Multi-Agent Path Finding Problem

CogRobo Cognitive Robotics Laboratory

Sabancı Üniversitesi

Multi-Agent Path Finding Problems

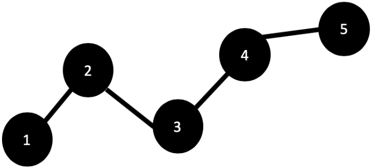
- Multi-Agent Path Finding (**MAPF**) Problem
  - Finding a plan for each agent in an environment
  - Without collisions
  - Constraints on plan length
- Optimization Variants
  - Maximum makespan
  - Total plan length
- Robotics [Lee and Yu, 2009],
  video games [Standley and Korf, 2011],
  autonomous aircraft towing vehicles [Morris *et al.*, 2016],
  traffic control [Dresner and Stone, 2008],
  autonomous warehouse systems [Wurman *et al.*, 2008].

[1] https://spectrum.ieee.org/robotics/robotics-software/three-engineers-hundreds-of-robots-one-warehouse
https://www.youtube.com/watch?v=6KRjuuEVEZs

Paths and Traversals

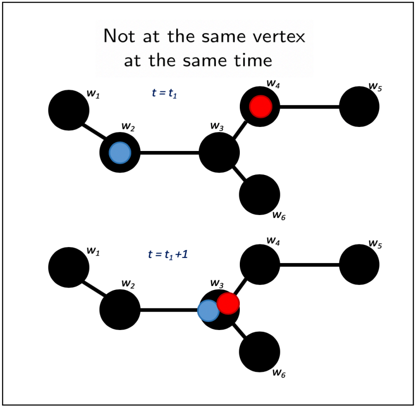
- A *path* P is a sequence of vertices $\langle w_1, w_2, \ldots, w_n \rangle$, such that every $w_i \in V$ is a vertex in a graph G and for every w_i, there is an edge to w_{i+1}.
- A *traversal f of a path* $P = \langle w_1, w_2, \ldots, w_n \rangle$ in a graph G within some time t ($t \in \mathbb{Z}^+$) is an onto function that maps every nonnegative integer less than or equal to t to a vertex in P, such that, for every w_i and w_j in P and for every $x < t$, if $f(x) = w_i$ and $f(x+1) = w_j$, then $w_j = w_i$ or $w_j = w_{i+1}$.

$$P = \langle 1, 2, 3, 4, 5 \rangle$$
$$f(P) = \langle 1, 1, 2, 3, 4, 4, 4, 5, 5 \rangle$$

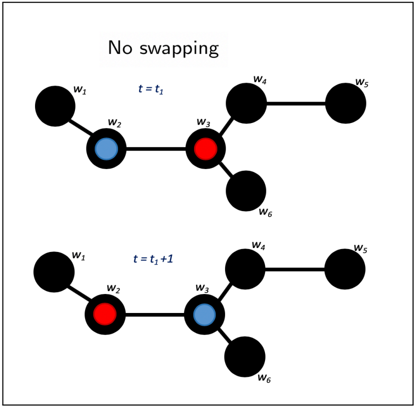


No Collisions of Traversals

- Let f_i and f_j be traversals of two different paths P_i and P_j, respectively, in a graph G within some time t.
- We say that the traversals f_i and f_j *do not collide with each other* within time t if,
  - for every $x, x' \leq t$, if $f_i(x) = f_j(x')$ then $x \neq x'$
  - for every time $x < t$, if $f_i(x) = f_j(x+1)$ then $f_i(x+1) \neq f_j(x)$



