular pentagon circumscribed about this same circle, we again find the upper and lower limit to value of π. As we continue this process adding more sides to our inscribed and circumscribed polygons, we can find a more and more precise estimation of π.

Archimedes was able to estimate the value accurately by going all the way up to a polygon with **96 sides**. $3\frac{10}{71} < \pi < 3\frac{1}{7}$

History

The progress of π can be thought of in three different periods

1) When it was studied geometrically
2) The development of calculus
3) The age of digital computers

Geometrical period

That the ratio of the circumference to the diameter of a circle is the same for all circles and that it is slightly more than 3 was known to ancient Egyptian, Babylonian, Indian and Greek geometers.

Early estimates included…

25/8 (Babylonia)
256/81 (Egypt)
339/108 (India)

Biblical Estimate

It is claimed that Bible states that π is exactly three based on 1 Kings 7:23 and 2 Chronicles 4:2

> "Now he made the sea of cast metal ten cubits from brim to brim, circular in form, and its height was five cubits, and thirty cubits in circumference." – 1 Kings 7:23
>
> "Also he made the cast metal sea, ten cubits from brim to brim, circular in form, and its height was five cubits and its circumference thirty cubits." – 2 Chronicles 4:2

Is the Biblical inaccurate? No… it's not trying to give the exact value, simply an estimation.

The Great Pyramid of Giza was built with a ratio of its height to circumference as 2π. It can be concluded that although they couldn't define, they knew it in practice.

Following on the work of Archimedes, Ptolemy of the 2nd Century AD did the same Archimedes exercise but with 360-gons.

Middle Ages

Until 1,000 π was known to fewer than 10 decimal digits only.

The proof that π was irrational was not discovered in Europe until 1761 so people thought there could be a pattern, etc.

Like many proofs of irrationality, the proof that π is irrational is done with an argument that proceeds by reduction ad absurdum

A Persian mathematician was able to calculate it 16 digits in the 15th century (1400's)

16th – 19th centuries

Around 1600, a German mathematician calculated π to the first 35 decimal places and so pleased with that achievement, he had these numbers inscribed on his tombstone

A very famous mathematician, especially in the area of calculus, named Gottfried Liebniz, developed a method of estimating Pi in the 17th century.

William Shanks, spent 20 years calculating π, and in 1873 he calculated it to 707 decimal places. He would calculate new digits all morning and would then spend all afternoon checking his morning's work. However, only the first 527 were correct. Although this was the longest expansion of π until the computer. (thus a trillion)