Not at the same vertex at the same time



No swapping

MAPF Problem

Input

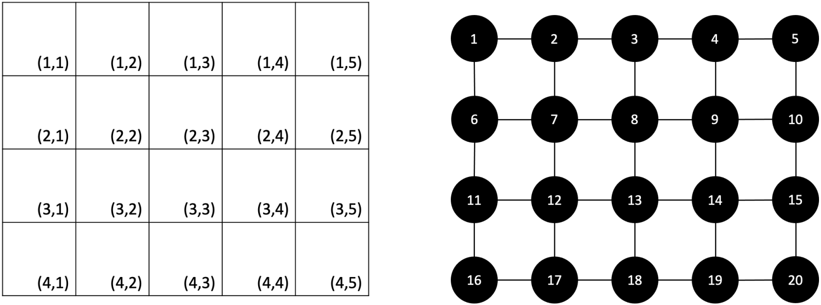
- A nonempty set $A = \{a_1, \ldots, a_n\}$ of agents $(n > 0)$.
- A graph $G = (V, E)$ (environment abstraction)
- A function $\mathit{init} : A \mapsto V$ (initial locations of agents)
- A function $\mathit{goal} : A \mapsto V$ (goal locations of agents)
- A set $O \subseteq V$ (obstacles)
- A positive integer τ (maximum makespan—plan length)

Output

For every agent $a_i \in A$, for some positive integer $u \leq \tau$,

- a path $P_i = \langle w_{i,1}, \ldots, w_{i,n_i} \rangle$ of length n_i $(n_i \leq u)$
  - that the agent a_i will follow to reach its goal location from its initial location (i.e., $w_{i,1} = \mathit{init}(a_i)$ and $w_{i,n_i} = \mathit{goal}(a_i)$),
  - without colliding with any obstacles (i.e., $w_{i,j} \in V \setminus O$), and
- a traversal f_i of the path P_i within time u, such that
  - for every other agent $a_j \in A$ with a path P_j and its traversal f_j within u, $f_i(P_i)$ and $f_j(P_j)$ do not collide with each other.

MAPF Example



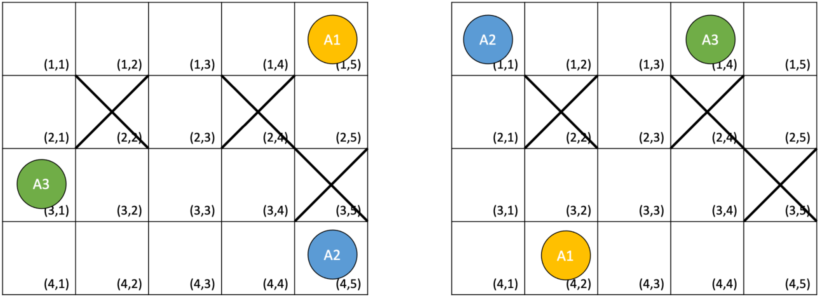
(1,1)	(1,2)	(1,3)	(1,4)	(1,5)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)

(a) A 4x5 grid

(b) Graph representation of the grid

Figure: How to represent a grid as a graph?

MAPF Example

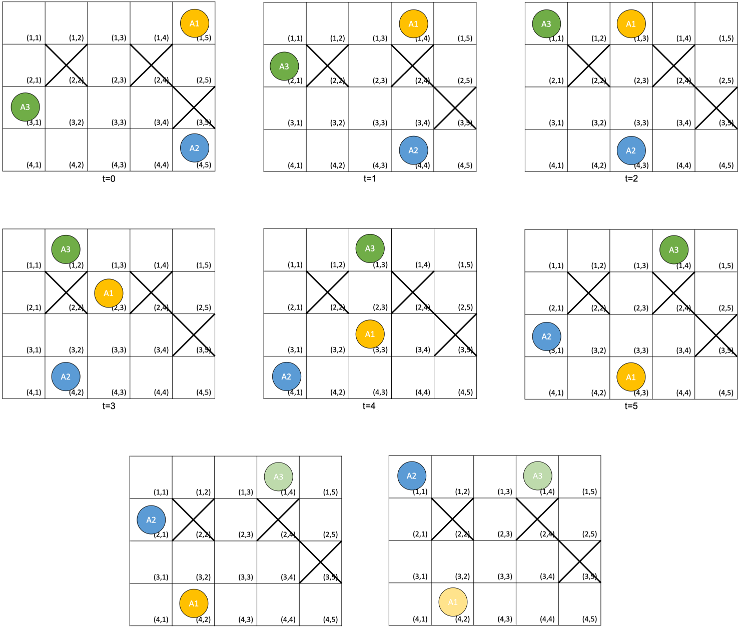


(a) Initial locations

(b) Goal locations

Figure: An example MAPF instance with 3 agents and initial & goal locations. Crossed cells denote cells with obstacles.

MAPF Example



t=0 t=1 t=2

t=3 t=4 t=5

t=6 t=7

MAPF-ASP

- *Requirements:* Python 3, Clingo[1]
- *Input:* input file name, output file name (optional)
- *Output:* an output file with plans and optimization values (default name: plan.txt)

Running:

```
python3 MAPF-ASP.py input.txt
python3 MAPF-ASP.py input.txt -o out.txt
```

[1] https://potassco.org/clingo/

MAPF-ASP: Input File Format

MAPF input:

- Graph (Grid size)

Input file format:

```
s <row_size> <column_size>
```

MAPF-ASP: Input File Format

MAPF input:

- Graph (Grid size)
- Maximum makespan

Input file format:

```
s <row_size> <column_size>
m <makespan>
```

MAPF-ASP: Input File Format

MAPF input:

- Graph (Grid size)
- Maximum makespan
- Agents

Input file format:

```
s <row_size> <column_size>
m <makespan>
a <agent_count>
```

MAPF-ASP: Input File Format

MAPF input:

- Graph (Grid size)
- Maximum makespan
- Agents
- Initial & goal locations of agents

Input file format:

```
s <row_size> <column_size>
m <makespan>
a <agent_count>
```

For each agent:

```
<init_r> <init_c> <goal_r> <goal_c>
```

MAPF-ASP: Input File Format

MAPF input:

- Graph (Grid size)
- Maximum makespan
- Agents
- Initial & goal locations of agents
- Obstacles

Input file format:

`s <row_size> <column_size>`

`m <makespan>`

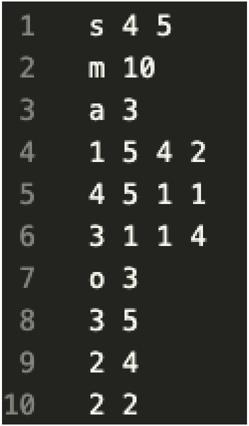
`a <agent_count>`

For each agent:
`<init_r> <init_c> <goal_r> <goal_c>`

`o <obstacle_count>`

For each obstacle:
`<obs_row> <obs_col>`

MAPF-ASP: Input File Format



```
s 4 5
m 10
a 3
1 5 4 2
4 5 1 1
3 1 1 4
o 3
3 5
2 4
2 2
```

Input file format:

`s <row_size> <column_size>`

`m <makespan>`

`a <agent_count>`

For each agent:
`<init_r> <init_c> <goal_r> <goal_c>`

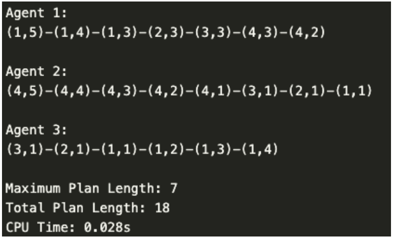
`o <obstacle_count>`

For each obstacle:
`<obs_row> <obs_col>`

MAPF-ASP: Output File Format

The output file includes:

- Plans for each agent.
  - Agent 1 is at (1,5) at time step 0.
  - Agent 2 is at (4,3) at time step 2.
- Maximum plan length.
- Total plan length.
- Computation time.



```
Agent 1:
(1,5)-(1,4)-(1,3)-(2,3)-(3,3)-(4,3)-(4,2)

Agent 2:
(4,5)-(4,4)-(4,3)-(4,2)-(4,1)-(3,1)-(2,1)-(1,1)

Agent 3:
(3,1)-(2,1)-(1,1)-(1,2)-(1,3)-(1,4)

Maximum Plan Length: 7
Total Plan Length: 18
CPU Time: 0.028s
```

Experiments

- How does the computation time change with the grid size?
- How does the computation time change with the number of agents?
- How does the makespan change with the number of agents?
- How does the total plan length change with the number of obstacles?

  …

Experiments

Input				*Output*		
Grid Size	# Agents	Maximum Makespan	# Obstacles	Maximum Plan Length	Total Plan Length	Time (s)
	⋮	⋮	⋮	⋮	⋮	⋮

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